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Gauge theories of strong interactions, spectral-function sum rules, and chiral-symmetry breaking*

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Using the explicit form of the strong-interaction Hamiltonian provided by a certain class of gauge theories, and the technique of the Bjorken-Johnson-Low expansion, we derive sum rules relating the vector, axial-vector, scalar, and pseudoscalar spectral functions and the chiral-symmetry-breaking parameters. Consistency of the sum rules in the pseudoscalar- (PS) meson sector demands that there should be a heavy nonet of PS mesons. It is shown that the nature of chiral-symmetry breaking depends on the character of these heavy mesons. The two modes of symmetry breaking resulting from this are studied. Using the assumption of pole dominance for the spectral functions, we evaluate the chiral-symmetry-breaking parameters in one case. We also evaluate decay widths of ρ, ω, and ϕ mesons to lepton pairs.

I. INTRODUCTION

An interesting outcome of the recent investigations[1] into spontaneously broken gauge theories has been the suggestion that strong interactions may be mediated by a "color" octet of gluons, coupled to three quartets of quarks.[2] In such models, violations of parity and strangeness selection rules are computable and have been shown to be small[3] (of order G_F for $\Delta S = 1$ and P-violating processes and of order G_F^2 for $\Delta S = 2$ processes, in agreement with experiment). Further impetus to the study of such models has been provided by the recent observation that a subclass of such theories are asymptotically free[4] and may, therefore, provide an explanation of observed Bjorken scaling in deep-inelastic electroproduction. In this note, we investigate the structure of hadron symmetry breaking and the spectrum of low-lying hadron states in such models.

It has been pointed out[5] that the basic symmetry of strong interactions in gauge models of the type described in Ref. 2 is $U(3)_L \otimes U(3)_R$ (in the space of observed particles) broken by quark mass terms, which transform as $(3, 3^*) \oplus (3^*, 3)$ under this group. An analysis of the nature of symmetry breaking in theories with $U(3)_L \otimes U(3)_R$ symmetry structure has been done by Mathur and Okubo,[6] using spectral-function sum rules and the assumption of pole dominance. In view of the great relevance of such models in the context of gauge theories, we would like to reexamine their work using the spectral-function sum rules that can be *derived* in gauge models of the type mentioned above. For this purpose we study the general question of spectral-function sum rules in gauge theories. Such sum rules had been studied earlier by Weinberg[7] and by Das, Mathur, and Okubo,[7] but in both these papers they were postulated on the basis of plausible assumptions such as asymptotic symmetry, and their validity was tested by the kind of results they yielded in the pole approximation. However, within the framework of gauge theories of strong interactions, our knowledge of the explicit structure of the strong-interaction Hamiltonian enables us to derive such sum rules, provided we assume that the behavior of two-point functions in the asymptotic domain in momentum space is correctly described by the Bjorken-Johnson-Low (BJL) expansion. We find that there are modifications to the sum rules usually assumed (the so-called second sum rules). We then attempt to saturate them with low-lying states, in order to get information about their spectrum and decay widths. We find that consistency of the sum rules in the pseudoscalar- (PS) meson sector leads us to a degenerate mass spectrum for the observed PS mesons in the pole approximation, unless there exists a heavy nonet of PS mesons[8] (above 1 GeV). This extra nonet could be thought of either as a convenient parameterization of the continuum contribution to the spectral functions or as a genuine particle nonet. We adopt the latter viewpoint and make some comments about the nature of such particles and their couplings. We point out that the nature of the chiral-symmetry breaking is intimately related to the nature of these heavy mesons. In particular, the magnitude of the coupling of the heavy pion to the axial-vector current determines whether SU(3) is a better symmetry than $U(2)_L \otimes U(2)_R$ or vice versa. It turns out that, when the above coupling is such that $U(2)_L \otimes U(2)_R$ is a better symmetry than SU(3), one can follow the Mathur-Okubo method to determine the value of the chiral-symmetry-breaking parameters. To estimate this breaking as well as the vector-meson spectrum and decay properties, we find the need to assume the so-called first spectral-function sum rules. Incidentally, our techniques do not shed any light on these sum rules, which are statements about the nature of Schwinger terms, and we *assume* their validity in our work.

The paper is organized as follows: In Sec. II, we derive the spectral-function sum rules; in Sec. III, we study the pseudoscalar-meson sector and point out the necessity for the heavy PS mesons. In Sec. IV, we study the relation between the nature of chiral-symmetry breaking and the properties of the heavy pion. In Sec. V, we study the numerical solution of the equations to estimate the chiral-symmetry-breaking parameters and discuss the lepton-pair decay widths of vector mesons. In Sec. VI, we speculate on the nature of the heavy PS mesons. In Sec. VII, we conclude with a discussion of the nature of symmetry breaking and the question of $\eta\to 3\pi$ decay puzzle in such schemes. Finally, in an appendix, we derive analogous sum rules, using a short-distance expansion and Weinberg's bridge theorem (instead of the BJL technique).

II. DERIVATION OF THE SUM RULES

Consider the spectral representation for the commutator of vector currents, i.e.,

$$\langle 0|[V_\mu^i(x), V_\nu^j(y)]|0\rangle = \int_0^\infty dm^2\left[\delta_{\mu\nu}\rho_1^{ij}(m^2,V) - \frac{\rho_2^{ij}(m^2,V)}{m^2}\partial_\mu\partial_\nu\right]\Delta(x-y,m^2), \tag{1}$$

and also a similar representation for the axial-vector currents in terms of the spectral functions $\rho_1^{ij}(m^2,A)$ and $\rho_2^{ij}(m^2,A)$. Denoting

$$\langle 0|[V_4^i(x), \partial_\nu V_\nu^j(y)]|0\rangle_{x_0=y_0}$$

by $K_{ij}\delta^3(x-y)$, we obtain[6,9]

$$K_{ij} = \int_0^\infty dm^2[\rho_2^{ij}(m^2,V) - \rho_1^{ij}(m^2,V)], \tag{2}$$

and corresponding to the axial-vector currents we get

$$I_{ij} = \int_0^\infty dm^2[\rho_2^{ij}(m^2,A) - \rho_1^{ij}(m^2,A)]. \tag{3}$$

