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Space-time code. IV

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A successor to quantum mechanics is studied. It extends atomism from matter to physical process and unites certain quantum and relativity principles with stricter finiteness, operationality, and locality. A specific kinematics is given. In it particle processes are discrete networks of elementary quantum processes, monads, of a binary nonunitary kind with specific laws of combination. The general dynamics and several examples are given. The dynamical law is not differential but algebraic. The interpretation involves a small constant time τ. Familiar space-time and field-theory concepts, the Hilbert-space metric, and the Riemannian pseudometric emerge in the approximation $\tau \to 0$, and are semimicroscopic statistical objects. Microscopic Lorentz invariance survives and implies four monads mating two by two. Operationally nonlocal concepts such as energy-momentum, gauge fields, and coordinates are absent from the microscopic theory. In the simplest model all elementary processes transfer both charge and spin, the familiar neutrality of the long-range fields being an average one analogous to that of an electrolyte. e, γ, and ν codes with correct laws of transformation and propagation are suggested. Electromagnetic, gravitational, weak, and strong interactions are considered within this framework. A heuristic argument estimates $\tau \sim \hbar/40$ GeV. The form of the theory has been determined by internal qualitative requirements and has not been subjected to external quantitative test. The developments needed for this are mentioned.

I. INTRODUCTION

The central hypothesis of this study is that a physical process is a finite network of finite elementary processes. This extension of atomism from matter to process makes necessary the development of a statistical geometry, in which the classical (c) space-time continuum emerges as a semimicroscopic statistical construct from a deeper quantum (q) structure. A step forward in the present study is the resolution of the contradiction between the noncompact Lorentz group of relativity and the compact unitary group of any finite q entity, and the formulation of a finite relativistic q mechanics in operational terms.

This attempt at theory making is guided by process ideas, locality, and simplicity.

(1) The process ideas include:

(a) The primitive entities of the physical world are not physical objects but physical operations, processes.

(b) Observational operations we actually do should be represented by the theory in their actual relations.

(c) Intrinsically impossible operations are excluded from the deep structure of the theory (operationality).

(2) By locality I mean that the law of nature relates each event only to those in its immediate causal neighborhood (Einstein).

(3) An operationally defined entity determined by operations restricted to the immediate causal neighborhood of an event I call local; all others nonlocal. The deep structure of present physical theories contains many nonlocal elements, entities whose operational determination reaches out for large distances: *Energy-momentum* requires a large periodic structure, such as a diffraction grating. A *gauge field,* such as the electromagnetic potential, requires measurements of the effect of transport over large paths through such instruments as flux loops or electron interferometers. *Space-time coordinates* are gauge variables in general relativity and indeed require large space-time frameworks. It is the practice to permit such entities in the deep structure nevertheless and to seek a kind of locality by means of gauge invariance. This invariance makes the noninvariant entities intrinsically unobservable and violates operationality [paragraph 1(c)].[1] The world might be that way, but the trend of physics inclines me to study the alternative that the law of nature admits a simple local formulation.

(4) Operationality leads one to interpret a state vector not as a probability amplitude, the probability of a single event not being an operational concept, but as a description of a process of preparation or detection (Sec. III).

(5) Operationality makes space-time geometry emergent from an underlying dynamics. The operational dependence of geometry on dynamics was recognized in the discovery of special relativity but never implemented, for until now successful theories have had an inverted or at best a sandwich structure, with the observed geometry arising out of a deeper dynamical structure itself resting on an unobserved still deeper geometry. Such a deeper geometry violates operationality as well

as making for the well-known infinities.[2] Here I take process itself as primitive entity (Sec. IV) and space-time as statistical, emergent.

(6) This q mechanics, like c and cq before it, consists of *kinematics*, the formal theory of the general process (Sec. V); *dynamics*, the formal theory of the natural process (Sec. VII); and *semantics*, the assignment of meanings to the formal descriptions (Sec. VI). As in cq mechanics the general process factors into subprocesses of creation-destruction, propagation, and interaction, respectively described by diagrams with 1, 2, and 3 or more external lines. In cq mechanics there are elementary processes, processes of zero duration, with 1, 3, 4, ... lines, incorporating the atomic hypothesis, but processes with 2 lines are indefinitely decomposable and have continuously variable duration. In q mechanics all processes are constructed by finite quantum logical techniques from elementary quantum processes[3] of creation alone as networks[4] described by plexors.[5]

(7) I have committed a violation of Lorentz invariance at the microscopic level until now,[5] a more serious objection to a theory than a violation of translational invariance. The linear space of stators for the elementary creation process was taken to be finite-dimensional for the sake of finiteness itself, and there is no finite-dimensional unitary representation of the Lorentz group. This clash of symmetries is like that between the Galilean and Lorentz groups: the unitary of q logic versus the Lorentz of relativity. In Sec. III the operational meaning of a nonunitary q logic is worked out. The elementary dynamical process is chosen to be an entity described by a nonunitary relativistic q logic from the start.[6]

(8) Increasing the symmetry group from $U(2, C)$ to $GL(2, C)$ makes it easier to assign meanings to operators, doubles the number of basic representations, and gives a natural description of electromagnetic and gravitational processes as aspects of one nonintegrable transport. There is also a possibility opened by this framework that every elementary process transfers both charge and spin (the neutrality of the electromagnetic and gravitational fields emerging only for distances $\gg\tau$, and neutral spinor and charged vector processes making use of a kind of composition that exists in a network theory). This leads to a heuristic estimate for the size of the quantum time $\tau \sim \hbar/40$ GeV. The framework suggests simple microscopic structures for e, γ, and ν processes. The proper time cq Dirac equation in a statistical geometry has been derived for the e code, and a proper-time Maxwell equation for the γ. The code for an electrodynamic interaction suggests itself immediately, and has to be shown to yield c quantum electrodynamics for $\tau \to 0$.

II. PRINCIPLES OF A q MECHANICS

The mechanics developed here retains the following elements of relativity:

(9) a single relativistic entity, an elementary process (event), relativistic in that the invariance group of its logic is the Lorentz group; and

(10) the causal or chronological connection of such processes, represented by the incidence relation of an algebraic topological complex formed of elementary processes (locality [paragraphs (2) and (3)] is assumed), and retains the following elements of cq mechanics:

(11) quantum logic, using a complex linear space to describe processes and the logical relations between them, and

(12) coherence, in the sense that this linear space is not a reducible or incoherent direct sum.

And I add a principle of process atomism:

(13) Every physical process is a finite combination of finite elementary processes, *monads*.[3]

(14) All of relativity that depends on the continuum nature of space-time and all of quantum logic that depends on an absolute unitary structure is dropped.

III. RELATIVISTIC QUANTUM LOGIC

(15) It has not been easy to bring relativity and quantum principles together because there is not a clear-cut contradiction between them. No reaction takes place between the two world pictures like that between Newtonian mechanics and Maxwellian electromagnetism. Infinities are not enough.

The reaction is catalyzed by a suitable third principle. There is a clear contradiction [see paragraph (7)] between the three principles (Q) that physical processes obey unitary logic, (R) that the elementary process is relativistically (Lorentz-) invariant [see paragraph (9)], and (F) finitism [see paragraph (13)]. One of these must be given up, usually F.

(16) I give up part of Q, the absolute metric of the linear space of stators (state vectors) of quantum mechanics, and with it unitarity, the requirement that invariance transformations be unitary or antiunitary operators. The invariance group is now the nonsingular linear or antilinear transformations of the space of stators or the projective group. The generalized quantum logic I call relativistic quantum (rq) logic, referring at first to the relativity of the Hilbert-space metric, but soon to the relativity of time [paragraph (42)]. I formulate rq logic here in parallel with the usual

unitary quantum (uq) logic, in order to apply it to elementary processes in Secs. V–VII.

