Peripherality of tensor and vector exchanges in an approximately unitarized dual $\pi\pi$ model. I*

Louis A. P. Balázs
Department of Physics, Purdue University, Lafayette, Indiana 47907
(Received 19 June 1973)

Recent experimental evidence tends to favor a picture in which vector exchanges (ρ, ω, K^*_{888}) are peripheral, as predicted by the dual absorptive model, whereas tensor exchanges (f^0, A_2, K^*_{1420}) are not. We show that these features arise fairly naturally in $\pi\pi$ scattering if we simply saturate finite-energy sum rules (FESR) with an approximately unitarized Lovelace-Veneziano model. Exchange degeneracy and two-component duality are not assumed *a priori* for the output effective Regge residues in the FESR, although we find that they are approximately satisfied anyway for $t = 0$. In this calculation, Pomeranchukon parameters are taken from experiment or calculated from simple models.

I. INTRODUCTION

In the last couple of years there has been a considerable amount of interest in the dual absorptive model (DAM) of Harari.[1] In this approach the t dependence of Regge exchanges follows from two-component duality and the assumed peripheral character of direct-channel resonances. The latter assumption is abstracted from the prominent role played by peripheral resonances like the ρ and f^0 at lower energies. Indeed it is possible to reproduce all the DAM predictions for $\pi\pi$ scattering by saturating finite-energy sum rules (FESR) with the ρ and f^0 alone.

The main success of DAM has been the correlation of the zeros of the imaginary parts of the s-channel helicity-nonflip amplitudes for vector exchanges (ρ, ω, K^*_{888}) with those of $J_0(r\sqrt{-t})$ for $r \approx 1$ fermi. Exchange degeneracy, which is built into DAM, would predict the same peripheral J_0 structure for tensor exchanges (f_0, A_2, K^*_{1420}). The experimental evidence, however, points to a rather less peripheral structure in this case.[2,3] Since DAM can be understood in terms of FESR, it might therefore be interesting to see how these could be modified in order to be consistent with tensor nonperipherality. We shall see that this can be done fairly naturally in $\pi\pi$ scattering if we saturate the FESR with an approximately unitarized Lovelace-Veneziano model.

In Sec. II we discuss our unitarization procedure, which is essentially a simplified version of the usual K-matrix approach.[4] In Sec. III we use the FESR to calculate effective Regge residues. In this calculation the parameters of the Pomeranchukon P are taken from experiment. In Sec. IV, however, we calculate the effective P residue by applying a generalized version of our method above the $\rho\rho$ threshold, while in the Appendix we obtain the P trajectory by assuming a simple J-plane structure abstracted from a broad class of multiperipheral models.

II. APPROXIMATE UNITARIZATION OF THE LOVELACE-VENEZIANO MODEL BELOW THE $\rho\rho$ THRESHOLD

In the Lovelace K-matrix method[4] a partial-wave amplitude T_J is given by

$$T_J(s) = [V_J^{-1}(s) + \rho(s)]^{-1} , \tag{1}$$

where $V_J(s)$ is the partial-wave projection of the Lovelace-Veneziano model[5]

$$V^{I=2} = V(t, u) , \tag{2a}$$

$$V^{I=1} = V(s, t) - V(s, u) , \tag{2b}$$

$$V^{I=0} = \tfrac{3}{2}[V(s, t) + V(s, u)] - \tfrac{1}{2}V(t, u) , \tag{2c}$$

with

$$V(s, t) = -\bar{\beta}\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} , \tag{2d}$$

where s, t, u are the usual Mandelstam variables, $\alpha(t) = \frac{1}{2}(1 + t/m_\rho{}^2)$ is the Regge trajectory, and $\bar{\beta}$ is given in terms of the ρ width Γ_ρ. We will take $\bar{\beta} = 0.6$, which corresponds to $\Gamma_\rho = 125$ MeV. To guarantee unitarity we must have

$$\mathrm{Im}\rho(s) = -2q/\sqrt{s} , \tag{3}$$

where q is the c.m. 3-momentum.

In our approach we will only be interested in average values of $\mathrm{Im}T_J$. In the $I = 0$, 1 states these are dominated by resonances. In such a situation, unless a resonance is very broad, it is always a good approximation to set $\mathrm{Im}T_J \simeq \mathrm{Im}V_J$, which is what we shall do in what follows. The only exception is the elastic ϵ, for which we are forced to use Eq. (1) in the interval $0 < s < 2m_\rho{}^2$.

