PHYSICAL REVIEW D VOLUME 9, NUMBER 7 1 APRIL 1974

Speculations on the breakdown of scaling at 10^{-15} cm*

Michael S. Chanowitz and Sidney D. Drell
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 12 October 1973)

We discuss the hypothesis that scaling in deep-inelastic electron-nucleon scattering is a "preasymptotic" phenomenon, which will be broken at energies large enough to probe the structure of the constituents of the nucleon. In particular, we consider a model in which the constituents are light (i.e., $M \lesssim M_p$) and are bound by a very heavy gluon ($M_G \gg M_p$), which induces a small "size" in the constituents of order M_G^{-1}. The experimental implications of this hypothesis are discussed, primarily for the region of momentum transfers $M^2 \ll Q^2 \ll M_G^2$. In a bound-state model, using the Bethe-Salpeter equation in ladder approximation, we show that the deviations from simple scaling behavior in spacelike scattering and timelike annihilation processes are correlated and measure the "size" of the constituent. Finally, we show that the hypothesis that the constituent has structure is not inconsistent with local current algebra and, in particular the Adler sum rule for neutrino-nucleon scattering.

I. INTRODUCTION

The observed scaling behavior of the structure functions for deep-inelastic electron scattering has led to major new concepts and techniques in the study of the hadron and its interactions. The proton has been analyzed as an assemblage of incoherently scattering pointlike "partons",[1] as a relativistic bound state of pointlike constituents,[2] and in terms of the singularities of products of local current operators near the light cone.[3]

In these analyses it is typically assumed that scaling reflects the fact that one is probing, with high resolution, the asymptotic, short-distance structure of the internal constituents of the nucleon. Here we wish to propose an alternative framework. We shall also assume that scaling is connected with the existence of constituents inside the nucleon, but the fundamental difference in our point of view is this: We assume that scaling reflects not the asymptotic but rather the *preasymptotic* structure of the nucleon's constituents.[4] Our hypothesis is that in the range of Q^2 and ν probed until now, previous electroproduction experiments have been too coarse to resolve the structure of the constituents. Thus it is the bluntness of the probe which is responsible for the observation of simple scaling, and there is no reason to expect that the constituents are themselves simple objects. Scaling does not represent the fact that we have probed inside the structure cloud of the constituent, but, to the contrary, it represents the fact that we have not yet even begun to probe its structure cloud.

In this paper we shall illustrate these ideas with a simple model in which the nucleon is a weakly bound system of light constituents and the binding force is supplied by the exchange of a massive gluon. That such a model is a *consistent* dynamical proposition is a conjecture on our part which may or may not be supported by further investigation.[5] However, independent of the particular model we have chosen to display in this paper, we wish to emphasize that the predicted scale-breaking effects in the deep-inelastic and lepton-annihilation experiments depend pivotally on our view that *scaling is a preasymptotic phenomenon* and that scaling may be observed under limited kinematic conditions between nonscaling regions. The prediction of primary experimental importance is that there are "hints" in the present data suggesting that we are on the verge of seeing the next scale of length at which simple scaling will fail.

We have already briefly described these ideas in a recent letter,[6] and here we wish to present a more complete discussion. The plan of the paper is as follows.

In Sec. II we discuss heuristically a model of the nucleon as a bound state of light constituents (quarks ?) bound together by very massive ($M_G \gg 1$ GeV) gluons. In Sec. III, we discuss the experimental hints (or "prejudices") which lead us to conjecture that the length at which scaling fails is $\sim 10^{-15}$ cm. Of especial importance is our prediction of the correlation between the deviations from scaling laws for spacelike and timelike momentum transfers. In Sec. IV we return to the model of Sec. II presenting a formal analysis using the Bethe-Salpeter equation in the ladder approximation. In Sec. V we discuss the consequences for current algebra (and, in particular, for the Adler sum rule) of the hypothesis that the nucleon constituents are not pointlike. We explore the use of smeared "almost-equal-time" commutators to derive current-algebra sum rules which are truncated at finite energies. Finally, in Sec. VI we make some concluding remarks, discussing in particular the alternative possibility that the nucleon's constituents are very heavy.

II. A BOUND-STATE MODEL OF THE PROTON

In a relativistic bound-state model of the proton as a composite structure[2] the notion of pointlike constituents bound by a gluon sea is introduced. From this point of view an *exact* scaling behavior would be a revolutionary departure from all that has been learned on the atomic and nuclear scales. Indeed, in the nuclear case, the pseudoscalar and vector mesons that bind the nucleons together into a nucleus give rise via their radiative self-effects to nucleon structure and consequently to deviations from scaling behavior. If we (unimaginatively) pursue the atomic and nuclear analogies to higher energies and momentum transfers, we are led to expect similarly that the gluons give rise to structure for the constituents of the nucleon so that, at best, we will observe approximate scaling laws that are valid over limited intervals of Q^2 and ν. Deviations from scaling will be observed in the kinematic regions when the Q^2 are large enough so that the electromagnetic currents are probing within the structure of the individual constituents or when the ν are large enough so that we are above the threshold to produce gluons in the final state. In between such nonscaling regions there may be individual scaling plateaus.

On the atomic scale, the scaling law is obeyed up to momenta and energies ($\approx$ tens of MeV) when the nucleus can no longer be treated as pointlike. A transition region with no scaling, which sets in while we start probing nuclear structure, persists until at $Q^2 \approx (400\ \mathrm{MeV}/c)^2$, as in Fig. 1, the nucleus responds as an assemblage of incoherently scattering individual nucleons.[7] However, in this case the "would-be" scaling is violated before it begins

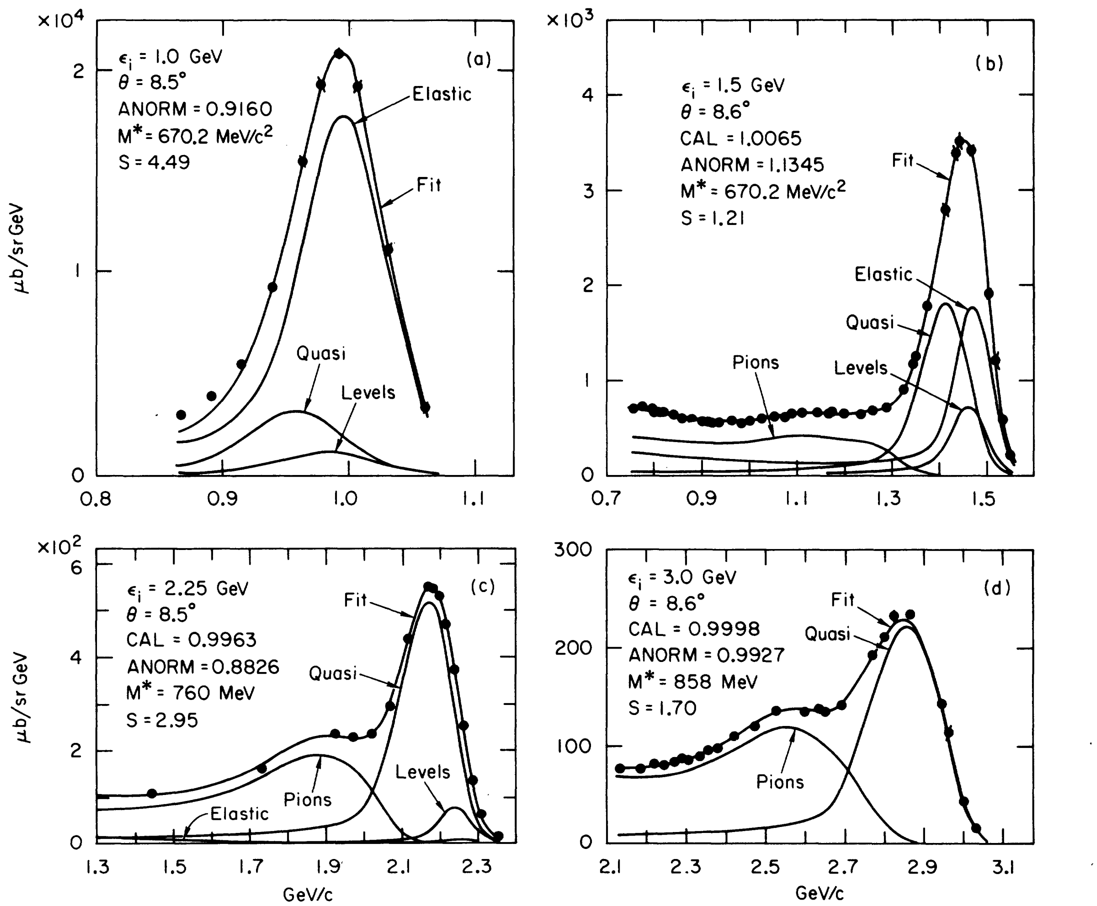

FIG. 1. Inelastic scattering of electrons from carbon, taken from Ref. 7: the differential cross section plotted against the energy of the final electron (initial electron energy is indicated as ϵ_i). The invariant momentum transfer at the peak, in MeV/c, is as follows: (a) ~150, (b) ~210, (c) ~320, (d) ~450.

