any of the cases we considered in this paper. For a discussion of this case and a general discussion of regional stability see, for example, N. Minorsky, *Nonlinear Oscillations* (Van Nostrand, Princeton, N. J., 1962).

[21]We emphasize that this result does not depend on the representation content of the fermions or scalar fields. For example, if we had a set of scalars transforming according to the (N, M) representation of $O(N)\times O(M)$ then we might expect $g_1g_2^2$ and $g_2g_1^2$ terms in the equations for $\beta_i(g_i)$ from graphs of type shown in Fig. 1(c). However, simple calculation shows that the coefficient vanishes because of the vanishing of the trace over internal symmetry matrices.

[22]See, for example, K. Johnson and M. Baker, Phys. Rev. D 8, 1110 (1973) and the references contained therein. This remark clearly does not apply to models where $SU(2)\times U(1)$ is embedded in a larger group with no U(1) factors.

[23]K. Symanzik, Nuovo Cimento Lett. 6, 77 (1973). However, a refined argument against $\lambda<0$, using the renormalization-group equation, has been given by S. Coleman and E. Weinberg [Phys. Rev. D 7, 1888 (1973)].

[24]In the notation of Ref. 7, $A=2s_1+s_2+2s_3$, and $B=6s_1+3s_2$.

[25]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); H. Pagels, Phys. Rev. D 7, 3689 (1973); R. Jackiw and K. Johnson, *ibid.* 8, 2386 (1973); J. Cornwall and R. Norton, *ibid.* 8, 3338 (1973).

[26]T.-M. Yan (private communication). We would like to thank Professor Yan for this remark.

[27]See Ref. 13. See also S. Weinberg, Phys. Rev. Lett. 31, 494 (1973).

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Classical direct interstring action*

Michael Kalb and P. Ramond
Physics Department, Yale University, New Haven, Connecticut 06520
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We generalize the classical action-at-a-distance theory between point particles to include one-dimensionally extended objects (strings) in space-time. We build parametrization-invariant couplings which lead to equations of motion for strings in each others' influence. The direct coupling of the area elements of the world sheets of the strings is considered in detail, from which we define an antisymmetric adjunct field. We find that, for a given interaction, the nature of the forces depends on the type of strings involved, that is, open- vs closed-ended. Our coupling can be understood in terms of states appearing in the Veneziano and Shapiro-Virasoro models in 26 dimensions. However, we find an additional massive pseudovector field which arises from the interaction between the "Reggeon" and "Pomeron" sectors of this dual model.

I. INTRODUCTION

The dual resonance models,[1] whose aim is a self-contained description of the strong interactions, have of late been understood in terms of a strikingly simple and beautiful picture. On the one hand, the states of motion of a one-dimensionally extended object (string)[2] with open ends are identified with the mesonic resonances which mediate the strong interactions,[3] while the "background" (Pomeron) is to be related to the states of motion of strings which close on themselves. On the other hand, Mandelstam[4] has shown that the Veneziano amplitudes can be obtained by breaking and joining open strings, thus completing the description. It is therefore rather unfortunate that such conceptual simplicity is spoiled by the presence of tachyons, and long-range forces, all in a 26-dimensional space-time.[5] Still, these problems appear only in the quantization procedure, and do not subtract from the appeal of the classical description. A difficulty in overcoming these defects is that the strings have so far been described in terms of their world sheets rather than by the fields associated with them. One may hope therefore that the development of a more powerful formalism might alleviate and perhaps solve the aforesaid problems.

Nevertheless, at the classical level this remains a very beautiful theory which does not make use of a field description. In this light, it seems natural to try to understand Mandelstam's interaction as being generated by direct interstring forces. One already knows that Maxwell's theory can be described in terms of such forces, as shown by Feynman and Wheeler.[6] It is our aim in this paper to generalize action-at-a-distance theories to include direct interstring interactions. As a first step, we limit ourselves to a specific type of interaction obtained by analogy to their work. Thus we concern ourselves, in what follows, with a tiny subset of all the possible direct interstring interactions, although we work out the general constraints they must satisfy.

We find that fundamental differences exist between open and closed strings and that the specific coupling we choose has strikingly different physical consequences depending on the string's nature. A special feature of our approach is that it gives us a glimpse into the interaction between closed and open strings.

In Sec. II we remind the reader of the action-at-a-distance formalism for point particles,[7] which we generalize to strings. Section III is devoted to the study of a specific coupling between the various types of strings; in it we also show how to abstract the fields associated with the particles which eventually appear in the quantized version of the theory. Our results are summarized in Sec. IV, where we present our concluding remarks. In the Appendix the free-field theory for an antisymmetric massless field, as well as its quantization, is presented.

II. ACTION AT A DISTANCE

In this section we review the general classical treatment of the relativistic interaction of point particles by means of action-at-a-distance forces. The formalism is then extended to describe the action-at-a-distance interaction of one-dimensionally extended objects (strings). Although no satisfactory quantization of such theories yet exists, they can still be used to abstract the "fields" that would mediate such interactions. These fields can then be quantized in the usual manner.

A. Direct interparticle action

Every point particle traces out in space-time a world line x^μ with length squared

$$ds_a{}^2 = g_{\mu\nu}\, dx_a^\mu dx_a^\nu \equiv dx_a \cdot dx_a \ , \tag{2.1}$$

where the subscript a indicates the particle label; $g_{\mu\nu}$ is the Lorentz metric ($g_{00} = -g_{ii} = 1$). Introduce a scalar parameter λ_a as a monotonically increasing label along the world line, in terms of which we define

$$u_a^\mu \equiv \frac{dx_a^\mu}{d\lambda_a} \ . \tag{2.2}$$

The free action for this point particle is given by

$$\begin{aligned} S_{fa} &= m_a \int ds_a \\ &= m_a \int (dx_a \cdot dx_a)^{1/2} \\ &= m_a \int_{\lambda_{ai}}^{\lambda_{af}} d\lambda_a (u_a \cdot u_a)^{1/2} \ , \end{aligned} \tag{2.3}$$

where we have taken $c=1$. By assuming that S_{fa} is stationary under the variation

$$\begin{aligned} &x_a^\mu \to x_a^\mu + \delta x_a^\mu \ , \\ &\delta x_a^\mu(\lambda_{ai}) = \delta x_a^\mu(\lambda_{af}) = 0 \ , \end{aligned} \tag{2.4}$$

we obtain the equations of motion

$$m_a \frac{d}{d\lambda_a} \frac{u_a^\mu}{(u_a \cdot u_a)^{1/2}} = 0 \ . \tag{2.5}$$

To understand the way point particles interact in this formalism, we postulate an action of the form

$$S = \sum_a m_a \int ds_a + \sum_{\substack{a,b \\ a<b}} \int d\lambda_a \int d\lambda_b R_{ab}(x_a, x_b, u_a, u_b) \ . \tag{2.6}$$

Note the absence of self-interaction terms, and the (standard) limitation of the dependence of R_{ab} to first-order derivatives. Further, we ask that

$$R_{ab} = R_{ba} \ , \tag{2.7}$$

to preserve the symmetry of the action. The equations of motion are obtained by demanding that S be stationary under the variation (2.4); these are

$$m_a \frac{d}{d\lambda_a}\left[\frac{u_a^\mu}{(u_a \cdot u_a)^{1/2}}\right] = \sum_{b\neq a} \int d\lambda_b \left(\frac{\partial R_{ab}}{\partial x_{a\mu}} - \frac{d}{d\lambda_a}\frac{\partial R_{ab}}{\partial u_{a\mu}}\right), \quad a = 1, 2, \ldots \ . \tag{2.8}$$