This suppresses the ϵ contribution considerably. So far we have not specified the real part of $\rho(s)$. Different authors have picked different forms for this function; the average $\mathrm{Im}T_J$ should not be too sensitive to this, however. We shall simply take $\mathrm{Re}\rho = 0$. It turns out that the resulting $\mathrm{Im}T_0$ is then a good approximation to the value obtained from the experimental $I=0$, $J=0$ phase shifts below about 1 GeV, at least on the average.[6]

In the exotic $I=2$ state, which does not contain any resonances, $\mathrm{Im}T_J$ is relatively small compared with the unitarity bound. In this case Eq. (1) gives

$$\mathrm{Im}T_J \simeq (2q/\sqrt{s})|V_J|^2 , \tag{4}$$

a form which can also be obtained directly from unitarity as long as V_J is a good first approximation to T_J.[7] We have assumed, of course that elastic unitarity is valid for $s \lesssim 4m_\rho^2$. This should be quite reasonable for $I=2$, where intermediate two-body states like $\pi\omega$ and $K\bar{K}$ do not come in. It then turns out that $\mathrm{Im}T_J$ is negligible for $J>0$. If we therefore combine Eq. (4) for $I=2, J=0$ with the resonant $\mathrm{Im}T_J$ of the preceding paragraph for $I=0, 1$, we can calculate the full absorptive part A for any I by using the partial-wave expansion

$$A = \sum_J (2J+1)\mathrm{Im}T_J P_J(1 + t/2q^2) . \tag{5}$$

In this sum the ρ and f^0 contributions will naturally be the most peripheral in an impact-parameter sense. All the other contributions will be referred to as "central" in what follows; since none of them seem to show up as prominent narrow peaks,[6,8] we can also think of them as "background" terms.

III. EFFECTIVE ρ AND f REGGE RESIDUES

We will now make the Harari assumption that the high-energy absorptive part (but not necessarily $\mathrm{Re}T$) can be approximated by the effective Regge behavior

$$A_{\mathrm{Reg}} = \sum_i b_i(t) s^{\alpha_i(t)} , \tag{6}$$

with $\alpha_i(t) = \alpha_i(0) + \alpha_i' t$. This assumption is approximately consistent with experiment and leads to the FESR:

$$\int_0^N ds(A - A_{\mathrm{Reg}}) = 0 , \tag{7}$$

where N will always be taken halfway between resonances at low energies and at channel thresholds at higher energies. We are assuming that fixed poles do not play an important role for $\pi\pi$ scattering at small t, a result which is suggested by the Lovelace-Veneziano model.

We will consider states of definite isospin I_t in the t channel. These are related to the s-channel absorptive parts A_{I_s} of Eq. (5) through

$$A_{I_t} = \sum_{I_s} \beta_{I_t I_s} A_{I_s} , \tag{8}$$

where β is the usual $\pi\pi$ crossing matrix

$$\beta = \begin{pmatrix} \frac{1}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} & -\frac{5}{6} \\ \frac{1}{3} & -\frac{1}{2} & \frac{1}{6} \end{pmatrix} . \tag{9}$$

In Eq. (6) we only keep $i=\rho$ for $I_t=1$ and $i=f$, P for $I_t=0$, with $\alpha_P(0)=1$. We will not assume any exchange degeneracy between the effective residues b_ρ and b_f, but will continue to take $\alpha_\rho(t) = \alpha_f(t)$, with $\alpha_\rho(0) = \frac{1}{2}$ and $\alpha_\rho' = 1/2m_\rho^2$.

If we apply Eq. (8) with $I_t=1$ and $N=4m_\rho^2$, which is both the $\rho\rho$ threshold and halfway between the f^0 and g resonances, we obtain $b_\rho(0) = 0.69$.[9] We also find that, because of the elements of the second row of Eq. (9), the $I=2$ background approximately cancels the $I=0, 1$ central T_J as calculated in Sec. II. Thus the peripheral ρ and f^0 resonances survive as the dominant contributions to A in Eq. (7). Not surprisingly we then find a zero in $b_\rho(t)$ at $t \simeq -0.32\ \mathrm{GeV}^2$, which corresponds to a zero of $J_0(r\sqrt{-t})$ with $r \simeq 0.84$ F, in agreement with the dual absorptive model.