The chiral-symmetry structure of the strong-interaction Hamiltonian is enough to enable us to evaluate I_{ij} and K_{ij}. In gauge theories with quarks and a "color" octet of gluons[2] we know the detailed structure of the Hamiltonian, and it can be written as follows [in the SU(3) subspace]:

$$\mathcal{H} = \mathcal{H}_0(q, B^A) + \mathcal{H}_1(B^A,\sigma) + \mathcal{H}_{\text{mass}}, \tag{4}$$

where

$$\mathcal{H}_0 = \bar{q}\gamma_k(\partial_k - if\lambda^A B_k^A)q, \tag{4$'$}$$

$$\mathcal{H}_{\text{mass}} = \epsilon_0 U_0 + \epsilon_8 U_8 + \epsilon_3 U_3, \tag{4$''$}$$

and $\mathcal{H}_1$ contains[10] the contribution of the gauge gluons, Higgs mesons, etc. An important thing to note is that the sum $\mathcal{H}_0 + \mathcal{H}_1$ is invariant under the chiral $U(3)_L \otimes U(3)_R$ group and $\mathcal{H}_{\text{mass}}$ transforms like the $(3,3^*)\oplus(3^*,3)$ representation under this group. As in Ref. 6, using Eq. (4), we obtain

$$K_{ij}=3\gamma ab f_{8ik} f_{8jk}, \tag{5}$$

$$I_{ij}=\tfrac{3}{2}\gamma[(\tfrac{2}{3})^{1/2}\delta_{ik}+2ad_{8ik}]d_{jkr}\frac{\langle 0|U_r|0\rangle}{\langle 0|U_0|0\rangle}, \tag{6}$$

where

$$\gamma=-\tfrac{2}{3}\epsilon_0\langle 0|U_0|0\rangle,$$
$$b=\langle 0|U_8|0\rangle/\langle 0|U_0|0\rangle\sqrt{2}, \tag{7}$$
$$a=\epsilon_8/\sqrt{2}\epsilon_0.$$

For future reference, we write the I's and K's explicitly[6]:

$$K_{44}=\tfrac{9}{4}\gamma ab,$$
$$I_{33}=\gamma(1+a)(1+b),$$
$$I_{44}=\gamma(1-\tfrac{1}{2}a)(1-\tfrac{1}{2}b), \tag{8}$$
$$I_{-2,-2}=\gamma(1-2a)(1-2b),$$
$$I_{-1,-1}=\gamma(1+a)(1+b)=I_{33},$$

where $I_{-1,-1}$ and $I_{-2,-2}$ correspond to the currents $(\sqrt{2}A_\mu^0+A_\mu^8)/\sqrt{3}$ and $(-\sqrt{2}A_\mu^8+A_\mu^0)/\sqrt{3}$, respectively.[11]

Now, let us look at the spectral representation for the vector-current propagator in terms of the functions $\rho_1(m^2,V)$ and $\rho_2(m^2,V)$ defined earlier in Eq. (1):

$$\Delta_{\mu\nu}^{V,ij}(q)=i\int e^{iq\cdot x}d^4x\langle 0|T(V_\mu^i(x)V_\nu^j(0))|0\rangle$$
$$=\int_0^\infty\frac{dm^2}{q^2+m^2}\left[\delta_{\mu\nu}\rho_1^{ij}(m^2,V)+\frac{q_\mu q_\nu}{m^2}\rho_2^{ij}(m^2,V)\right]-\delta_{\mu4}\delta_{\nu4}\int_0^\infty dm^2\,\frac{\rho_2^{ij}(m^2,V)}{m^2}, \tag{9}$$

and also in a similar manner for the axial-vector currents. Assuming that Bjorken expansion correctly describes the behavior of $\Delta_{\mu\nu}^{V,ij}(q)$ for large spacelike q^2, we find [using $H=\int d^3y\,\mathcal{H}(y)$]:

$$\lim_{q^2\to\infty} q^2\Delta_{\mu\mu}^{V,ij}=\int d^3x\langle 0|[[V_\mu^i(x),H]_{\rm ET},V_\mu^j]_{\rm ET}|0\rangle, \tag{10}$$

where $\mathcal{H}$ is defined in Eq. (4) and the subscript ET denotes an equal-time commutator. Using the explicit form for $\mathcal{H}$ [see Eqs. (4), (4′), (4″)], we get the following sum rule for the spectral functions

$$\int_0^\infty dm^2[4\rho_1^{ij}(m^2,V)-\rho_2^{ij}(m^2,V)]=C_1\delta_{ij}+C_2d_{8ij}-K_{ij}+3I_{ij}, \tag{11}$$

where C_1, C_2, and C_3 are unknown constants. Similarly, for the axial-vector currents we get

$$\int_0^\infty dm^2[4\rho_1^{ij}(m^2,A)-\rho_2^{ij}(m^2,A)]=C_1\delta_{ij}+C_2d_{8ij}+3K_{ij}-I_{ij}. \tag{12}$$

Using Eqs. (2), (3), (11), and (12), we finally obtain the following sum rules:

$$\int_0^\infty dm^2\rho_1^{ij}(m^2,V)=\tfrac{1}{3}C_1\delta_{ij}+\tfrac{1}{3}C_2d_{8ij}+I_{ij}, \tag{11′}$$

$$\int_0^\infty dm^2\rho_1^{ij}(m^2,A)=\tfrac{1}{3}C_1\delta_{ij}+\tfrac{1}{3}C_2d_{8ij}+K_{ij}. \tag{12′}$$

Subtracting Eq. (11′) from Eq. (12′), and again using Eq. (2) and Eq. (3), we get the analog of Weinberg's second sum rule,[7] i.e.,

$$\int_0^\infty dm^2[\rho_2^{ij}(m^2,A)-\rho_2^{ij}(m^2,V)]=0. \tag{13}$$

Next, let us consider the spectral representation for the pseudoscalar densities[9];

$$\Delta^{ij}(q,P)=i\int e^{iq\cdot x}d^4x\langle 0|T(\mathcal{V}^i(x)\mathcal{V}^j(0))|0\rangle$$
$$=\int_0^\infty dm^2\rho^{ij}(m^2,P)/(q^2+m^2). \tag{14}$$

Employing the same technique as before, we get

$$\int_0^\infty\rho^{ij}(m^2,P)dm^2=D_1\delta_{ij}+D_2d_{8ij}+I_{ij}. \tag{15}$$

For the corresponding scalar densities, we get

$$\int_0^\infty\rho^{ij}(m^2,S)dm^2=D_1\delta_{ij}+D_2d_{8ij}+K_{ij}. \tag{16}$$

In Eqs. (15) and (16), D_1 and D_2 are unknown parameters representing the vacuum expectation values of complicated bilinear local operators in quark fields. Subtracting Eq. (16) from Eq. (15), we get

$$\int_0^\infty dm^2[\rho^{ij}(m^2,P)-\rho^{ij}(m^2,S)]=I_{ij}-K_{ij}. \tag{17}$$

Here, we would like to point out that Mathur *et al.*[12] assume a similar sum rule in their work on chiral symmetries, but with the right-hand side equal to zero. We therefore see that in the quark model their sum rule does not hold, and their analysis will have to be modified, as we will see in Sec. V.