It is interesting to note that the contraction of the equation of motion (2.8) with $u_{a\mu}$ reduces to

$$\frac{d}{d\lambda_a}\left[\left(1 - u_a^\mu \frac{\partial}{\partial u_a^\mu}\right) \sum_{b\neq a} \int d\lambda_b R_{ab}\right] = 0, \quad a = 1, 2, \ldots \ . \tag{2.9}$$

Hence not all expressions for R_{ab} give rise to consistent equations of motion. In fact Eq. (2.9) can also be obtained by demanding that S be stationary under an arbitrary change of parameters

$$\lambda_a \to \lambda_a + \delta\lambda_a, \quad a = 1, 2, \ldots \ . \tag{2.10}$$

In what follows we will always consider R_{ab}'s which satisfy Eq. (2.9). Then, we are at liberty to choose the most convenient interpretation for λ_a, that of the path length s_a. This leads to

$$u_a \cdot u_a = 1 \ , \tag{2.11}$$

and λ_a becomes the proper time, τ_a.

The action-at-a-distance (AD) form of dynamics came into its own when Feynman and Wheeler[6] showed that the classical electrodynamics of Maxwell could be understood in terms of AD forces provided that radiation was always interpreted as the transmission of energy (and momentum) between point particles. They started from the general form of the action considered above, Eq. (2.6), with

$$R_{ab}=e_a e_b u_a \cdot u_b \delta(s_{ab}{}^2) , \tag{2.12}$$

where e_a and e_b are dimensionless coupling constants and

$$s_{ab}{}^2=(x_a-x_b)\cdot(x_a-x_b) . \tag{2.13}$$

The equations of motion are

$$m_a \ddot{x}_a^\mu = e_a \sum_{b\neq a} F_b^{\mu\nu}(x_a)\dot{x}_{a\nu} , \tag{2.14}$$

with

$$\dot{x}_a^\mu = \frac{d}{d\tau_a} x_a^\mu, \quad \ddot{x}_a^\mu = \frac{d}{d\tau_a}\dot{x}_a^\mu , \tag{2.15}$$

$$\dot{x}_a \cdot \dot{x}_a = 1 , \tag{2.16}$$

and the field constructs are written in terms of the world lines of the remaining particles:

$$F_b^{\mu\nu}(x)=\left[\frac{\partial}{\partial x_\mu}A_b^\nu(x)-\frac{\partial}{\partial x_\nu}A_b^\mu(x)\right] . \tag{2.17}$$

Here the "vector potential" due to particle b is given by

$$A_b^\mu(x)=e_b\int dx_b^\mu \delta((x-x_b)^2) . \tag{2.18}$$

Note that Eq. (2.14) differs from the usual Lorentz force expression in that the field constructs contain both the advanced and the retarded signals. This can best be seen in Eq. (2.18) by noting that

$$A_b^\mu(x)=\tfrac{1}{2}[A_b^{\mu\,\mathrm{ret}}(x)+A_b^{\mu\,\mathrm{adv}}(x)] . \tag{2.19}$$

In Maxwell's electrodynamics, we would only have $A_b^{\mu\,\mathrm{ret}}(x)$. It was the great contribution of Feynman and Wheeler to show that the extra terms (due to the advanced signals) could be reduced to the radiation reaction term that appears in Dirac's treatment,[8] provided that one assumes that no net radiation exists in the system. Then causality can be restored. It is clear from this interpretation that it is contrary to the AD formalism of electrodynamics to talk of free radiation. Even though a free-field theory does not exist, it can be abstracted in the following way: We see from Eq. (2.18) that $A_b^\mu(x)$ obeys

$$\frac{\partial^2}{\partial x_\rho \partial x^\rho}A_b^\mu(x)=-4\pi j_b^\mu(x) . \tag{2.20}$$

Here

$$j_b^\mu(x)=e_b\int dx_b^\mu \delta^{(4)}(x-x_b) \tag{2.21}$$

is the current generated by charge b. Further,

$$\frac{\partial}{\partial x^\mu}A_b^\mu(x)=0 . \tag{2.22}$$

Hence $A_b^\mu(x)$ has all the earmarks of the electromagnetic potential in the Lorentz gauge. It follows that

$$\frac{\partial}{\partial x^\mu}F_b^{\mu\nu}(x)=-4\pi j_b^\nu(x) . \tag{2.23}$$

It is now important to realize that we can abstract from the Feynman-Wheeler action the form of Maxwell's equation. Still, in this formulation the quantities associated with the fields are not independent degrees of freedom, but rather constructs from the world lines of the particles in the theory. One may then drop the action-at-a-distance formalism and develop from the field equations a free-field theory, which can then be quantized according to the usual methods. This procedure will be detailed in Sec. III.

B. Direct interstring action

Instead of a point particle which traces out a world line in space-time, we consider a one-dimensionally extended object which traces out a world sheet in space-time, $x_a^\mu(\tau_a,\xi_a)$, where τ_a and ξ_a are the invariant parameters needed to describe the world sheet.

The surface tensor element of the sheet is

$$d\sigma_a^{\mu\nu}=d\tau_a d\xi_a \sigma_a^{\mu\nu} , \tag{2.24}$$

with

$$\sigma_a^{\mu\nu}=(\dot{x}_a^\mu x_a'^\nu - x_a'^\mu \dot{x}_a^\nu) \tag{2.25}$$

and

$$\dot{x}_a^\mu=\frac{\partial x_a^\mu}{\partial\tau_a} , \quad x_a'^\mu=\frac{\partial x_a^\mu}{\partial\xi_a} . \tag{2.26}$$

Just as the action for a free point particle is the length of its world line, the action for a free string is taken to be the area of its world sheet,[3,9]

$$S_{fa}^{(s)}=-\mu_a{}^2\int_{\tau_a=\tau_{ai}}^{\tau_a=\tau_{af}}\int_{\xi_a=0}^{\xi_a=l}(-d\sigma_a^{\mu\nu}d\sigma_{a\mu\nu})^{1/2} ; \tag{2.27}$$

the minus sign appearing in the square root indicates that we are only considering spacelike surfaces. The integration range is designed so as to have only strings of finite spatial extent; here l is a parameter with dimensions of length and μ_a is chosen to make the action dimensionless in natural units. This action is manifestly parametrization-independent and remains so even while the string is in interaction.

