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Peripheral partial waves in K^+p elastic scattering and the accelerated-convergence-expansion analysis

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Predictions of high partial waves from a recent accelerated-convergence-expansion (ACE) partial-wave analysis of K^+p scattering are examined in detail. Comparison with the conflicting results of Alcock and Cottingham and of Fox and Griss reveal agreement with the former and disagreement with the latter. Possible reasons for this discrepancy are discussed.

Peripheral partial-wave amplitudes of meson-nucleon scattering have been studied recently.[1,2] Asymptotically in angular momentum, these amplitudes are controlled by long-range forces which for pion-nucleon and kaon-nucleon scattering come from t-channel pion exchange. Alcock and Cottingham[1] (AC) used the two-pion-exchange box graph together with Mandelstam's unitarity equations to calculate the s-t double spectral functions at small t. Values of these double spectral functions allowed them to estimate the peripheral partial waves for πN, KN, and $\bar{K}N$ scattering. In an alternate study, Fox and Griss[2] (FG) calculated the pion-exchange pole contribution to the process $KN \to K\pi N$ and found inelasticities of high partial waves for KN elastic scattering. FG also used a model for K^* production at high energy[3] to calculate the same quantities. Results of the two methods did not differ greatly, and FG concluded that their high-angular-momentum amplitudes were in error by at most 18%. These two groups, unfortunately, disagree with each other by an amount far in excess of 18%.

Resolution of this discrepancy is necessary since peripheral partial waves are important both as tests of t-channel exchange models and for use in phase-shift analyses. Conventional phase-shift analyses have either ignored high partial waves or calculated them from models such as the Veneziano model. The accelerated-convergence-expansion[4] (ACE) method provides a means to relate high partial waves to low partial waves using analyticity and conformal mapping. In principle, this method provides a model-independent way to calculate high partial waves. In practice, it was found that data and analyticity alone did not provide unambiguous results. Previous ACE analyses, ACE 1 (see Ref. 5) and ACE 2 (see Ref. 6), of KN elastic scattering resulted in high partial waves which were much too inelastic in comparison with model calculations.[2] This situation has been improved. In the recent ACE analysis ACE 3 (see Ref. 7) theoretical information on high partial waves was included. Results of this analysis found high partial waves in agreement with AC and in disagreement with FG. Here we present a detailed analysis of these high partial waves at one energy. For ease of comparison with FG, we choose $P_{\text{lab}} = 1.80$ GeV/c.

In ACE 3 the Born term was refined. For real parts of the scattering amplitudes, we added to $\Lambda + \Sigma$ exchange in the u channel $\rho + A_2$ exchange in the t channel. Parameters for this term were extracted from fits to high-energy data[8] using the

Regge form to extrapolate to lower energies. For simplicity we required $A'_\rho(t=0)=0$ and took the A_2 to be exchange-degenerate with the ρ. Additional terms representing $f^0+\omega$ exchange did not significantly improve the fits and were not included. We modified the lower partial waves according to a standard absorption-model prescription by substituting

$$\mathrm{Re}[a_J(s)] \to \mathrm{Re}[a_J(s)](1-2\,\mathrm{Im}[a_J(s)])\,.$$

For the imaginary part, we enlarged the conformal ellipse[4] to include the nearby part of the t-channel branch cut in its interior. The discontinuity across this cut was described using the results of AC. We included, in addition, a term to account for short-range diffraction scattering. For this, we chose the simplest possible (non-Reggeized) "Pomeron" with a slope parameter given by an optical-model approximation. Couplings to invariant amplitudes were found using the total cross section at high energy and s-channel helicity conservation. To give no diffraction scattering below the inelastic threshold we included an energy-dependent form factor in our Pomeron coupling which was zero below $W<1.7$ GeV and increased exponentially towards unity at higher energies. Thus, in GeV units, this short-range absorption (SRA) term is

$$B_{\mathrm{SRA}} = \begin{cases} 0 & (\sqrt{s}<1.7) \\ 46.5i\{1-\exp[2(1.7-\sqrt{s})]\}\exp[2.25g(t)] & (\sqrt{s}>1.7) \end{cases}$$

$$A_{\mathrm{SRA}} = 0\,,$$

where $g(t)\simeq t$ in the physical region[9] but remains finite and negative at large $|t|$ [$g(t)$ possesses a "gentle" four-pion branch cut]. Finally, for the low partial waves, we required

$$\mathrm{Im}[a_J(s)] \le \tfrac{1}{2}\,.$$

Mathematical details concerning both real and imaginary parts will be published in a subsequent article.[7]