For $I_t=0$ the situation is complicated by the presence of two exchanges in Eq. (6), the f and the P, which cannot both be determined by Eq. (7). In this section we will therefore take the P parameters from experiment, although in Sec. IV some of them are calculated from a model. In particular we determine $b_P(0)$ by using the f-coupled Pomeron hypothesis,[10] according to which the ratio

$$R = b_P(0)/b_f(0) \tag{10}$$

is independent of the external particles. From πp and pp scattering data we then obtain $R \approx 1$. If we now apply Eq. (7) at $t=0$ with $N=4m_\rho^2$, we obtain $b_P(0) = 1.08$, which corresponds to an asymptotic cross section $\sigma_{\mathrm{tot}} = 14.4$ mb. This should be compared with the value $\sigma_{\mathrm{tot}} \approx 15$ mb extracted, via factorization, from the experimental πp and pp cross sections. Combining with the previous paragraph, we also obtain $b_f(0)/b_\rho(0) = 1.57$. Exchange degeneracy, which would predict a value of 1.5 for this ratio, thus appears to be approximately satisfied at $t=0$.

For $t \neq 0$ we take the P to be structureless, at least for $t \lesssim 1\ \mathrm{GeV}^2$, with the form

$$b_P(t) = b_P(0)e^{at} . \tag{11}$$

To determine a, we assume it is the same in πp and pp scattering. This would be true for a broad class of multiperipheral models and also follows from factorization and the observed approximate

equality of the πp and pp diffraction widths. Experimentally, the average pp width parameter $w = d(\ln d\sigma/dt)/dt$ is about 12.5 GeV^{-2} at $s = 3000$ GeV^2, i.e., at ISR (CERN Intersecting Storage Rings) energies,[11] although taking a smaller value like $w \simeq 10$ does not affect our results very much. Equations (6) and (11) then give $2a \simeq (w - 16.0\alpha_P')$. The average slope α_P' for $|t| \lesssim 0.6$ GeV^2 is less well determined but certainly lies in the interval $0 < \alpha_P' \lesssim 0.4$ GeV^{-2}. We will see that any value in this interval will lead to essentially the same results.

If we now apply Eq. (7) with $N = 4m_\rho^2$ we find that the P contribution to A_{Reg} in the FESR falls off fairly rapidly with t. On the other hand, the elements of the first row of the crossing matrix (9) are such that all the central T_J contributions to A tend to add (instead of cancel) and become comparable with the resonance contributions. Since these central T_J terms are nonperipheral, A then falls off fairly slowly with t. As a result, the f contribution to A_{Reg} in Eq. (7) must also fall off fairly slowly with t so that there is no way in which b_f can develop a zero for small t. In fact with $\alpha_P' = 0.5$ $\alpha_\rho' = 0.425$ GeV^{-2}, for example, the first zero occurs at $t \simeq -0.81$ GeV^2 in complete disagreement with DAM. For smaller α_P' the zero is pushed out to even larger $|t|$, although the change is not a very large one.

The only way in which we can get a zero at a smaller value of $|t|$ is to take an abnormally large value of α_P'. In this case the P contribution to A_{Reg} in Eq. (7) falls off less rapidly with t. With the right value of α_P', it will have about the same average t dependence as the central T_J background of A and we might expect to have the classic two-component situation in which the P is dual to the background, and the f to the peripheral ρ and f^0 resonances. Indeed, with $\alpha_P' = \alpha_\rho'$ we find that $b_f = 0$ at $t \simeq -0.29$ GeV^2, which is about the value predicted by DAM. Unfortunately, this value of α_P' is much larger than that inferred from pp scattering.[2,11] Similar conclusions were inferred on purely phenomenological grounds by Barger, Geer, and Halzen.[2]

It should perhaps be emphasized that our conclusions were based primarily on certain general features combined with the properties of the $\pi\pi$ crossing matrix (9), rather than on the details of our model. As a further test of this we repeated the calculation with the Lovelace-Veneziano f^0 width replaced by the experimental value of 150 MeV. This gave $b_\rho(0) = 0.82$, $b_f(0)/b_\rho(0) = 1.39$, and $\sigma_{\text{tot}} = 15.1$ mb, which are not too different from the values obtained above. The zeros of $b_\rho(t)$ and $b_f(t)$ were hardly affected at all, being within 5% or less of the previous values.