by the production of pions and by the nucleon form factors which vary with Q^2. This is because the electromagnetic current is already probing well within the structure clouds of the individual nucleons by the time Q^2 is large enough for them to be scattering incoherently. At still larger ν and Q^2 we emerge once again onto a scaling plateau when the composite nucleon structure scatters as an assemblage of independent pointlike constituents. In contrast with the nuclear case, we actually do see scaling occur in this case (Bjorken scaling) which means that the constituents of the nucleon, if not actually pointlike, must be much smaller than the nucleons themselves. The fact that the onset of scaling occurs at such small values of Q^2 suggests that the constituents of the nucleon may be relatively light and weakly bound ($\approx$ few hundreds of MeV).

The notion of weak binding of light quarks ($M_Q \sim 300$ MeV) to form the nucleon is in accord with analyses of baryon spectra and transition amplitudes which are generally computed with considerable success on the basis of a nonrelativistic quark model.[8] The basic problem of why we do not "see" free, individual quarks or partons persists in this approach and we have nothing to add to the resolution of this problem. Among the "excuses" for nonobservation of quarks, Johnson's[9] proposal of a dynamical mechanism for creating a confining self-consistent potential is closest to our view and might be adopted. We must also keep in mind, in adopting this point of view, that the three SU(6) quarks which constitute the nucleon are related by a nontrivial transformation to the quark fields which determine the leading singularities of products of hadronic currents.[10] However, even if relativistic effects as contained in such a transformation are important in the nucleon, we would still expect the constituents to have such structure due to their gluon clouds.[11]

Once again, as we further increase Q^2 and ν the electromagnetic current probes for internal structure of these constituents. There is the possibility that none will be found and the Bjorken scaling behavior is exact. In this case we will have reached the ultimate constituents or the inner most layer of particle structure in nature and there will be no higher mass scale separating us from the light cone. Alternatively, pursuing the atomic, nuclear, and nucleon analogies one more round, the constituents of the nucleon may themselves have structure,[12] and deviations from scaling will be observed when Q^2 and ν grow to values that excite their internal dynamics and probe their gluon cloud structure. However, the very fact that we have found scaling to occur to a good ($\approx \pm 15\%$) approximation in the region $1.5 < Q^2 < 10$ GeV^2 and $2 < \nu < 20$ GeV means that we have evidently not yet seen the form factor of the constituent, nor have the gluons that bind them and give them structure been produced. These facts can be accounted for by asserting that the gluons are very heavy, and their mass defines a scale of new physics.

In light-cone language, this picture corresponds to successive hierarchies of masses separating us from the light cone (see in particular Wilson's photon symposium talk, Ref. 4). Approximate scaling laws will be valid whenever there is an interval between adjacent mass or binding energy scales E_i and E_{i+1} such that

$$E_i \ll Q^2, \qquad \sqrt{s} \ll E_{i+1}.$$

In contrast, in the field theory and parton models with superconvergent behavior, scaling behavior[13] emerges from the formalism because there are no masses larger than the nucleon's, $M_p \approx 1$ GeV. In the deep-inelastic Bjorken region the electromagnetic current has already seen through the structure cloud "dressing" the constituents and is scattering from the pointlike bare constituents themselves. In these models the constituent form factor is a constant in the scaling region. Corrections to the scaling behavior and to the constancy of $F_c(Q^2)$ are proportional to $\approx M_p^2/Q^2$ and are negligible in the Bjorken limit.

Very simply then, the question is whether the presently observed scaling represents the asymptotic pointlike core of the nucleon constituent or whether it represents a preasymptotic behavior in which one has not yet begun to see the structure of the constituent. We are here advocating the latter alternative as the more conservative explanation of the origin of scaling.

A striking and unexpected property of the observed Bjorken scaling is its "precocity," i.e., the fact that the scaling is realized for surprisingly small values of Q^2 and ν. Precocity has a natural explanation in our model of the proton as a weakly bound system, since we expect the impulse approximation to apply and scaling to occur when $Q^2 \gg mE_B$, where m is the constituent mass and E_B is the binding energy. The smaller E_B the more precocious the scaling. However, it is also a consequence of such a weak binding model that we would then expect to see a quasielastic peak, which does occur in the nuclear case, Fig. 1, but is not evident in the proton structure functions. Now the shape of the nucleon's structure functions is also assumed to reflect the fact that the nucleon is not composed of a fixed number of constituents but in fact the nucleon wave function is a sum of amplitudes involving different numbers of constituents. For small ω the virtual photon probes the part of the wave function dominated by small numbers of constituents with relatively weak binding and we expect precocious scaling. For very large ω, on the other hand, the amplitudes for large numbers of constituents, each bearing a small momentum fraction $x=1/\omega$, are important. In this case their binding must be stronger, and we do not expect as precocious an approach to scaling. Of course we do not expect the nuclear analogy to be a reliable guide in the nucleon case for very large values of ω. However, if one looks at the neutron-proton difference, one finds what looks like a quasielastic peak centered near $\omega \sim 3$. This suggests that the multiconstituent amplitudes which dominate at large ω are largely isoscalar, so that the n-p difference is dominated by the parts of the amplitude containing few constituents (i.e., the "valence quarks") for which the nuclear analogy should be a more reliable guide.

If these ideas are correct, then as we increase ω from small to moderate values, we would expect the approach to scaling for the proton to become less precocious. The best scaling data now available is from the small-ω region, $\omega \lesssim 4$, and it will be interesting to study the approach to scaling as accurate data are accumulated at larger ω.

To summarize, imitative thinking by analogy has led to a simple qualitative model of the proton: a weakly bound system of light constituents (perhaps quarks) with their strong interaction carried by massive gluons. We cannot, however, advocate this picture as a complete theoretical basis for understanding nucleon structure because

(1) we have no dynamical theory relating the constituent and gluon masses and the interaction strength to the nucleon radius and mass, and so we have no assurance that such a model can be realized in a consistent dynamical system; and

(2) we have no explanation for the nonappearance of the constituents. Our interest in the model is primarily that it affords a simple example of how scaling might be realized as a preasymptotic phenomenon. In Sec. III we shall utilize this picture to discuss the breakdown of scaling. We conjecture that the picture of the breakdown of scaling which we abstract from the "preasymptotic nature" of the model may be correct even if it turns out that this particular model does not provide a tenable description of nucleon structure.

III. CONSEQUENCES OF CONSTITUENT STRUCTURE AND EXPERIMENTAL HINTS

In models such as those discussed in Sec. II, perturbation theory leads us to expect that the charge structure of the nucleon's constituent will be of the form

$$F_c(q^2) \sim 1 + f^2(q^2/M_G^2)\{\ln[M_G^2/(-q^2)] + c\}\,, \quad (3.1)$$

where $M^2 \lesssim |q^2| \ll M_G^2$, M_G is the gluon mass, M is the (light) constituent mass, f is the dimensionless gluon-constituent coupling constant, and c is a model-dependent constant.