It is convenient at this point to introduce the linear differential operator

$$D_a^\mu = x_a'^\mu\frac{\partial}{\partial\tau_a}-\dot{x}_a^\mu\frac{\partial}{\partial\xi_a} , \tag{2.28}$$

which has the remarkable property

$$D_a^\nu f(x_a) = \sigma_a^{\mu\nu} \frac{\partial}{\partial x_a^\mu} f(x_a) \ . \tag{2.29}$$

In particular

$$D_a^\nu x_a^\mu = \sigma_a^{\mu\nu} \ . \tag{2.30}$$

The equations of motion of the free string are obtained by demanding that the action (2.27) be stationary under the variation

$$x_a^\mu(\tau_a, \xi_a) \to x_a^\mu(\tau_a, \xi_a) + \delta x_a^\mu(\tau_a, \xi_a) \ , \tag{2.31a}$$

$$\delta x_a^\mu(\tau_{ai}, \xi_a) = \delta x_a^\mu(\tau_{af}, \xi_a) = 0 \ . \tag{2.31b}$$

Effecting the variation yields

$$\begin{aligned} 0 &= \delta S_{fa}^{(s)} \\ &= 2\mu_a{}^2 \int_0^l d\xi \left[\frac{\sigma_a^{\mu\nu}}{(-\sigma_a \cdot \sigma_a)^{1/2}} x'_{a\nu} \delta x_{a\mu}\right]_{\tau_{ai}}^{\tau_{af}} + 2\mu_a{}^2 \int_{\tau_{ai}}^{\tau_{af}} d\tau_a \left[\frac{\sigma_a^{\mu\nu}}{(-\sigma_a \cdot \sigma_a)^{1/2}} \dot{x}_{a\mu} \delta x_{a\nu}\right]_{\xi_a=0}^{\xi_a=l} \\ &\quad -2\mu_a{}^2 \int_{\tau_{ai}}^{\tau_{af}} d\tau_a \int_0^l d\xi_a \, \delta x_a^\mu D_a^\nu \left[\frac{\sigma_{a\mu\nu}}{(-\sigma_a \cdot \sigma_a)^{1/2}}\right] \ . \end{aligned} \tag{2.32}$$

Note that, in contrast with the point-particle case, we have two surface terms to consider. The first one vanishes because of the nature of the variation, but the second surface term is of additional interest. If the string closes upon itself, i.e., if

$$x_a^\mu(\tau_a, 0) = x_a^\mu(\tau_a, l) \tag{2.33}$$

and

$$\delta x_a^\mu(\tau_a, 0) = \delta x_a^\mu(\tau_a, l) \ , \tag{2.34}$$

the vanishing of the second surface term is automatically achieved. On the other hand, for an open string where

$$x_a^\mu(\tau, 0) \neq x_a^\mu(\tau_a, l) \tag{2.35}$$

and

$$\delta x_a^\mu(\tau_a, 0) \neq \delta x_a^\mu(\tau_a, l) \ , \tag{2.36}$$

the vanishing of this surface term is ensured only if we impose

$$\begin{aligned} &\frac{1}{[-\sigma_a \cdot \sigma_a(\tau_a, 0)]^{1/2}} \sigma_a^{\mu\nu}(\tau_a, 0) \dot{x}_{a\mu}(\tau_a, 0) \\ &\quad = \frac{1}{[-\sigma_a \cdot \sigma_a(\tau_a, l)]^{1/2}} \sigma_a^{\mu\nu}(\tau_a, l) \dot{x}_{a\mu}(\tau_a, l) = 0 \ . \end{aligned} \tag{2.37}$$

The meaning of this requirement is nicely understood by squaring the boundary conditions (2.37); one then finds

$$\dot{x}_a \cdot \dot{x}_a(\tau_a, 0) = \dot{x}_a \cdot \dot{x}_a(\tau_a, l) = 0 \ , \tag{2.38}$$

which means that the end points trace out null geodesics[3]; they must therefore move with the speed of light.

Still, in either case, we obtain the equations of motion for the free string

$$2\mu_a{}^2 D_a^\nu \left[\frac{\sigma_{a\mu\nu}}{(-\sigma_a \cdot \sigma_a)^{1/2}}\right] = 0 \ . \tag{2.39}$$

If we wish to consider a number of strings interacting via AD forces, we start from an action

$$S^{(s)} = \sum_a S_{fa}^{(s)} + \sum_{\substack{a,b \\ a<b}} \int d\tau_a d\xi_a \int d\tau_b d\xi_b R_{ab}^{(s)}(x_a, x_b, x'_a, \dot{x}_a, x'_b, \dot{x}_b), \tag{2.40}$$

with the dependence of the interaction term on up to first-order derivatives, and

$$R_{ab}^{(s)} = R_{ba}^{(s)} \ , \tag{2.41}$$

from symmetry requirements. By demanding that $S^{(s)}$ be stationary under the variation (2.31), we obtain the equations of motion

$$2\mu_a{}^2 D_a^\nu \left[\frac{\sigma_{a\mu\nu}}{(-\sigma_a \cdot \sigma_a)^{1/2}}\right] = \sum_{b \neq a} \int d\tau_b d\xi_b \left[\frac{\partial R_{ab}^{(s)}}{\partial x_a^\mu} - \frac{\partial}{\partial \tau_a} \frac{\partial R_{ab}^{(s)}}{\partial \dot{x}_a^\mu} - \frac{\partial}{\partial \xi_a} \frac{\partial R_{ab}^{(s)}}{\partial x'^\mu_a}\right], \tag{2.42}$$

and, when a represents an open string, we have the additional requirements at the end points

$$2\mu_a{}^2 \frac{\sigma_{a\mu\nu}}{(-\sigma_a \cdot \sigma_a)^{1/2}} \dot{x}_a^\nu = \sum_{b \neq a} \int d\tau_b d\xi_b \frac{\partial R_{ab}^{(s)}}{\partial x'^\mu_a} \quad \text{at } \xi_a = 0 \text{ and } l \ . \tag{2.43}$$

It shows that the end points have an interaction of their own with the "fields" created by the other strings. Hence they do not necessarily move with the speed of light in this case.

By multiplying the equation of motion (2.42) by $\dot{x}_a^\nu$ and x'^ν_a, respectively, we obtain, after a little bit of algebra,

$$0 = \left[\frac{\partial}{\partial \tau_a}\left(1 - \dot{x}_a^\alpha \frac{\partial}{\partial \dot{x}_a^\alpha}\right) - \frac{\partial}{\partial \xi_a} \dot{x}_a^\alpha \frac{\partial}{\partial x'^\alpha_a}\right] \times \sum_{b \neq a} \int d\tau_b d\xi_b R_{ab}^{(s)}, \tag{2.44}$$

$$0=\left[\frac{\partial}{\partial \xi_a}\left(1-x_a'^{\alpha}\frac{\partial}{\partial x_a'^{\alpha}}\right)-\frac{\partial}{\partial \tau_a}x_a'^{\alpha}\frac{\partial}{\partial \dot{x}_a^{\alpha}}\right]\times\sum_{b\neq a}\int d\tau_b d\xi_b R_{ab}^{(s)}\ . \tag{2.45}$$

A similar procedure for the boundary conditions on an open string yields

$$0=\dot{x}_a^{\alpha}\frac{\partial}{\partial x_a'^{\alpha}}\sum_{b\neq a}\int d\tau_b d\xi_b R_{ab}^{(s)}\quad \text{at } \xi_a=0,\ l\ , \tag{2.46}$$

$$\mu_a{}^2(-\sigma_a\cdot\sigma_a)^{1/2}=x_a'^{\alpha}\frac{\partial}{\partial x_a'^{\alpha}}\times\sum_{b\neq a}\int d\tau_b d\xi_b R_{ab}^{(s)}\quad \text{at } \xi_a=0,\ l. \tag{2.47}$$

Just as in the point-particle case, Eqs. (2.44)–(2.47) can be obtained by demanding that the action be invariant under the reparametrization

$$\tau_a\to\tau_a+\delta\tau_a\ ,\qquad \xi_a\to\xi_a+\delta\xi_a\ . \tag{2.48}$$

Assuming this invariance to hold [i.e., by choosing $R_{ab}^{(s)}$'s which obey Eqs. (2.44)–(2.47)], we are free to find the most convenient set of τ_a and ξ_a. We will consider the choice, corresponding to an orthogonal parametrization,[3]

$$\dot{x}_a\cdot x_a'=0\ , \tag{2.49a}$$

$$\dot{x}_a\cdot\dot{x}_a+x_a'\cdot x_a'=0\ , \tag{2.49b}$$

in which case both τ and ξ have the same units.