III. POLE DOMINANCE OF SPECTRAL FUNCTIONS AND PSEUDOSCALAR-MESON SPECTRUM

In order to use the sum rules derived in Sec. II to get information about the nature of chiral-symmetry breaking and the hadron spectrum, we will make the assumption that the spectral functions are dominated by the lowest-lying particles and one can neglect the continuum contribution. In this section, we will examine the sum rules that involve only the spin-zero part of the spectral functions, i.e., the ones in Eqs. (2), (3), (8), (15), and (16), and show that the assumption of saturation by known low-lying states is not consistent with these sum rules. We further see that to restore consistency it is sufficient to assume that there exists a heavy nonet of pseudoscalar mesons with nonzero coupling to the axial-vector currents and the pseudoscalar densities. Let us define the following:

$$(2k_0V)^{1/2}\langle 0|A_\mu^3|\pi^0\rangle = \frac{i}{\sqrt{2}} f_\pi k_\mu,$$

$$(2k_0V)^{1/2}\langle 0|A_\mu^{-1}|\eta\rangle = \frac{i}{\sqrt{2}} f_\eta k_\mu, \tag{18}$$

$$(2k_0V)^{1/2}\langle 0|A_\mu^{-2}|\eta\rangle = \frac{i}{\sqrt{2}} g_\eta k_\mu.$$

We similarly define f_K for the K meson, f_κ for the κ meson, and f_X, g_X and f_E, g_E for the X and E mesons, respectively. Now, saturating the last line of Eq. (8) by η, X, E, and π we find

$$\sum_{i=\eta,X,E} f_i^{\,2} m_i^{\,2} = f_\pi^{\,2} m_\pi^{\,2}. \tag{19}$$

We next observe that, using Eq. (15), we can derive the following equation:

$$\int_0^\infty dm^2 \rho^{-1,-1}(m^2,P) = \int_0^\infty \rho^{33}(m^2,P)dm^2. \tag{20}$$

Now, observing that $\mathfrak{V}^{-1} = \partial_\mu A_\mu^{-1}/\epsilon_0'(1+a)$ and $\mathfrak{V}^3 = \partial_\mu A_\mu^3/\epsilon_0'(1+a)$, where

$$\epsilon_0' = \tfrac{2}{3}\epsilon_0, \tag{21}$$

saturation of Eq. (20) leads to the following sum rule:

$$\sum_{i=\eta,X,E} f_i^{\,2} m_i^{\,4} = f_\pi^{\,2} m_\pi^{\,4}. \tag{22}$$

It is easy to see from Eq. (19) and Eq. (22) that either one has

$$f_i \neq 0 \text{ for all } i \text{ and } m_X = m_E = m_\eta = m_\pi \tag{23}$$

or if any of the f_i's is zero the corresponding particle has a mass not degenerate with the pion. Since we know that both f_π and m_π are nonzero, Eqs. (19) and (22) imply that there must be at least one pseudoscalar meson which is degenerate in mass with the pion. The reason why this is happening is clearly that the sum rule (20) presumably converges rather slowly. This can be understood if, for example, one assumes that the ϵ_0 and ϵ_8 are small and the pseudoscalar mesons become Goldstone bosons in the limit $\epsilon \to 0$ (i.e., their masses go to zero in that limit), because the last line of Eq. (8) can be written as

$$\int_0^\infty \rho^{-1,-1}(m^2,P)\frac{dm^2}{m^2} = \int_0^\infty \rho^{33}(m^2,P)\frac{dm^2}{m^2}. \tag{24}$$

Therefore, in Eq. (24) [which gives rise to Eq. (19) after pole dominance] the pole terms are of order $(1/\epsilon)$ as compared to the continuum contribution, which is of order 1 and is therefore small compared to the pole term. On the other hand, in Eq. (20) both the pole and continuum contributions are of the same order (i.e., of order 1). Therefore, *a priori*, it is probable that in such a case the continuum contribution could be relevant in Eq. (20). Below, we will see how we can get around this difficulty by postulating an extra heavy nonet of pseudoscalar mesons.

Now, suppose that there exists a heavy octet or nonet of pseudoscalar mesons. Denoting the corresponding parameters by a prime Eq. (19) becomes

$$\sum_{i=\eta,X,E,\ldots} f_i^{\,2} m_i^{\,2} = f_\pi^{\,2} m_\pi^{\,2} + f_{\pi'}^{\,2} m_{\pi'}^{\,2}, \tag{19$'$}$$

where the dots on the left-hand side stand for any contribution coming from the eighth and ninth members of the heavy nonet. Similarly Eq. (22) becomes

$$\sum_{i=\eta,X,E,\ldots} f_i^{\,2} m_i^{\,4} = f_\pi^{\,2} m_\pi^{\,4} + f_{\pi'}^{\,2} m_{\pi'}^{\,4}, \tag{22$'$}$$

and clearly the mass degeneracy mentioned earlier no longer exists. The question of the relative magnitude of f_π and $f_{\pi'}$ and its relevance to the manner of chiral-symmetry breaking will be discussed in Sec. IV. Here, we would only like to stress that whatever their relative magnitude is, it is possible by adjusting the value of the heavy-meson masses to resolve the difficulty of mass degeneracy observed earlier.

We would like to point out here another amusing point that comes out of the assumption of pole dominance of the above sum rules. Suppose all the equations involving A_μ^{-1} and $\mathfrak{V}^{-1}$ were saturated only by η and X (i.e., no other isosinglet pseudoscalar object coupled to them). This would then imply that $m_\eta = m_X$, regardless of whether there is a heavy pion or not. To see this, observe that one gets from Eqs. (8) and (15)

$$I_{-1,-2}=0$$

and (25)

$$\int_0^\infty \rho^{-1,-2}(m^2,P)dm^2=0.$$

Saturating them with poles, we get

$$g_\eta f_\eta m_\eta{}^2+g_X f_X m_X{}^2=0$$
$$=g_\eta f_\eta m_\eta{}^4+g_X f_X m_X{}^4. \tag{26}$$

It is clear from this that if g_η, g_X, f_η, and f_X are nonzero, one has $m_\eta=m_X$. This degeneracy is, however, removed when one includes another isosinglet PS meson, which incidentally could belong to (and probably ought to belong to) the heavy-meson nonet. There are other sum rules which also follow from the work of Sec. II, but we do not write them down since they involve many unknown parameters and hence are not very useful at the moment.