In the remainder of this section we assume $f \sim 1$, and the dependence on f is suppressed.[14] However, it is worth noticing that our principal speculations really apply to the ratio f/M_G, so that a light, weakly coupled gluon is also a possibility.[15] We also ignore the logarithmic variation in approximating (3.1) for $Q^2 \ll M_G^2$;

$$F_c(q^2) \sim 1 - Q^2/M_G^2\,, \quad (3.2)$$

where henceforth M_G is an "effective" gluon mass and $Q^2 \equiv -q^2 > 0$ for scattering processes.

For $Q^2 \ll M_G^2$, so that the approximations (3.1) and (3.2) to the constituent's charge form factor are valid, one might expect intuitively that parton-model results (and other results obtained assuming pointlike constituents) would be modified by replacing pointlike vertices by form factors $F_c(Q^2)$, viz.,

$$\nu W_2(\nu, Q^2) \cong \mathfrak{F}_2(\omega)\left(1 - 2\frac{Q^2}{M_G^2}\right) \quad (3.3)$$

for $M^2 \lesssim Q^2 \ll M_G^2$, where $\omega \equiv 2M_p\nu/Q^2$ and the factor 2 enters with the square of (3.2). In Sec. IV we shall justify Eq. (3.3) in a Bethe-Salpeter model of the nucleon. Here we discuss the experimental consequences.

At this time there is no definite evidence of the failure of scaling of the form (3.3) but we can ask what limits can be put on possible values of M_G or the parton size. As we discuss below, present experimental limits leave open the possibility that $M_G \gtrsim 10$ GeV. An unambiguous interpretation of the data using Eq. (3.3) is not now possible because in the kinematical regions that have been experimentally studied, corrections due to the approach to scaling $\sim M_P^2/Q^2$ are likely to be of the same order of magnitude as the possible scaling violations $\sim Q^2/(10\text{ GeV})^2$. To resolve the ambiguity, accurate data at large Q^2 are needed. But one can use the available data to get a rough idea of the possible magnitude of scale-breaking effects. Bloom[16] has analyzed the moment integrals of the scaling functions as derived from the Wilson operator expansions

$$B_n = \int_{1+M^2/Q^2}^{\infty} \frac{d\omega'}{(\omega')^{2n+2}}\,[\nu W_2(\omega', Q^2)]$$

in terms of the scaling variable $\omega' \equiv \omega + M^2/Q^2$. With the presently available data and a constant fit to the ratio of longitudinal to transverse cross sections that is consistent with the data $R \equiv \sigma_s/\sigma_T = 0.168$, he found no evidence for parton size up to masses $M_G \gtrsim 12$ GeV.

On the other hand, looking at the νW_2 directly as extracted from a mesh of existing data points in the two variables ν and ω, Riordan in his thesis[17] made a scaling study in terms of ω which shows a slight falloff of νW_2 with increasing Q^2 that can be fitted with a parton size in the range $M_G \sim 8$ GeV in (3.3). In terms of the Bloom-Gilman variable ω',[18] on the other hand, fits can be achieved without requiring any scale-breaking effects. The difference between ω and ω' is only important during the approach to scaling, so this is a particular instance of our remark that present data do not allow an unambiguous separation of corrections due to the approach to scaling from scale-violating effects. Since $d\mathfrak{F}_2/d\omega > 0$ for $\omega \lesssim 4$ the Bloom-Gilman proposal also accounts qualitatively for the "observed" decrease of $\nu W_2(\omega \approx 2, Q^2)$ as Q^2 increases. To decide between their interpretation and ours, it will be sufficient to have accurate $W_1 - \nu W_2$ separated data for $\omega > 4$(where $d\mathfrak{F}_2/d\omega \sim 0$, so that according to Bloom and Gilman the effect should disappear) and/or for larger Q^2 values (where according to Bloom and Gilman the effect diminishes while according to our hypothesis, it becomes more pronounced). Hopefully the crucial data for larger ω and Q^2 values will be available before long from experiments now in progress.

We turn next to the behavior of the elastic electromagnetic form factor of the proton at high Q^2 for a hint of the scale of "new physics." Here there is presently more data to refer to in search of such hints[19] but any interpretation in terms of possible constituent structure relies on specific theoretical models. The experimental facts are summarized in Fig. 2, which contains all data

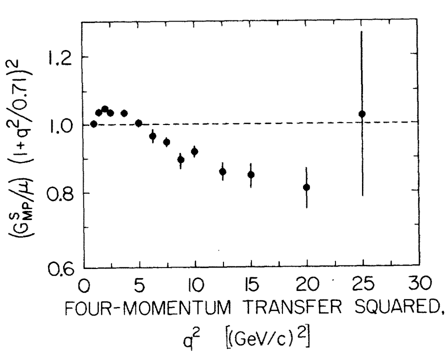

FIG. 2. The proton magnetic form factor as a function of Q^2 taken from Ref. 18.

for the magnetic form factor of the proton $G_M(Q^2)$ plotted relative to a dipole form $(1+Q^2/0.71\ \mathrm{GeV}^2)^{-2}$. A scaling relation is assumed to hold between the electric and magnetic form factors in Fig. 2, i.e., $G_M(Q^2) = 2.79\, G_E(Q^2)$, but the large Q^2 data are very insensitive to this assumption as the electric scattering is relatively very small. The dipole form has *per se* no fundamental theoretical significance. Furthermore the exact nature of the falloff and the quantitative behavior of G_M for large Q^2 cannot be specified accurately or uniquely due to the limited data for $Q^2 \gtrsim 10$ GeV^2. Fits to these data over the entire experimental range can be achieved by introducing complicated analytic forms (see the resume in Ref. 19); however, if we use simple pole models, a large mass parameter, ~5–10 GeV, has to be introduced. As emphasized by Massam and Zichichi,[20] a fit based on the vector-dominance model, including the effects of the ρ, ω, and ϕ propagators, as well as their vector-dominated nucleon form factors, must be modified by introducing a heavy vector meson of mass $M_V = 7.7 \pm 1.1$ GeV to give the over-all electromagnetic form factor a more rapid falloff with increasing Q^2. Alternatively, a modification of the dipole formula in Fig. 3 by a multiplicative factor $(1 - Q^2/M_G^2)$ fits the data for $Q^2 > 5$ GeV^2 for $M_G \sim 10$ GeV. Finally, if one makes a 3-parameter fit to G_M with the trial form

$$G_M = \frac{1}{(1+Q^2/M_1^2)(1+Q^2/M_2^2)(1+Q^2/M_3^2)}\ ,$$

it is possible to find a good χ^2 over the entire range[21] of measured Q^2 in terms of two masses, $M_1, M_2 \sim 1 \pm 0.3$ GeV, and with one large mass $M_3 \gtrsim 5$ GeV.

Independent of a specific theoretical interpretation the appearance of a large mass $M_G \sim 10$ GeV suggests the possibility of a new scale of large masses or short distances on which qualitatively new behavior may occur. In particular a relativistic bound state of two pointlike constituents satisfying a Bethe-Salpeter equation in the ladder model and bound by a Yukawa-type potential, so that the wave function is not singular at the origin,

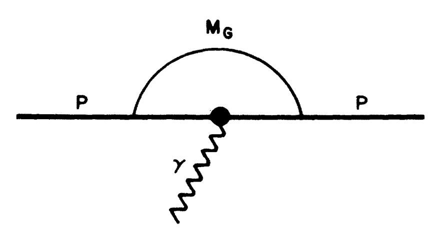

FIG. 3. The contribution of the constituent form factor due to the second-order vertex correction.

leads to a form factor $G_M^D(Q^2)$ with a $(1/Q^2)^2$ fall-off at large Q^2. A result of the form

$$G_M(Q^2) \cong G_M^D(Q^2)\left(1-\frac{Q^2}{M_G^2}\right) \tag{3.4}$$

for $Q^2 \ll M_G^2$ would arise if the constituents were not pointlike but had themselves a structure, as in Eq. (3.2). Typically $G_M^D(Q^2)$ reaches its asymptotic form when Q^2 is large compared with the binding energy which, as in Sec. II, we take to be no larger than ~ 1 GeV. In this case, to fit Eq. (3.4) to the data, we are forced to choose $M_G > 5$ GeV. In Sec. IV we will give a theoretical discussion of Eq. (3.4).