(17) In both uq and rq logic I take as primitive the operational ideas of *channel* $|\,)$, a process taking in signals from us and producing certain entities, and *cochannel* $)\,|$, a process taking in such entities and putting out signals to us. The symbols $|1)$, $)2|$ may be written in any direction; $|1) = (1|$, $)2| = |2($, etc.

(18) The duality between channel and cochannel is represented by the familiar categorical duality of mathematics for which we use the prefix co: domain and codomain, vector and covector, rank and corank of a tensor, and so forth. Channel and cochannel together take the place of the single c logical, self-dual concept of class.

(19) A pair $|1)$, $)2|$ of a channel and a cochannel is called a transition.

(20) The signals of relativistic channels are binary, on or off. Those of unitary channels are numbers called counts regarded as controlling the number of systems put out and in. The difference is that between a galvanometer and an ammeter: rq channels permit only the primitive null judgment $|1)\perp)2|$, no system from $|1)$ passes $)2|$, $|1)$ *excludes* $)2|$. uq channels admit also the less primitive universal judgment $|1)\subset)2|$, all systems from $|1)$ pass $)2|$, $|1)$ *is included* in $)2|$, which requires comparing the counts of $|1)$ and $)2|$ through some auxiliary physical link called the counting channel in distinction to the system channels. Transitions [paragraph (19)] of those two kinds are called *forbidden* and *compulsory*. Table I shows the domains and relations I define. For example, we can speak of a channel either excluding or including another in uq logic, but only inclusion is defined between channels in rq logic. In a clear sense rq logic has half the structure of uq. Operationally it is more primitive to judge a transition forbidden than compulsory. rq logic is the logic of the forbidden alone. (What is not forbidden is *allowed*.)

The exclusion relation $\perp$ is a form of the Sheffer stroke of c logic.

(21) The exclusion relation between dual channels defines inclusion relations between similar channels:

$$|1)\subset|2) \equiv \text{for all })3|,\ |2)\perp)3| \text{ implies } |1)\perp)3|,$$

$$)1|\subset)2| \equiv \text{for all } |3),\ |3)\perp)2| \text{ implies } |3)\perp)1|;$$

and *dualities*, inclusion-reversing 1-1 maps $)\!\sim\!($ and $(\sim)$ transforming channels into cochannels and back:

TABLE I. Domains and relations in rq and in uq logic. For instance the first entry shows that rq channels obey a logic without negation, a well-known idea of Indian logic.

	Channels	Transitions	Cochannels
rq	$\subset$	$\perp$	$\subset$
uq	$\subset,\perp$	$\subset,\perp$	$\subset,\perp$

$$|1)\sim(= |2($$

$$\equiv \text{for all })3|,\ |1)\perp)3| \text{ if and only if })3|\subset)2|,$$

$$(\sim)2| = (1|$$

$$\equiv \text{for all } |3),\ |3)\perp)2| \text{ if and only if } |1)\perp)3|.$$

Simply put, $|1)\sim($ is the greatest cochannel excluding $|1)$; $(\sim)2|$ is the greatest channel excluded by $)2|$.

The axioms of rq logic are:

(22) The channels with the partial ordering by inclusion [paragraph (21)] form an abstract finite-dimensional projective geometry, the cochannels its dual geometry, and the dualities are inverses: $(\sim)\sim(= (1($ the identity map. The coefficient rings of these geometries are the complex numbers C. Here the coefficient ring of a projective geometry is its von Staudt division ring of marks.[7]

(23) The operational meanings of the axioms of paragraph (22) are known. It follows[8] there is an essentially unique complex linear space L whose projective geometry is the projective geometry of channels. I call vectors of L stators. Each nonzero stator represents a pure or singlet channel. General channels are subspaces in L, cochannels are subspaces in the dual space L^T. A stator will be written (ψ^m) or $|\rangle$ or $\langle|$, a costator (ψ_m) or $\rangle\,|$ or $|\,\langle$. There is no natural map between stators and costators in rq logic. The exclusion $|1)\perp)2|$ holds if and only if every stator $|1\rangle$ in $|1)$ nullifies $\rangle 2|$: $|1\rangle 2| = 0$.

(24) All rq logical relations are invariant under the linear group GL(L) and complex conjugation, which generate the *antilinear* group AL(L).

(25) A *classification* of an rq entity is a set of channels whose join, in the sense of the projective geometry, is the identity channel I and whose pairwise meets are all the null channel $\varnothing$. A *coordinate* Z is a classification with distinct complex numbers z_i assigned as labels to its channels. In rq logic each coordinate Z has a unique linear operator $\langle Z\langle$ such that $\langle Z\langle\ | = \langle\ |z$ holds just for $\langle\ |$ in the z channel. The $\langle Z\langle$'s are the diagonalizable operators. A classification assigning only the two complex numbers 0, 1 is called a class and has a *projection*, a coordinate $\langle Z\langle$ with $\langle Z\langle Z\langle = \langle Z\langle$. Coclasses are defined dually but are naturally iso-

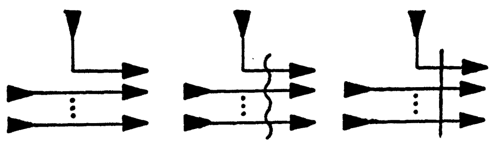

FIG. 1. Creators of quanta in a sequence, a diagonal sequence, and a set (Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac ensemble) demonstrating their relativistic invariance. The wave indicates symmetrization, the stroke skew-symmetrization. Each arrow designates a unit tensor. See R. Penrose, Ref. 4. The creator is a three-channel tensor. One channel gives the quantum being created, one the n quanta acted upon, one the $n+1$ quanta resulting.

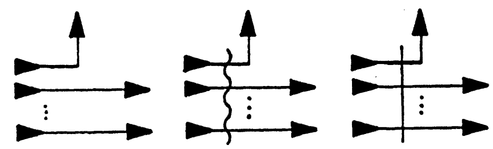

FIG. 2. Destroyers corresponding to Fig. 1. The relativistic invariance is indicated by the absence of any metric.

morphic to classes.

In rq logic the same channel therefore occurs with many classes differing in the choice of complementary channel, while in uq logic there is a unique class for every channel.

(26) Any linear operator that generates an algebra containing no nilpotents (roots of 0) is diagonalizable and is hence a coordinate. Every projection is diagonalizable and defines a class. These are well known.

(27) Those concepts of uq logic which are purely linear, nonmetric, are also relativistic. One requiring mention is the creator-destroyer formalism. For the Maxwell-Boltzmann ensemble or sequence[5] seq$S$, the stator space seq$L = \sum_n L^n$ is the linear space of Fock stators

$$|a) = \oplus_n a(1, \ldots, n)\, |1\rangle\, |2\rangle \cdots |n\rangle .$$

(We let the sum over n range indefinitely, not specifying the upper limit here. Each physical process provides its own finite upper limit for n.) If $|\,\rangle$ is any stator of L, there is an obvious definition of $|\,\rangle\, |a) = |b)$, defining thus for each $|\,\rangle$ a linear operator called the $|\,\rangle$ creator formed from $|\,\rangle$ with a three-index tensor (Fig. 1). Dually any costator $\rangle\,|$ defines a $\rangle\,|$ destroyer formed with the three-index tensor of Fig. 2. The creators and destroyers obey the identity of Fig. 3. Since only unit tensors occur in these figures, they are relativistic.