In addition, τ_a may be taken to be the proper time, which yields

$$\dot{x}_a\cdot\dot{x}_a=1=-x_a'\cdot x_a'\ . \tag{2.50}$$

In this case, we have

$$\sigma_a^{\mu\nu}\sigma_{a\mu\nu}=-2\ , \tag{2.51}$$

to be compared with Eq. (2.16) for point particles. To push the analogy even further, we might assume that $R_{ab}^{(s)}$ depends on $\dot{x}_a^{\mu}$ and $x_a'^{\mu}$ only through the combination $\sigma_a^{\mu\nu}$. Under these restrictions, we obtain homogeneity conditions similar to Eq. (2.9):

$$\frac{\partial}{\partial \tau_a}\left(1-\sigma_a^{\mu\nu}\frac{\partial}{\partial\sigma_a^{\mu\nu}}\right)\sum_{b\neq a}\int d\tau_b d\xi_b R_{ab}^{(s)}=0\ , \tag{2.52}$$

$$\frac{\partial}{\partial \xi_a}\left(1-\sigma_a^{\mu\nu}\frac{\partial}{\partial\sigma_a^{\mu\nu}}\right)\sum_{b\neq a}\int d\tau_b d\xi_b R_{ab}^{(s)}=0\ . \tag{2.53}$$

Further, for an open string, Eq. (2.46) is automatically satisfied, while Eq. (2.48) reads

$$\mu_a{}^2(-\sigma_a\cdot\sigma_a)^{1/2}=\sigma_a^{\mu\nu}\frac{\partial}{\partial\sigma_a^{\mu\nu}}\sum_{b\neq a}\int d\tau_b d\xi_b R_{ab}^{(s)}\quad \text{at } \xi_a=0,\ l\ . \tag{2.54}$$

The equations of motion also take a very appealing form, viz.,

$$2\mu_a{}^2D_a^{\nu}\left[\frac{\sigma_{a\mu\nu}}{(-\sigma_a\cdot\sigma_a)^{1/2}}\right]=\sum_{b\neq a}\int d\tau_b d\xi_b\left[\frac{\partial R_{ab}^{(s)}}{\partial x_a^{\mu}}-2D_a^{\nu}\frac{\partial R_{ab}^{(s)}}{\partial\sigma_a^{\mu\nu}}\right]\ , \tag{2.55}$$

and for the end points

$$\mu_a{}^2\frac{\sigma_{a\mu\nu}}{(-\sigma_a\cdot\sigma_a)^{1/2}}\dot{x}_a^{\nu}+\sum_{b\neq a}\int d\tau_b d\xi_b\frac{\partial R_{ab}^{(s)}}{\partial\sigma_a^{\mu\nu}}\dot{x}_a^{\nu}=0\ , \tag{2.56}$$

where Eq. (2.55) is like Eq. (2.8) for point particles under the replacements

$$\frac{d}{d\tau_a}\to 2D_a^{\mu}\ ,\qquad u_a^{\mu}\to\sigma_a^{\mu\nu}\ . \tag{2.57}$$

We now give several examples of interactions which satisfy the above requirements:

$$R_{ab}^{(s)}=g_ag_b[(-\sigma_a\cdot\sigma_a)]^{1/2}[(-\sigma_b\cdot\sigma_b)]^{1/2}G(s_{ab}{}^2)\ , \tag{2.58}$$

where G is some symmetric Green's function. It is easy to see that it corresponds to scalar potentials.

One can form the tensor interaction quite easily by taking

$$R_{ab}^{(s)}=g_ag_b\frac{\sigma_a^{\mu\nu}\sigma_{a\nu}{}^{\rho}}{(-\sigma_a\cdot\sigma_a)^{1/2}}\frac{\sigma_{b\mu\alpha}\sigma_{b\rho}{}^{\alpha}}{(-\sigma_b\cdot\sigma_b)^{1/2}}G(s_{ab}{}^2)\ . \tag{2.59}$$

This is exactly the stress tensor of the string. It therefore describes a gravitonlike exchange if the Green's function G is chosen to correspond to zero mass. Note that the interaction between the trace terms of the stress tensor is precisely given by Eq. (2.58).

Now, if we want to take the replacements (2.57) at face value, the electromagnetic coupling of point particles should have as a string analog

$$R_{ab}^{(s)}=g_ag_b\sigma_a^{\mu\nu}\sigma_{b\mu\nu}G(s_{ab}{}^2)\ . \tag{2.60}$$

This antisymmetric second-rank tensor coupling represents a new type of object previously unknown in the point-particle construction. It can nevertheless be understood as an analogy to the vector interaction of point particles. Its remarkable properties will be discussed in Sec. III.

III. ANTISYMMETRIC COUPLING

In this section we explicitly consider the action

$$S=-\sum_a \mu_a{}^2\int(-d\sigma_a\cdot d\sigma_a)^{1/2} + \sum_{\substack{a,b\\ a<b}} g_a g_b \int d\sigma_a^{\mu\nu} d\sigma_{b\mu\nu} G(s_{ab}{}^2)\ , \tag{3.1}$$

where G is some Green's function describing time-symmetric interactions. Note that the coupling constants g_a have the units of mass. The sums are over all the strings, both closed and open. In view of the very different physical interpretation of the above coupling depending on the nature of the strings, we are going to consider all possible cases separately in order of increasing complexity.

A. Coupling between two closed strings

In this case the variation of the action yields only one type of equation of motion, i.e.,

$$2\mu_a{}^2 D_{a\nu}\left[\frac{\sigma_a^{\mu\nu}}{(-\sigma_a\cdot\sigma_a)^{1/2}}\right]=g_a\sum_{b\neq a} F_b^{\mu\rho\sigma}(x_a)\sigma_{a\rho\sigma}\ . \tag{3.2}$$

Note the great similarity with the Lorentz force equation (2.14). The analog of the electromagnetic field tensor is given by

$$F_b^{\nu\rho\sigma}(x)=\left[\frac{\partial}{\partial x_\nu}\phi_b^{\rho\sigma}(x)+\frac{\partial}{\partial x_\rho}\phi_b^{\sigma\nu}(x)+\frac{\partial}{\partial x_\sigma}\phi_b^{\nu\rho}(x)\right], \tag{3.3}$$

where $\phi_b^{\rho\sigma}$ is the tensor potential construct due to string b. Its explicit dependence on the world sheet of string b is given by

$$\phi_b^{\rho\sigma}(x)=g_b\int d\sigma_b^{\rho\sigma} G((x-x_b)^2)\ . \tag{3.4}$$

As in the Feynman-Wheeler theory, we expect to recover causality by requiring that no net radiation exists. This procedure will then yield the radiative reaction force on the string due to our coupling. Further, we find by explicit calculation that

$$\frac{\partial}{\partial x^\rho}\phi_b^{\rho\sigma}(x)=0 \tag{3.5}$$

only when b refers to a closed string.