If the spin-$\frac{1}{2}$ constituents of the nucleon develop an electromagnetic structure from their gluon interaction as we have proposed, then in general they will also acquire an anomalous magnetic moment. We may ask how big this Pauli moment will be and how this will affect the deep-inelastic cross section. For instance, if we assume a fermion constituent with a constant Pauli moment form factor when $Q^2 \ll M_G^2$, then not νW_2 but rather W_2 should scale which might pose a disastrous disagreement with the observed scaling behavior. However, estimating $F_2^p(0)$ from the vertex correction, Fig. 3, we find $F_2^p(0) \sim (1/3\pi)M^2/M_G^2$. For massive gluons with $M_G/M \sim 10$, this is far below the experimental upper bound.[22]

In addition to deviations from scaling as in Eq. (3.3), the most striking experimental consequence of these speculations is for the behavior of the total cross section for electron-positron annihilation into hadrons in the single-photon approximation. As we discussed already in our previous paper, we predict for $M^2 \ll s \ll M_G^2$ (in the annihilation channel $s \equiv q^2 > 0$)

$$\sigma_{e^+e^-\to\gamma\to X}(s) \propto \frac{1}{s}\left(1+2\frac{s}{M_G^2}\right). \tag{3.5}$$

To leading order in s/M_G^2 the rise in (3.5) above the pointlike behavior has the same slope as the decrease below scaling behavior in the scattering region. Physically the correction due to the constituents' form factor is introduced, as discussed in Ref. 6, because the production time of the constituents, $\sim (1/s)^{1/2}$, is not short compared with their interval of free particle propagation before they rescatter to form the final hadrons, i.e., $\sim 1/M_G \ll (1/s)^{1/2}$. Also as noted earlier, if the gluons have the same quantum numbers as the photon, i.e., vector gluons with unitary octet indices [perhaps due to SU(3) breaking], then the correction in (3.5) may grow to a resonance form $\sim 1/(1-s/\mathfrak{M}^2)^2$. Thus a sizable increase in the annihilation cross sections would be observed as $s \to \mathfrak{M}^2$, while at the same time the corrections to scaling for the scattering experiments would remain much smaller. The actual position of the resonance is, however, unknown since M_G, as already commented at the outset of this section, is an "effective" mass in terms of coupling strengths and particle masses; therefore we cannot identify $\mathfrak{M}^2$ with M_G^2 defined by the effective-range expansion in (3.2).

The same correction factor in (3.5) also modifies the scaling behavior predicted for one-body inclusive cross sections $e+\bar{e}\to h+X$, as well as the massive lepton pair production $p+p(n)\to\mu\bar{\mu}+X$ (or $e\bar{e}+X$) for finite ratio Q^2/s, where Q^2 is the invariant squared mass of the lepton pair and s is the total reaction (energy).[2] We also recall from Ref. 6 the implications for a nonscaling increase in deep-inelastic electron and neutrino cross sections when we are at energies ν above the gluon production threshold.

These predictions are the main experimental implications of our suggestion that a larger mass scale remains between the energies at which present data have been obtained and the light cone. Concerning the production of gluons in purely hadronic processes, we have already conjectured[6,23] on the possible implications for recent CERN ISR data.

To summarize this section, we have suggested that existing data on electromagnetic form factors and deep-inelastic structure functions of the proton "hint" at a possible appearance of a new large mass scale, $M_G \sim 10$ GeV and of new physics at energies $\sim M_G$. It would hardly be surprising to encounter one (or more) such large mass scales between our present electromagnetic probes with $Q^2 \sim (\text{few GeV})^2$ and the light cone. Fortunately the reality of the existing "hints" can be experimentally tested in the near future.

IV. FACTORIZATION OF THE FORM FACTOR

In an impulse-approximation analysis the form factors and structure functions of the nucleon factor into a product of two terms. The first term is just what we would expect if the constituents were themselves pointlike; the second term describes the structure of a free constituent. Physically this approximation corresponds to ignoring the effects of the binding of the constituents to one another within the nucleon on their electromagnetic interactions.

The analogous result is familiar in the analysis of nuclear scattering. For example, the interpretation of neutron structure from electron-deuteron scattering is based on the similar factorization of nuclear and nucleon form factors.[24]

In this section we use a relativistic bound-state model of the nucleon to derive this factorization property, which was introduced in Sec. III, in the kinematic region under consideration, i.e., for $M^2 \ll Q^2 \ll M_G^{\,2}$, where M and M_G are, respectively, the light constituent and heavy gluon masses. We also restrict ourselves to the region below the threshold for producing these massive gluons. For $Q^2 \ll M_G^{\,2}$, this is also the Bjorken deep-inelastic region of $Q^2/2M_p\nu$ finite, since the gluon production threshold occurs at $2M_p\nu \approx M_G^{\,2} \gg Q^2$. In this model we assume that the physical proton p is composed of a spin-$\frac{1}{2}$ particle P and a neutral scalar meson X, forming a bound state given by the solution of a Bethe-Salpeter equation in the ladder approximation. The binding potential is generated by the exchange of a neutral gluon of mass M_G which couples to the spin-$\frac{1}{2}$ and -0 constituents with strength f. The important property of the bound state that we shall make use of in this model is this: The Bethe-Salpeter wave function remains finite for vanishing space-time interval between the constituents. This property is derived for scalar gluon exchange and can also be assured for exchange of vector gluons with conserved vector couplings if the gluon propagator is modified by subtraction of its most singular term.[25]

We shall discuss the deep-inelastic structure functions; an analogous treatment can be given for the elastic form factors. In this model factorization is equivalent to the statement that the inelastic-scattering amplitude is dominated by the diagram of Fig. 4(a), in which the shaded blob represents the fully dressed P-P-photon irreducible vertex. Figure 4(a) will factorize into two terms as discussed above, provided that the off-shell corrections to the virtual intermediate spinor line are negligible. Figure 4(b), in which a gluon is exchanged between the scalar meson and the P-P-photon irreducible vertex, is a diagram which violates the impulse approximation. So do rescattering graphs of the type shown in Fig. 4(c). The complete sum of all possible photon insertions as shown in Fig. 4, including the self-energy parts, must be included in order to protect the Ward identity and hence current conservation. Our task is to show that except for Fig. 4(a), which contributes to the structure of a free constituent, all other contributions of Fig. 4 are negligible when $M^2 \ll Q^2 \ll M_G^{\,2}$; and furthermore that the off-shell corrections to the constituent form factor in Fig. 4(a) are negligible.

The general power-counting analysis of Ref. 2 can be repeated to verify that graphs such as Figs. 4(b) and 4(c) can be neglected in an order-by-order perturbation analysis. We find that the wave functions at the bound-state vertex provide the needed powers of momenta that, up to logarithmic factors, converge the added integration loop for momenta exceeding $\sim M$, where for simplicity in describing this model we assume that the constituent masses and the binding energy are all comparable, $\sim M$. Hence the additional massive gluon propagator introduces factors $\approx M_g^{\,2}$ in the denominators and these contributions are typically smaller[26] by $\sim f^2M^2/M_G^{\,2} \ll 1$.