IV. THE NATURE OF CHIRAL-SYMMETRY BREAKING

In this section, we will point out that the nature of chiral-symmetry breaking will depend on $f_{\pi'}$ and $m_{\pi'}$. To see this, let us write down the first three lines of Eq. (8) in the pole-dominance approximation:

$$\begin{aligned}
&f_{\pi'}{}^2m_{\pi'}{}^2+f_\pi{}^2m_\pi{}^2=2\gamma(1+a)(1+b),\\
&f_{K'}{}^2m_{K'}{}^2+f_K{}^2m_K{}^2=2\gamma(1-\tfrac{1}{2}a)(1-\tfrac{1}{2}b),\\
&f_K{}^2m_K{}^2=\tfrac{9}{2}\gamma ab.
\end{aligned}\tag{27}$$

Depending on the relative magnitudes of $f_{\pi'}{}^2m_{\pi'}{}^2, f_{K'}{}^2m_{K'}{}^2$ and $f_\pi{}^2m_\pi{}^2, f_K{}^2m_K{}^2$ we will have two situations.

Case (i). $f_{\pi'}{}^2m_{\pi'}{}^2\lesssim f_\pi{}^2m_\pi{}^2$ *and* $f_{K'}{}^2m_{K'}{}^2\lesssim f_K{}^2m_K{}^2$. In this case, Eq. (27) becomes

$$\beta f_\pi{}^2m_\pi{}^2\simeq 2\gamma(1+a)(1+b)$$

and (28)

$$\beta' f_K{}^2m_K{}^2\simeq 2\gamma(1-\tfrac{1}{2}a)(1-\tfrac{1}{2}b) \text{ with } \beta\approx\beta' \text{ (say).}$$

Taking their ratio and noting the experimental fact that $f_\pi\simeq f_K$, we obtain

$$\frac{m_\pi{}^2}{m_K{}^2}\simeq\frac{(1+a)(1+b)}{(1-\frac{1}{2}a)(1-\frac{1}{2}b)}\,. \tag{29}$$

On examining this equation, it becomes clear that either $a\simeq-1$ or $b\simeq-1$ since $m_\pi{}^2/m_K{}^2\simeq0.08$. However, $b\simeq-1$ is unacceptable on physical grounds because this would imply that the vacuum is not at all invariant under SU(3), and it will be difficult to understand the successes of SU(3). Therefore, we ought to have $a\simeq-1$. As we will see in Sec. V, if $a\simeq-1$, b can be small, consistent with SU(3) being a good symmetry of the vacuum. In summary, if $f_{\pi'}{}^2m_{\pi'}{}^2\lesssim f_\pi{}^2m_\pi{}^2$ and $f_{K'}{}^2m_{K'}{}^2\lesssim f_K{}^2m_K{}^2$, we must have the chiral-symmetry breaking as follows:

$$U(3)_L\otimes U(3)_R\to U(2)_L\otimes U(2)_R\to SU(2)\otimes U(1).$$

As is well known,[5,13] in this case one must have a fourth light PS meson (with mass of order m_π) corresponding to the spontaneous breaking of the nonobserved approximate symmetry associated with the current A_μ^{-1}. It can also be seen by looking at Eq. (19′). The right-hand side of this equation is small by our assumption. Therefore, at least one of the particles i contributing to the left-hand side must have $f_i\approx f_\pi$ and $m_i\approx m_\pi$ and the rest must have $f_j\approx0$ and m_j arbitrary. Incidentally, if the original symmetry was $SU(3)_L\otimes SU(3)_R$, such a situation will not arise, as is clear by looking at Ref. 11, because $\langle 0|S|0\rangle$ could be quite large. Therefore, even when $f_\pi{}^2m_\pi{}^2+f_{\pi'}{}^2m_{\pi'}{}^2\approx0$, the left-hand side of the modified Eq. (19′) need not be small.

Another interesting point in this case is that we may restrict a to be exactly equal to -1. This does not mean that $m_\pi=0$, but that m_π arises purely out of electromagnetism and ϵ_3U_3 terms. The advantage of this point is that one can understand the $\eta\to3\pi$ decay[14] in this case.

Case (ii). $f_{\pi'}{}^2m_{\pi'}{}^2\gg f_\pi{}^2m_\pi{}^2$; $f_{K'}{}^2m_{K'}{}^2\approx f_{\pi'}{}^2m_{\pi'}{}^2\gtrsim f_K{}^2m_K{}^2$. In this case, Eq. (29) becomes

$$\frac{f_{\pi'}{}^2m_{\pi'}{}^2}{f_{K'}{}^2m_{K'}{}^2}\approx1\simeq\frac{(1+a)(1+b)}{(1-\frac{1}{2}a)(1-\frac{1}{2}b)}\,. \tag{29′}$$

There is no reason why in this case a or b must be close to -1; in fact both of them can be small (e.g., $a\simeq-0.1$, $b\simeq+0.1$) and consistent with Eq. (29′). This corresponds to symmetry breaking as follows:

$$U(3)_L\otimes U(3)_R\to SU(3)\otimes U(1)\to SU(2)\otimes U(1).$$

There is no conflict with the observed spectrum, and also, as has been pointed out by Drell,[8] this is a realization of the weak-PCAC (partial conservation of axial-vector current) idea of Brandt and Preparata.[15] In this case also, one can argue that corrections to PCAC results are small. However, as pointed out recently,[14] it is difficult to understand the $\eta\to3\pi$ decay in this case.