We have not put any restriction on the Green's function G except that it represents a time-symmetric interaction. The general expression for G is then

$$G(s^2)=\delta(s^2)-\Theta(s^2)\frac{m}{2s}J_1(ms)\ , \tag{3.6}$$

where $J_1(x)$ is the Bessel function of order 1. G obeys

$$\left(\frac{\partial^2}{\partial x_\rho\partial x^\rho}+m^2\right)G(x^2)=-4\pi\delta^{(4)}(x)\ , \tag{3.7}$$

where m is to be identified (eventually) with the mass of the field. It follows from the above that

$$\left(\frac{\partial^2}{\partial x_\rho\partial x^\rho}+m^2\right)\phi_b^{\alpha\beta}(x)=-4\pi j_b^{\alpha\beta}(x)\ , \tag{3.8}$$

where $j_b^{\alpha\beta}$ is the "matter current" construct

$$j_b^{\alpha\beta}(y)=g_b\int d\sigma^{\alpha\beta}\delta^{(4)}(y-x(\tau,\xi))\ . \tag{3.9}$$

In the case b is a closed string; it is conserved as can be seen from Eq. (3.5):

$$\frac{\partial}{\partial x^\alpha}j_b^{\alpha\beta}(x)=0\ . \tag{3.10}$$

These equations are now sufficient for us to abstract a free-field theory which corresponds to the same forces being exerted on the strings. This is the more popular view of matter (strings) interaction being mediated by fields. So as to render the procedure completely obvious, let us apply it to the classical electrodynamics of Feynman and Wheeler. There the aim, of course, is to obtain Maxwell's equations.

The first steps consist in remarking that the field constructs only couple to point particles in the combination

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ , \tag{3.11}$$

which is invariant under the transformation

$$A_\mu\to A_\mu+\partial_\mu\Lambda\ . \tag{3.12}$$

However, since we have in the action-at-a-distance formalism

$$\partial_\mu A^\mu=0\ , \tag{3.13}$$

it follows that

$$\partial_\mu\partial^\mu\Lambda(x)=0\ . \tag{3.14}$$

Also, we require A_μ to obey the homogeneous equivalent of the equations of motion (2.20), i.e.,

$$\partial_\mu\partial^\mu A_\nu(x)=0\ . \tag{3.15}$$

Our aim is now to build a Lagrangian density for A_μ which is invariant under the transformation (3.12), subject to the condition (3.13), and which leads to the equations of motion (3.15). Such a Lagrangian density is that introduced by Fermi to quantize the electromagnetic field:

$$\mathcal{L}_{\text{e.m.}}=-\tfrac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A^\nu)\ , \tag{3.16}$$

which yields the free Maxwell's equations in the Lorentz gauge only. It is remarkable that the action-at-a-distance theory gives only the Lagrangian density that is suitable for covariant quantization. Further, the gauge invariance (3.12) does not allow for the introduction of any mass term, thus requiring A_μ to describe a massless vector field. Then the field-theory equivalent of the Feynman-Wheeler action would be

$$S=m\int(dx\cdot dx)^{1/2}+e\int dx_\mu A^\mu(x)+\int d^4x\,\mathcal{L}_{\text{e.m.}}\ . \tag{3.17}$$

(Here we have not yet associated a field with the matter.) We follow exactly the same procedure in the case of our string-string interaction and proceed to build a Lagrangian density for the antisymmetric tensor potential $\phi_{\mu\nu}(x)$.

First we observe that coupling to matter only takes place through the combination

$$F^{\alpha\beta\gamma}=(\partial^\alpha\phi^{\beta\gamma}+\partial^\beta\phi^{\gamma\alpha}+\partial^\gamma\phi^{\alpha\beta})\ , \tag{3.3$'$}$$

which is seen to be left invariant by the (gauge) transformation

$$\phi^{\alpha\beta}\to\phi^{\alpha\beta}+\partial^\alpha\Lambda^\beta-\partial^\beta\Lambda^\alpha\ . \tag{3.18}$$

In addition, since we have

$$\partial_\mu\phi^{\mu\nu}=0\ , \tag{3.5$'$}$$

the Λ^μ's must satisfy

$$\partial_\mu(\partial^\nu\Lambda^\mu-\partial^\mu\Lambda^\nu)=0\ , \tag{3.19}$$

which is the Proca equation for a massless spin-one field. The Lagrangian density that is invariant under the transformation (3.18), subject to Eq. (3.5$'$), is given by

$$\mathcal{L}=+\tfrac{1}{12}F_{\alpha\beta\gamma}F^{\alpha\beta\gamma}+\tfrac{1}{4}(\partial_\alpha\phi^{\alpha\beta}\partial^\gamma\phi_{\gamma\beta}+\partial_\alpha\phi^{\beta\alpha}\partial^\gamma\phi_{\beta\gamma})\ . \tag{3.20}$$

One can check that the variation of this Lagrangian density with respect to $\phi^{\mu\nu}$ leads to the equations of motion

$$\Box\phi_{\mu\nu}(x)=0\ . \tag{3.21}$$

Hence, we see that if we want to write a Lagrangian density that is left invariant by the transformation (3.18), we must have zero mass. This is of course not unexpected. Further, the Lagrangian density we obtain is that suited for covariant quantization, and it is the analog of the Fermi Lagrangian (3.16). Of course we assume we still are in the specific gauge (3.5).

Although we have written $\mathcal{L}$ in this specific covariant gauge, at the classical level, we need not restrict ourselves to it, and we can start from

$$\mathcal{L}=\tfrac{1}{12}F_{\alpha\beta\gamma}F^{\alpha\beta\gamma}\ , \tag{3.22}$$

which has the full gauge invariance (3.18) and leads to the equations of motion

$$\Box\phi^{\rho\sigma}+(\partial^\rho\partial_\mu\phi^{\sigma\mu}+\partial^\sigma\partial_\mu\phi^{\mu\rho})=0\ ; \tag{3.23}$$

then we choose the divergence of $\phi_{\mu\nu}$ to be anything we wish by means of the gauge transformation (3.18).

The action we obtain through this procedure is (for one closed string, say)

$$S=-\mu^2\int(-d\sigma\cdot d\sigma)^{1/2}+g\int d\sigma_{\mu\nu}\phi^{\mu\nu}+\int d^4x\,\mathcal{L}\ , \tag{3.24}$$

where $\mathcal{L}$ is given either by Eq. (3.20) or (3.22).

The parity of the potential $\phi_{\mu\nu}$ is fixed through its coupling to the string. We define the parity transformation P by

$$P:\begin{cases}x_0(\tau,\xi)\to x_0(\tau,\xi)\ ,\\ \vec{x}(\tau,\xi)\to-\vec{x}(\tau,\xi)\ .\end{cases} \tag{3.25}$$

The above coupling is left invariant if we take

$$P:\begin{cases}\phi^{0i}\to-\phi^{0i}\ ,\\ \phi^{ij}\to+\phi^{ij}\ ,\end{cases} \tag{3.26}$$

where

$$i,j,\ =1,2,3\ .$$

The free-field theory of this tensor potential is examined in greater detail in the Appendix, where it is quantized along conventional lines. For our purposes, it suffices to say that it represents one scalar degree of freedom of mass zero. This is most easily seen by going into momentum space and taking a specific frame for the lightlike 4-vector k_μ. Its null-plane coordinates are

$$\begin{aligned}&k_1=k_2=0\ ,\\ &k_-\equiv(1/\sqrt{2})(k_0-k_3)=0\ ,\\ &k_+\equiv(1/\sqrt{2})(k_0+k_3)\neq0\ .\end{aligned} \tag{3.27}$$

Then, under a gauge transformation,

$$\begin{aligned}&\tilde\phi_{+-}\to\tilde\phi_{+-}+k_+\tilde\Lambda_-\ ,\\ &\tilde\phi_{+i}\to\tilde\phi_{+i}+k_+\tilde\Lambda_i\ ,\\ &\tilde\phi_{-i}\to\tilde\phi_{-i}\ ,\\ &\tilde\phi_{12}\to\tilde\phi_{12}\ ,\end{aligned} \tag{3.28}$$

where the tilde denotes the Fourier transform. This shows that only ϕ_{-i} and ϕ_{12} are gauge-independent. However, the gauge condition (3.5) yields

$$\begin{aligned}&k_+\tilde\phi_{-i}=0\ ,\\ &k_+\tilde\phi_{-+}=0\ ,\end{aligned} \tag{3.29}$$

which shows that of the six possible degrees of freedom contained in $\phi_{\mu\nu}$, only one survives. We should emphasize that this is not true for the massive case, where three degrees of freedom survive, giving rise to a pseudovector field.