The modifications of this for the Bose-Einstein ensemble dia$S$ and the Fermi-Dirac ensemble set S are routine.[5] Instead of the product, the commutator or anticommutator of Fig. 3 gives 1.

Thus, the usual appearance of Hermitian conjugation in the Bose-Einstein or Fermi-Dirac commutation relations is spurious and unnecessary. One may use instead the more basic relativistic concept of the destroyers dual to a basis of creators.

The rq logic reduces to uq in the presence of an inclusion relation $\subset$ between channels and cochannels or, equivalently, one exclusion relation $\perp$ between channels and one between cochannels. The uq logic adds to paragraph (22) the axiom:

(28) An exclusion relation $\perp$ between channels is given defining an orthocomplemented projective geometry.

It is known[9] that such a $\perp$ is always representable by a Hermitian conjugation,[10] a nonsingular antilinear operator from stators or costators, h:

$$\psi^A \to \psi^h_A = h_{A\dot{B}}\psi^{\dot{B}} \equiv (h\psi^C)_A ,$$

where $\psi^C = (\psi^{\dot{B}})$ is the complex conjugate stator of $\psi = (\psi^B)$. (The dot notation of van der Waerden extends from special linear tensors in two complex dimensions to general linear tensors in n complex dimensions in an obvious way.) The stators ψ and ψ^C belong to different spaces L and L^C. We may make $h_{A\dot{B}}$ a positive-definite Hermitian symmetric form or *metric* merely by a numerical factor. One channel ψ excludes another φ if $\psi^h\varphi = 0$. When one metric $H = (H_{A\dot{B}})$ is singled out as absolute, the linear space L becomes a Hilbert space (L, H).

(29) I designate the original stators $|\,\rangle$ of L by $|\to\rangle$.

There are three other related kinds of spinor: ψ_A, $\psi^{\dot{A}}$, $\psi_{\dot{A}}$. Of these only a $\psi^{\dot{A}}$ can be made from ψ^A naturally, as the skew tensor ϵ_{AB} is not a GL(2, C)-invariant. But there are natural maps among the *algebras* of these four kinds of spinor. If $q^A{}_B$ is in the algebra of linear operators of the (ψ^A), then the transpose $(q^T)_A{}^B = q^B{}_A$ is in that of the (ψ_A), the complex conjugate $(q^C)^{\dot{A}}{}_{\dot{B}} = (q^A{}_B)^C$ is in that of the $(\psi^{\dot{A}})$, and the Hermitian conjugate $(q^H)_{\dot{A}}{}^{\dot{B}} = (q^B{}_A)^C$ is in that of the $(\psi_{\dot{A}})$. It is sometimes convenient to call these four kinds of spinor

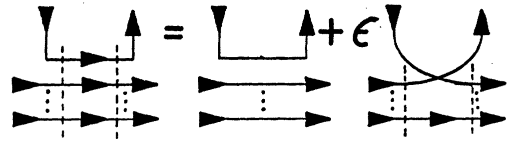

FIG. 3. Algebraic relation between creator and destroyer. The dashed line either blank, wavy, or straight and the coefficient ϵ is 0, +1, −1 for sequence, diagonal sequence, set, respectively. Note the relativistic invariance of these relations as well, which involve only unit tensors.

I, T, C, or H spinors. Spinors of GL(2, C) can be called *general* spinors to separate them from the more familiar *special* spinors of SL(2, C) and *unitary* spinors of SU(2, C). The notation is summarized in the first three columns of Table II.

An h has the structure $\rangle \overset{h}{-} \langle$. There are two unit operators, $\rangle \rightarrow \rangle$ and $\rangle \leftarrow \rangle$, for the four spinors I, T, C, H.

(30) There is a four-group consisting of the four involutions of paragraph (29), the identity I, the transpose T, the complex conjugate C, and the Hermitian conjugate H, with

$$I = T^2 = C^2 = H^2$$

$$TC = H, \quad CH = T, \quad HT = C \, .$$

Since the notation tells correctly how I, C, T, H are permuted by the four-group, TC = H, HI = H, etc., separate symbols will not be needed for the four kinds of spinor and the four group elements. These involutions enter into the familiar discrete symmetries of space, time, and charge. As maps of algebras, T and H are order-reversing.

(31) All rq logical relations among binary systems may be expressed as GL(2, C)-invariant linear relations among spinors of the four kinds I, T, C, H. If the four skew-symmetric tensors ϵ_{AB}, $\epsilon_{\dot{A}\dot{B}}$, ϵ^{AB}, $\epsilon^{\dot{A}\dot{B}}$ appear in such a relation, they may be eliminated.

(32) Each concept of uq logic that uses the absolute negation operation ~ or the absolute Hermitian conjugation ψ^H also defines a concept of rq logic relative to a metric h, the negation $\sim_h$, and the Hermitian conjugate ψ^h. In particular we may speak of a *normal* or *h-normal classification*, one whose channels not only have null meets but also are orthogonal, and of normal or h-normal operators and projections likewise.

(33) The probability or expectation value formulas of uq logic then follow from the eigenvalue principle for large assemblies and an assumption of continuity, just as in c logic of finite sets.

(34) In any case where a metric is added to rq logic we must ask where it comes from. General relativity has a conditional causal structure, its pseudometric; rq mechanics, a conditional logical structure as well, its metric.

(35) The general process will be a collection of pairs of creation and destruction. We may enumerate all possible essentially distinct kinds of pairs formed from one relativistic quantum S:

$$SS, \quad SS^T, \quad SS^C, \quad SS^H.$$

(36) The *pair SS* is familiar from logic. Its only intrinsic (projective-invariant) channels are the symmetric and skew-symmetric, of multiplicity $\frac{1}{2}n(n+1)$ and $\frac{1}{2}n(n-1)$, and the trivial 0, 1. For $n > 2$ there is no intrinsic singlet. When a group acts on S the infinitesimal generators G of SS are formed from those g of S by duplication:

$$G = g \oplus g \, .$$

(37) *The dual pair* SS^T has one intrinsic singlet,

$$\rangle \rightarrow \rangle = (\delta^A{}_B) \, ,$$

one intrinsic (n^2-1)-plet, the subspace of traceless stators

$$\{\psi^A{}_B \mid \psi^A{}_A = 0\} \, ,$$

and no other invariant channels but 0, 1. Its generators G are formed from those g of A by subtracting the transpose, forming the direct sum

$$G = g \oplus -g^T \, .$$

A linear operator $T^A{}_B$ of S can be interpreted as the stator of a dual pair. The dual pair is the only one whose stators themselves form a natural algebra, and is a natural choice for the concept of a q mapping. The singlet $\rangle \rightarrow \rangle$ represents the *identity channel* of this q mapping.

(38) *The conjugate pair* SS^C has no intrinsic multiplet but 0, 1. For $n = 2$ only there is a relative scalar Hermitian form

$$\psi^m \varphi = \epsilon_{\dot{A}\dot{B}} \, \epsilon_{CD} \, (\psi^{A\dot{C}}) \, \varphi^{DB}$$

of signature 1–3, with which the Hermitian symmetric stators $\psi^{A\dot{B}} = (\psi^{B\dot{A}})^C$ become a Minkowski four-space.

(39) *The Hermitian pair* SS^H has its complex conjugate $S^C S^T$ naturally isomorphic to its dual $S^T S^C$ by the exchange X of S and S^H: $(S^C S^T)^X = S^T S^C$. Therefore the linear space $\psi^A{}_{\dot{B}}$ has the h-invariant Hermitian form $\psi^h \varphi = (\psi^A{}_{\dot{B}})^C \varphi^B{}_{\dot{A}}$ of signature $\frac{1}{2}n(n+1) - \frac{1}{2}n(n-1) = n$ (and hence not a metric). This pair has no invariant multiplets but 0, 1.