In the above calculation we assumed that the effective P we use at lower energies is the same as the one measured at the CERN ISR. This is almost certainly not exactly true. However, our results are not sensitive to the precise parameters of the P. We have already seen that a smaller w does not change any of our qualitative conclusions. Neither does the precise value of α_P', as long as $\alpha_P' \lesssim 0.5$ $\alpha_\rho' \approx 0.5$ GeV^{-2}. Experimentally, α_P' has not been pinned down yet for $p_{\text{lab}} \lesssim 30$ GeV/c. Perhaps the cleanest process for studying the P in this range is $\gamma p \to \varphi p$ (Ref. 2), which gives $\alpha_P' = 0.03 \pm 0.13$ GeV^{-2} and is therefore consistent with our conclusions. It is true that certain analyses lead to $\alpha_P' \simeq 0.7$ GeV^{-2} in "exotic" processes like K^+p and pp scattering.[2] However, these analyses assume exact exchange degeneracy for the lower-lying trajectories, which, as we have seen, may not be true for $t \neq 0$. On the other hand, a peripheral vector and nonperipheral tensor exchange would require a smaller α_P' in order to reproduce the same data.

IV. GENERALIZATION OF OUR METHOD ABOVE THE $\rho\rho$ THRESHOLD

In Sec. III the parameters of the P were essentially taken from experiment. In the present section the parameters $b_P(0)$ and a will be calculated instead by applying a generalized version of our method above the $\rho\rho$ threshold. Now in the exotic $I_s = 2$ state, where no resonances are present, it should be reasonable to set $T \approx T^{\text{Regge}}$ as a first approximation. This is in fact a good approximation for the Veneziano amplitude V even at fairly low energies. We can then generate the absorptive part A by using the approximate unitarity relation

$$\text{Im}T_J = \sum K |T_J^{\text{Regge}}|^2 , \tag{12}$$

where K is an appropriate kinematic factor and the sum is over all possible intermediate channels.[7]

The above method can be readily generalized to nonexotic channels, provided we do not apply it to states and energy regions which contain narrow resonances. Now both experiment and studies of dual models suggest that such resonances should lie only on leading trajectories at high energies. Daughter resonances, on the other hand, are generally broad and show up only as Argand-diagram loops. This suggests that they may simply be Schmid loops which arise when we make a partial-wave projection of Regge exchange.[12] In any case we shall assume that we can continue to set $T \approx T^{\text{Regge}}$ in such waves at least as far as Eq. (12) is concerned.[7]

In the region $4m_\rho^2 < s < (m_\rho + m_f)^2$ we assume that the dominant intermediate states are $\pi\pi$ and $\rho\rho$. Diagrammatically, the right-hand side of Eq. (12)

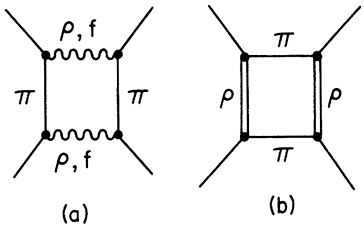

FIG. 1. The contributions to the absorptive part from (a) the $\pi\pi$ and (b) the $\rho\rho$ intermediate state (all our external lines are pions). Production experiments suggest that the horizontal Regge lines of (b) are dominated by pion exchange.

is then given by Fig. 1. We shall assume that $\rho\epsilon$ and $\epsilon\epsilon$ are negligible, since the ϵ is suppressed relative to the ρ both by unitarity, as we saw in Sec. II, and by the usual Dürr-Pilkuhn off-shell corrections to the pion exchange of Fig. 1(b). Alternatively, we could also have something like πA_2. This is actually dual to $\rho\rho$ so that we cannot have both at the same time, at least in any given region of phase space. We will pick $\rho\rho$ mainly because the known resonances such as the g[13] and the recently discovered ρ' tend to decay primarily into $\rho\rho$.[8] Our choice is also the one consistent with the Amati-Bertocchi-Fubini-Stanghellini model.[14]

