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Inconsistency of a Duffin-Kemmer-Petiau model for meson decay rates

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We confront a Duffin-Kemmer-Petiau model of pseudoscalar mesons (as proposed by Fischbach *et al.*), with data on thirteen independent ratios of meson decay rates. With five adjustable singlet-octet ratios as parameters and SU_3-symmetric couplings, this formalism is able to fit only eight of the ratios, while it is inconsistent with experiment on the remaining five ratios. We conclude that this description is not a viable alternative to the usual Klein-Gordon formalism.

In recent letters the Duffin-Kemmer-Petiau equation (DKP) has been proposed as a serious alternative to the Klein-Gordon equation (KG) for spin-zero particles.[1-3] Evidence offered is a large negative theoretical value for the K_{l3} parameter ξ,[1] improvement of the Cabibbo angle,[2] and resolution of the "SU_3 puzzle" $\Gamma(\eta\to\gamma\gamma)/\Gamma(\pi^0\to\gamma\gamma)$.[3]

We wish to demonstrate that in spite of these apparent successes, the DKP model leads to many inconsistencies and creates more problems than it solves. It already has been pointed out[4] that the DKP formulation *per se* does not lead to the results of Fischbach *et al.*[1-3] Instead the latter's conclusions are based upon keeping only covariant forms which seem most natural in the DKP framework. In order to test this postulate, we shall put this "DKP model" to the logical task of trying to fit all available meson-decay data. We find five examples which are in complete contradiction with experiment and assumed SU_3 symmetry at the decay vertex. This result is a consequence of the fact that the KG theory and SU_3 symmetry successfully describe *all* meson-decay data,[5,6] and any tampering with this pattern in one decay mode is bound to cause problems in other modes.

We consider the meson decays $P\to\gamma\gamma$, $V\to PP$, $T\to PP$, $T\to VP$, $V\to P\gamma$, and $P\to V\gamma$, where T, V, P refer to tensor, vector, and pseudoscalar mesons. Decay kinematics for KG pseudoscalar fields and SU_3 symmetry at the decay vertex[7] implies that the rate for two-body decay of a meson of mass m is $\Gamma\sim|T|^2p/m^2$, where T is the relativistically invariant matrix element. This implies that

$$\begin{aligned}&\Gamma_{P\to\gamma\gamma}\sim d_{P\gamma\gamma}{}^2p^3, \quad \Gamma_{V\to PP}\sim f_{VPP}{}^2p^3/m_V{}^2,\\ &\Gamma_{T\to PP}\sim d_{TPP}{}^2p^5/m_T{}^2, \quad \Gamma_{T\to VP}\sim f_{TVP}{}^2p^5, \\ &\Gamma_{V\to P\gamma}\sim \tfrac{1}{3}d_{VP\gamma}{}^2p^3, \quad \Gamma_{P\to V\gamma}\sim d_{VP\gamma}{}^2p^3,\end{aligned} \tag{1}$$

where d_{ijk} and f_{ijk} are the SU_3 structure constants, and p is the magnitude of the momentum of either decay product in the rest frame of the decaying particle. If the DKP equation is used to describe the pseudoscalar particles, these rates in (1) must be modified by the mass-dependent factors of m_P for each pseudoscalar particle, $|\frac{1}{2}[(1/m_1)+(1/m_2)]|^2$ for each KG meson coupling of the form $\phi_1\overleftrightarrow{\partial}_\mu\phi_2$, and $(m_1m_2)^{-2}$ for each KG coupling $\phi_1\partial_\mu\partial_\nu\phi_2$.[8] This leads to the additional factors of $(m_1+m_2)^2(4m_1m_2)^{-1}$ for $V\to P_1P_2$, $(m_1m_2)^{-1}$ for $T\to P_1P_2$, and m_P for $P\to\gamma\gamma$, $T\to VP$, $V\to P\gamma$, and $P\to V\gamma$, to be included in (1). One might argue that one should use DKP rather than KG fields for vector and tensor mesons. However, it is easy to see that the large differences come from the large mass splittings in the pseudoscalar nonet, and ratios of vector or tensor masses could not resolve them.