(40) An ensemble stator $|E)$ is said to *link* a pair of entities S, T when an ST pair stator can be factored out of $|E)$, but not individual S and T stators. Only SS and SS^T pairs can be invariantly linked.

TABLE II. Four kinds of spinors and their quantum numbers.

			Q	R
ψ^A	I	$\vert\rightarrow\rangle$	+1	+1
ψ_A	T	$\rangle\rightarrow\vert$	−1	−1
$\psi^{\dot{A}}$	C	$\vert\leftarrow\rangle$	−1	−1
$\psi_{\dot{A}}$	H	$\rangle\leftarrow\vert$	+1	−1

Relations to special relativity

(41) It has been suggested that the world process is a discrete network of discrete processes in much the way that the path of a checker is a sequence of moves, and that the duration of a process measures the number of monads[3] in it. Then a counting channel for monads, at least in large numbers, must be a clock. This identification is permitted by three well-known circumstances:

(a) The Lorentz group is isomorphic to the group of a binary rq logic, the antiprojective group in two complex homogeneous coordinates.[10]

(b) Metrics $h_{A\dot{B}}$ in complex two-space are naturally isomorphic to future vectors h^{μ} in Minkowski space-time as convex linear sets.

(c) The invariance subgroup of a timelike vector, the rotation group $SO(3, R)$, is isomorphic to the invariance subgroup of a metric, $SU(2, C)/Z$, where $Z = \{1, -1\}$ is the center of $SU(2, C)$.

These lead me to take for the elementary quantum process an rq binary entity, not a unitary one as before.

(42) The quantity previously identified with time[5] was the total number of monads. This is the relativistic invariant $\psi^m \psi_m$ for a Bose-Einstein (diagonal) sequence, for example, where the ψ_m are two creators corresponding to two basis vectors ψ_0, ψ_1, and the ψ^n are two annihilators corresponding to covectors in the reciprocal basis. Now that I identify the projective group of rq logic with the Lorentz group of relativity, I cannot identify this invariant with a coordinate time, but only with proper time. The unit for these times is always the yet-to-be-determined τ.

(43) Designate by $\hat{2}$ a projective binary quantum and by $\hat{2}^C$ the related one with complex conjugate stators. Then the classical limit of dia$\hat{2}\hat{2}^C$ as a causal space is a Minkowski space-time. From this projective ensembles can be made which approach the familiar unitary representations of the Poincaré group. This suggests that c relativity and c quantum theory study two emergent aspects of a plexus made of one basic relativistic binary quantum in which they meet.

IV. PROCESS STATORS

In rq mechanics the amplitudes for experimental processes will be of the form $|D)E|$, where $|D)$ is a plexor embodying the dynamical law and $)E|$ is one expressing the experimental situation. I develop these concepts next. Before giving the kinematics and dynamics of rq mechanics in terms of $|D)$'s and $)E|$'s I give nonrelativistic cq mechanics in this language with space, time, and matter as prior concepts. I leave the reader to translate c mechanics into these terms. There, $|D)$'s and $)E|$'s are sets in phase space.

(44) Let discrete time $t = n\Delta t$ be described by the one-parameter unitary group $U^n (n = \ldots, -1, 0, 1, \ldots)$ on a Hilbert space L. Then any transition amplitude has the form

$$a = \varphi^H U U \cdots U \psi \,.$$

Here φ, ψ are in L. The unitary operator U is a vector in $L \otimes L^T$, the product of L and the dual L^T, a stator linking [see paragraph (40)] one creation and one destruction. Since there is no interference between different t, the case $n = 3$ is sufficiently typical and I drop the ellipsis. In terms of the usual matrix elements $U_{A\dot{B}}$,

$$a = \sum \varphi^{\dot{A}} U_{A\dot{B}} U_{B\dot{C}} U_{C\dot{D}} \psi^D$$

$$= \sum \varphi^{\dot{A}} \delta_{\dot{A}B} U_{B\dot{C}} \delta_{\dot{C}D} U_{D\dot{E}} \delta_{\dot{E}F} U_{F\dot{G}} \delta_{\dot{G}H} \psi^H \,.$$

Here A labels an orthonormal basis $|A\rangle$ for the Hilbert space L, $\dot{A}$, one for the complex conjugate linear space L^C, and $\delta_{A\dot{B}}$ is the Hilbert-space metric, a Kronecker δ.

(45) Now I unzip[5] this amplitude into two parts, both process stators: $)E|$ characterizing the particular experimental situation, and $|D)$ involving only universal statements about dynamical processes of the given duration. In nonrelativistic mechanics the causal network is always a trivial one, a topological line, and L is infinite-dimensional, expressing the assumed possibility of causal connections between all space points.

Evidently the creator ψ^A and annihilator $\varphi^{\dot{B}}$ belong to $)E|$. If $U_{A\dot{B}}$ is fixed, if ambient fields are not varied in the processes considered, then the factors $U_{A\dot{B}}$ can be put in $|D)$. [If U varies for the processes considered, it is put in $)E|$. Such arbitrariness is common in phenomenological theories.] Then $|D)$ is (the stator with components) $U_{\dot{A}B} U_{\dot{C}D} U_{\dot{E}F}$ and $)E|$ is the costator $\varphi^{\dot{A}} \delta^{B\dot{C}} \delta^{D\dot{E}} \psi^F$. $|D)$ involves 6 processes, is a stator of three conjugate pairs of processes. $)E|$, likewise, involves 6 processes. The amplitude a is given by $a = |D)E|$.

This $|D_3)$ stator means: Evolve. Evolve. Evolve. This $)E_3|$ stator means: Actuate channel ψ. Wait. Wait. Actuate channel φ^C.

(46) The usual time parameter counts the number of linked pairs in either $|D_3)$ or $)E_3|$. Both $|D_3)$ and $)E_3|$ are singlets. If we connect φ and ψ we can think of $)E_3|$, too, as made of three pairs, two of which are linked. I conventionally assign $\frac{1}{2}\Delta t$ to each process of a pair.

(47) Each such process in $|D_3)$ mates with a unique process in $)E_3|$. To form a we connect each process in $|D_3)$ to its mate in $)E_3|$ as if we were closing a zipper. If the zipper does not close, the amplitude is zero. For comparison, the Feynman

amplitude is the summand in the expression for the transition amplitude,

$$U_{A\dot{B}}\, U_{B\dot{C}}\, U_{C\dot{D}}$$

(no summation). While each of the factors has a simple quantum logical meaning, the above product is not a geometric object at all because of the repeated but unsummed indices. This is not a problem in the usual use but only in the special context of a q theory of processes. The dynamical stator $|D)$ can be regarded merely as the Feynman amplitude with the index identifications suspended.

