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Bounds on chiral-symmetry breaking and K_{l3} slope parameters*

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Using the rigorous bounds of Li and Pagels and Okubo and the broken SU(3) × SU(3) model of Glashow and Weinberg and Gell-Mann, Oakes, and Renner, we derive bounds on the symmetry-breaking parameters which measure the invariance of the Hamiltonian and the vacuum under SU(3). We also obtain bounds on Λ_0, the slope of the spin-0 form factor in K_{l3} decay. Our bounds on Λ_0, though inconsistent with the experimental value obtained by a quadratic fit to the K_{l3} form factors, are consistent with the value obtained by a linear fit. Some theoretical implications of our results are discussed.

Li and Pagels[1] and Okubo[2] have derived some rigorous bounds on the K_{l3} decay parameters:

$$f_+^2(0) \leq \frac{\Delta(0)}{N} , \tag{1}$$

$$\left| 4\left(\frac{m_K+m_\pi}{m_\pi}\right)^2 \Lambda_0 - 2 + \frac{(4m_K m_\pi)^{1/2}}{[(m_K)^{1/2}+(m_\pi)^{1/2}]^2} \right| \leq \left[\frac{\Delta(0)}{Nf_+^2(0)} - 1\right]^{1/2} , \tag{2}$$

where

$$N = \frac{3}{256\pi} (m_K - m_\pi)^2 (m_K + m_\pi)[(m_K)^{1/2} + (m_\pi)^{1/2}]^2 . \tag{3}$$

Here $f_+(0)$ is the usual spin-1 form factor, $\Delta(0)$ is the propagator function for the divergence of the strangeness-changing vector current, and Λ_0 is the slope of the spin-0 form factor for the divergence of the current, all quantities evaluated at the zero momentum transfer. Apart from the standard assumptions of analyticity, unitarity, and crossing symmetry, the most crucial assumption under which these bounds are derived is that the spin-0 form factor satisfies an unsubtracted dispersion relation.[1,2] The parameter Λ_0 has been measured experimentally, and for a quadratic fit to the form factors the world average value[3] is

$$\Lambda_0 = -0.11 \pm 0.03 . \tag{4}$$

On the other hand, a previously reported value[4] obtained by assuming that the form factors depend linearly on momentum transfer is given by

$$\Lambda_0 = -0.024 \pm 0.02 . \tag{5}$$

In order to compute the bounds, one needs to know $\Delta(0)$ or an upper bound to $\Delta(0)$ in (1) and (2). An estimate has been made for $\Delta(0)$ using the broken-SU(3)×SU(3) model of Glashow and Weinberg[5] and Gell-Mann, Oakes, and Renner[5] (GWGMOR) where the symmetry-breaking terms transform as the $(3,3^*)+(3^*,3)$ representation. One gets $\Delta(0) \simeq 2.04\, m_\pi^2 F_\pi^2$, where F_π is the pion decay constant, $F \simeq 94$ MeV. From (1) and (2), one then obtains $f_+(0) \lesssim 1.01$ and $0.011 \leq \Lambda_0 \leq 0.024$ [for $f_+(0) = 0.85$], the bounds for Λ_0 being in contradiction with the experimental results (5) and (4). Although the experimental situation at the present time is quite uncertain,[6] it is desirable from the theoretical point of view to reexamine the evaluation of $\Delta(0)$. Among various assumptions, the estimate of $\Delta(0)$ mentioned above is based on the pole-dominance approximation. *A priori*, it is hard to see how a refinemant in the value of $\Delta(0)$, which we do not expect to be drastic, can lead to bounds for Λ_0 consistent with the experimental result (4). This is confirmed by the work of Okubo[7] and others[7] who have made estimates of $\Delta(0)$ under different approximations. Thus, if relatively large negative values like (4) are confirmed by further experiments, one may have to give up one or more fundamental premises underlying the derivation of (1) and (2) or conclude that the GWGMOR model is incorrect.

The purpose of the present note is twofold. First, using (1) in conjunction with the GWGMOR model we derive rigorous bounds on the symmetry-breaking parameters. It turns out that these bounds allow both the GMOR-type solution[5] as well as the type advocated by Brandt and Preparata (BP).[8] We find, however, that in the case of the BP-type solution where the Hamiltonian is approximately SU(3)-invariant the vacuum cannot be approximately SU(3)-invariant. Secondly, on the basis of the GWGMOR model we obtain an upper bound for $\Delta(0)$ by taking into consideration the departures from the pole-dominance approximation, to see if small negative values such as (5) are consistent with the theoretical bounds in (1) and (2).