Finally the $\eta-\eta'$, $\omega-\phi$, and $f-f'$ mixing parameters must be taken into account. We define the states as

$$\begin{aligned}|\eta,\omega,f\rangle &= \cos\theta_{P,V,T}|\eta_8,\omega_8,f_8\rangle \\ &\quad -\sin\theta_{P,V,T}|\eta_0,\omega_0,f_0\rangle\,, \\ |\eta',\phi,f'\rangle &= \sin\theta_{P,V,T}|\eta_8,\omega_8,f_8\rangle \\ &\quad +\cos\theta_{P,V,T}|\eta_0,\omega_0,f_0\rangle.\end{aligned} \tag{2}$$

The DKP model insists on linearly mixed states[3,4] and the meson mass formulas then imply $\theta_P\cong-24°$, $\theta_V\cong-53°$, $\theta_T\cong-62°$.[9] Mixing leads to five additional parameters which are needed to describe the various decays in (1). These singlet-octet ratios are usually defined as

$$\begin{aligned}&S_{\gamma\gamma}=\langle\eta_0|\gamma\gamma\rangle/\langle\eta_8|\gamma\gamma\rangle\,,\\ &S_{V\gamma}=\langle\eta_0|V_3\gamma\rangle/\langle\eta_8|V_3\gamma\rangle\,,\\ &S_{P\gamma}=\langle V_0|P_3\gamma\rangle/\langle V_8|P_3\gamma\rangle\,,\\ &S_{TP}=\langle\eta_0|T_3P_3\rangle/\langle\eta_8|T_3P_3\rangle,\\ &S_{PP}=\langle T_0|P_3P_3\rangle/\langle T_8|P_3P_3\rangle\,,\end{aligned} \tag{3}$$

and one should keep in mind their U_3 quark-model values of

$$S_{V\gamma}=S_{P\gamma}=S_{TP}=S_{PP}=\tfrac{1}{2}S_{\gamma\gamma}=\sqrt{2} \tag{4}$$

TABLE I. Comparison of predictions from KG and DKP descriptions with experimental values for ratios R_1, R_2, and R_3.

	Expt.	KG	DKP
R_1	0.043 ± 0.004	0.040	0.028
R_2	9.7 ± 2.5	8.25	29.6
R_3	0.34 ± 0.09	0.29	1.03

when computing their phenomenological values in the KG and DKP formalisms.

However, before becoming involved with these singlet-octet ratios, we can make three comparisons which are independent of singlet-octet ratios:

$$R_1 = \Gamma_{\phi \to K^+K^-}/\Gamma_{K^* \to K\pi},$$

$$R_2 = \Gamma_{K_N \to K\pi}/\Gamma_{A_2 \to K\bar{K}},$$

$$R_3 = \Gamma_{K_N \to K\rho}/\Gamma_{K_N \to K^*\pi}.$$

The results are given in Table I and we see that the KG formalism is compatible with experiment,[10] while the additional mass factors in the DKP model cause its predictions to be 4, 8, and 8 standard deviations off from the central values of $R_{1,2,3}$, respectively.

Turning to processes involving mixing parameters, Fischbach *et al.*[3] attempt to explain the large $\eta \to \gamma\gamma/\pi^0 \to \gamma\gamma$ ratio by the DKP enhancement factor of $m_\eta/m_\pi \sim 4$ and a small singlet-octet ratio of $S_{\gamma\gamma} \sim 0.6$ rather than the KG value of $S_{\gamma\gamma} \sim 3.5$, which is near the quark-model value of $2\sqrt{2}$. This in turn would seem to imply that the other four DKP ratios in (4) are $\sim \frac{1}{2} S_{\gamma\gamma} \sim 0.3$, which will fit only five of the thirteen decay ratios to be considered.

If we instead drop the quark-model constraint (4) and allow the four remaining ratios to take on independent values, the situation is not much improved. The $T \to PP$ ratios,

$$R_4 = \Gamma(A_2 \to \eta\pi)/\Gamma(A_2 \to K\bar{K}) = 2.8 \pm 0.05,$$

$$R_5 = \Gamma(A_2 \to \eta'\pi)/\Gamma(A_2 \to K\bar{K}) < 0.15 \quad \text{(Ref. 11)},$$

are fitted by the KG value of $S_{TP} = 0.7 \pm 0.3$, while the DKP model needs $S_{TP} = -0.6$ or -3.9 to fit R_4, which in turn predicts $R_5 = 0.13$ or 4.5, one of which may be acceptable. However, tensor mixing can also be probed in the $T \to PP$ decays

$$R_6 = \Gamma(f' \to \pi\pi)/\Gamma(f' \to K\bar{K}) \leq 0.2,$$

$$R_7 = \Gamma(f \to K\bar{K})/\Gamma(f \to \pi\pi) = 0.056 \pm 0.019.$$

The ratio R_6 implies the value $S_{PP} \sim 1.8$ in both the KG and DKP formalisms. This value predicts $R_7 = 0.059$ for KG and $R_7 = 0.0046$ for DKP, and again we see the DKP model is in trouble.