In order to illustrate that Fig. 4(a) gives a factorized structure function, we calculate the lowest-order perturbation contribution in the case in which the current interaction with the spin-$\frac{1}{2}$ constituent P includes the single-vector-gluon vertex correction illustrated in Fig. 3. For $M^2 \ll Q^2 \ll M_G^{\,2}$ this diagram gives the constituent P a charge form factor

$$F_c(q^2) = 1 - \frac{f^2}{24\pi^2}\frac{Q^2}{M_G^{\,2}} \ln\frac{M_G^{\,2}}{Q^2}\,, \tag{4.1}$$

provided that both P legs are on the mass shell. Now in calculating the correction to the structure functions, we must consider an off-shell constituent, and to the same approximation we represent Fig. 4(a) by the lowest-order correction, Fig. 5. To this must be added all possible orderings of the current insertion to protect the Ward identity at the electromagnetic vertex. However, building on the preceding paragraph and Ref. 2, it is sufficient for us to write the amplitude

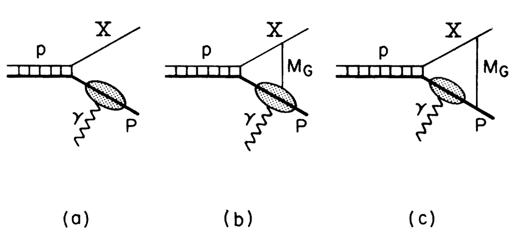

FIG. 4. Inelastic scattering from the bound state. The shaded blobs represent effects of constituent structure.

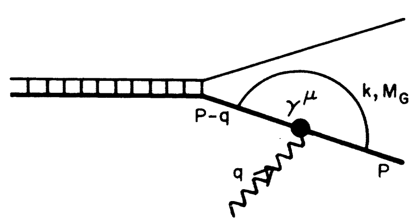

FIG. 5. Inelastic scattering from the bound state, with the constituent form factor treated in second-order perturbation theory.

$$I_G^\mu = \bar{u}_P g(u) \frac{1}{\not{P}-\not{q}-M}\left(f^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-M_G^2} \gamma_\beta \frac{1}{\not{P}-\not{q}+\not{k}-M} \gamma^\mu \frac{1}{\not{P}+\not{k}-M} \gamma^\beta\right) u_p \tag{4.2}$$

and recognize that we are interested only in the correction to the electromagnetic vertex γ_μ that is proportional to q^2. In (4.2) the factor $g(u)$ is the bound-state wave function and $u \equiv (P-q)^2$; for large u (compared with the binding energy $\sim M$), $g(u) \propto u^{-1}$. Adding (4.2) to the lowest-order point electromagnetic vertex I_0^μ gives the factorized form to leading order in f^2 and $Q^2/M_G^2 \ll 1$, $Q^2/M^2 \gg 1$ for the coefficient of γ_μ:

$$I_0^\mu + I_G^\mu \cong \bar{u}_P g(u) \frac{1}{\not{P}-\not{q}-M} \gamma^\mu u_p [F_c(q^2)]$$

$$\cong I_0^\mu F_c(q^2). \tag{4.3}$$

The corrections to the electromagnetic vertex arising due to the fact that the virtual spinor line is off-shell are reduced relative to (4.3) by additional powers of (M/M_G). These additional powers express the fact, as analyzed in Ref. 2, that the bound-state wave function $g(u)$ which falls as $1/u$ for $|u| \gg M^2$ restricts the intermediate fermion line to virtual masses $|u| \lesssim M^2$; in contrast, the corrections to the on-shell behavior of the electromagnetic vertex in Fig. 5 are measured on the scale of masses $M_G^2 \gg M^2$. This same statement is expressed in space-time language by observing that the lifetime of the intermediate state at the electromagnetic vertex of the spinor is $\tau_\gamma \sim 1/M_G$, which is much shorter than the lifetime of the intermediate spinor state $\tau_s \sim 1/M$. Therefore, the corrections $\tau_\gamma/\tau_s \sim M/M_G \ll 1$ can be neglected and the intermediate spinor can be treated as if on its mass shell in calculating its electromagnetic structure. The same power-counting arguments in $1/M_G$ apply for higher-order perturbative contributions to the blob in Fig. 4(a).

In considering the processes illustrated in Fig. 4, we have gone beyond the framework of the ladder model since gluons are emitted and absorbed by the same constituent in dressing the electromagnetic vertex. The question naturally arises as to what happens to the bound-state structure itself if we go beyond the ladder model and dress the strong-interaction vertices also in the ladders in Fig. 4. Here we only know how to answer in terms of a perturbation analysis of the corrections to the bound state $g(u)$. Again resorting to power-counting methods, we find that aside from renormalizing the coupling constants, the corrections are of order M^2/M_G^2, since the wave functions converge the added integration loops for momenta exceeding $\sim M$, whereas the additional gluon propagators introduce factors $\sim M_G$ in the denominators. In the space-time language, the constituent propagators have virtual lifetimes $\tau_c \sim 1/M$ which are very long compared with the lifetimes of the gluons in the radiative corrections, $\tau_G \sim 1/M_G$. Therefore, the individual strong vertices can be approximated "on shell" and the gluons renormalize their coupling constants.

In this context—namely, bound-state models of the proton based on a ladder approximation to the Bethe-Salpeter equation with wave functions that remain finite for vanishing interparticle separation—it is seen that the form factors factorize as claimed to leading order in $Q^2/M_G^2 \ll 1$; $M^2/Q^2 \ll 1$.

V. CURRENT COMMUTATORS AND THE ADLER SUM RULE

In this section, we will discuss the implications for local current algebra of our speculations concerning the structure of nucleon constituents. In particular, we consider the Adler sum rule for neutrino-nucleon scattering,[27] with the momentum carried by the currents restricted to $M^2 \ll Q^2 \ll M_G^2$. Specifically, we ask whether or not the ideas of constituent structure developed earlier may be compatible with the validity of the Adler sum rule. We discuss separately the energy domains below and above the threshold for the production of the heavy gluon. The discussion of Sec. IV and in particular our conjecture about scale breaking in deep-inelastic scattering,

$$\nu W_2(\nu, Q^2) \cong F_2(x)\left(1-2\frac{Q^2}{M_G^2}\right), \tag{5.1}$$

are restricted to the "preasymptotic" kinematic region below the gluon threshold, $\nu \ll M_G^2/2M_p$ and to $M^2 \ll Q^2 \ll M_G^2$, where we are probing only the mean-square-radius parameter of the constituents. These considerations are independent of dynamical details of specific models. However, to discuss current-algebra sum rules, we are compelled in this section to give some consideration to the region above the heavy-gluon threshold, which means speculating in more detail into the heavy-gluon dynamics.

First we consider $\nu < M_G^2/2M_p$. Here Eq. (5.1) may be formulated in configuration space as a current commutator[28]

$$[J^\mu(x), J^\nu(0)] \underset{x^2 \ll M^2}{\doteq} \left(1-2\frac{\Box}{M_G^2}\right)\left\{\frac{1}{2\pi}(\partial_\rho \epsilon(x^0)\delta(x^2))[\bar{\psi}(x)Q^2\gamma^\mu\gamma^\rho\gamma^\nu\psi(0)-\bar{\psi}(0)Q^2\gamma^\nu\gamma^\rho\gamma^\mu\psi(x)]\right\}, \tag{5.2}$$

where ψ is the triplet of quark fields and Q the quark charge matrix. The notation $\doteq$ is intended to emphasize that (5.2) is not an operator statement but is only assumed valid when evaluated between single-particle states and with photon momenta satisfying $M^2 \ll Q^2 \ll M_G^2$ and $\nu < M_G^2/2M_p$. The right-hand side is the usual light-cone commutator except for the factor $-2\Box/M_G^2$ which gives the effect of the quark form factor in the radius approximation. Equation (5.2) is not scale-invariant in Wilson's sense,[29] but that is no surprise since the presence of a large mass means we are not in the asymptotic region where scale invariance is conjectured to be valid. Taking (5.2) between nucleon states and calculating the Fourier transform, we recover (5.1).

