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Meson decays and the DKP equation*

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It has been argued that the Duffin-Kemmer-Petiau (DKP) description of mesons fails in predicting meson decay rates and the strong-interaction D/F ratio, while the Klein-Gordon (KG) description succeeds. It is shown here that these arguments are deficient in three respects: (a) The various dynamical assumptions used in comparing the DKP and the KG descriptions preclude a rigorous test of their relative merits at present, except in some particularly simple cases; (b) if the same phenomenological freedom were used in the two descriptions the results could be made similar; and (c) actually, when worked out systematically on the basis of a Lagrangian formalism, it is the DKP rather than the KG description which can extract a consistent strong-interaction D/F ratio.

We and others[1-6] have recently raised the question whether the Duffin-Kemmer-Petiau (DKP) description of mesons is preferable to the Klein-Gordon (KG) description in situations where there is symmetry breaking. In addition to discussing formal aspects of the problem[3,4] we have obtained, on the basis of the DKP description and several auxiliary assumptions, results for K_{l3} decays,[1,6] the Cabibbo angle,[2] and the decay-rate ratio[5] $R \equiv [\Gamma(\eta \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)]$ which agree better with experiment[7,8] than the corresponding KG results.

However, the preceding paper by Scadron and Thews[9] (ST) criticizes our work by purporting to show a difficulty in fitting a number of meson decay rates to a phenomenological model based on the DKP description. We will show, however, that the meson-decay analysis of ST in no way proves either that the DKP description of mesons is invalid or that the KG description is valid. We will do this by concentrating on three aspects of the ST analysis.

(a) Interpretation of the ST phenomenology. The meson decays considered[10] by ST, in contradistinction to the K_{l3} and $P \to \gamma\gamma$ decays previously considered by us, necessitate the introduction of a large number of dynamical assumptions which make any meaningful comparison between the DKP and KG descriptions extremely difficult. (In this paper P = pseudoscalar, V = vector, and T = tensor mesons; B = baryon.) As is now generally agreed, the DKP and KG descriptions are alternative *kinematical frameworks* in terms of which various dynamical assumptions can be cast. Consequently no meaningful test of the DKP or the KG description is possible unless the underlying dynamics of the particular decay is itself very well understood. In $P \to \gamma\gamma$ decay and in K_{l3} decay, where there is at most one hadron in the initial or final state, the dynamics are sufficiently simple so that a comparison of the predictions of the two descriptions with experiment is possible. However, this is no longer the case for the $V \to PP$, $T \to PP$, and $T \to VP$ decays considered by ST.

Contrary to the assertion of ST, KG-description decay kinematics and SU(3) do *not* necessarily imply that the meson decay rates have the simple forms given in their Eq. (1). In fact, for the multiple-meson vertices entering into the $V \to PP$, $T \to PP$, and $T \to VP$ decays, final-state interactions and centrifugal-barrier effects generate a large number of possible alternative expressions depending on the details of the strong interactions. Even for the meson electromagnetic decays $V \to P\gamma$ and $P \to V\gamma$, the correct expressions for the decay rates could differ from those given by their Eq. (1) by an arbitrary function[11] $F(m_V/m_P)$, where m_V and m_P are the vector and pseudoscalar masses, respectively, without conflicting with SU(3) or any other accepted principles (see also Ref. 12). All of this is in sharp contrast to the $P \to \gamma\gamma$ decays considered previously by us,[5] where the exact expressions for the decay-rate ratios can be written down immediately, in either the KG or the DKP description, on the basis of what reduces to simple dimensional considerations and SU(3). Thus, whereas the $P \to \gamma\gamma$ decays may test the formulation of SU(3) in the DKP or the KG description, the other decays considered by ST really test a combination of SU(3) and various strong-interaction assumptions (in either the DKP or the KG description) from which one would be hard pressed to extract any firm conclusions.[13]