We start with the Hamiltonian density for the GWGMOR model

$$H(x)=H_0(x)+\epsilon_0 u^0(x)+\epsilon_8 u^8(x)\,, \tag{6}$$

where $H_0(x)$ is invariant under $SU(3)\times SU(3)$ and $u^i(x)$ $(i=0,1,\ldots,8)$ are the scalar densities which belong to the $(3,3^*)+(3^*,3)$ representation of $SU(3)\times SU(3)$. We define the following two-point functions:

$$\begin{aligned} I_{\alpha\beta}&=i\int d^4x\langle 0|T(\partial_\mu A_\mu^\alpha(x)\partial_\nu A_\nu^\beta(0))|0\rangle\,,\\ K_{\alpha\beta}&=i\int d^4x\langle 0|T(\partial_\mu V_\mu^\alpha(x)\partial_\nu V_\nu^\beta(0))|0\rangle\,, \end{aligned} \tag{7}$$

where $A_\mu^\alpha(x)$ and $V_\mu^\alpha(x)$ $(\alpha=1,2,\ldots,8)$ are the usual octet of axial-vector and vector currents. As noted elsewhere,[9] from Eqs. (6) and (7) the following exact relations can be derived:

$$\begin{aligned} I_{33}&=\gamma(1+a)(1+b)\,,\\ I_{44}&=\gamma(1-\tfrac{1}{2}a)(1-\tfrac{1}{2}b)\,,\\ I_{88}&=\gamma(1-a-b+3ab)\,,\\ K_{44}&\equiv\Delta(0)=\tfrac{9}{4}\gamma ab\,, \end{aligned} \tag{8}$$

where

$$a=\frac{\epsilon_8}{\sqrt{2}\,\epsilon_0}\,,\quad b=\frac{\xi_8}{\sqrt{2}\,\xi_0}\,,\quad \gamma=-\tfrac{2}{3}\epsilon_0\xi_0\,, \tag{9}$$

with $\xi_0=\langle 0|u^0(0)|0\rangle$ and $\xi_8=\langle 0|u^8(0)|0\rangle$. We also note that the GMOR solution for the symmetry-breaking parameter a is given by

$$a=\frac{2(m_\pi^{\,2}-m_K^{\,2})}{m_\pi^{\,2}+2m_K^{\,2}}\simeq -0.89\,. \tag{10}$$

Separating out the single pion and kaon contributions to I_{33} and I_{44}, respectively, we write

$$I_{33}=F_\pi^{\,2}m_\pi^{\,2}(1+\delta_\pi)\,, \tag{11}$$

$$I_{44}=F_K^{\,2}m_K^{\,2}(1+\delta_K)\,, \tag{12}$$

where δ_π and δ_K are related to the multiparticle intermediate-state contributions, and F_K is the kaon decay constant. Since the contribution of any intermediate state to I_{33} and I_{44} is positive-definite, we conclude that $\delta_\pi\geq 0$, $\delta_K\geq 0$. Now from well-known arguments suggesting dominance by nearby singularities, and the success of the hypothesis of partially conserved axial-vector currents (PCAC), we believe, especially for I_{33}, that pole dominance is perhaps a reasonable assumption. To be generous, however, we shall adopt in the following the weaker assumption that the continuum contribution does not overwhelm the contribution from the single-particle states. In particular, we assume that $\delta_\pi\leq 1$, $\delta_K\leq 1$. We emphasize that this is a rather weak assumption, and in practice one does not expect δ_π and δ_K to be much more than 10–20%. With considerable reasonableness, we then expect

$$1\geq\delta_\pi\geq 0\,,\quad 1\geq\delta_K\geq 0\,. \tag{13}$$

We now proceed to derive bounds on the symmetry-breaking parameters a and b. Eliminating γ and b from the relations for I_{33}, I_{44}, and $\Delta(0)$, we get

$$\Delta(0)=\frac{3a}{2}\left(\frac{I_{33}}{1+a}-\frac{2I_{44}}{2-a}\right)\,. \tag{14}$$