In generalizing (5.2) to the full SU(2) or SU(3) current densities we shall assume that the constituent's charge radius is SU3×SU(3)-invariant—i.e., the same factor $(1-2\Box/M_G^2)$ prefaces all light-cone commutators as in (5.2); viz. for the SU(2) currents

$$[J_+^\mu(x), J_-^\nu(0)] \underset{x^2 \ll M^2}{\doteq} \left(1-2\frac{\Box}{M_G^2}\right)\left\{\frac{1}{2\pi}(\partial_\rho \epsilon(x^0)\delta(x^2))[\bar{\psi}_{\mathcal{P}}(x)\gamma^\mu\gamma^\rho\gamma^\nu\psi_{\mathcal{P}}(0)-\bar{\psi}_{\mathfrak{N}}(0)\gamma^\nu\gamma^\rho\gamma^\mu\psi_{\mathfrak{N}}(x)]\right\}, \tag{5.3}$$

where

$$J_i^\mu(x) \equiv \bar{\psi}(x)\gamma^\mu \frac{\tau_i}{2}\psi(x),$$

$$J_\pm^\mu \equiv J_1^\mu \pm i J_2^\mu,$$

and $\psi_{\mathcal{P}}$ and $\psi_{\mathfrak{N}}$ are the fields of the proton and neutron quark.

We now wish to apply current-algebra techniques to study the implications of (5.3). We immediately encounter a difficulty since the usual current-algebra techniques involve predictions about physics at arbitrarily large energies, whereas (5.3) is only expected to apply when $\nu < M_G^2/2M_p$. In fact this difficulty is more than just a technical problem which faces us because of the limitations of our particular model: It is also a conceptual problem inherent in all the usual applications of local current algebra. The equal-time current algebra only implies statements about an undefined asymptotic region; it does not itself tell us at what energies sum rules should be satisfied (or even if they converge). Since experiments are limited to finite energies, the predictions of local current algebra are physically ambiguous, though they may be well defined mathematically. Even if a sum rule appears to be satisfied experimentally there is never a guarantee that future measurements at still higher energies will not reveal the earlier agreement to have been fortuitous.

In this sense an hypothesis about the structure of a local equal-time commutator is a mathematical idealization. Physically it is more precise to consider "almost-equal-time" commutators smeared over an interval $\Delta t \sim 1/E$, where E is the largest available energy. In this way we can construct sum rules involving finite-energy domains as we show below. For us this technique is essential since we are here discussing current algebra in a theory in which there are two important lengths, M and $M_G \gg M$, and we wish to exhibit separately the contributions from energies below the gluon production threshold and those from above the threshold which may not yet have been experimentally probed.

Before discussing sum rules obtained from almost-equal-time commutators, we very briefly review the $P\to\infty$ technique for deriving fixed-q^2 sum rules from local, equal-time current algebra.[30] One evaluates the quantity

$$\lim_{\vec{P}\to\infty}\left\{\langle P|\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}[J_a^\mu(\vec{x},0), J_b^\nu(0)]|P\rangle\right\} \tag{5.4}$$

by inserting a complete set of states and commuting the limit $\vec{P}\to\infty$ with the sum over the intermediate states. The expression (5.4) may then be written as

$$\frac{M}{P^0}\int_{-\infty}^{\infty} d\nu A_{ab}^{\mu\nu}(P,q)\big|_{q^2 \text{ fixed}}, \tag{5.5}$$

where $M\nu \equiv P\cdot q$, the direction of $\vec{p}$ is chosen so that $\vec{p}\cdot\vec{q}=0$, the value of q^2 is determined by the choice of $\vec{q}$ in (5.4), namely, $q^2=-\vec{q}^2$ in the infinite-momentum frame, $P^0\to\infty$, when $(q^0)^2 = (M\nu/P^0)^2 \ll Q^2$, and $A_{ab}^{\mu\nu}$ is the absorptive part of the forward current-hadron scattering amplitude.[31] Decomposing $A_{ab}^{\mu\nu}$ into Lorentz-invariant amplitudes and introducing assumptions about their high-energy behavior, (5.5) yields a family of sum rules, among them the Adler sum rule for neutrino-nucleon scattering.

Now consider a modified version of this procedure. The equal-time commutator in (5.4) is replaced by a smeared almost-equal-time commutator:

$$\lim_{P_0\to M\Omega}\frac{1}{N}\left\{\left\langle P\right|\int_{-\infty}^{\infty}dt\,e^{-\Delta^2t^2}\int d^3x\,e^{-i\vec{q}\cdot\vec{x}}\times[J_a^\mu(\vec{x},t),\,J_b^\nu(0)]\left|P\right\rangle\right\},\tag{5.6}$$

where $N\equiv\int_{-\infty}^{\infty}dt\,e^{-\Delta^2t^2}$ is a normalization factor. Proceeding as before, we find in place of (5.5) an integral over a finite range[32] of ν:

$$\frac{1}{\Omega}\int_{|\nu|<|\nu_{\max}|}d\nu\,A_{ab}^{\mu\nu}(P,q)|_{q^2}\quad ,\tag{5.7}$$

where $\nu_{\max}\equiv\Omega\Delta$ and Ω is chosen large enough that $\Omega^2\gg\nu_{\max}{}^2/Q^2$, so that $q^2\cong-\vec{q}^{\,2}$. It is the integral in (5.7), not that in (5.5), which is experimentally measurable; and therefore it is the smeared almost-equal-time commutator in (5.6), not the equal-time commutator in (5.4), which is actually the object of physical investigation.

Notice that for fixed $\nu_{\max}$, as $\Omega\to\infty$ we have $\Delta\to0$, so that larger and larger values of t become important in (5.6). Thus in the $P\to\infty$ frame we are not probing small times at all, which corresponds to the fact that q^0 vanishes in the $P\to\infty$ frame. Performing a Lorentz transformation to the laboratory we find that the important times are

$$t_{\rm lab}\cong\frac{M}{\nu_{\max}}=\frac{1}{(q^0{}_{\max})_{\rm lab}}\quad ,$$

as indeed they must be according to the uncertainty principle.

It is necessary in the discussion which follows to replace the idealization $P_0=M\Omega\to\infty$ by the condition

$$M^2\ll\Delta^2=\frac{\nu_{\max}{}^2}{\Omega^2}\ll Q^2.\tag{5.8}$$

The lower bound, $\Omega^2\gg\nu_{\max}{}^2/Q^2$, is imposed, as remarked above, to guarantee the conditions $Q^2/P_0^2\ll1$ and $Q^2\cong\vec{q}^{\,2}$ (we always choose $\vec{q}\cdot\vec{P}=0$), which are necessary to derive the sum rule in its covariant form. The upper bound, $M\Omega\ll\nu_{\max}$, is imposed, in contrast with the usual choice $M\Omega=P_0\to\infty$, so that the important times being probed in (5.6) are restricted by $t\lesssim\Delta^{-1}\ll M^{-1}$. This restriction, together with causality, ensures that we are only making use of the unequal-time commutator in the region within M^{-2} of the light cone where we are prepared to conjecture about its structure.

We may now derive the sum rule for the region below the gluon threshold. We proceed by substituting (5.2) into (5.6). Inserting a complete set of states and choosing $P_0=M\Omega$ as in (5.8), we find, with the analog of the usual assumptions about the Lorentz-invariant amplitudes, that the left-hand side of the sum rule becomes

$$P^0\int_{Q^2/2M}^{\nu_{\max}}d\nu\,[W_{2V}^{\bar{\nu}p}(\nu,Q^2)-W_{2V}^{\nu p}(\nu,Q^2)],\tag{5.9}$$

where W_{2V} denotes the vector-current part of the structure function and we have chosen $\nu_{\max}<M_G{}^2/2M$ and $M^2\ll Q^2\ll M_G{}^2$.