For both real and imaginary parts of the scattering amplitudes, we required the asymptotic behavior $t^{\alpha_c(s)}$ for fixed s and $|t|\to\infty$ (this point lies on the boundary of the conformal ellipse[4]). Here, $\alpha_c(s)$ is the trajectory corresponding to that j-plane branch point which controls backward K^-p scattering and which we determined from two-Reggeon exchange. Forcing this behavior upon the scattering amplitude gave a significantly improved fit. The resulting partial waves (except for the lowest three partial waves in the $J=L+\frac{1}{2}$ sequence) were somewhat more elastic. We remark that this information as input is accessible only to ACE methods of analysis since the point at $t=\infty$, s finite, can affect scattering amplitudes only through analytic continuation.

In Fig. 1, we illustrate the real part of the Born term. Also shown are the results of AC and of ACE 3. For the latter, we include the uncertainties resulting from our fit to the data. These errors are certainly an underestimate for the high partial waves, since no considerations of theoretical bias are made. Except for the lowest partial waves, ACE 3 and the Born approximation agree well, a remarkable result in view of the simplicity of the latter. Marginal improvement could be made by including an exchange-degenerate, helicity-conserving f^0-ω exchange (this has partial waves with $J=L+\frac{1}{2}$ carrying the same sign as and dominating in magnitude those with $J=L-\frac{1}{2}$). For the case of AC, we find agreement with ACE 3 for $L-$ ($J=L-\frac{1}{2}$) partial waves where Λ exchange is small and disagreement for $L+$ ($J=L+\frac{1}{2}$) partial waves where Λ exchange is significant and where strong interference effects exist between forward and backward contributions. Agreement in the latter case improves with increasing J as forward effects predominate and where AC becomes more accurate. We compare the quantities, $\chi_J=\frac{1}{4}(2J+1)(1-\eta_J^2)$, which are directly related to the partial inelastic cross sections instead of the imaginary parts of partial waves (for large J, they are the same). In Fig. 2 we plot the contributions of AC and SRA terms to χ_J as well as their sum. In Table I we list the results of fits to the data using different models of inelasticity. The best fit was found using the sum of the above two contributions.

Also in Fig. 2 we present the results of ACE 3. The central value is the result of the fit using the Born term shown. Error bars represent uncertainties resulting from our fit to the data as well as theoretical uncertainties in the Born approximation. We estimated the latter by fitting with different values of the SRA parameters as well as with a variable over-all multiplicative factor for AC amplitudes. The uncertainties given in Fig. 2 represent approximately a one-standard-deviation increase in X^2 when each parameter was varied independently (coincidentally, the parameters which we used to represent central values