Now for the physical solution we expect $-1\leq a\leq 0$. Equations (14) and (1) then give the following relation:

$$a^2[3(2I_{44}+I_{33})-2Nf_+^{\,2}(0)] +2a[3(I_{44}-I_{33})+Nf_+^{\,2}(0)]+4Nf_+^{\,2}(0)\leq 0\,. \tag{15}$$

Using Eqs. (11) and (12) we obtain from (15) the following exact bounds for a:

$$-\frac{D+B}{2A}\leq a\leq\frac{D-B}{2A}\,, \tag{16}$$

where

$$\begin{aligned} A&=\tfrac{3}{2}[2m^2\alpha^2f_+^{\,2}(0)(1+\delta_K)+1+\delta_\pi]-Rf_+^{\,2}(0)\,,\\ B&=3[m^2\alpha^2f_+^{\,2}(0)(1+\delta_K)-1-\delta_\pi]+Rf_+^{\,2}(0)\,, \end{aligned}$$

and

$$\begin{aligned} D=3\{&[m^2\alpha^2f_+^{\,2}(0)(1+\delta_K)+1+\delta_\pi-Rf_+^{\,2}(0)]^2\\ &-4(1+\delta_\pi)(1+\delta_K)m^2\alpha^2f_+^{\,2}(0)\}^{1/2}\,, \end{aligned} \tag{17}$$

with

$$m=\frac{m_K}{m_\pi}\,,\quad R=\frac{N}{F_\pi^{\,2}m_\pi^{\,2}}\,,\quad \alpha=\frac{F_K}{F_\pi f_+(0)}\,. \tag{18}$$

Using the experimental value $\alpha = 1.28$, $m = 3.659$ and the fact that the Ademollo-Gatto theorem[10] requires $f_+(0)$ to be close to unity, in the exact pole-dominance approximation ($\delta_K = \delta_\pi = 0$) we get the following bounds for a:

$$-0.925 \leq a \leq -0.071 , \quad (19)$$

where we have taken $f_+(0) = 1.0$. However, if we use instead the weaker assumption (13) for δ_π and δ_K, we get the absolute bounds

$$-0.964 \leq a \leq -0.034 . \quad (20)$$

Note that both the GMOR and BP solutions, where $a = -0.89$ and $a = -0.17$, respectively, are consistent with (19) and (20).

To obtain bounds on b we eliminate γ between I_{33} and $\Delta(0)$ and use (1) and (11); we get

$$\frac{9ab}{4(1+a)(1+b)} \geq \frac{Rf_+^2(0)}{1+\delta_\pi} . \quad (21)$$

The physical solution for b is expected to lie between 0 and -1; we can therefore write (21) as

$$b \leq \frac{4(1+a)Rf_+^2(0)}{9a(1+\delta_\pi) - 4(1+a)Rf_+^2(0)} . \quad (22)$$

Given δ_π, $f_+(0)$, and a the bounds on b can be calculated from (22). We have calculated the bounds on b by taking $f_+(0) = 1.0$, taking two different values for δ_π (0 and 0.1, respectively), and varying a between 0 and -1. The allowed values for b are given by the shaded area in Fig. 1. The shaded area increases as we increase δ_π from 0 to 0.1. In particular, for the GMOR model ($a = -0.89$) we get

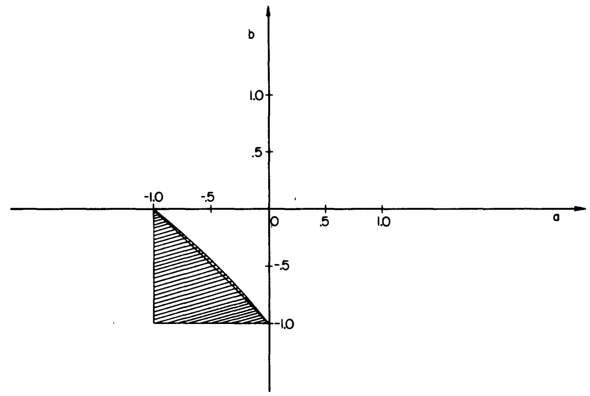

FIG. 1. The physically allowed values of a and b are indicated by the shaded region. The allowed region increases as we increase δ_π from 0 to 0.1.

$$b \leq -0.104, \quad \delta_\pi = 0$$
$$b \leq -0.095, \quad \delta_\pi = 0.1 \quad (23)$$

and for the BP model ($a = -0.17$) we get

$$b \leq -0.823, \quad \delta_\pi = 0$$
$$b \leq -0.809, \quad \delta_\pi = 0.1 . \quad (24)$$

We note that in the BP model where the Hamiltonian is approximately SU(3)-invariant the vacuum *cannot* be approximately SU(3)-invariant.