V. CHIRAL-SYMMETRY BREAKING AND LEPTON-PAIR DECAY MODES OF VECTOR MESONS

In this section, we will restrict ourselves to case (i) of Sec. IV and evaluate the symmetry-breaking parameters γ, a, and b, using the assumption of pole dominance. Our evaluation of the above parameters will differ from that given in Ref. 12 because, as remarked earlier, the sum rule used in Ref. 12 for such an evaluation, i.e., $\int\rho^{ij}(m^2,S)dm^2=\int\rho^{ij}(m^2,P)dm^2$, is not valid within our framework and the modified version of this sum rule given in Eq. (17) is not helpful anymore, since it involves a new unknown parameter ϵ_0'. However, we have new constraints coming from the vector-meson sector [i.e., Eq. (13)], and using these we can evaluate the five parameters γ, a, b, (f_K/f_π), and f_κ, assuming $m_\kappa\simeq 1050$ MeV and the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[16] $G_\rho{}^2=f_\pi{}^2m_\rho{}^2$. We will elaborate on this point now. Dominating the sum rules in Eq. (8) by lowest-lying poles and neglecting the contribution of heavy mesons to this equation [note that this is the case (i) of Sec. IV], we get

$$\begin{aligned} f_\pi{}^2m_\pi{}^2 &= 2\gamma(1+a)(1+b),\\ f_K{}^2m_K{}^2 &= 2\gamma(1-\tfrac{1}{2}a)(1-\tfrac{1}{2}b),\\ f_\kappa{}^2m_\kappa{}^2 &= \tfrac{9}{2}\gamma ab. \end{aligned}\tag{30}$$

We supplement the above information by the following relation obtained by Mathur and Okubo[6] and by Gell-Mann, Oakes, and Renner[17] from general variational arguments, i.e.,

$$\frac{m_\pi{}^2}{m_K{}^2}=\frac{1+a}{1-\frac{1}{2}a}.\tag{31}$$

This equation immediately provides us with an evaluation of the parameter a:

$$a\simeq-0.88.\tag{32}$$

Furthermore, Eq. (13) yields in the pole-dominance approximation the relation

$$G_{K^*}{}^2-G_{K_A^*}{}^2=\tfrac{1}{2}(f_K{}^2m_K{}^2-f_\kappa{}^2m_\kappa{}^2),\tag{33}$$

where we define

$$(2k_0V)^{1/2}\langle 0|V_\mu^i|V^i(k)\rangle=G_{Vi}\,\delta_{ij}\epsilon_\mu(k),$$

and similarly for the axial-vector particles.

We will at this point assume Weinberg's first sum rules for chiral SU(3)⊗SU(3). These sum rules depend on the nature of Schwinger terms, and the quark model does not shed any light on this. However, it is presumably not in conflict with it. In any case, this is an extra assumption, one which is believed to be valid more generally. These sum rules are

$$\int_0^\infty\frac{\rho_2^{ij}(m^2,V\text{ or }A)}{m^2}dm^2=C\delta_{ij}+C'\delta_{i0}\delta_{j0}.\tag{34}$$

From this we get

$$\begin{aligned}\frac{G_\rho{}^2}{m_\rho{}^2}&=\frac{G_{K^*}{}^2}{m_{K^*}{}^2}+\tfrac{1}{2}f_\kappa{}^2\\ &=\frac{G_{K_A^*}{}^2}{m_{K_A^*}{}^2}+\tfrac{1}{2}f_K{}^2.\end{aligned}\tag{34$'$}$$

Using Eqs. (30), (32), (33), and (34) and the KSRF relation $G_\rho{}^2=f_\pi{}^2m_\rho{}^2$, we get (for $m_\kappa\simeq 1000-1050$ MeV) $a\simeq-0.88$, $b\simeq-0.1$, $(f_K/f_\pi)^2\simeq1.16$; $f_\kappa{}^2m_\kappa{}^2=1.83f_\pi{}^2m_\pi{}^2$, $\gamma=4.6f_\pi{}^2m_\pi{}^2$.

The value of f_K/f_π obtained above is in reasonable agreement with experiment. From Eq. (34), one obtains $f_\kappa{}^2=0.036f_\pi{}^2$, which is quite reasonable since it implies that SU(3) is a good symmetry of hadrons and that the vacuum is nearly SU(3)-invariant. Furthermore, the values of a and b (as in Ref. 12) are also consistent with the general belief that chiral U(2)⊗U(2) is an approximate Goldstone symmetry, pions being the corresponding Goldstone bosons.

Before passing on to the considerations of leptonic decay modes of the ρ, ω, and ϕ mesons, we would like to remark that Eqs. (13) and (32) imply the following mass relation between the A_1, ρ, and π mesons [note that we assume $f_{\pi'}m_{\pi'}\ll f_\pi m_\pi$ as in case (i) of Sec. IV; for an analysis of the mass relation $m_{A_1}\simeq\sqrt{2}m_\rho$ in case (ii), see Sec. VI]:

$$m_{A_1}{}^2=2m_\rho{}^2+m_\pi{}^2.\tag{35}$$

Thus the correction to the famous Weinberg mass relation is very small in case (i). To obtain the leptonic decay modes of ρ, ω, and ϕ mesons we again stay within case (i) of Sec. IV so that we can neglect the contribution of the heavy mesons to the sum rules considered below. We get the first relation among the coupling constants and masses by looking at Eq. (11) in the pole approximation:

$$\begin{aligned}G_\rho{}^2+3(G_\omega{}^2+G_\phi{}^2)-4G_{K^*}{}^2=\tfrac{1}{2}f_\pi{}^2m_\pi{}^2+3I_{88}\\ -2f_K{}^2m_K{}^2,\end{aligned}\tag{36}$$

where

$$\begin{aligned}I_{88}&=(1-a-b+3ab)\\ &\simeq 9.84f_\pi{}^2m_\pi{}^2,\end{aligned}\tag{37}$$

and we define

$$\begin{aligned}(2k_0V)^{1/2}\langle 0|V_\mu^8|\omega\rangle&=\epsilon_\mu(k)G_\omega,\\ (2k_0V)^{1/2}\langle 0|V_\mu^8|\phi\rangle&=\epsilon_\mu(k)G_\phi.\end{aligned}\tag{31$'$}$$

Moreover, Eq. (34) yields

$$\frac{G_\rho{}^2}{m_\rho{}^2}=\frac{G_\omega{}^2}{m_\omega{}^2}+\frac{G_\phi{}^2}{m_\phi{}^2}.\tag{38}$$

From Eqs. (36), (37), and (38), we get

$$\frac{G_\omega{}^2}{G_\rho{}^2}=0.37\text{ and }\frac{G_\phi{}^2}{G_\rho{}^2}=1.11.\tag{39}$$