In conclusion, we see that our coupling between two closed strings corresponds to the exchange of a massless scalar meson. It is remarkable that exactly such a state makes its appearance in the quantized version of the theory (Virasoro-Shapiro[10] model) as well as in the Pomeron sector of the Veneziano model.[11] This is very encouraging since the mechanisms that cause the particle to be massless are of a very different nature in both theories (at least superficially).

B. Coupling between two open strings

In this case the situation is much more interesting because we have to take into account the boundary conditions at the end of each string. While the forces on the body of each string are still given by (3.2), we have the additional equations

$$2\mu_a{}^2 \frac{\sigma_{a\mu\nu}}{(-\sigma_a\cdot\sigma_a)^{1/2}}\,\dot{x}_a^{\nu} = -g_a \sum_{b\neq a} \phi_{b\mu\nu}\,\dot{x}_a^{\nu} \quad \text{at } \xi = 0,\, l\ . \tag{3.30}$$

These mean that the coupling to matter does not take place exclusively in the combination (3.3). Thus we see that the end-point couplings break our original gauge invariance.

A further difference is encountered in the divergence of $\phi_b^{\mu\nu}(x)$. We find explicitly that

$$\partial_\mu \phi_b^{\mu\nu}(y) = -g_b \int_{\tau_i}^{\tau_f} d\tau \int_0^l d\xi\, D^\nu G((y-x)^2). \tag{3.31}$$

Integration by parts yields

$$\partial_\mu \phi_b^{\mu\nu}(y) = g_b \int_{\tau_i}^{\tau_f} d\tau \{\dot{x}_b^{\nu}(\tau,\xi) G([y - x_b(\tau,\xi)]^2)\}_{\xi=0}^{\xi=l}. \tag{3.32}$$

The right-hand side is, up to a coupling constant, the same as a vector field construct generated by the end points of string b, each contributing with opposite "charge." [In the case where G corresponds to a zero-mass interaction, it is exactly like the electromagnetic vector-potential construct of Eq. (2.18).] Another way of stating this is to notice that with the Green's function (3.6), the equations of motion for $\phi_b^{\mu\nu}(x)$ still are

$$(\Box + m^2)\phi_b^{\mu\nu}(y) = -4\pi g_b \int d\sigma_b^{\mu\nu} \delta^{(4)}(y - x_b(\tau,\xi))\ , \tag{3.8$'$}$$

but, because of (3.32), the antisymmetric current $j_b^{\mu\nu}(y)$ is no longer conserved, i.e.,

$$\partial_\alpha j_b^{\alpha\beta}(y) = g_b \int_{\tau_i}^{\tau_f} d\tau [\dot{x}_b^{\beta}(\tau,\xi)\delta^{(4)}(y - x_b(\tau,\xi))]_{\xi=0}^{\xi=l}\ , \tag{3.33}$$

the amount of nonconservation corresponding to "leakage" from the end points of the string. This unwieldy situation can be rectified in the following way.

First we remark that the gauge invariance of Sec. III A is spoiled only by end-point couplings. Thus it is natural to ask whether or not it can be restored by adding end-point couplings between the two open strings. This is a standard procedure for generating additional interactions.

In order to get a hint as to what to add, consider the change of the action

$$S_{\text{int}} = g_a \int d\sigma_a^{\mu\nu} \phi_{\mu\nu}(x_a) \tag{3.34}$$

under the gauge transformation (3.18). It is

$$\delta S_{\text{int}} = 2g_a \int_{\tau_i}^{\tau_f} d\tau_a \int_0^l d\xi_a D_{a\mu} \Lambda^\mu(x_a)\ , \tag{3.35}$$

which is exactly the form of a vector interaction. It is easy to see that it vanishes when string a is closed. This leads us to add to the action (3.1), between open strings, the additional interaction

$$-\sum_{\substack{a,b\\a<b}} 2e_a e_b \int d\tau_a d\xi_a D_a^\mu \int d\tau_b d\xi_b D_{b\mu} G(s_{ab}{}^2)\ , \tag{3.36}$$

where now e_a and e_b are dimensionless coupling constants.[12,13] This extra term vanishes when either string a or b is closed, so it will not affect the "body" equations (3.2); it will only alter the end-point equations as follows:

$$2\mu_a{}^2 \frac{\sigma_{a\mu\nu}}{(-\sigma_a\cdot\sigma_a)^{1/2}}\,\dot{x}_a^{\nu} = -\sum_{b\neq a}[g_a\phi_{b\mu\nu} + e_a(\partial_\mu B_{b\nu} - \partial_\nu B_{b\mu})]\dot{x}_a^{\nu} \quad \text{at } \xi = 0,\, l\ , \tag{3.37}$$

where the extra vector-potential constructs are given by

$$B_b^\mu(y) = e_b \int_{\tau_i}^{\tau_f} d\tau \{\dot{x}_b^{\mu}(\tau,\xi) G([y - x_b(\tau,\xi)]^2)\}_{\xi=0}^{\xi=l}\ , \tag{3.38}$$

that is, in view of Eq. (3.32)

$$\partial_\mu \phi_b^{\mu\nu}(x) = \frac{g_b}{e_b} B_b^\nu(x)\ . \tag{3.39}$$

By the antisymmetry of $\phi_b^{\mu\nu}$, we see that

$$\partial_\mu B_b^\mu(x) = 0\ . \tag{3.40}$$

Furthermore, this vector potential obeys

$$(\Box + m^2) B_b^\mu(y) = -4\pi j_b^\mu(y)\ , \tag{3.41}$$

where the conserved current $j_b^\mu(y)$ is given by

$$j_b^{\mu}(y) = e_b \int d\tau [\dot{x}_b^{\mu}(\tau, \xi)\delta^{(4)}(y - x(\tau, \xi))]_{\xi=0}^{\xi=l} \quad (3.42)$$

or

$$j_b^{\mu}(y) = \frac{e_b}{g_b} \partial_\nu j_b^{\nu\mu}(y) \; . \quad (3.43)$$

This shows that the leakage can be understood in terms of another conserved current. At this point it is important to realize that if we had considered *only* the coupling (3.36) between two open strings, the sole "equations of motion" would have been Eqs. (3.37) with no $\phi_{\mu\nu}$ term. Thus we would have recovered the usual electromagnetic-type gauge invariance, since the right-hand side is just like the Lorentz force. By a reasoning similar to that of the Feynman-Wheeler theory, we would have been led to consider a massless vector field. This, of course, corresponds to the massless vector state encountered in the Veneziano model. It is again satisfying to see that we can understand the Virasoro constraints[14] in terms of gauge invariances on local fields.