Relativistic cq system

(48) In principle, relativistic cq mechanics is a special case of the above. However, this formulation then puts observer-dependent factors into $|D)$ rather than $)E|$. In relativity "Wait" is ambiguous. Two relatively moving experimenters can both be waiting but they are doing something different. In the relativistic case it is more natural to replace the δ links in $)E|$ by factors defining the special features of the process the observer calls waiting. The only relevant feature is the periodic tick of her laboratory clock which marks off a space-time period n^μ or

$$n^{A\dot{B}} = \sigma_\mu^{A\dot{B}}\, n^\mu \ .$$

(When we assume a single n^μ for use throughout the system we are turning off gravity and storing up problems for the future.) Indeed for a two-component neutrino, the amplitude

$$a = \sum \int \varphi^*(x_1)\, U(x_1 - x_2)\, U(x_2 - x_3)\, U(x_3 - x_4)\, \psi(x_4)$$

unzips naturally into

$$a = \sum \int \begin{array}{llll} \cdots & K_{\dot{B}C}(x_1 - x_2) & \cdots & \\ \varphi^*(x_1)\, d\sigma n^{A\dot{B}} & \cdots & d\sigma n^{C\dot{D}}\, \delta(t_2 - t_3 - \Delta t) & \end{array}$$

$$\times \begin{array}{lllll} K_{\dot{D}E}(x_3 - x_4) & \cdots & K_{\dot{F}G}(x_5 - x_6) & & \\ \cdots & d\sigma n^{E\dot{F}}\, \delta(t_4 - t_5 - \Delta t) & \cdots & d\sigma \psi^G(x_6) & \end{array} ,$$

where $K_{\dot{B}C}$ is an invariant propagator

$$K_{\dot{B}C}(x) = \partial D(x) / \partial x^{C\dot{B}} \ .$$

Here

$$|D_3) = K \otimes K \otimes K$$

and

$$)E_3| = \varphi^*\, n\, d\sigma \otimes \delta(\cdots)\, n\, d\sigma \otimes \delta(\cdots)\, n\, d\sigma \otimes \psi \ ,$$

much as before.

(49) Now the unitary nature of $|D_3)$ by itself is lost, meaningless. The Hilbert-space metric is first provided by the experimental stator $)E_3|$, and is a product of more primitive metrics belonging to each of the channels joining $|D)$ and $)E|$, involving the laboratory clock.

$|D)$ and $)E|$ acquire meaning only relative to a particular mating of $|D)$ channels and $)E|$ channels, which can be given by a diagram, indices, etc. The rules of connection give $|D)$ the additional structure of a plexor rather than a mere tensor product; in particular the sequential product $(\odot K)^3 = K \odot K \odot K$.

(50) Now I treat a process as a physical system and consider some of its coordinates. Its stators (in the above example with $n = 3$ pairs) are superpositions of products $K_1 \otimes K_2 \otimes K_2$ of three dual-pair stators [see paragraph (37)].

(51) Every group e^{gt} that can act on the stator $|\rangle$ with generator $\rangle g \rangle$, $\delta |\rangle = |\rangle g \rangle\, \delta t$, also acts on any costator $\rangle\ |$ with $-\rangle g \rangle$:

$$\begin{aligned} \delta(|\rangle\ |) &= 0 \\ &= (\delta |\rangle) \cdot \rangle\ | + |\rangle \cdot \delta \rangle\ | \\ &= (|\rangle g \rangle) \cdot \rangle\ | \delta t + |\rangle \cdot \delta \rangle\ | \ , \end{aligned}$$

so

$$\delta \rangle\ | = -\rangle g \rangle\ | \delta t \ . \qquad (4.1)$$

Likewise the generator acts on the six-process stator with a six-term generator

$$G = g_1 - g_2^T + g_3 - g_4^T + g_5 - g_6^T \ ,$$

appropriate factors of $1 \otimes$ being always understood. What does G mean?

The eigenvalues of G give the changes the process can cause in the eigenvalues of the object g with suitable assistance from the experimenter. For example if g is a rotation σ_z and the object is a spin $\frac{1}{2}$, the process can cause a change of 3 provided each time the dynamics makes a change of 1 ($\sigma_z = -\frac{1}{2} \rightarrow +\frac{1}{2}$) the experimenter collects this, resetting the spin for the next dynamical evolution. Identity links cause no change in any of the object properties.

This coordinate G is a measure of nonconservation of g and will be called the *g increment* of the process and written symbolically

$$G = \Delta g. \tag{4.2}$$

(52) Related to an increment much as relative momentum is to center-of-mass momentum is a *transfer* given for 3 dual pairs by

$$G' = \tfrac{1}{6}(g_1 + g_2^T + g_3 + g_4^T + g_5 + g_6^T) \equiv \text{Avg}$$

and in general for N pairs by

$$\text{Avg} \equiv \left(\sum_{n=1}^{N} g_n + \sum_{n=1}^{N} g_n^T\right) \bigg/ 2N\,. \tag{4.3}$$

(53) These definitions of Δg and Avg, the increment and transfer of g, are uniquely determined by their composition properties under ⊙ and ⊕ and the familiar forms to which they are required to reduce for dynamics $|D)$ invariant under g.

(54) The process undergone by a system is a more complex entity then the system itself. Δg and Avg have more complex spectra than g. cq mechanics must put this complexity in the object of the process because it ignores most of the process. Is this why operationally nonlocal quantities [paragraph (3)] continually appear as system quantities in the deep structure of cq mechanics? I believe so. Let us see if we can locate this complexity in the process where it belongs, eliminate operationally nonlocal quantities from the deep structure, and simplify the theory.

(55) The omission of this possibility from cq mechanics was natural for historical reasons. In cq mechanics we usually treat systems of nearly definite rest mass. The complementary variable, proper time, is then highly indeterminate. For such a system the detailed proper-time evolution is not of operational but merely formal interest, since the new physical possibilities it admits are taken away soon after they are given. Today the rapidly growing need to understand large mass spectra in a unified way makes developments like the present one natural as well.

(56) A coordinate illustrating the complexity of the process is s, the duration of a sequential process, the number coordinate $N = \oplus\, 1$ (a direct sum of unit operators over all the elementary processes in the sequence) times the duration $\frac{1}{2}\tau$ of the elementary process: $s = \oplus\, \frac{1}{2}\tau$. Even for the singulary[5] system 1, there is no coordinate ∂_s conjugate to the duration s in the sense of the commutation relation $[\partial_s, s] = 1$, because s has non-negative spectrum. However, there is the well-known creator C conjugate in the different sense that $sC = C(s+1)$. When s/τ is large and uncertain,

$$s \gg \Delta_s \gg \tau\,,$$

the shift operator $\delta_s = C(C^H C)^{-1/2}$ is defined and nearly unitary and $(\delta_s - 1)/i$ nearly satisfies the defining relation of ∂_s. Thus the simplest object system, the singulary, undergoes a process complex enough to model one pair of canonically conjugate coordinates. The binary models Minkowski space-time as well.

(57) The law of dynamics is no longer to be given by a Hamiltonian but by a class D of dynamically allowed processes, a projection in the linear space of the world plexors. But in a coherent world [paragraph (12)], dynamics can be pure. I assume that in q dynamics *D is a singlet.*

This is a natural extension of our past experience. The Feynman amplitude for a process of given duration t is a single vector defining a singlet dynamical law D_t for cq dynamics even though the similar dynamical law D_t of c dynamics is an infinite multiplet, a distinct path though each initial point of phase space. The transition cq → c is an averaging process. I assume the transition q → cq is of the same kind, that the infinite multiplet $D = \cup_t D_t$ of cq dynamics comes from a singlet $D =)D|D)$ of q dynamics.

(58) Therefore every physical prediction is expressible in terms of amplitudes of the form

$$a = |D)E|\,,$$

where $|D)$ is the dynamic stator and $)E|$ is an experimental or kinematic stator. This too is a natural extension of Feynman's path-amplitude theory. The universal stator $|D)$ gives the detailed description of the intrinsic dynamical development. A specific $)E|$ gives the experimental environment, including input and output channels and ambient fields, and everything else extrinsic affecting the experiment. $|D)$ and $)E|$ are plexors, usually sums over many plexi. The classes D, E also define costators $)D|$, $|E)$ [see paragraph (25)].