If we assume that the electromagnetic current does not have an isosinglet piece in it, then from the above equations we predict that

$$\frac{\Gamma(\omega\to e^+e^-)}{\Gamma(\rho\to e^+e^-)}\simeq 0.12,\quad \frac{\Gamma(\phi\to e^+e^-)}{\Gamma(\rho\to e^+e^-)}\simeq 0.17. \tag{40}$$

These appear to be in reasonable agreement with experiment.[18] If we examine Eq. (11′) as well as Eq. (34) a little carefully, we find that there are in all six independent constraints among coupling constants after the unknown parameters C_1, C_2, C, and C' are eliminated. However, there are only five unknowns, i.e., G_{K^*}, G_ω, G_ϕ, σ_ϕ, and σ_ω, where we define

$$(2k_0V)^{1/2}\langle 0|V^0_\mu|\omega(k)\rangle=\epsilon_\mu(k)\sigma_\omega,$$

and similarly for σ_ϕ. The equations are, apart from Eqs. (34′), (36), and (38),

$$\frac{G_\omega\sigma_\omega}{m_\omega^{\,2}}+\frac{G_\phi\sigma_\phi}{m_\phi^{\,2}}=0, \tag{41}$$

which follows from Eq. (34) on taking $i=8$ and $j=0$, and

$$G_\omega^{\,2}+G_\phi^{\,2}=\sigma_\omega^{\,2}+\sigma_\phi^{\,2}-\frac{1}{\sqrt{2}}(\sigma_\omega G_\omega+\sigma_\phi G_\phi)+\left(I_{88}+\frac{1}{\sqrt{2}}I_{08}-I_{00}\right), \tag{42}$$

$$G_\rho^{\,2}=\sigma_\omega^{\,2}+\sigma_\phi^{\,2}+\frac{1}{\sqrt{2}}(\sigma_\omega G_\omega+\sigma_\phi G_\phi)+\left(I_{33}-I_{00}-\frac{1}{\sqrt{2}}I_{08}\right). \tag{43}$$

Even though the system appears overdetermined, there is one set of solutions for σ_ω/G_ρ and σ_ϕ/G_ρ which, together with Eq. (39), is consistent with all the constraints obtained. Those solutions are

$$\frac{\sigma_\phi}{G_\rho}\simeq -0.79 \text{ and } \frac{\sigma_\omega}{G_\rho}\simeq 0.77. \tag{44}$$

If the electromagnetic current can be written as

$$J^{\text{em}}_\mu=V^3_\mu+xV^8_\mu+yV^0_\mu, \tag{45}$$

then

$$\frac{\Gamma(\phi\to e^+e^-)}{\Gamma(\rho\to e^+e^-)}=\left[x\left(\frac{G_\phi}{G_\rho}\right)+y\left(\frac{\sigma_\phi}{G_\rho}\right)\right]^2\frac{m_\rho^{\,3}}{m_\phi^{\,3}},$$
$$\frac{\Gamma(\omega\to e^+e^-)}{\Gamma(\rho\to e^+e^-)}=\left[x\left(\frac{G_\omega}{G_\rho}\right)+y\left(\frac{\sigma_\omega}{G_\rho}\right)\right]^2\frac{m_\rho^{\,3}}{m_\omega^{\,3}}. \tag{46}$$

These reduce to Eq. (40) for $x=1/\sqrt{3}$ and $y=0$.

VI. HEAVY PION

In this section, we will discuss the implications of a heavy pion[8] in both the situations described in Sec. IV. Clearly, the presence of the heavy pion modifies the PCAC equation to the following form:

$$\partial_\mu A^i_\mu=\frac{1}{\sqrt{2}}f_\pi m_\pi^{\,2}\Phi^i_\pi+\frac{1}{\sqrt{2}}f_{\pi'}m_{\pi'}^{\,2}\Phi^i_{\pi'}. \tag{47}$$

To see how the two cases described in Sec. IV arise, let us look at the experimental deviation[19] from the Goldberger-Treiman relation, Δ, in terms of f_π, $f_{\pi'}$, etc. If we assume that the entire deviation arises because of the heavy pion π', then we have

$$\Delta\simeq\frac{g_{NN\pi'}f_{\pi'}}{g_{NN\pi}f_\pi}\simeq 0.1. \tag{48}$$

It is easy to see from here that if $g_{NN\pi}\approx\frac{1}{2}g_{NN\pi'}$, then $f_{\pi'}\approx 0.05f_\pi$, and the situation corresponding to case (i) arises. However, as pointed out in Ref. 8, if for some reason $g_{NN\pi'}/g_{NN\pi}\approx 0.1$, we can have a situation where $f_\pi\approx f_{\pi'}$, and since $m_{\pi'}\gg m_\pi$, we arrive at case (ii) of Sec. IV. Hand-waving arguments can be provided to the effect that corrections to PCAC results are small in both the cases.[8] One may now ask how big the corrections are to $\pi^0\to 2\gamma$ decay due to the off-shell effects in case (ii). It was argued in Ref. 8 that they can be quite large (roughly a factor of 3). Since we are working within the framework of the three-triplet model,[20] we cannot tolerate a large off-shell correction. In fact we observe that if we take over the analysis of Ref. 8 in estimating the off-shell effect, the extrapolation factor is indeed very sensitive to the mass of the heavy pion, and if we choose it to around 1.2 GeV, the correction remains small. Below, we will provide further arguments in support of this value for the mass from considerations of Weinberg sum rules and the A_1 mass. As far as case (i) is concerned, the off-shell effects clearly remain small since $f_{\pi'}/f_\pi\approx 0.1$.

We would also like to point out here that there exists a lower bound for the $m_{\pi'}$ if we look at Eqs. (19′) and (22′). If η is the lowest-mass particle contributing to the left-hand sides of Eqs. (19′) and (22′), we find

$$\frac{f_\pi^{\,2}m_\pi^{\,4}+f_{\pi'}^{\,2}m_{\pi'}^{\,4}}{f_\pi^{\,2}m_\pi^{\,2}+f_{\pi'}^{\,2}m_{\pi'}^{\,2}}>m_\eta^{\,2}. \tag{49}$$