TABLE II. Values of $S_{V\gamma}$ required in the KG and DKP descriptions to fit experimental values of R_9, R_{10}, and R_{11}.

	KG	DKP
R_9	1.08 ± 0.76	2.82 ± 0.39
	8.12 ± 0.76	6.32 ± 0.39
R_{10}	1.76 ± 0.30	$0.75^{+0.31}_{-0.32}$
	-0.89 ± 0.30	$0.11^{+0.32}_{-0.20}$
R_{11}	$1.1^{+4.2}_{-3.4}$	$-0.6^{+2.1}_{-1.7}$
	$-5.7^{+3.4}_{-4.2}$	$-4.0^{+1.7}_{-2.1}$

Finally the radiative decays $V \to P\gamma$ and $P \to V\gamma$ can be analyzed in terms of the two ratios $S_{V\gamma}$ and $S_{P\gamma}$:

$$R_8 = \Gamma(\phi \to \pi\gamma)/\Gamma(\omega \to \pi\gamma) \leq 0.017,$$

$$R_9 = \Gamma(\phi \to \eta\gamma)/\Gamma(\omega \to \pi\gamma) = 0.127 \pm 0.061,$$

$$R_{10} = \Gamma(\eta' \to \rho\gamma)/\Gamma(\omega \to \pi\gamma)$$

$$= 0.139 \pm 0.079 \text{ KG} \quad \text{(Ref. 12)}$$

$$= 0.056^{+0.16}_{-0.056} \text{ DKP} \quad \text{(Ref. 12)},$$

$$R_{11} = \Gamma(\omega \to \eta\gamma)/\Gamma(\omega \to \pi\gamma) = 0.01^{+0.04}_{-0.01} \quad \text{(Ref. 13)}.$$

For R_8 to be small, both KG and DKP require $1.2 \leq S_P \leq 1.7$ (we shall use the quark-model value of $\sqrt{2}$). In Table II we see that the KG value of $S_{V\gamma} \sim 1.7$ can fit R_9, R_{10}, and R_{11}, but that DKP can only fit R_{10} and R_{11} simultaneously with $0 < S_{V\gamma} < 1$, while R_9 cannot be fitted with this value.

The net result is that the KG formalism is able to fit all thirteen decay ratios (including the two $P \to \gamma\gamma$ ratios) with five (U_3) parameters, while the DKP formalism fits only eight of the ratios with five parameters and fails on the other five. If the DKP model (and SU_3) is to agree with experiment, it must somehow account for all of the thirteen ratios by departing from the traditional SU_3 rates (1) in some *consistent* fashion. Even if final-state interaction effects or rescaling of the SU_3 couplings by additional mass factors are used, we do not see how the DKP rates could agree with all of the data. It is important to stress that both the linear KG and quadratic KG SU_3 rates are always within one standard deviation of experiment. This fact in itself is a compelling reason for insisting upon (1) without recourse to final-state interaction or rescaling effects.

In conclusion, we believe that the DKP model is *not* an acceptable alternative to the KG equation.

Note added in proof. A recent new measurement $\Gamma(\eta \to \gamma\gamma) = 374 \pm 60$ eV (Ref. 14) is almost a factor of three less than previous values. If this is correct, it removes the original "SU_3 puzzle" which was the subject of Ref. 3. The conventional KG approach can still accommodate this new value in either the quadratic or linear mixing schemes with reasonable singlet-octet ratios.[15]

There has also been a new measurement of K_{l3} decay which gives the ξ parameter a value of ≈ 0.[16] This is in disagreement with the older values of ≈ -1, and disagrees with the DKP approach which "explains" the old value (Ref. 1).