The importance of centrifugal-barrier effects in meson decays, which ST ignore, has been emphasized recently by von Hippel and Quigg[14] (vHQ). In fact, these authors cite as apparent evidence for centrifugal-barrier effects the deviations of the $V \to PP$ widths from their SU(3) symmetric values (see Table I of vHQ). Although the experimental value of the ρ width is still uncertain at present, the extensive analysis of vHQ makes it apparent that centrifugal-barrier effects must in general be taken into account in strong particle interactions. In view of the work of vHQ we must regard any agreement between the predictions of the KG-based phenomenology of ST and experiment, however interesting, as conceivably fortuitous since ST make no attempt to show that the inclusion of centrifugal-barrier effects will not modify their results.

The dynamical uncertainties in the phenomenology of ST are not limited to the possible importance of the centrifugal-barrier effects. When considering decays involving vector mesons, the entire complex of problems associated with "ω-φ mixing" must be confronted. ω-φ mixing is even more complicated than η-η' mixing due to the fact that vector-meson fields can be identified with hadronic currents via the current-field identity.[15,16] Notwithstanding the complexity of the ω-φ system, ST quote an apparently unique mixing angle $\theta_V \simeq -52°$ taken from *their* linear mass formula [see Sec. b below]. By contrast, however, Kroll, Lee, and Zumino[16] (KLZ) construct three different models for ω-φ mixing, each depending on at least two (and sometimes three) mixing angles. An identical analysis to that of KLZ can be carried out in the DKP description, and we conclude that the number of different models is again rather large. Evidently, then, one cannot fairly deduce any failure of the DKP description from the ST model of ω-φ mixing, without first demonstrating that no reasonable model of ω-φ mixing of the KLZ variety works, and this ST have not done.

(b) Aspects of the ST phenomenology. We question the phenomenological methodology of ST and of Rotelli and Scadron[17] (RS) in *comparing* the KG and the DKP descriptions of meson decays. In short, it appears to be somewhat prejudicial. For example, ST assume the freedom to use either linear or quadratic mass mixing in the KG description and RS, in addition, scale vertex functions with factors depending on the physical masses of the particles involved.

Following Feynman[18] we note that since the KG, DKP, and Dirac Lagrangian mass terms are

$$\mathcal{L}^{\mathrm{KG}} = m^2\varphi^*\varphi, \quad \mathcal{L}^{\mathrm{DKP}} = m\bar{\psi}\psi, \quad \mathcal{L}^{\mathrm{Dirac}} = m\bar{\chi}\chi, \qquad (1)$$

mass mixing is expected to be quadratic (linear) in the KG (DKP and Dirac) descriptions. Though ST use the "unnatural" linear mixing with KG they do not consider the symmetrical possibility of unnatural quadratic mixing with DKP.

Furthermore, ST do not attempt an analysis of DKP models using mass scaling factors as described in RS. RS insert mass factors $(m_1 m_2)^{1/2}$ or $\frac{1}{2}(m_1+m_2)$ into vertices involving baryons with masses m_1 and m_2 and field dimensions $(\mathrm{mass})^{3/2}$. [RS use this freedom to give a consistent account of $(\frac{3}{2})^+$ baryon–$(\frac{1}{2})^+$ baryon–meson vertices.] However, if one is granted this phenomenological freedom for $(\mathrm{mass})^{3/2}$-dimension baryon fields, then one should be also granted the freedom to insert analogous mass factors into vertices involving $(\mathrm{mass})^{3/2}$-dimension DKP meson fields. In particular, this freedom could be used to alter the ST-DKP results in the direction of having them yield the corresponding ST-KG results, if so desired.

For the above reasons, in addition to those given in (a), the confrontation of ST's particular DKP model with experiment does not warrant the conclusion that the DKP description is inferior to the KG description.