In the $4m_\rho^2 < s < 6m_\rho^2$ region the only prominent resonance is the g at $s \simeq 5m_\rho^2$. Since this is an $I=1$ resonance (as is the ρ'), we can avoid all prominent narrow resonances by restricting ourselves to the $I_s = 0, 2$ states only. All waves are then given by Eq. (12) or Fig. 1. We will sum these via Eq. (5) and denote the contributions of Figs. 1(a) and 1(b) as $A^{\pi\pi}$ and $A^{\rho\rho}$, respectively. Note that simple isospin considerations give

$$A^{\rho\rho}_{I_s=0} = 4A^{\rho\rho}_{I_s=2} \ , \tag{13}$$

a result which is independent of detailed dynamics and follows from the fact that all the lines of Fig. 1(b) are $I=1$ systems. It will therefore continue to hold even if we add other $I=1$ exchanges, such as the A_2 trajectory, to the horizontal lines of Fig. 1(b).

It is rather difficult to evaluate Fig. 1(a) for general t. We shall therefore restrict ourselves to the value and derivative at $t=0$. In that case[15]

$$A^{\pi\pi}(s,0) = 2\frac{q}{\sqrt{s}}\int_0^1 dz\,|T^{\text{Regge}}(s,t)|^2 \tag{14}$$

and

$$\left[\frac{\partial}{\partial t}A^{\pi\pi}(s,t)\right]_{t=0} = 2\frac{q^3}{\sqrt{s}}\int_0^1 dz(1-z^2)\left|\frac{\partial}{\partial t}T^{\text{Regge}}(s,t)\right|^2 , \tag{15}$$

where $z = 1 + t/2q^2 = \cos\theta$ and θ = c.m. scattering angle. We have used the fact that the integrands have to be symmetric in z about $z=0$. If we use the Lovelace-Veneziano model then both Eqs. (14) and (15) have the approximate behavior $cs^{2\alpha(0)-1}$, except for logarithmic factors, which we shall ignore. To fix c we simply evaluated Eqs. (14) and (15) numerically at $s = 4m_\rho^2$, setting

$$T^{\text{Regge}}_{I_s=2}(s,t) \simeq V(t,u) \tag{16}$$

and

$$T^{\text{Regge}}_{I_s=0}(s,t) \simeq \tfrac{1}{2}[3e^{-i\pi\alpha(t)} - 1]V(t,u) \ , \tag{17}$$

which follows from Eq. (2) at large s.

To obtain our constraints on b_P we now take $N = 6m_\rho^2$ with $A = A^{\pi\pi} + A^{RR}$ in Eq. (7), and subtract from this the sum rule (7) with $N = 4m_\rho^2$, which was already enforced separately in Sec. III. We then have

$$\int_{4m_\rho^2}^{6m_\rho^2} ds[(\tfrac{1}{3}b_f + \beta_{I_s 1}b_\rho)s^\alpha + \tfrac{1}{3}b_P s^{\alpha_P} - A^{\pi\pi}_{I_s} - A^{\rho\rho}_{I_s}] = 0 \ . \tag{18}$$

We have neglected any kind of low-lying $I_t = 2$ Regge exchange, which should be quite reasonable for $s > 4m_\rho^2$. By applying Eq. (18) for both $I_s = 0$ and $I_s = 2$ and using Eq. (13) to eliminate the $A^{\rho\rho}$ between them, we get a single equation involving only $A^{\pi\pi}$. For simplicity we only considered this equation and its t derivative at $t=0$, using the results of the preceding paragraph to evaluate the contributions of the $A^{\pi\pi}$.

If we now combine these two constraints with those of Sec. III we obtain $b_f(0) = 1.25$, $b_P(0) = 0.93$, and $a = 5.72 - 6.50\ \alpha'_P$. These values do not change any of the general peripherality conclusions of Sec. III. Our asymptotic cross section is now $\sigma_{\text{tot}} \simeq 12.4$ mb, which should be compared with the experimental value of 15 mb. Our ratio of Eq. (10) is $R = 0.75$, which is also somewhat smaller than the value $R \simeq 1$ suggested by πp and pp experiments, although it must be kept in mind that this ratio is difficult to extract experimentally. On the other hand, our ratio $b_f(0)/b_\rho(0) = 1.81$ is somewhat lar than the exchange-degeneracy prediction of 1.5. However, both ratios are fairly sensitive in our calculation. Thus if we turn our problem around and require $\bar{\beta}$ to be such that exchange degeneracy is satisfied exactly we find a $\bar{\beta} \simeq 0.578$, which is not too different from the value $\bar{\beta} \simeq 0.6$ we have been assuming. If instead we replace the Lovelace-Veneziano f^0 width by the experimental value, we obtain $R = 1.62$ and $b_f(0)/b_\rho(0) = 1.05$.