When both couplings (3.1) and (3.36) are considered simultaneously, we do not lose the whole gauge invariance, for the coupling to matter is still left invariant under the joint transformation

$$\phi_{\mu\nu} \to \phi_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \; , \quad (3.44a)$$

$$B_\mu \to B_\mu - \frac{g}{e} \Lambda_\mu \; , \quad (3.44b)$$

since only the combination

$$\psi_{\mu\nu} \equiv \phi_{\mu\nu} + \frac{e}{g}(\partial_\mu B_\nu - \partial_\nu B_\mu) \quad (3.45)$$

occurs in the equations.

Just as in the previous cases, we proceed to construct the Lagrangian density which is left invariant by the transformations (3.44). If we ignore the gauge conditions (3.39) and (3.40), the Lagrangian density is given by

$$\mathcal{L}_P = \tfrac{1}{12} F_{\mu\nu\rho} F^{\mu\nu\rho} - \tfrac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \tfrac{1}{4}\frac{g^2}{e^2}\phi^{\mu\nu}\phi_{\mu\nu} - \frac{1}{2}\frac{g}{e}\phi^{\mu\nu}(\partial_\mu B_\nu - \partial_\nu B_\mu) \; . \quad (3.46)$$

We emphasize that $\mathcal{L}_P$ reflects the physical situation at hand only when the gauge constraints are satisfied. To see this explicitly, note that the equations of motion for B_μ are

$$\Box B^\nu - \partial^\nu(\partial_\mu B^\mu) = -\frac{g}{e}\partial_\mu \phi^{\mu\nu} \; , \quad (3.47)$$

which in view of the gauges become

$$\Box B_\mu = -\left(\frac{g}{e}\right)^2 B_\mu \; , \quad (3.48)$$

so that, contrary to a superficial reading of $\mathcal{L}_P$, B_μ is a massive vector field with mass

$$m = \frac{g}{e} \, . \quad (3.49)$$

Further, it follows from (3.39) and (3.48) that

$$\partial_\mu \psi^{\mu\nu} = 0 \; . \quad (3.50)$$

In view of this, we can rewrite $\mathcal{L}_P$ in terms of $\psi_{\mu\nu}$ as

$$\mathcal{L}_P = \tfrac{1}{12} F_{\mu\nu\rho} F^{\mu\nu\rho} - \frac{1}{4}\frac{g^2}{e^2}\psi_{\mu\nu}\psi^{\mu\nu} \; , \quad (3.51)$$

which is to be understood in conjunction with the constraint (3.50). We have used the fact that the vector field B_μ makes no contribution to the kinetic part of $\mathcal{L}_P$ due to $\phi_{\mu\nu}$.

The equations of motion are now

$$\Box \psi_{\mu\nu} = -\left(\frac{g}{e}\right)^2 \psi_{\mu\nu} \; . \quad (3.52)$$

This, together with the constraint (3.50), shows that we now have a massive pseudovector field. Its interaction with the string would be described by the action

$$s = -\mu^2 \int (-d\sigma \cdot d\sigma)^{1/2} + g\int d\sigma_{\mu\nu}\psi^{\mu\nu} + \int d^4x \, \mathcal{L}_P \; . \quad (3.53)$$

In order to understand the meaning of this field, we note that it is produced by the gauge-invariant superposition of two types of couplings, each of which represents a *bona fide* interaction between strings. In fact, the fields B_μ and $\phi_{\mu\nu}$, when taken individually, are associated with massless particles which mediate the long-range forces between open and closed strings, respectively.

On the other hand, the combined effect of these fields produces a massive pseudovector interaction between open strings. Let us recall that the antisymmetric potential $\phi_{\mu\nu}$ acquires two degrees of freedom when developing a mass, these being provided by the massless vector field B_μ. So we have a peculiar situation in which the vector field (photon) gets absorbed, in contradistinction with the more familiar Higgs-type mechanisms.[15,16]

Before closing this section, let us remark that the coupling between open and closed strings can be analyzed in terms of the exchange of a massive pseudovector.

IV. SUMMARY AND CONCLUSIONS

We have seen that a direct interstring action can be constructed in analogy to the action-at-a-distance formalism for point particles. It lies in the correspondence between the line element of the world line of the point particle and the surface element of the world sheet of the string in space-time. The concept of parametrization invariance was generalized for the string case, leading to restrictions on the form of interactions, just as it does for point particles. It is precisely this notion which allowed for the elimination of ghost states in the quantized version of the theory.[3,17] We then presented some interactions, which we suggested could be linked to some specific spin exchange between strings.

In particular we studied the coupling between strings which had the greatest similarity with the Maxwell interaction between point particles. We found that we had to consider separately strings with open and closed ends because two types of coupling were taking place; we had interactions involving only the end points of the (open) strings, as well as interactions that coupled only points on the body of the strings. Thus we found that our coupling between two *closed* strings led to Lorentz-type equations of motion for the string bodies and that the forces it produced could be derived (in field-theoretic language) from the exchange of a massless scalar field, the masslessness being the result of the invariance of the equations of motion. When looking at the case of two open strings, we found that our coupling was capable of describing two possible physical situations. When the interaction proceeded solely through the end points, it was seen to reduce to the electromagnetic-type interaction between point charges located at the ends of the strings, thus likening it to the exchange of a massless vector field. However, we found to our surprise that when interaction between two open strings took place via both "body" and "end point" mechanisms, a new solution emerged which was shown to correspond to the exchange of a massive pseudovector field (in field-theoretic language, of course). Further, we found that the mass of this pseudovector was determined in terms of the ratio of the coupling constants appearing in the aforementioned couplings. This is very suggestive of a bootstraplike structure; it is made possible by the fact that the coupling constants for body interactions have dimensions of mass. It is interesting to note that in the Veneziano model (for open strings) and in the Virasoro-Shapiro model (for closed strings) all these states make their appearance with the correct masses (in 26 dimensions, of course). We see then that our formalism provides a natural unification between closed and open strings, since the *same* interaction produces different spectra, depending on the nature of the strings they link. Further, our work suggests that the massive pseudovector mediates the interaction between closed and open strings, thus leading to yet another spectrum.

It would be interesting to see if the same type of structure obtains when we look at more complicated interactions. However, it is a much more difficult task, because higher-spin field constructs, in our approach, interact with the strings in a nonlinear way. In particular the spin-two interaction poses interesting challenges.

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APPENDIX

Here we will consider the free-field theory for $\phi_{\mu\nu}$ and its quantization.

We begin by considering the Lagrangian density

$$\mathcal{L} = \tfrac{1}{12} F_{\alpha\beta\gamma} F^{\alpha\beta\gamma} , \tag{A1}$$

where

$$F^{\alpha\beta\gamma} \equiv \partial^\alpha \phi^{\beta\gamma} + \partial^\beta \phi^{\gamma\alpha} + \partial^\gamma \phi^{\alpha\beta} . \tag{A2}$$

This Lagrangian is invariant under the gauge transformation

$$\phi_{\mu\nu} \to \phi_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu , \tag{A3}$$

and leads to the equations of motion

$$\partial_\alpha F^{\alpha\beta\gamma} = 0 \tag{A4a}$$

and

$$\partial^\mu \tilde{F}_\mu = 0 , \tag{A4b}$$

where

$$\tilde{F}_\mu \equiv \tfrac{1}{6} \epsilon_{\mu\alpha\beta\gamma} F^{\alpha\beta\gamma} \tag{A4c}$$

is the dual field. In terms of the potentials we have

$$\partial_\alpha \partial^\alpha \phi^{\beta\gamma} + \partial^\beta \partial_\alpha \phi^{\gamma\alpha} + \partial^\gamma \partial_\alpha \phi^{\alpha\beta} = 0 . \tag{A5}$$

We can write Eqs. (A4) in terms of a rotation subgroup decomposition. Define

$$K \equiv \tilde{F}_0 , \quad G_i \equiv \tilde{F}_i \quad (i = 1, 2, 3) . \tag{A6}$$

Thus we have

$$\vec{\nabla} \times \vec{G} = 0 , \tag{A7a}$$

$$\frac{\partial}{\partial t} K + \vec{\nabla} \cdot \vec{G} = 0 , \tag{A7b}$$

$$\frac{\partial}{\partial t} \vec{G} + \vec{\nabla} K = 0 . \tag{A7c}$$

From these equations it is easy to see that

$$\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)K=0\ , \tag{A8a}$$

$$\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)\vec{G}=0\ . \tag{A8b}$$

In addition, a solution to these equations is[18]

$$K=\frac{\partial}{\partial t}\phi,\quad \vec{G}=-\vec{\nabla}\phi\ , \tag{A9}$$

provided that

$$\Box\phi=0\ . \tag{A10}$$

Thus, these equations are satisfied by a scalar potential.