As a last comment about mixing we note that the nonet mass formulas determine only the *magnitudes* of the mixing angles so that, e.g., with linear mixing,

$$|\theta_P| = 23.8°, \quad |\theta_V| = 36.8°, \quad |\theta_T| = 28°. \qquad (2)$$

Thus, the *magnitudes* of all the angles in Eq. (2) are fairly close to the magnitude of the ideal mixing angle $= \sin^{-1}(\frac{1}{3})^{1/2} = 35°$. As has been previously emphasized,[5] the experimentally required *negative* value of $(\tan\theta_P)(\langle\gamma\gamma\|\eta_1\rangle/\langle\gamma\gamma\|\eta_8\rangle)$ is consistent with $\theta_P < 0$ and $(\langle\gamma\gamma\|\eta_1\rangle/\langle\gamma\gamma\|\eta_8\rangle) > 0$ or $\theta_P > 0$ and $(\langle\gamma\gamma\|\eta_1\rangle/\langle\gamma\gamma\|\eta_8\rangle) < 0$. From a theoretical point of view, the sign of θ_P cannot be determined without making further assumptions and for this reason the last two sentences in Ref. 9 of ST must be based on a misunderstanding. (Note that ST take θ_V and θ_T to be positive and then subtract 90° from them, while θ_P is taken to be negative.)

(c) ST-DKP formalism. In ST a set of "rules" is given for the conversion from a KG description of a meson process to a DKP description of the same process. These rules are based on an *extrapolation* of some previously obtained rules for

such conversions but do not agree, *in general*, with KG → DKP conversion rules obtained from a systematic Lagrangian treatment of the DKP formalism. Thus, a specific counter-example to the ST-DKP conversion rules is found in the treatment of the meson decay $\kappa \to K\pi$, where κ is an $(I, J^P) = (\frac{1}{2}, 0^+)$ meson. For this process the ST-DKP conversion rules predict for the decay rate

$$\Gamma^{\text{ST-DKP}} = (m_\kappa m_K m_\pi)\Gamma^{\text{KG}}(g^{\text{DKP}}/g^{\text{KG}})^2. \quad (3)$$

However, the DKP expression obtained from a systematic Lagrangian treatment of the DKP formalism is

$$\Gamma^{\text{DKP}} = \frac{m_\kappa[m_\kappa{}^2 - (m_K + m_\pi)^2]^2}{m_K m_\pi}\Gamma^{\text{KG}}(g^{\text{DKP}}/g^{\text{KG}})^2, \quad (4)$$

where in Eqs. (3) and (4) the ratio $(g^{\text{DKP}}/g^{\text{KG}})$ has dimensions $(\text{mass})^{-3/2}$. In general, one can obtain the appropriate consequences of the DKP meson description in processes where more than one meson is involved *only* on the basis of a systematic Lagrangian treatment of the DKP formalism.

A second point regarding the ST-DKP conversion rules involves the decay of a spin-two (tensor) meson into two spin-zero (pseudoscalar) mesons: $T \to PP$. For the TPP effective Lagrangian in the DKP case ST take as a "natural" choice $T_{\mu\nu}\bar{\psi}\beta_\mu\beta_\nu\psi$ and are unable to render a consistent account of the various $T \to PP$ decays. ST also comment that another "natural" choice for a TPP effective Lagrangian in the DKP case, $T_{\mu\nu}\bar{\psi}\partial_\mu\partial_\nu\psi$, yields even worse results. However, it has been shown recently[19] that certain TPP coupling-constant sum rules are well satisfied with the $\beta_\mu\beta_\nu$ Lagrangian. [Note that the effective Lagrangian $T_{\mu\nu}\bar{\psi}\partial_\mu\partial_\nu\psi$ yields KG → DKP conversion rules that are somewhat different from those of ST-DKP by comparing the ratio of Eq. (47) to Eq. (43) in Ref. 19 with what is expected from the ST-DKP conversion rules.] This apparent discrepancy deserves further analysis as does the circumstance that, in neither of the $T \to PP$ treatments mentioned, is a DKP-type formalism used for the spin-two meson.

It is worth emphasizing that the whole question of the appropriate description of mesons with spin greater than one in the presence of symmetry breaking is as yet only little investigated and, in particular, practically nothing is known about the DKP-type description of spin-two mesons.