Therefore, in case (i) Eq. (49) gives

$$\frac{m_{\pi'}^{\,2}}{m_\pi^{\,2}}>\frac{m_\eta}{m_\pi}\frac{f_\pi}{f_{\pi'}}\simeq 40, \tag{50}$$

which implies

$$m_{\pi'}>6.3m_\pi.$$

In case (ii), Eq. (49) implies $m_{\pi'}>m_\eta$, consistent with $m_{\pi'}\approx 1.2$ GeV mentioned earlier. It also turns out that in case (ii) Weinberg's first and second sum rules for $U(2)_L\otimes U(2)_R$ [the latter suitably modified as in Eq. (13)], along with the observed mass relation $m_{A_1} \approx \sqrt{2} m_\rho$, imposes further constraints on $f_{\pi'}$ and $m_{\pi'}$ and gives us an idea as to how big $f_{\pi'}/f_\pi$ is. This can be seen if we observe that Eq. (13) and the Weinberg first sum rule imply, respectively,

$$G_A{}^2 - G_\rho{}^2 + \tfrac{1}{2} f_\pi{}^2 m_\pi{}^2 + \tfrac{1}{2} f_{\pi'}{}^2 m_{\pi'}{}^2 = 0 \tag{51}$$

and

$$\frac{G_A{}^2}{m_A{}^2} - \frac{G_\rho{}^2}{m_\rho{}^2} + \tfrac{1}{2}(f_\pi{}^2 + f_{\pi'}{}^2) = 0. \tag{52}$$

If we write the modified KSRF relation[16] as $G_\rho{}^2 = f_\pi{}^2 m_\rho{}^2 (1+\epsilon)$ (where ϵ is the effect of the heavy π'), to have $m_{A_1} \approx \sqrt{2} m_\rho$, we must have

$$\epsilon \simeq \left(\frac{f_{\pi'}}{f_\pi}\right)^2 \left(1 - \frac{m_{\pi'}{}^2}{2 m_\rho{}^2}\right). \tag{53}$$

Within experimental uncertainty $\epsilon \simeq -0.03$, and this implies that, for $m_{\pi'} \simeq 1.2$ GeV, $(f_{\pi'}/f_\pi) \simeq 0.5$. From Ref. 16, we observe that this value of ϵ implies that $G_{\pi'\rho\pi}/G_{\rho\pi\pi} \simeq 0.06$. These estimates, however, must be taken with caution in view of the uncertainties surrounding A_1.[18] [Note that Eq. (53) imposes strong constraints on $m_{\pi'}$ as well as $f_{\pi'}$. For example, if $m_{\pi'} \simeq 1.6$ GeV as in Drell's case, $(f_{\pi'}/f_\pi) \approx 0.1$, in apparent contradiction with the requirements that $f_{\pi'} \approx f_\pi$.]

VII. DISCUSSION

In conclusion, we wish to mention that in gauge theories one has two alternative ways in which one can break chiral $U(3)_L \otimes U(3)_R$ symmetry (Fig. 1).

(i) $\epsilon_8/\sqrt{2}\,\epsilon_0 \ll 1$. This situation occurs when

$$f_{\pi'} m_{\pi'} \approx f_{K'} m_{K'} \approx \chi f_K m_K \gg f_\pi m_\pi$$

(χ is between 2 and 3). Then SU(3) is a good symmetry, better than $U(2)_L \otimes U(2)_R$, and there is no conflict with particle spectrum. However, $\eta \to 3\pi$ decay remains a mystery[14] (see below). This case is a realization of the weak-PCAC idea of Brandt and Preparata.[15] Of course, the smallness of pion mass here is an accident and $m_{\pi'} \simeq 1.2$ GeV (lower path in Fig. 1).

(ii) $\epsilon_8/\sqrt{2}\,\epsilon_0 = -1$. This corresponds to the situation when $f_{\pi'} m_{\pi'} \ll f_\pi m_\pi$. With this alternative, one understands the $\eta \to 3\pi$ decay, but as mentioned in Refs. 5, 13, and 14 there must be an extra PS boson apart from the pion around the pion mass region, or if η is the corresponding particle one must understand what makes it so much heavier than the pion. In this case, the success of SU(3) symmetry can be understood since $\langle 0 | U_8 | 0 \rangle / \sqrt{2} \langle 0 | U_0 | 0 \rangle \simeq -0.1$, i.e., vacuum is approximately SU(3)-invariant. However, the baryon masses must arise from some other source (other than the $\epsilon_0 U_0$ term).

We would like to remark that there may be significant off-shell corrections to $\eta \to 3\pi$ decay results in case (i), and in particular the current-algebra constraints on $M(\eta \to 3\pi)$ now appear as constraints on the

$$M(\eta \to 3\pi) + (f_{\pi'}/f_\pi) M(\pi' \to \eta\pi\pi)$$

amplitude. In absence of any knowledge about the magnitude of $M(\pi' \to \eta\pi\pi)$, it will not be possible to say whether the $\eta \to 3\pi$ difficulty encountered[14] in this case is avoided or not. If, however, it turns out that $M(\pi' \to \eta\pi\pi)$ is large, $\eta \to 3\pi$ difficulty can be avoided as in the case of weak PCAC.[15]

We would like to conclude with a few remarks about the π' meson. Our investigation shows that if $f_{\pi'} \approx f_\pi$ [case (ii) of Sec. IV], then we conclude from an investigation of the deviations to the Goldberger-Treiman relation and the KSRF relation that it must be weakly coupled to the hadrons (i.e., for $f_{\pi'}/f_\pi \approx 0.5$, $g_{NN\pi'}/g_{NN\pi} \simeq 0.16$, $g_{\pi' \to \rho\pi}/g_{\rho \to \pi\pi} \simeq 0.06$, etc.). Also, we find that its mass ought to be around 1.2 GeV. On can understand π' in the language of quark model as a radial excitation of the $q\bar{q}$ bound state with $L=0$. Further experimental implications of the heavy pion are presently under investigation.

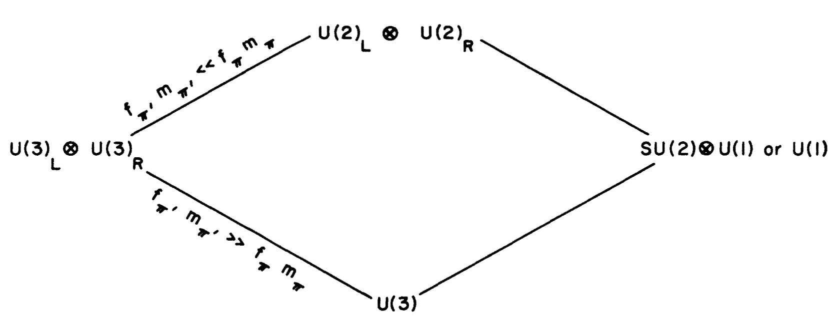

FIG. 1. Two ways of breaking chiral symmetry.