The remaining task is to evaluate the right-hand side of (5.2) when substituted into (5.6). The D'Alembertian is evaluated by an integration by parts: The Laplacian gives rise to terms proportional to $\vec{q}^{\,2}/M_G{}^2$, which become $Q^2/M_G{}^2$ because of (5.8), while the time derivatives give rise to terms of order $\Delta^2/M_G{}^2$, which according to (5.8) may be neglected. The result of a straightforward calculation is that the right-hand side of (5.2) yields[33]

$$\left(1-2\frac{Q^2}{M_G{}^2}\right)P^0\left\{\int_{Q^2/2M\nu_{\max}}^{1}+\int_{-1}^{-Q^2/2M\nu_{\max}}d\alpha\,A(\alpha)\right\},\tag{5.10}$$

where $A(\alpha)$ is the Fourier transform of the matrix element of the bilocal operator appearing in (5.2). That is,

$$A(\alpha)\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}d(P\cdot x)\,e^{-i\alpha(P\cdot x)}\tilde{A}(P\cdot x),\tag{5.11}$$

where

$$\langle P|\bar{\psi}_P(x)\gamma^\rho\psi_P(0)-\bar{\psi}_N(0)\gamma^\rho\psi_N(x)|P\rangle\equiv P_\rho\tilde{A}(P\cdot x).\tag{5.12}$$

In (5.12) we have also neglected a possible contribution to the bilocal of the form $x_\rho\tilde{B}(P\cdot x)$. With the same assumptions on the small α behavior of $B(\alpha)$ which are necessary in the usual equal-time $P\to\infty$ derivation of the sum rule, we find, using (5.8), that the contribution of $\tilde{B}$ is negligible compared to the contribution of $\tilde{A}$.

As a check on the calculation, we observe that in the limit $M_G\to\infty$ and $\nu_{\max}\to\infty$, which corresponds to the conventional equal-time derivation of the sum rule, (5.10) becomes

$$P^0\int_{-1}^{1} d\alpha\, A(\alpha) = P^0\,, \tag{5.13}$$

where we use the property, deduced from (5.2) in the limit $x\to 0$, that $A(0)=\int_{-1}^{1} d\alpha A(\alpha)=1$. Thus we recover the usual sum rule.[34]

$$\int_{\nu_t}^{\infty} d\nu\,[W_{2\nu}^{\bar\nu p}(\nu,Q^2)-W_{2\nu}^{\nu p}(\nu,Q^2)]=1\,. \tag{5.14}$$

With M_G and $\nu_{\max}$ finite, additional dynamical information is required to evaluate (5.10). In the simplest model we treat the nucleon as an elementary particle and include its interaction with massive gluons in second-order perturbation theory. In this case in (5.10) we have $A(\alpha)=\delta(1-\alpha)$ so that the sum rule becomes[35]

$$\int_{Q^2/2M}^{\nu_{\max}<M_G^2/2M} d\nu[W_2^{\bar\nu p}(\nu,Q^2)-W_2^{\nu p}(\nu,Q^2)] = 2\left(1-2\frac{Q^2}{M_G^2}\right). \tag{5.15}$$

Due to the simplicity of the model, (5.15) is saturated by the elastic contribution and $\nu_{\max}$ is free to vary between $Q^2/2M$ and $M_G^2/2M$.

In a bound-state model as discussed in Sec. IV, in which the nucleon is a bound state of light constituents interacting via massive gluons and the structure of the constituents is treated in second-order perturbation theory, the contribution to (5.10) and (5.15) comes from the quasielastic peak. However, the value of the integral in (5.10) depends on $\nu_{\max}$ through the limits of integration. In the particular model developed in Ref. 2, $A(\alpha)$, which is simply $A(\alpha)=W_1(\alpha)=\nu W_2/2\alpha M_p$, is constant for small $\alpha\to 0$ and therefore the brackets in (5.10) differ from unity by terms of order $(Q^2/2M_p\nu_{\max})\gtrsim Q^2/M_G^2$.[36]

We turn finally to the kinematic region above the gluon threshold. Our purpose is to investigate whether our speculations on constituent structure contradict the Adler sum rule, which is derived on the basis of local equal-time algebra plus generally accepted assumptions on the high-energy limiting behavior of the forward virtual Compton amplitude. In particular it is assumed in deriving the Adler sum rule that no subtractions are required for the odd amplitude under crossing, in accord with standard Regge asymptotic arguments. In this investigation we must resort to specific dynamical models for describing gluon production. This means going beyond the general notion of a constituent size that appears as a correcting factor in (5.15). We must compute in specific models whether the contribution to the sum rule for $\nu>M_G^2/2M_p$ when added to that from $\nu<M_G^2/2M_p$ exactly adds to 2, as we found in the M_G, $\nu_{\max}\to\infty$ limiting case (5.14).

First as a simple illustrative example we consider in second-order perturbation theory an elementary spin-zero nucleon which exists as an SU(2) doublet (p,n) and interacts with a scalar gluon that is an isoscalar. We then find that the elastic contribution to the sum rule from Fig. (3) is given by (5.15), where now Q^2/M_G^2 is a mnemonic for

$$\frac{Q^2}{M_G^2}\equiv\frac{G^2}{96\pi^2}\frac{Q^2}{M_G^4}\left(\ln\frac{M_G^2}{Q^2}-\frac{1}{6}\right) \quad \text{for } M_p^2\ll Q^2\ll M_G^2\,. \tag{5.16}$$

In (5.16), G is the gluon-nucleon coupling constant and has the dimensions of a mass. To this we must add the contribution of the gluon-radiation diagrams, Fig. (6), which we denote by $\delta(\nu W_2)$. The contribution to the Adler sum rule is calculated directly to be (for $M^2\ll Q^2\ll M_G^2$)

$$\int_{M_G^2/2M_p}^{\infty} d\nu\,[\delta(W_2^{\bar\nu p}(\nu,Q^2))-\delta(W_2^{\nu p}(\nu,Q^2))] = 4\frac{Q^2}{M_G^2}\,, \tag{5.17}$$

where Q^2/M_G^2 is the identical mnemonic as in (5.16). Combining (5.15) and (5.17), we recover the exact Adler sum rule[37]

$$\int_{\nu_t}^{\infty} d\nu\,[W_2^{\bar\nu p}(\nu,Q^2)-W_2^{\nu p}(\nu,Q^2)]=2 \quad \text{for } M^2\ll Q^2\ll M_G^2\,. \tag{5.18}$$

This same result can also be verified in the familiar scaling region $M^2, M_G^2\ll Q^2$.

Returning to the bound-state model discussed in Sec. IV we again confirm the Adler sum rule. As in the earlier analysis, we work to leading order in Q^2/M_G^2 and compute in second-order perturbation theory the massive-gluon radiation as well as its contribution to the constituent structure. This suffices to show how the Adler sum rule may remain valid even though the structure functions themselves do not scale, as a consequence of modifications (5.1) and (5.2). Since this is the point we want to illustrate—and not the validity of the particular model being used—we do not carry the calculations (which become very tedious)

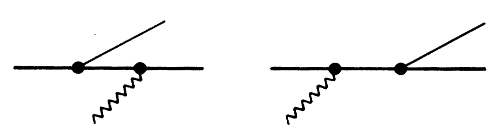

FIG. 6. Gluon radiation in second-order perturbation theory.

beyond this lowest approximation. We shall "embed" the above perturbative calculation within the bound state, and to this end we summarize (5.15) to (5.18) in the useful form

$$W_2(Q^2,\nu')=2\int\frac{M d^3P}{E_P}\,\delta^4(P-q-(p-X)_s)|F_1(Q^2)|^2+\int\frac{M d^3P}{E_P}\,\frac{d^3K}{2\Omega_K}\times\delta^4(P+K-q-(p-X)_s)\,\mathfrak{a}(q,P,K)\,,$$

$$\int_0^\infty W_2(Q^2,\nu')d\nu'=2\,, \tag{5.19}$$

where the kinematics corresponds to Fig. (7); $\nu'\equiv(p-X)_s\cdot q/M_p$, $E_p\equiv(P^2+M^2)^{1/2}$, $\Omega_K\equiv(K^2+M_G^2)^{1/2}$; $F_1(Q^2)$ is the charge form factor, and $\mathfrak{a}\,(q,P,K)$ denotes additional factors in the inelastic gluon production amplitude which depend on the spin of the particle. $(p-X)_s$ denotes the 4-vector of an on-shell nucleon with momentum $\vec{p}-\vec{X}$, and (5.19) is just the statement of (5.18) and Ref. 37.