A recent examination of SU_3 symmetry for meson-baryon couplings in the DKP formalism[17] claims success for the pseudovector DKP coupling scheme, contrary to a previous criticism.[18] However, this hinges on acceptance of a low value for the $KN\Lambda$ coupling. We wish to mention three new and independent evaluations with small associated errors which favor larger values of $g_{KN\Lambda}$ in agreement with the KG approach,[19] and also with the original work of Kim.[20] Furthermore, Ref. 17 relies heavily upon the analysis of Martin and Sakitt,[21] who in fact concluded that the $KN\Lambda$ coupling uncertainty found by their approach is large enough to encompass both extreme values.

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[1]E. Fischbach *et al.*, Phys. Rev. Lett. 26, 1200 (1971).

[2]E. Fischbach *et al.*, Phys. Rev. Lett. 27, 1403 (1971).

[3]E. Fischbach *et al.*, Phys. Rev. Lett. 29, 1046 (1972).

[4]R. S. Willey, P. Winternitz, and Tsu Yao, Phys. Rev. D 7, 3540 (1973).

[5]P. Rotelli and M. D. Scadron, Nuovo Cimento 15A, 648 (1973). This is valid for linear or quadratic mixed states, although linear may be preferred (see Refs. 6 and 11). We do not insist upon correlating the quadratic mass formula with KG or the linear mass formula with DKP. The DKP model is inconsistent with linear or quadratic mixing, as our first three decay ratios will demonstrate. We use linear mixing for KG in order to make a direct comparison with the linearly mixed DKP predictions.

[6]F. D. Gault, H. F. Jones, and M. D. Scadron, Nucl. Phys. B51, 353 (1973).

[7]We follow the traditional approach that the medium-strong interaction shifts masses and mixes the η-η', ϕ-ω, and f-f' states but still conserves SU_3 at three-particle vertices. See M. Gourdin, *Unitary Symmetry* (North-Holland, Amsterdam, 1967), pp. 53, 95, and 101.

[8]These factors are derived from vector and tensor couplings $\bar{\psi}\beta_\mu\psi$ and $\bar{\psi}\beta_\mu\beta_\nu\psi$ in the DKP formalism. See N. G. Deshpande and P. C. McNamee, Phys. Rev. D 5, 1389 (1972), and Ref. 1. The other natural choice for tensor couplings, $\bar{\psi}\overleftrightarrow{\partial}_\mu\overleftrightarrow{\partial}_\nu\psi$, leads to even greater disagreement with experiment.

[9]We take the angles to be negative in agreement with the quark model. For convenience we use the canonical Okubo angle $\tan\theta_V = -\sqrt{2}$ corresponding to $\theta_V = -55°$. In Ref. 3 the conventional $\theta_V = \frac{1}{2}\pi + \theta_V^{us}$ is used, giving $\theta_V = 35°$. However, this is *not* close to the pseudoscalar angle of $\theta_P = -24°$ as implied in Ref. 3.

[10]All rates unless otherwise referenced are taken from the Particle Data Tables [Particle Data Group, Phys. Lett. 39B (1972)].

[11]L. Eisenstein *et al.*, Phys. Rev. D 7, 278 (1973).

[12]It is important to realize that the total η' width is different in the two formalisms: $\Gamma_{\eta'}^{KG}(L) = 0.43 + 0.19$ MeV and $\Gamma_{\eta'}^{DKP} = (17^{+54}_{-17})$ keV as given in Ref. 3. The experimental width is known to be < 1.9 MeV [D. M. Binnie *et al.*, Phys. Lett. 39B, 275 (1972)]. The quadratic mixing solution of $\Gamma_{\eta'}^{KG}(Q) = 2.7 \pm 1.1$ MeV is indeed in trouble, but the linear and the DKP values are consistent with experiment.

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[15]F. D. Gault, H. F. Jones, M. D. Scadron, and R. L. Thews, Nuovo Cimento (to be published).

[16]G. Donaldson *et al.*, Phys. Rev. D (to be published).

[17]F. T. Meiere *et al.*, Phys. Rev. D 8, 4209 (1973).

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[20]J. K. Kim, Phys. Rev. Lett. 19, 1079 (1967).

[21]B. R. Martin and M. Sakitt, Phys. Rev. 183, 1352 (1969). See the abstract and particularly Table V.