Let us define the canonical momenta,

$$\pi_{\mu\nu}\equiv\frac{\partial\mathcal{L}}{\partial(\partial^0\phi^{\mu\nu})}=F_{0\mu\nu}=-\pi_{\nu\mu}\ . \tag{A11}$$

We see that the momenta π_{ij} $(i,j=1,2,3)$ are well-defined conjugates to ϕ_{ij}, but the π_{0i} vanish because of the antisymmetry of $F_{\alpha\beta\gamma}$. Thus, the Lagrangian, in this form, cannot be used for canonical quantization.

The classical Poisson brackets, the antisymmetry of $\phi_{\mu\nu}$, and the usual postulates of quantum mechanics lead to the canonical equal-time commutators

$$[\pi_{\rho\sigma}(\vec{x},t),\phi^{\mu\nu}(\vec{x}',t)]=-\tfrac{1}{2}i(g_\rho{}^\mu g_\sigma{}^\nu-g_\rho{}^\nu g_\sigma{}^\mu)\delta^3(\vec{x}-\vec{x}')\ , \tag{A12}$$

However, by (A11) these commutators are inconsistent with the Lagrangian (A1) and the resulting equations of motion. We can restore consistency by assuming that we are in the Lorentz gauge, defined by

$$\partial_\mu\phi^{\mu\nu}=0\ , \tag{A13}$$

and adding the "Fermi term"

$$\tfrac{1}{4}(\partial_\mu\phi^{\mu\nu}\partial^\rho\phi_{\rho\nu}+\partial_\mu\phi^{\nu\mu}\partial^\rho\phi_{\nu\rho}) \tag{A14}$$

to the Lagrangian, thus obtaining (3.19). This procedure leads to a covariant quantization. However, we can quantize (A1) directly by using noncanonical commutators, thus choosing a specific gauge. The corresponding situation in electromagnetism is to quantize in the Coulomb gauge rather than using the Gupta-Bleuler formalism.

Let us define the commutators

$$[\pi_{0i},\phi^{0j}]=[\pi_{0i},\phi^{jk}]=[\pi_{ij},\phi^{0k}]=0\ , \tag{A15a}$$

$$[\phi_{\mu\nu},\phi^{\rho\sigma}]=[\pi_{\mu\nu},\pi^{\rho\sigma}]=0\ , \tag{A15b}$$

$$[\pi_{ij}(\vec{x},t),\phi^{kl}(\vec{x}',t)]=-i\delta^*_{ij}{}^{kl}(\vec{x},\vec{x}'), \tag{A15c}$$

where

$$i,j,k,l,=1,2,3\ .$$

The special singular object, $\delta^*_{ijkl}(\vec{x}-\vec{x}')$, must be defined such that the equations of motion are consistent with these commutators.

From the equations of motion we find that

$$\partial_i\pi^{ij}=(\vec{\nabla}\times\vec{G})_j=0\ ; \tag{A7a$'$}$$

thus, we must have

$$\partial_i\delta^{*ijkl}(\vec{x}-\vec{x}')=0\ . \tag{A16}$$

Define

$$\delta^{*ijkl}(\vec{x}-\vec{x}')\equiv\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot(\vec{x}-\vec{x}')}\times[G^{ijkl}-H^{ijkl}(q)]\ , \tag{A17}$$

where

$$G^{ijkl}\equiv\tfrac{1}{2}(\delta^{ik}\delta^{jl}-\delta^{il}\delta^{jk}) \tag{A18}$$

and

$$H^{ijkl}(q)\equiv\frac{1}{2q^2}[(q^iq^k\delta^{jl}-q^iq^l\delta^{jk})-(q^jq^k\delta^{il}-q^jq^l\delta^{ik})]\ , \tag{A19}$$

with $q^2=\vec{q}\cdot\vec{q}$, which has property (A16) and satisfies

$$\partial'_k\delta^{*ijkl}(\vec{x}-\vec{x}')=0 \tag{A20}$$

as well. This means that $\partial_k\phi^{kl}$ is a c number. Also from (A15), we see that ϕ^{0i} is a c number. By appropriate choice of gauge, we can then have

$$\phi^{0i}=0 \tag{A21a}$$

$$\partial_k\phi^{kl}=0\ . \tag{A21b}$$

This situation is similar to choosing the Coulomb gauge in the quantization of the electromagnetic field, and is consistent with the Lorentz conditions.

The meaning of this special δ function becomes clear if we define the fields

$$\phi^{kl}\equiv\frac{1}{\sqrt{2}}\epsilon^{klm}\psi^m,\quad \pi^{ij}\equiv\frac{1}{\sqrt{2}}\epsilon^{ijl}\omega^l\ . \tag{A22}$$

The commutator becomes

$$[\omega^i(\vec{x},t),\psi^j(\vec{x}',t)]=-i\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot(\vec{x}-\vec{x}')}\frac{q^iq^j}{q^2}\equiv-i\delta^{ij}_{\text{long}}(\vec{x}-\vec{x}')\ , \tag{A23}$$

and

$$\partial^k\phi^{kl}=0\rightarrow\vec{\nabla}\times\vec{\psi}=0\ , \tag{A24a}$$

$$\partial^i\pi^{ij}=0\rightarrow\vec{\nabla}\times\vec{\omega}=0\ . \tag{A24b}$$

Thus, the massless $\phi_{\mu\nu}$ has one degree of freedom. This can be chosen as $\phi_{12}\sim\psi_3$. The longitudinal commutator is a direct result of the antisymmetry of the field.

Equation (A9) shows us that $K=\partial_0\phi$, but using the above solution we have that

$$K=\partial_3\phi_{12}\ . \qquad \text{(A25)}$$

Thus, ϕ and ϕ_{12} have opposite parity, and must represent independent solutions to the equations of motion. In addition, ϕ is only a solution to the free equations, but the $\phi_{\mu\nu}$ will be a solution to the inhomogeneous equations as well.

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[1]See, for instance, J. H. Schwarz, Phys. Rep. 8C, 269 (1973), and references contained therein.

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[15]While this work was being completed, we received a report by E. Cremmer and J. Scherk [NYU Report No. NYU/TR5/73 (unpublished)] in which a similar mechanism is found. However, their approach is different from ours, as they find the massive field arising from Pomeron loop effects; we also differ in the parity assignment for this new state.

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[18]We thank Professor S. W. MacDowell for pointing out this solution to us.