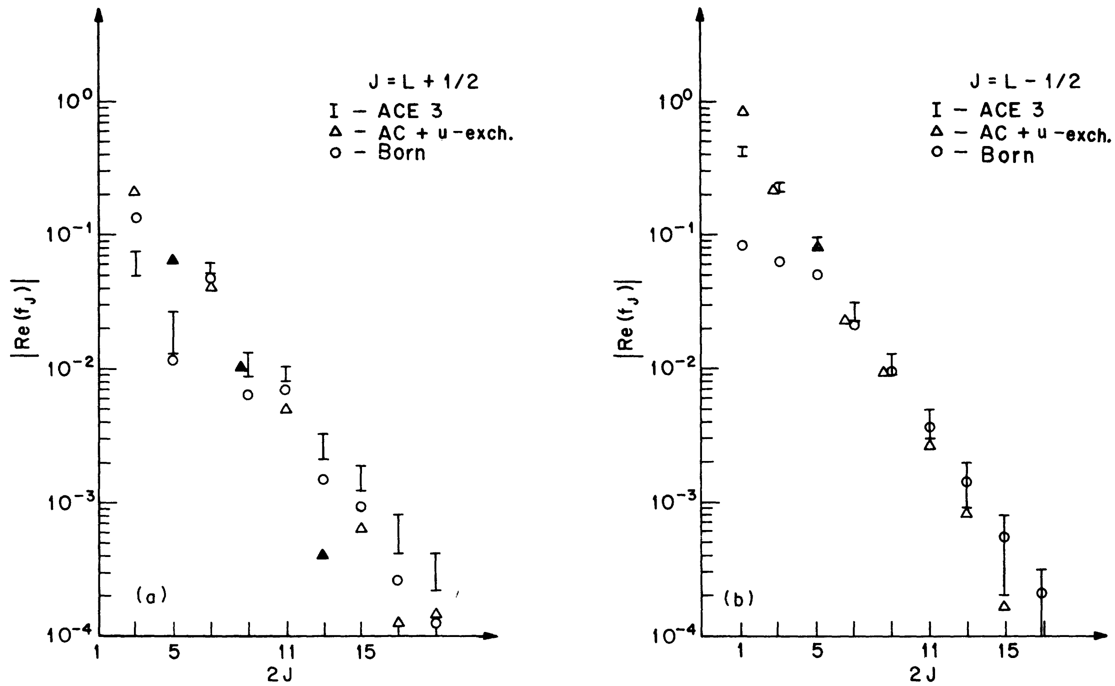

FIG. 1. (a) $\mathrm{Re}(f_J)$ vs $2J$. For the case of AC, we add u-channel exchange from our Born term to their results. Filled-in triangles indicate opposite sign. (b) $-\mathrm{Re}(f_J)$ vs $2J$.

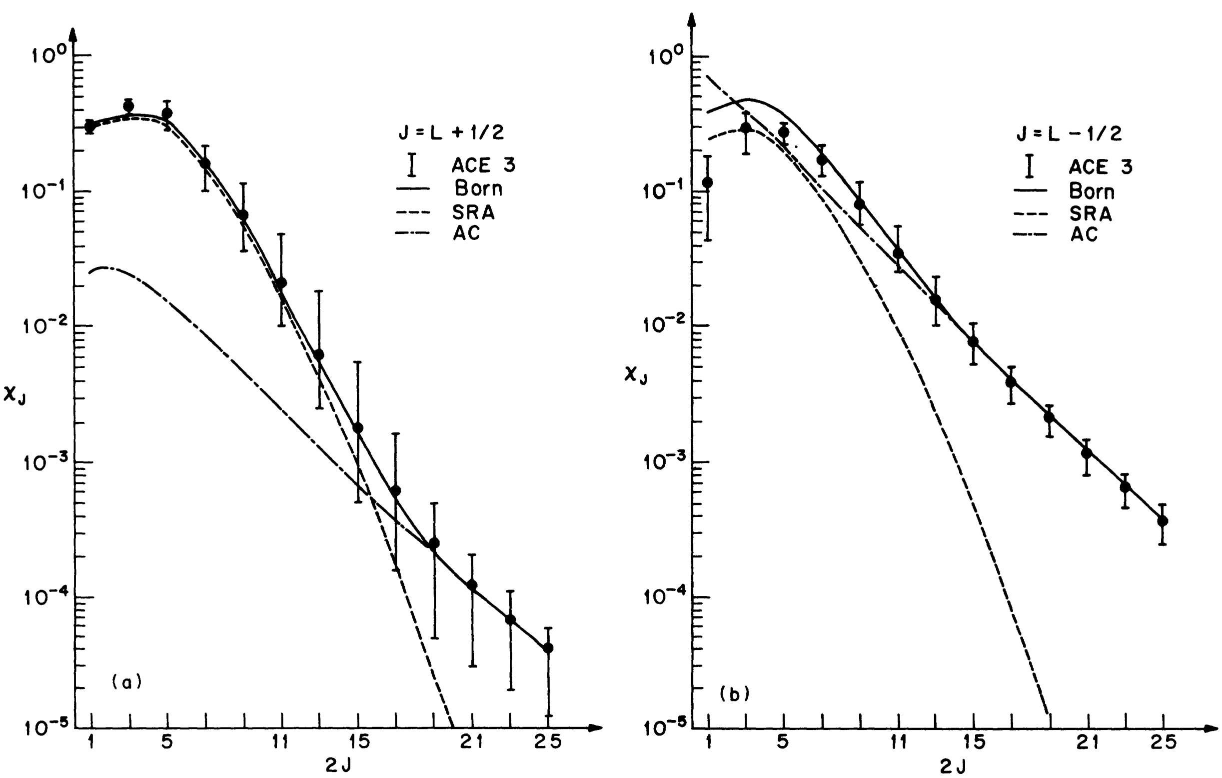

FIG. 2. Inelastic cross sections vs $2J$. For AC, $\chi_J = \mathrm{Im}(f_J)$. For all others, $\chi_J = \frac{1}{4}(2J+1)(1-\eta_J{}^2)$.