(59) In q mechanics *kinematics* is defined by giving the plexor space of $)E|$'s, $)D|D)$ defines a *dynamics*, and the rule for translating operational descriptions of a real experimental situation into $)E|E)$'s defines the *semantics*.

(60) The Feynman path amplitude, particle propagators, the Hamiltonian operator, and the Heisenberg S matrix are projections of $|D)$ containing less information than $|D)$. For instance, a propagator $K^A{}_B(x^\mu)$ for a particle of the α type is the

amplitude

$$K^A{}_B(x^\mu) = |D)\,\alpha, 1_B, 1^A, x^\mu\,|,$$

where $)\alpha, 1_A, 1^B, x^\mu|$ is the $)E|$ describing an α channel of spin stator 1_A and an α cochannel of spin costator 1^B with a space-time separation x^μ.

The model called the nuon dynamics[5] is not pure but has steadily growing multiplicity and entropy. It represents chaos exploding at the speed of light. While it can still serve as a q picture of the future light cone (as it did in Ref. 5), it is not a dynamical process but a random walk, we now see.

V. KINEMATICS

(61) A maximal (singlet) description of a kinematically possible process is a plexor $)E|E)$ over the four kinds of spinor I, C, T, H of paragraph (30). This gives the causal and quantum logical elements of the kinematics in the language set up for that purpose in paragraph (17) and Ref. 5. We may turn at once to interpretation.

(62) There might be several elementary processes with stators $\psi^A, \varphi^A, \ldots$ transforming alike under SU(2, C) but differently under GL(2, C) or other groups. For simplicity a construction based on a single kind of elementary quantum process will be studied. This is sufficient for a quantum electrodynamics and can be generalized if the other interactions demand.

VI. SEMANTICS

Here I tie some familiar operational concepts of physics to terms in the kinematics of Sec. V. Meaning formation is necessarily an informal open-ended process, but translation of the successful parts of one theory to another can be formal and analytic. I begin with the former but go to the latter as quickly as possible.

(63) Localized parts of a process, subprocesses such as a production process in an emulsion, correspond to small subnetworks of the process plexor. The invariant chronological order among nonoverlapping subprocesses corresponds to the partial order of the plexor. The proper time between localized subprocesses is a statistical description of lines of the network joining the subprocesses. For single lines, proper time s is proportional to the number N of elementary processes along the line

$$s = \tfrac{1}{2}N\tau\ .$$

(64) The relation between stable physical processes and networks, like that between organic molecules and their diagrams, is not 1-1. One stable process is generally a superposition of many networks and one network represents a superposition of many processes, if for no other reason than the complementarity of mass and proper time.

(65) Further correspondences are systematized by identifying physical symmetries with mathematical symmetries of the kinematic theory. This is done for the local Lorentz and electromagnetic groups. The effect is to identify Q of Sec. V with electric charge, and the quotient of GL(2, C) by its center with the proper Lorentz group.

Continuous symmetries

(66) The only GL(2, C) invariants are Kronecker deltas $\delta^A{}_B$, $\delta^{\dot A}{}_{\dot B}$. Therefore any invariant amplitude $|D)E|$ is expressible in terms of products like $\delta^A{}_B\delta^B{}_C\cdots\delta^Z{}_A$, defining a closed sequence of processes $(A, B', B, C', \ldots, Z, A')$ in $|D)$ and $)E|$. This sequence consists of links [see paragraph (40)] alternately in $|D)$ and $)E|$. Monads in $|D)$ whose duals in $)E|$ are links occur at a *junction* in $|D)$ and are called *joined* in $|D)$.

(67) Thus a plexor may have two distinct discrete algebraic topologies: a pure q one determined by links and junctions, and a causal one determined by the chronological order. For simplicity [paragraph (62)] and locality [paragraph (3)], I assume these topologies coincide for $|D)$ and opt in favor of the quantum over the chronological element of deep structure [paragraph (5)]. Incidence between different loops stays but now causal succession is replaced by q linkage and the over-all causal direction, time's arrow, will be provided by the macroscopic experimental situation. It appears that plexor algebra, like the quantization algorithm, will not be an element of the q theory but a rope to be discarded after reaching q from cq.

(68) Let us designate the singlet coupling $\delta^A{}_B$ by a link $\cdot IT\cdot$, showing its indices by dots. There are thus two kinds of cycles in $)E|D)$, made of either $\cdot IT{:}IT\cdot\ldots$ or $\cdot CH{:}CH\cdot\ldots$. While a strand of letters without punctuation like $\ldots ITITI\ldots$ would have no definable direction, these punctuated strands have two distinguishable directions. The direction that goes from a T to an I or H to C within a link will be shown by a large arrow $\twoheadrightarrow$. The distinction between IT rings and CH rings will be shown by a small arrow $\rightarrow$ parallel to the large for IT and antiparallel for CH. Both arrows form closed loops in invariant plexors.

(69) Two conserved vector currents arise from GL(2, C) invariance because of locality. Their full discussion requires a generalization of Abelian gauge theory from continua to networks presently incomplete. Here the continuity of these currents is established.

(70) Q is the quantum number giving the law of

transformation $e^{iQ\theta}$ under the subgroup $\psi^A \to e^{i\theta}\psi^A$ of GL(2, C), a subgroup U(1, C). I identify U(1, C) with the electromagnetic gauge group, Q with electric charge. R gives the law of transformation under the real Abelian subgroup $\psi^A \to r\psi^A$. The quantum numbers assigned to the four chronons are shown in the last two columns of Table II.

The small arrow is then the electric current. The large arrow gives R current, whose meaning is left open.

(71) It is a familiar idea that the one group GL(2, C) thus contains both the group of the causal order and the gauge group of electromagnetism. Here it implies that every elementary process transfers charge 1 and spin $\frac{1}{2}$, that all the fields thought to be neutral have only a statistical neutrality, and that each spinor index on a cq field indicates a transfer of charge shown in Table II.

(72) For example, the c electromagnetic coupling $j^\mu A_\mu$ seems to involve no charge transfer between current j^μ and field A_μ. But in spinor form this coupling is $j^{\dot{B}A}A_{A\dot{B}}$, and I take this to mean the q process involves two lines, not one. If the two lines are even one τ apart somewhere, they form a microscopic current loop, and the electromagnetic field is not microscopically neutral.

(73) A similar discussion applies to the gravitational field with its four spinor indices, but it requires a generalization of non-Abelian gauge theory to q networks. Generally speaking, the sources of the long-range fields correspond to extensive coordinates of star subnetworks and their fluxes correspond to extensive coordinates of loop subnetworks measuring nonintegrability of transport.

(74) If the electromagnetic vector potential exists, then magnetic monopoles do not. The beauty and success of electrodynamics based on the vector potential sometimes makes magnetic monopoles seem an *ad hoc* disfigurement. But in the present framework, locality [paragraph (3)] separately excludes vector potentials from the deep structure and the kinematics (Sec. V) automatically includes magnetic monopoles. Their arbitrary elimination would therefore seem *ad hoc*, especially in view of the possible relevance to strong interactions.[11]

VII. DYNAMICS

(75) We have not determined $|D)$ yet. We now begin to do this by assigning codes to the simpler elementary particles. These give us $)E|$'s of large amplitude in $|D)$. Then the law L generating $|D)$ is sought.

These early assignments of internal structure are not successes of a particular q dynamical theory. I have drawn on the principles F, Q, R [paragraph (15)] to infer the alphabet of the code and the simpler rules of syntax. Now I add what is known about elementary particles to infer some words $|E($ in this alphabet. Much of this is input to the theory, not output.