If we insert our calculated value of a into Eqs. (11) and (6) we can obtain the diffraction-peak width parameter w at $s = 3000$ GeV^2. With $\alpha'_P = 0$ we have a value $w = 11.4$ GeV^{-2}, which is only slightly smaller than the value $w = 12.5$ GeV^{-2} ob-

served at ISR for pp scattering, while $\alpha_P' = 0.5\alpha_\rho'$ gives a value $w = 16.1$ GeV^{-2}. Naturally $\alpha_P' = \alpha_\rho'$ would give a considerably larger magnitude for w, which may be an additional reason for not taking such a large value for α_P'. These values of w are not affected much by replacing the Lovelace-Veneziano f^0 width by the experimental value. (See also the Appendix.)

V. CONCLUSION

We have considered a simple model for the low- and intermediate-energy absorptive part which, in addition to peripheral resonances, also contains a central T_J generated dynamically by combining unitarity with certain simple duality ideas. This was then used to calculate effective Regge residues through FESR. We then found that, as a result of a natural cancellation of the central T_J in the $I_t = 1$ state, ρ exchange is peripheral, as predicted by DAM. The central T_J in the $I_t = 0$ state, on the other hand, tended to add. They also had a slower t dependence than P exchange in the FESR. This breakdown of Pomeranchuk-"background" duality led in turn to a nonperipheral f exchange. Our arguments, moreover, are fairly general and do not seem to depend too much on the details of our model.

Our results correspond to a violation of exchange degeneracy and DAM for $t \neq 0$, even though these are still approximately valid for $t = 0$. This conclusion, of course, applies to effective Regge residues, which might include cuts, rather than to any kind of primordial Regge-pole residues which might come into a full-fledged dual theory. It should perhaps be emphasized, however, that if the only cuts are (Regge)-(Pomeranchuk) cuts, our conclusion will naturally apply to the primordial residues also.

In our calculation we assumed that the Pomeranchuk singularity has a relatively simple structure. If it were more complicated, with an effective α_P' which is larger at lower energies than it is at ISR energies, it may be possible to rescue exchange degeneracy. However, such a structure would make any kind of Regge approach much less useful for analyzing data. Besides, our scheme, which involved relatively low energies, led to results which were very similar to those of, say, Barger, Geer, and Halzen,[2] who did their analysis at much higher energies. This argues against any such complicated picture of Pomeranchuk exchange.

In addition to the zero structure of the ρ and f exchanges, our approach also enabled us to calculate the numerical values of various Regge and diffraction parameters, particularly when it is extended above the $\rho\rho$ threshold, as in Sec. IV. These values are in reasonable agreement with the ones inferred from πp and pp data.

Finally, there is no reason why some generalized version of our methods could not be applied to other processes such as πN and KN scattering. In such a situation, there is also no reason why we should always expect to find a peripheral vector and nonperipheral tensor exchange, as we do in the case of $\pi\pi$ scattering and the processes discussed in Refs. 2 and 3. Indeed, certain processes can still be explained in terms of a peripheral tensor exchange (see, for example, Loos and Matthews, Ref. 3). If this indeed turns out to be the case, it would be interesting to see whether our methods can actually predict when a particular tensor exchange is peripheral and when it is not.

ACKNOWLEDGMENTS

Part of this work was begun during a visit to the Brookhaven National Laboratory. The author would like to thank the high-energy theory group there for their hospitality.

APPENDIX: MULTIPERIPHERAL MODEL CONSTRAINTS

Up to now $\alpha_P(t)$ was simply inferred from experiment. In this appendix we will calculate it by assuming a simple J-plane structure consistent with a broad class of multiperipheral models. In all such models the $I_t = 0$ projected absorptive part can be written as[16]

$$A(J, t) = \bar{b}(t)/D(J, t) , \tag{A1}$$

where D has zeros at $J = \alpha_P, \alpha_f$ and a logarithmic branch point at $J = \alpha_c(t) = 2\alpha_P(\frac{1}{4}t) - 1$. We can therefore write a dispersion relation in J with a double subtraction at $J = \alpha_f$ (Ref. 17):