TABLE I. Models of Inelasticity.

Model	Data	Energy smoothing	Convergence test function	Total
AC + SRA	248.0	2.2	4.1	254.3
AC(no S-wave) + SRA	248.4	2.6	3.7	254.8
AC only	247.6	3.1	13.9	264.6
SRA only	250.6	2.7	3.6	256.9
$\eta^2_{Born} = 1$	250.2	3.6	17.0	270.9
FG(K^* + Δ)	249.8	3.5	9.4	262.7
FG(K^* + Δ) + SRA	250.3	3.3	2.3	255.8
FG(π-exch.)	249.1	3.0	11.8	263.9
FG(π-exch.) + SRA	257.9	1.7	2.9	262.5

produced the best fit at this energy but were chosen by *a priori* arguments and checked at other energies).

It is surprising that our Born approximation "predicts" inelasticity as well as it does. Although for partial waves of the highest angular momenta the theoretical bias is certainly strong, theory does not completely determine these partial waves. This may be seen in Fig. 3, where we plot the predictions of FG as well as the result of using their values to determine the inelastic part of the Born term. For both models of FG, amplitudes resulting from fits to data were much closer to our results, especially for $L+$ waves. From Table I we see that use of FG amplitudes gave higher X^2 values. Regarding these values, we remark that our convergence test function was primarily sensitive to the intermediate partial waves $3 \leq L \leq 6$. Furthermore, for a given Born approximation, the total is more significant than the individual contributions, because the latter tend to be negatively correlated. We conclude that data and the methods of ACE partially determine high partial waves and find better agreement