Some provisional assignments present themselves at once:

Electron. (76) The simplest process transferring charge 1 and spin $\frac{1}{2}$ (diagram e of Fig. 4) is $\ldots:TI:TI:\ldots$ and its complex conjugate. In later work[12] it has been shown that this process generates the proper-time Dirac equation with the proper time of paragraph (63). This is satisfactory.

Photon. (77) The simplest process transferring charge 0 and spin 1 (diagram γ of Fig. 4) is the spin-1 subspace of

$$\begin{array}{l} TI:IT:TI:IT: \\ \ldots IT:TI:IT:TI:\ldots \quad , \end{array}$$

where the aligned colons form one tetradic junction. This generates a proper-time Maxwell equation as $\tau \to 0$.

Electrodynamic interaction. (78) The simplest

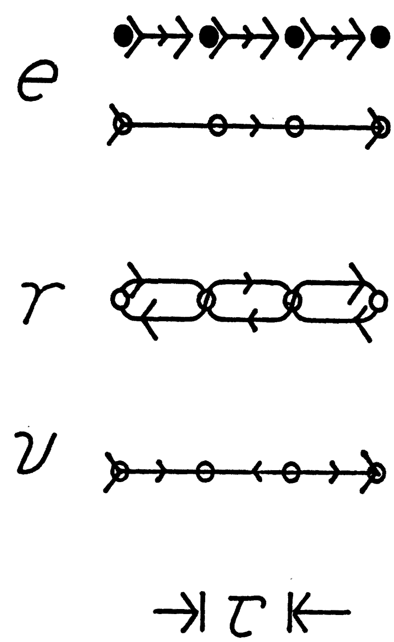

FIG. 4. Simplest sequence of elementary quantum processes with correct charge and spin for the electron, photon, and neutrino. The two diagrams shown for e are synonymous, and the simplification introduced in the second is used for the succeeding diagrams. Each two-ended arrow stands for a pair of spinor indices and the unit tensor $\delta^A{}_B$ or $\delta^{\dot{A}}{}_{\dot{B}}$. Each circle ○ represents a causal junction. The scale of this discrete structure is proposed to be $\tau \sim 5\times10^{-25}$ sec $\sim \hbar/40$ GeV.

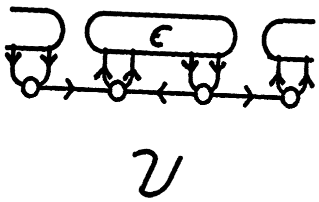

FIG. 5. Neutrino-process model showing junctions to external sources represented by tensors ϵ_{AB}, $\epsilon_{\dot{A}\dot{B}}$.

interaction connecting e and γ [paragraphs (76) and (77)] is (see Fig. 6 also)

$$\begin{array}{l} :TI:TI:TI\ldots \\ TI:IT:TI:IT \\ \ldots IT:TI:IT:TI:IT:IT:IT\ldots\ . \end{array}$$

(79) It is unreasonable to use a model constructed for electrodynamics and geometry to describe weak interactions, but since there are already indications of how to express gravitational and strong interactions in this simplest kinematics, I do so. Failure does not reflect on the adequacy of the e model for electrodynamics but refutes it as a fundamental theory.

Charged vector and neutral spinor processes are problematical. Their description within the simple kinematics [paragraph (62)] requires an innovation whose consequences have not been seen to be harmless:

Neutrino. (80) Either invariant process $\cdot TI\cdot$ or $\cdot HC\cdot$ transfers spin $\frac{1}{2}$ and charge ±1. Can we cancel the two charges against each other? Not in cq mechanics: the sum has no predictable Q at all, the product has $Q=0$ but integer spin. In q dynamics both of these options exist, the product as the ⊕ product of the two plexi, and are still useless, but there is also the sequential ⊙ product. The neutrino might be $\cdot IT:CH:\ldots:IT:CH$ (diagram ν of Fig. 4).

This possibility is explicit in the expression (4.2) for the transfer of a system property g. The charge transfer is AvQ. The empirical fact is that the charge transfer vanishes for ν processes of macroscopic duration. In cq mechanics all the terms in AvQ must be equal, since Q has an integer spectrum and is supposed to vary continuously with τ as $\tau\to 0$. Therefore, in cq mechanics each term in AvQ must vanish separately. In q mechanics the limit $\tau\to 0$ is interesting only as an approximation, and AvQ can vanish by the cancellation of successive pairs of terms. Then the limit $\tau\to 0$ is a bad approximation for certain physical processes.

One law L for such a ν involves noting the last pair in the strand and appending its complex conjugate in sequence. An even simpler program that, nevertheless, makes a time average neutral ν simply adds the two results of appending an $\cdot IT\cdot$ and a $\cdot CH\cdot$, or appends $\cdot IT\cdot + \cdot CH\cdot$. Then the ν is a sum of binomial sequences $\nu = \sum_n \odot(\cdot IT\cdot + \cdot CH\cdot)^n$.

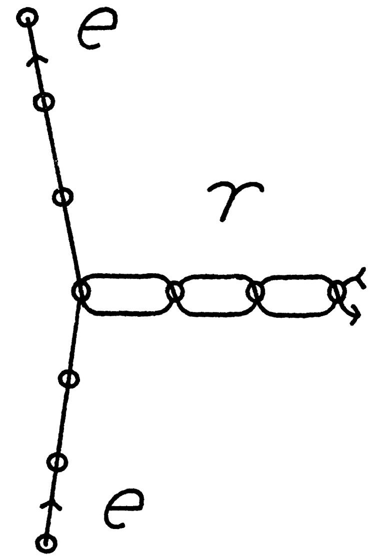

FIG. 6. Model of electrodynamic interaction process.

Continuity of charge. (81) The central pair of dots in $\cdot IT:CH\cdot$ represents a flow of charge $Q=2$ from this vertex of $|D)$ to $)E|$. As long as we keep the theory local and GL(2, C)-invariant there is, nevertheless, continuity of charge. Since the flow $Q=2$ at an $IT:CH$ vertex is compensated by a flow $Q=-2$ at a $CH:IT$ vertex only τ away, the environment of the neutrino need only carry the charge briefly and can have the same kind of average neutrality as ν. The charge jumps of the neutrino must connect to the charge fluctuations [paragraph (72)] of the long-range fields that guide its propagation. In particular, the tensor density ϵ^{AB} has the charge $Q=2$ and spin 0 that we need. It and its dual ϵ_{AB} are used as external fields in Fig. 5. The pairs $\epsilon^{AB}\epsilon_{CD}$ form the invariant δ^{AB}_{CD}.

Weak interactions. (82) As $\tau\to 0$ the neutrino must approach a neutral field. For finite τ weak short-range electromagnetic interactions remain. In the simplest model these are the weak interactions of the physical ν's. Then the weak range and strength provide two estimates of τ. According to a naive extrapolation of cq ideas at very high energies $E \geqslant \hbar/\tau$ the internal electric structure can be resolved by photons, the range of the weak interaction is $\sim c\tau$, and the weak coupling constant made dimensionless with this length is $\sim \alpha$, the electromagnetic coupling constant. This gives two preliminary estimates of τ. It is well known that these are consistent estimates.[13]

There is a second indication that the weak interactions set the scale of τ. The elementary quantum process lacks the discrete Lorentz invariances. Interactions in which a single process figures importantly should lack the corresponding conservation laws. Therefore, the interaction that breaks the discrete symmetries should set the scale of τ. It is just as well for this argument that the weak interactions have the shortest range.