We next turn to the derivation of bounds for Λ_0, the slope of the spin-0 form factor. For this purpose we need an upper bound for $\Delta(0)$ in (2). Using Eqs. (11), (12), and (14) we get

$$\Delta(0) = \frac{F_\pi^2 m_\pi^2 [3a(2-a)(1+\delta_\pi) - 6a(1+a)m^2\alpha^2 f_+^2(0)(1+\delta_K)]}{2(1+a)(2-a)} . \quad (25)$$

Now for the physical solution, since $a \leq 0$, the maximum value of $\Delta(0)$ is obtained when δ_K is maximum and δ_π is minimum. Using the bounds from (13) with $\delta_K = 1$ and $\delta_\pi = 0$, we get

$$\Delta_{\max}(0) = \frac{F_\pi^2 m_\pi^2 [3a(2-a) - 12a(1+a)m^2\alpha^2 f_+^2(0)]}{2(1+a)(2-a)} . \quad (26)$$

Numerically for the GMOR solution ($a = -0.89$) we obtain

$$\Delta_{\max}(0) = 26.051 F_\pi^2 m_\pi^2, \quad f_+(0) = 1.0 \quad (27)$$

$$\Delta_{\max}(0) = 18.749 F_\pi^2 m_\pi^2, \quad f_+(0) = 0.9 . \quad (28)$$

In order to obtain the bounds on Λ_0 we note that we can write (2) as

$$\left| 4(m+1)^2 \Lambda_0 + 2\left[\frac{\sqrt{m}}{(\sqrt{m}+1)^2} - 1\right] \right| \leq \left[\frac{\Delta_{\max}(0)}{Nf_+^2(0)} - 1\right]^{1/2} . \quad (29)$$

The relation (29) implies the following bounds for Λ_0:

$$\left[\frac{\Delta_{\max}(0)}{Nf_+^2(0)} - 1\right]^{1/2} \geq 4(m+1)^2\Lambda_0 + 2\left[\frac{\sqrt{m}}{(\sqrt{m}+1)^2} - 1\right] \geq -\left[\frac{\Delta_{\max}(0)}{Nf_+^2(0)} - 1\right]^{1/2} \quad (30)$$

Using the estimates of $\Delta_{\max}(0)$ from (27) and (28), we get from (30) the following bounds for Λ_0:

$$0.058 \geq \Lambda_0 \geq -0.022, \quad f_+(0) = 1.0 \quad (31)$$

$$0.057 \geq \Lambda_0 \geq -0.020, \quad f_+(0) = 0.9 . \quad (32)$$

Both the bounds (31) and (32) are inconsistent with the experimental value for Λ_0 in (4) and are barely consistent with that in (5). We could have obtained bounds for Λ_0 by choosing the BP value of a instead of the GMOR value. It can be readily checked that the BP estimate of a gives bounds on Λ_0 which are in even worse agreement with the experimental results in (4) and (5). Therefore, within our framework the only possibility for a relatively large and negative value for Λ_0 is when $\Delta_{max}(0)$ is about an order of magnitude larger than in Eqs. (27) and (28). This is very unlikely because this requires the kaon pole dominance of I_{44} to be very badly violated and δ_K to be much larger than 1. We can understand this by putting the experimental values of Λ_0 in (29) and then calculating $\Delta_{max}(0)$, which via (25) gives bounds on δ_K. We find

$$\delta_K \geq 1.12^{-1.02}_{+1.65}\,, \quad \Lambda_0 = -0.024 \pm 0.02 \tag{33}$$

and

$$\delta_K \geq 12.77^{-5.41}_{+6.85}\,, \; \Lambda_0 = -0.11 \pm 0.03\,. \tag{34}$$

Such large violations of kaon pole dominance as in (34) are unwarranted. Therefore, if future experiments confirm a relatively large and negative value for Λ_0 as in (4), we believe that we will have to give up either one or more of the assumptions used in the derivation of the bound on Λ_0 in (2), or the GWGMOR model on which our estimate of $\Delta(0)$ is based.

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