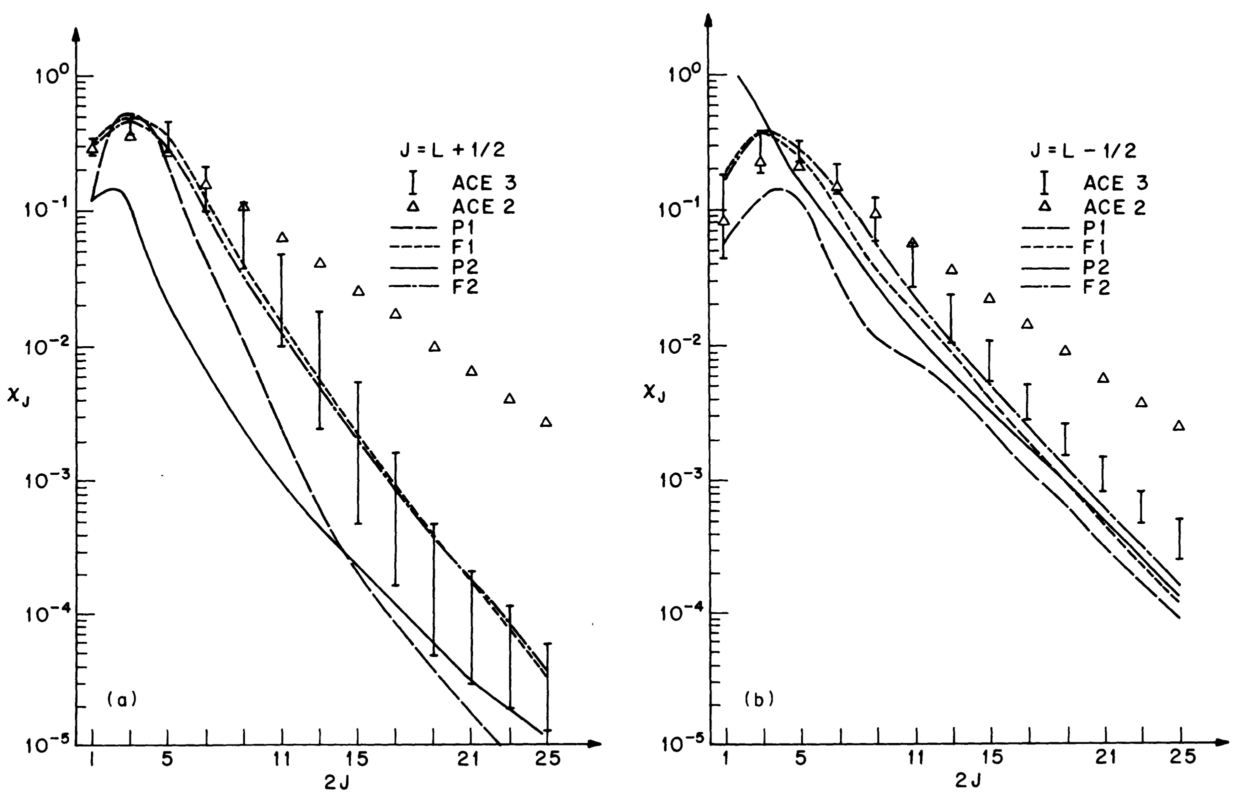

FIG. 3. Inelastic cross sections vs $2J$. $\chi_J = \frac{1}{4}(2J+1)(1-\eta_J{}^2)$. P denotes FG predictions; F denotes the results of fits to the data using these predictions. Model 1 is the K^* and Δ production model; model 2 is analytic pion exchange.

with those high partial waves calculated from AC and our SRA than with those from FG.

Some of the discrepancy between our results and those of FG can be understood by considering duality. Duality states that the imaginary part of the K^+p amplitude is produced by Pomeron exchange. Whatever the true nature of the Pomeron, at high energies we expect low and intermediate partial waves to be dominated by diffractive scattering. At low energies, we cannot expect this diffractive background to be given entirely by two-pion exchange, and we include the explicit SRA term together with the contribution from the ACE expansion. Double counting is avoided by our threshold factor. Referring to Table I, we see that the results of ACE fits support our theoretical conclusions. For both AC and FG, addition of our SRA term to their amplitudes resulted in better fits. Finally, the greater effect of the SRA term is in the $L+$ waves, and this is where we found the larger shift in FG between predictions and fits to the data (Fig. 3).

FG stated their results were accurate to at most 18% error. Especially in the case of the $L+$ sequence, we feel that this estimate is optimistic. Regge-exchange amplitudes are notoriously unreliable when extrapolated to lower energies. Although $P_{lab} = 1.80$ is an "intermediate" energy for KN elastic scattering, it certainly is not asymptotic for K^* and for Δ production, where the reaction thresholds are higher. Also, our need for a threshold factor to describe the Pomeron implies that the absorptive corrections in FG should be modified. Accordingly, we feel that the $K^* + \Delta$ production model cannot be taken seriously at these energies. For the case of analytic pion exchange, we dispute that two-pion-production contributions (not included in FG) are negligible. Since the threshold for K^* *and* Δ production lies well below the real axis, its effective distance (expressed by the norm of the complex center-of-mass momentum) is further away than a corresponding real threshold at the same energy. Thus centrifugal barrier effects may not be strong enough to eliminate contributions to partial waves of intermediate angular momentum from this process. We understand that FG are reevaluating their amplitudes and are including two-pion-production effects.[10] Finally, we remark that the $L+$ partial waves are not well determined in any case due to their small size in comparison with $L-$ partial waves.

The $I=\frac{3}{2}$ $K^-\pi$ S-wave cross section causes a large uncertainty in both FG and AC. As it is not directly accessible to experiment, this number is in doubt (especially since it may change with energy): AC used 20 mb; FG used a much smaller value. Let us assume that 20 mb is too large and consider the effect upon our analysis if we reduce this contribution to our Born term. A smaller cross section primarily affects AC partial waves by reducing their "background" π-K scattering.[1] This in turn could lower the total imaginary part of their partial waves (background + resonances) by as much as 40%. Also, their explicit S-wave model contributions[11] would become smaller (it is curious that results of *this* model agree, approximately, with FG for $L-$ partial waves). If we reduce AC by (say) 70% and use this as input to the Born term, the resulting fit will have a larger χ^2 and will have $L-$ waves which are slightly more elastic for $L \gtrsim 8$ than our error bars indicate but $L+$ waves which are not.

Although large uncertainties are still present in the magnitudes of inelasticity of high partial waves, it is to be emphasized that *all* of our fits produced low partial waves which agree with ACE 3. This is true even in the case of ACE 2 (see Fig. 3), where high partial waves are very inelastic. Thus the major results of ACE 3 do not seem to depend too much on models for high partial waves. As for the high partial waves themselves, large uncertainties still exist. Results from energy smoothing[7] give high partial waves which are more elastic, though *not* outside our error bars. Together with the difference between ACE 2 and ACE 3, this and the necessary additions and modifications needed in FG may resolve the disagreement between authors.

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