$$D(J, t) = [J - \alpha(t)]\{c - [J - \alpha(t)]\sigma(J, t)\} , \tag{A2}$$

where

$$\sigma(J, t) = \frac{1}{\pi}\int_{-\infty}^{\alpha_c} dJ' \frac{\mathrm{Im}D(J't)}{[J' - \alpha(t)]^2(J' - J)} \tag{A3}$$

and $c \simeq 1$ if the effect of the cut is in fact a small perturbation, as we are assuming. In the immediate neighborhood of $J = \alpha_c$, then

$$\sigma(J, t) \simeq -\xi(t)\ln[J - \alpha_c(t)] . \tag{A4}$$

We have neglected any J dependence in the numerator of Eq. (A1), an assumption which was checked explicitly in the case of a specific ABFST (Amati-Bertocchi-Fubini-Stanghellini-Tonin) model and found to be quite reasonable for $\frac{1}{2} \lesssim J \lesssim 1$. Moreover it can be shown that the addition of a certain amount of J dependence will not affect our results very much. From Eqs. (A1) and (A2) we

see immediately that $\bar{b}(t) = b_f(t)$, irrespective of the detailed form of σ. The upper zero of D now gives us $\alpha_P(t)$:

$$D(\alpha_P(t), t) = 0 , \tag{A5}$$

with

$$b_P(t) = \bar{b}(t) \Big/ \left[\frac{\partial}{\partial J} D(J, t)\right]_{J=\alpha_P} \tag{A6}$$

Since we are only interested in the properties of the f and P trajectories, we see that σ is needed only at $J = \alpha_P(t)$. This is indeed very close to $\alpha_c(t)$ when t is near the forward direction and means that we can use the approximation (A4).

If we now combine Eqs. (A5) and (A6) and their t derivative at $t = 0$ with the constraints of Sec. III, we obtain approximately the same results for $b_P(0)$ and $b_f(0)$ as in Sec. III. This time, however, we also obtain $\alpha_P(0) = 0.990$ and $\alpha_P' = 0.20$ GeV^{-2}. This value, as we saw in Sec. III, leads to a nonperipheral f exchange and is consistent with experiment. It also corresponds to a diffraction-peak width parameter w at $s = 3000$ GeV^2 of $w = 13.6$ GeV^{-2}, which is only slightly larger than the observed ISR pp value of $w \simeq 12.5$ GeV^{-2}.

---

*Work supported in part by the U. S. Atomic Energy Commission and from an Alfred P. Sloan Foundation Fellowship.

[1]H. Harari, Phys. Rev. Lett. 26, 1400 (1971); M. Davier and H. Harari, Phys. Lett. 35B, 239 (1971).

[2]V. Barger, K. Geer, and F. Halzen, Nucl. Phys. B49, 302 (1972).

[3]C. Michael, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 3, p. 165. See, however, J. S. Loos and J. A. J. Matthews, Phys. Rev. D 6, 2463 (1972).

[4]C. Lovelace, in *Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions, Argonne National Laboratory, 1969,* edited by F. Loeffler and E. D. Malamud (Argonne National Laboratory, Argonne, Ill., 1969), p. 562.

[5]C. Lovelace, Phys. Lett. 28B, 264 (1968).

[6]S. D. Protopopescu *et al.*, Phys. Rev. D 7, 1279 (1973).

[7]L. A. P. Balázs, Phys. Lett. 42B, 276 (1972).

[8]R. E. Diebold, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 3, p. 1.

[9]L. A. P. Balázs, Phys. Rev. D 6, 1038 (1972).

[10]R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. D 4, 3439 (1971).

[11]V. Barger, K. Geer, and R. J. N. Phillips, Phys. Lett. 36B, 343 (1971); 36B, 350 (1971).

[12]C. Schmid, Phys. Rev. Lett. 20, 689 (1968).

[13]N. M. Cason *et al.*, Phys. Rev. D 7, 1971 (1973).

[14]L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento 25, 626 (1962); D. Amati, A. Stanghellini, and S. Fubini, *ibid.* 26, 896 (1962).

[15]L. A. P. Balázs, Phys. Rev. D 4, 2364 (1971).

[16]G. F. Chew and D. R. Snider, Phys. Rev. D 3, 420 (1971).

[17]L. A. P. Balázs, Phys. Lett. 37B, 528 (1971).