Finally, ST referred to an earlier criticism of DKP due to Deshpande and McNamee[20] (DM). The main point of DM involves the claim that the DKP description cannot extract a consistent value for β, the strong-interaction $[D/(D+F)]$ ratio, from the experimentally based values of the $(\frac{1}{2})^+$ baryon–$(\frac{1}{2})^+$baryon–pseudoscalar-meson coupling constants g_{BBP}, while the KG description can. This claim, as several of us have recently shown, is not correct.[21,22]

First of all, it is the KG description, with either pseudoscalar (ps) or pseudovector (pv) BBP vertices, which does not yield a consistent β. Thus, with ps vertices, and using experimentally based values of the coupling-constant ratios, one gets[21] $[(g_{\Sigma\Sigma\pi}/g_{NN\pi}) \equiv \{\Sigma\Sigma\pi\}$, etc.]

$$\begin{aligned} &\beta^{\text{KG}}(\{\Sigma\Sigma\pi\}) \cong \beta^{\text{KG}}(\{\Sigma NK\}) \cong 0.6 \pm 0.1, \\ &\beta^{\text{KG}}(\{\Sigma\Lambda\pi\}) \cong \beta^{\text{KG}}(\{\Lambda NK\}) \cong 1.0 \pm 0.1. \end{aligned} \quad (5)$$

On the other hand, the DKP description with pv BBP vertices does yield a consistent β. Here the experimentally based values of the coupling-constant ratios give[21]

$$\begin{aligned} \beta^{\text{DKP}}(\{\Sigma\Sigma\pi\}) &\cong \beta^{\text{DKP}}(\{\Sigma NK\}) \\ &\cong \beta^{\text{DKP}}(\{\Sigma\Lambda\pi\}) \\ &\cong \beta^{\text{DKP}}(\{\Lambda NK\}) \cong 0.7 \pm 0.1. \end{aligned} \quad (6)$$

DM did not consider pv BBP coupling, which yields the result

$$[BB'P \text{ pv vertex}]^{\text{DKP}}/[BB'P \text{ ps vertex}]^{\text{KG}} = (m_B + m_{B'})/m_P{}^{1/2} \quad (\text{and not } m_P{}^{1/2}),$$

and agrees with experiment.

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[3]E. Fischbach, M. M. Nieto, and C. K. Scott, Phys. Rev. D 6, 726 (1972).

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[5]E. Fischbach, M. M. Nieto, H. Primakoff, and C. K. Scott, Phys. Rev. Lett. 29, 1046 (1972).

[6]E. Fischbach, M. M. Nieto, and C. K. Scott, Phys. Rev. D 7, 207 (1973).

[7]A new experiment by A. Browman, J. DeWire, B. Gittle-

man, K. Hanson, E. Loh, A. Silverman, and R. Lewis has been reported [Cornell Reports Nos. CLNS 224 and 242 (unpublished); Bull. Am. Phys. Soc. 18, 595 (1973)], which gives a value of $\Gamma(\eta\to\gamma\gamma)=(374\pm 60)$ eV, or about $\frac{3}{8}$ of the currently accepted value of Ref. 8, which was used by us in Ref. 5. If the correct value of $\Gamma(\eta\to\gamma\gamma)$ is that given by these authors, then different predictions emerge both from our and ST's analysis. In particular, if $R^{\text{expt}}=R^{\text{Cornell}}=48$, then the ratio (defined in Ref. 5) $Y\equiv|\langle\gamma\gamma\|\eta_1\rangle/\langle\gamma\gamma\|\eta_8\rangle|$ is 2.7 for KG and 0.47 for DKP [with $(\tan\theta)\times\langle\gamma\gamma\|\eta_1\rangle/\langle\gamma\gamma\|\eta_8\rangle$ negative (positive) for KG (DKP)]. This yields predictions for $\Gamma(\eta'\to\gamma\gamma)$ of 5.5 keV (KG) and 4.5 keV (DKP), and for $\Gamma_{\text{tot}}(\eta')$ of 283 keV (KG) and 232 keV (DKP). Thus, KG and DKP would both predict values of Y relatively close to unity and both would make similar predictions for $\eta'\to\gamma\gamma$. We wish to thank B. Gittleman for his kindness in discussing and showing the apparatus of this experiment to one of us (M.M.N.).