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APPENDIX

In this section, we will argue that the more fashionable short-distance expansion[21] methods may also lead to sum rules similar to the ones derived in Sec. II. We will follow Weinberg's method[22] of using bridge theorems to get the structure of operators appearing in the short-distance expansion of currents. We will work only with the vector currents, and in that case we find, using the method of Ref. 22, that

$$\int d\Omega_q \, \Delta^{ij}_{\mu\mu}(q) \underset{q^2\to\infty}{\sim} \langle 0|[-\bar{q}\,U^1_{ij}(q^2)q - \bar{q}\gamma_\mu U^2_{ij}(q^2)(\partial_\mu - if\lambda^A B^A_\mu)q - \tfrac{1}{4}F^A_{\mu\nu}F^A_{\mu\nu}U^3_{ij}(q^2)]|0\rangle + O(1/q^4). \tag{A1}$$

It becomes clear on inspection that the last two terms can be written as

$$\frac{a(q^2)}{q^2}\,\delta_{ij} + \frac{b(q^2)}{q^2}\,d_{8ij}\,,$$

where a and b are infinite series in powers of strong gauge coupling f and $\ln q^2$. Similarly, the first term can be written

$$\frac{d(q^2)}{q^2}K_{ij} + \frac{e(q^2)}{q^2}I_{ij} + \frac{a'(q^2)}{q^2}\delta_{ij} + \frac{b'(q^2)}{q^2}d_{8ij}\,,$$

where d, e, a', and b' are also power series in f and $\ln q^2$. Now, to get the spectral sum rules, we need the asymptotic behavior a, b, a', b', d, and e in q^2, and, in particular, our sum rules will hold if they go to constants for large $\ln q^2$. It is quite possible that in asymptotically free non-Abelian gauge models (e.g., non-Abelian gauge models with massless "color" octet of gluons, etc.), this situation occurs. (This may, of course, happen even in models which are not asymptotically free.) Then we will get an analog of Eq. (11), i.e.,

$$\int_0^\infty dm^2[4\rho_1^{ij}(m^2,V) - \rho_2^{ij}(m^2,V)] = C_1\delta_{ij} + C_2 d_{8ij} + d_0 K_{ij} + e_0 I_{ij}\,, \tag{A2}$$

where

$$d(q^2) \underset{q^2\to\infty}{\sim} d_0 + O\left(\frac{1}{\ln q^2}\right)$$

and

$$e(q^2) \underset{q^2\to\infty}{\sim} e_0 + O\left(\frac{1}{\ln q^2}\right). \tag{A3}$$

Similar arguments hold for the other sum rules. Note that (A2) and analogs of Eqs. (15) and (16) are weaker than Eq. (11). Nevertheless, the inconsistency mentioned in Sec. III still holds. However, we may lose the predictive power in the vector-meson sector. We may of course demand that this asymptotic behavior match the BJL-limit predictions. Then, obviously, we would get the sum rules described in the text.

Note added in proof. It has been stressed to the author by S. Borchardt, V. S. Mathur, and H. Pagels that $\Delta^{ij}(q,p)$ and $\Delta^{ij}(q,s)$ may or may not satisfy the unsubtracted spectral representation. We, of course, assume that they do. Also, for an alternative derivation of the second Weinberg sum rule in the framework of gauge theories, see S. Borchardt and V. S. Mathur, this issue, Phys. Rev. D 9, 2371 (1974). We would like to thank the above authors and S. C. Prasad for comments on the present work.

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[10]In models of the type discussed in Ref. 2, $\mathcal{H}_0$ is invariant under the $\mathrm{U}(4)_L \otimes \mathrm{U}(4)_R$ group and there is an extra $\epsilon_{15} U_{15}$ term in $\mathcal{H}_{\text{mass}}$. But, since we are concerned only with the $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ subspace of this, the $\epsilon_{15} U_{15}$ term is a singlet and is absorbed into the $\epsilon_0 U_0$ term.

[11]Note that in deriving the last two lines of Eq. (18) the $\mathrm{U}(3)_L \otimes \mathrm{U}(3)_R$ symmetry of $\mathcal{H}_0 + \mathcal{H}_1$ is crucial. For example, in a theory with broken $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ symmetry, one will have instead of the last two lines the following result:

$$I_{-2,-2} = \gamma(1-2a)(1-2b) + \tfrac{2}{3} \langle 0|S|0\rangle,$$

$$I_{-1,-1} = I_{33} + \tfrac{1}{3} \langle 0|S|0\rangle,$$

where

$$S = \left[Q_5^0, \left[Q_5^0, \int d^3x (\mathcal{H}_{\text{tot}} - \mathcal{H}_{\text{mass}}) \right] \right].$$

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Proof of the Weinberg sum rules in the Bars-Halpern-Yoshimura model

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Both sets of Weinberg spectral-function sum rules are proved in the context of the Bars-Halpern-Yoshimura model. We first discuss how to obtain the appropriate weak currents of the hadrons in such a gauge model, and then obtain the results by proving the equal-time commutation relations $[V_0^A, V_i^B] = [A_0^A, A_i^B]$, and $[\partial_0 V_i^A - \partial_i V_0^A, V_j^B] = [\partial_0 A_i^A - \partial_i A_0^A, A_j^B]$; the proof allows the spectral-function integrals involved to be different for the separate I-spin multiplets.

I. INTRODUCTION

Recently, models have been proposed for a unified theory of strong, weak, and electromagnetic interactions,[1,2] based on the ideas of local gauge invariance and the Higgs-Kibble mechanism.[3] In one of these,[1] the strong spin-1 gauge bosons are identified as the usual low-lying nonets of spin-1 mesons,[4] i.e., the ρ, A_1, etc., and (some of) the spin-0 mesons as the corresponding pseudoscalar and scalar particles, i.e., π, π_N, etc.

In such a model, it seems natural to check whether the Weinberg spectral-function sum rules[5] for the weak currents of the hadrons can be proved, since there is no convincing proof to date.[6] The very interesting algebra-of-fields approach employed by Lee, Weinberg, and Zumino[7] has the basic drawback of being based on a nonrenormalizable model. Accordingly, in the present paper, we use the renormalizable gauge model of Bars, Halpern, and Yoshimura as a convenient vehicle in which to prove the equality of the two sets of time-space equal-time commutators for the vector and axial-vector currents from which Lee *et al.*