VIII. RECAPITULATION

In these four papers we have formulated pure quantum concepts of geometry, kinematics, and dynamics of free propagation and interaction. The development spirals. The vague outlines of each structure are sketched in, the higher structures are eased into place and impose their stresses on the lower, and appropriate adjustments are made. The present structure is now self-consistent enough so that it will not collapse of internal stresses, and one can go on to the next level of structure. There is no operational difference between a variable that is continuous and one that is an integer in an unspecified unit. To work out the operational consequences of a $\tau \sim \hbar/40$ GeV requires further theoretical development: a particular $|D)$ and the detailed translation of scattering concepts into plexic terms begun in the next paper. Here are listed some of the rather unexpected conceptual unities revealed by the present incomplete theory. These unities are justifications for a further study rather than evidence for a theory and are presented primarily to do useful work against the resistance that each advance of atomism seems to meet.

Unity between operation and theory. Since Einstein we have known that space-time geometry arises out of underlying dynamical processes and not conversely and so I have put geometry in the surface structure, dynamics in the deep. There is no space-time point in the microscopic structure. Since von Neumann we have known that quantum theory implies a revised operational class logic and so I have put this into the deep structure. Preceding physical theories are upside down in these respects.

Unity of the world process. In previous theories, processes of production, interaction, and absorption are supposed to consist of indivisible elementary parts, while propagation processes are supposed to be infinitely divisible. Here the entire world process is treated in a uniform quantum way. The hypothesis of an underlying space-time continuum already seems to me as naive as the caloric fluid, the homunculus, and other important historical reifications.

Unity of quantum and relativity theories. The problem of unification of these two parts of physics is avoided by building the theory out of primitive parts which are both quantum and relativistic. That is the main point of this paper.

ACKNOWLEDGMENTS

I have benefited greatly from discussion with R. P. Feynman and C. F. von Weizsäcker in 1972. The hospitality of the Max Planck Institute (Erlebensbedingungen), Starnberg, is gratefully recorded. I thank Harriet Nachmann for her patient assistance in preparation.

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[1]Without operationality invariance is empty. Operationality is the distinction between the conventional, and hence irrefutable, classical logic and geometry of Poincaré and the operational geometry of Einstein and logic of Bohr.

[2]The polyhedral approximation to general relativity of T. Regge [Nuovo Cimento 19, 551 (1961)] is an interesting and suggestive c model without coordinates. It would seem worthwhile to add electromagnetism to his framework. See J. A. Wheeler, in *Relativity, Groups and Topology,* edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1964), p. 467, and R. Penrose, *An Analysis of the Structure of Space-Time,* Adams Prize Essay (Princeton Univ. Press, Princeton, 1967), p. 46.

[3]I call an ensemble of n elementary quantum processes an n-ad, and one such process, therefore, a monad. It is not proper to ask of the elementary process where and when it occurs or what undergoes it, both space-time and matter being higher level constructs.

[4]Network methods are urged by D. Bohm [in *S. N. Bose 70th Birthday Commemoration* (Birkbeck Inaugural Lecture), Calcutta, Vol. II, p. 279], and used by R. Penrose (Ref. 2); methods are also urged by R. Penrose, in *Combinatorial Mathematics and its Applications,* edited by D. J. A. Welch (Academic, London, 1971), p. 221. I am indebted to Penrose for prepublication copies of his work. To go from Penrose spin networks to the plexors I use, replace unitary by general spinors and add causal structure. For still earlier related attacks on the continuum, see C. F. v. Weizsäcker [Naturwissenschaften 38, 533 (1951)] and related work in his *Zum Weltbild der Physik* (Hirzel, Stuttgart, 1970), 11th ed. I am indebted to Weizsäcker for frequent discussions of these ideas and much encouragement.

[5]D. Finkelstein, Phys. Rev. D 5, 2922 (1972) for plexor algebra; 5, 320 (1972) for q logic.

[6]The steps from c to uq to rq logics look especially nat-

ural in algebraic terms. They are the steps from commutative * algebras to * algebras to algebras.

[7]A mark is an equivalence class of throws. A throw of a line L is an ordered quadruple of points $(P_0 P_1 P_\infty P)$ on L. See O. Veblen and J. W. Young [*Projective Geometry I* (Ginn, Boston, Mass., 1910), p. 157] for the operations of the ring of marks. A. N. Whitehead [*Axioms of Projective Geometry* (Cambridge Tracts in Mathematics and Mathematical Physics, Cambridge, 1906)] rather anticipates q logic; for him, "Geometry is the science of cross-classification." And he meant projective geometry especially.

[8]Cf. V. S. Varadarajan, *Geometry of Quantum Theory* (Van Nostrand Rheinhold, New York, 1968). I have adapted the standard term channel from J. N. Blatt and V. F. Weisskopf [*Theoretical Nuclear Physics* (Wiley, New York, 1952)], dropping purity but keeping idempotence. Elsewhere channels and cochannels are called states and tests, effectors and receptors, ...

[9]A. H. Taub and J. W. Givens [*Geometry of Complex Domains* (Princeton Univ. Press, Princeton, 1955)] are a good source for projective concepts. Every projective concept is also an rq logical one.

[10]Termed antipolarity in Ref. 9.

[11]J. Schwinger, *Particles, Sources and Fields* (Addison-Wesley, Reading, Mass., 1970).

[12]D. Finkelstein, G. Frye, and L. Susskind, following paper, Phys. Rev. D 9, 2231 (1974).

[13]T. D. Lee, Phys. Rev. Lett. 26, 801 (1971).

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Space-time code. V

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The concept of a quantum dynamics is recapitulated. The Dirac equation is obtained from a pure quantum dynamics as the limit of classical time. The theory is defective in projective gauge invariance and semantic consistency, but illustrates the relation between dynamical and experimental elements of q dynamics, and is finite, Lorentz-invariant, and local.

I. INTRODUCTION

In this work we recapitulate the present status of pure quantum (q) mechanics[1] (Sec. II) and show how the Dirac equation may be obtained as the mixed cq theory resulting from a q mechanics in the limit of classical time (Sec. III). The procedure is marred by a certain arbitrariness discussed in Sec. IV but provides a guide toward a fuller q dynamics with interactions.

The formulation of mechanics that emerges from these mathematical models is stable under the transition from classical mechanics to quantum mechanics and provides a plausible successor for quantum mechanics. It implies the following conception of the world:

(1) Both the classical space-time continuum and quantized fields are semimacroscopic statistical contructs, part of the surface structure of the world manifested in processes that are long compared to an elementary time τ.

(2) The deep structure contains neither space-time nor fields. The microscopic world is a discrete complex of discrete binary entities, elementary quantum processes. Such a world is not plenum but plexus, obeying Mach's principle in the strongest possible form: There is no space between matter, no spatial relations without interaction.

(3) The dynamical law is not a differential equation but one stator $|D)$ constructed by finite algebraic operations and yielding the Feynman amplitude in the appropriate limit $\tau \to 0$. The amplitude for any process E is the inner product of $|D)$ with a costator $)E|$.

(4) Particles are recognized by discrete chromosomelike patterns of elementary process. For example a most simple $|\dot{D})$ involving only a line complex (processes in simple series) gives rise to the Minkowski space-time and the proper-time Dirac equation for the electron as $\tau \to 0$, while a double strand is similarly related to Maxwell's equation and the photon.

II. q DYNAMICS

The basic entity is the q process. We start from a primitive q process or monad χ. (Here χ is only the name of a quantum, not an algebraic quantity of some sort.) Like any quantum, χ is associated