[8]Particle Data Group, Rev. Mod. Phys. 45, S1 (1973).

[9]M. D. Scadron and R. L. Thews, preceding paper, Phys. Rev. D 9, 2180 (1974), referred to as ST in the text.

[10]A phenomenological analysis similar in some ways to that of ST has been given earlier in the following articles: M. Gourdin, in *Proceedings of the European Physical Society International Conference on Meson Resonances and Related Electromagnetic Phenomena, Bologna, Italy, 1971*, edited by R. H. Dalitz and A. Zichichi (Editrice Compositori, Bologna, Italy, 1972), pp. 219–233; in *Hadronic Internactions of Electrons and Photons*, proceedings of the Scottish Universities Summer School, 1970, edited by J. Cumming and H. Osborn (Academic, London, 1971), pp. 395–471; G. H. Trilling, Nucl. Phys. B40, 13 (1972).

[11]The function $F(m_V/m_P)$ may represent the combination of centrifugal-barrier effects and/or electromagnetic final-state interactions. Although the latter are expected to be relatively small, the former may be comparable to the centrifugal-barrier effects arising in the strong decays $T\to PP$, etc.

[12]In their treatment of $V\to P\gamma$ and $P\to V\gamma$ decays in the ratios R_8-R_{11}, ST use only three independent "reduced" amplitudes. However, unless further restrictions are assumed (e.g., that the photon belongs to a self-adjoint representation), this cannot be correct. In fact, it is easily shown (first article of Ref. 10) that no fewer than five independent "reduced" $A^{[U\text{-spin}]}_{[\text{SU(3) mult. of } V][\text{SU(3) mult. of } P]}$ amplitudes ($A^0_{88}, A^1_{88}, A^0_{18}, A^0_{81}, A^0_{11}$) are required to express all of the $V\to P\gamma$ amplitudes. Specifically, the $\phi\to\eta\gamma$ amplitude, contained in the ratio that ST particularly emphasize, namely, R_9, depends on every one of the A's, so that their discussion is vitiated by the presence of two more adjustable parameters.

[13]In the case of K_{l3} decays, the decay amplitude consists of a sum of $J=1$ and $J=0$ partial waves in the dilepton center-of-mass system. Thus, all strong interaction effects are incorporated into two form factors, $f_+(t)$ and $f_0(t)$, respectively, which can then be analyzed in detail.

[14]F. von Hippel and C. Quigg, Phys. Rev. D 5, 624 (1972), referred to as vHQ in the text.

[15]M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).

[16]N. M, Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967), referred to as KLZ in the text.

[17]P. Rotelli and M. D. Scadron, Nuovo Cimento 15A, 648 (1973), referred to as RS in the text.

[18]See, for example, the discussion in Ref. 42 of J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963). Also note the comment by M. Gell-Mann in M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964), p. 8.

[19]E. Golowich and V. Kapila, Phys. Rev. D 8, 2180 (1973).

[20]N. G. Deshpande and P. C. McNamee, Phys. Rev. D 5, 1012 (1972), referred to as DM in the text.

[21]F. T. Meiere, E. Fischbach, A. McDonald, M. M. Nieto, and C. K. Scott, Phys. Rev. D 8, 4209 (1973).

[22]Although the extraction of the g_{BBP} from the experimental data is complicated by various strong-interaction effects, considerable attention has been given to these effects by a large number of authors over the past few years. Consequently, the comparison of the experimentally based g_{BBP} with SU(3) expectations using either the KG or the DKP description of P stands on a much firmer footing than does the analysis of the various meson decays given by ST.