Turning to the bound state we recall the normalization condition derived in Ref. 2 for pointlike constituents. With the kinematics in Fig. (8) we have the explicit expression for $W_2^{\text{B.S.}}$

$$\int\frac{M d^3P}{E_p}\,\frac{d^3X}{2\omega_X}\,\delta^4(P+X-p-q)|g(u)|^2f(p-X,q)\equiv W_2^{\text{B.S.}}(Q^2,\nu) \tag{5.20}$$

which satisfies the Adler sum rule,

$$\int_0^\infty d\nu\, W_2^{\text{B.S.}}(Q^2,\nu)=2, \tag{5.21}$$

where $E_p\equiv(P^2+M^2)^{1/2}$, $\omega_X\equiv(X^2+\mu^2)^{1/2}$, $\nu=p\cdot q/M_p$, $u\equiv(p-X)^2$, $g(u)$ is the bound-state vertex, and f contains the remaining factors for projecting out $W_2^{\text{B.S.}}$. Substituting (5.20) into (5.21) gives

$$2=\frac{M}{M_p}\int\frac{d^3X}{2\omega_X}\,|g(u)|^2f(u)\,\frac{1}{x}\,. \tag{5.22}$$

In obtaining (5.22) we use the fact that the bound-state wave function keeps $u/M^2\sim 0(1)$ so that in an infinite-momentum frame we may write $p-X=xp$, where x is the fraction of longitudinal momentum carried by the charged constituent in an infinite-momentum frame. (In our verification of the sum rule, the use of the infinite-momentum frame is convenient but not necessary.)

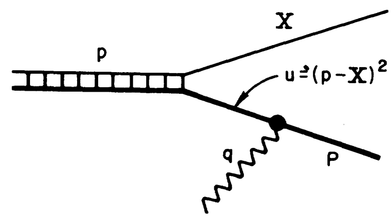

FIG. 8. Contribution to the Adler sum rule for a bound state with a pointlike constituent.

We now observe that if the charged constituent in the bound-state model emits virtual and real gluons as in Fig. (9) then instead of (5.20) we have

$$W_2^{\text{B.S.}}(Q^2,\nu)=\int\frac{d^3X}{2\omega_X}\,|g(u)|^2f(u)\,W_2\left(Q^2,\nu x\frac{M_p}{M}\right), \tag{5.23}$$

where W_2 is calculated in second-order perturbation as in (5.19), but now for a target of mass M carrying momentum $p-X=xp$. Using the validity of the Adler sum rule in perturbation theory (5.19) together with the normalization condition (5.22), we deduce the validity of the sum rule in the bound-state model,

$$\int_0^\infty d\nu\, W_2^{\text{B.S.}}(Q^2,\nu)=2\,. \tag{5.24}$$

Note that the quasielastic peak contributes $2|F_1(Q^2)|^2$ to the sum rule and the remaining portion comes from real gluon production. This completes our example and shows explicitly that the ideas discussed here are not incompatible with local current algebra and the Adler sum rule.

VI. CONCLUDING REMARKS

In this paper we have developed our view that *scaling is a preasymptotic phenomenon* in terms of a simple model of the nucleon as a weakly bound system of light constituents bound to one another by massive gluons. However, we have not offered

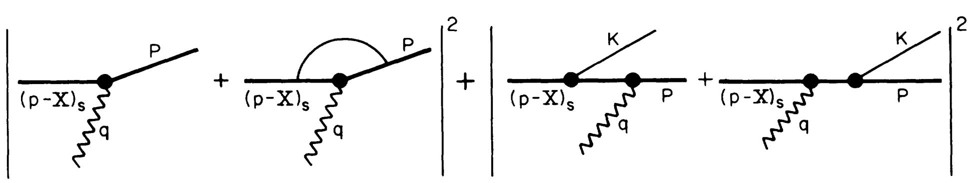

FIG. 7. Contributions to the Adler sum rule in second-order perturbation theory.

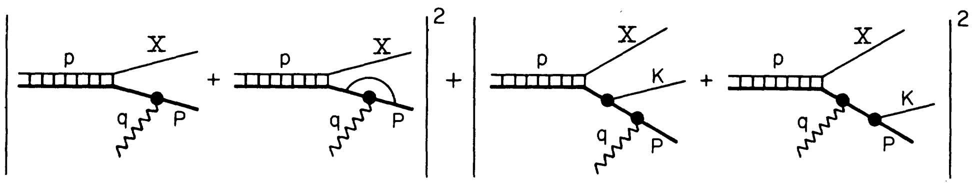

FIG. 9. Contributions to the Adler sum rule for a bound state with constituent structure treated in second-order perturbation theory.

a consistent dynamical basis for this theoretical picture. It is our conjecture that we can abstract a correct description of how scaling fails even if it turns out that the particular model we have utilized fails.

In the very near future there should be considerably more evidence bearing on the question of scaling, both for spacelike and timelike values of q^2. One qualitative feature is suggested by the data available at this moment: Deviations from scaling in the spacelike scattering region of $-q^2 \lesssim 10\ \text{GeV}^2$, indeed if present,[16] seem significantly less pronounced than the apparent enhancements above pointlike for the timelike annihilation region[38] of $+q^2 \lesssim 20\ \text{GeV}^2$. If this feature is verified by future experiments, to accommodate it within the context of our model we would have to assume that there is a resonant enhancement modifying (3.5) by $(1+2s/M_G^2 \to (1-s/\mathfrak{M}^2)^{-2}$, as described below (3.5) with $\mathfrak{M} \sim 8$ GeV. Additional data should indicate whether or not such an explanation is tenable.

If the hypothesis which we have discussed in this paper is correct, we would still be faced with a deepening mystery: Where are the light constituents? Why are they not observed? In this connection, it is interesting to consider the alternative hypothesis that the constituents (and the gluons) are very massive, say $\gg 10$ GeV. In this case one might not expect to see s^{-1} scaling behavior in e^+e^- annihilation until $s \gg 4M_{\text{constituent}}^2$ and the range of timelike momenta presently under experimental investigation might be too small to reveal any easily understood scaling behavior. In contrast, the effective mass of the constituent inside the nucleon could be small as a result of the strong binding forces. A proton bound state of low density would then allow the early onset of incoherence and "preasymptotic" scaling behavior as discussed in this paper. Turning once again to nuclear matter for a guide we find from the results of Stanfield[7] and the analysis of Moniz[39] that the nuclear forces cause a qualitative shift in the effective nucleon mass by as much as 30%, even for values of Q^2 limited within the region where incoherent scattering is observed. Due to the considerably stronger forces binding such massive constituents within the nucleon there could well be an even greater difference between effective bound-constituent masses and free masses. In this way we might hope to accommodate "preasymptotic" scaling for inelastic scattering measurements at precociously small values of spacelike q^2, without at the same time having simple pointlike behavior for the annihilation cross section at comparably small values of timelike q^2.

ACKNOWLEDGMENTS

We thank our colleagues at SLAC, and in particular J. D. Bjorken for valuable discussions.

*Work supported by the U. S. Atomic Energy Commission.

[1]R. P. Feynman, unpublished work and Phys. Rev. Lett. 23, 1415 (1969); and in *High Energy Collisions*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969); J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969). Field-theoretic deviations of the parton model are presented in S. D. Drell, D. J. Levy, and T.-M. Yan, *ibid.* 187, 2159 (1969); S. D. Drell and T.-M. Yan, Ann. Phys. (N.Y.) 66, 578 (1971). For a covariant formalism see P. V. Landshoff and J. C. Polkinghorne, Phys. Rev. D 6, 3708 (1972), with references to earlier work contained therein.

[2]S. D. Drell and T. D. Lee, Phys. Rev. D 5, 1738 (1972); C. H. Woo, Phys. Rev. D 6, 1127 (1972).

[3]Y. Frishman, Ann. Phys. (N.Y.) 66, 373 (1971); R. Brandt and G. Preparata, Nucl. Phys. B27, 541 (1971).

[4]K. Wilson, Phys. Rev. Lett. 27, 690 (1971); and in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, New York, 1972); S. D. Drell and T. D. Lee, Ref. 2; T. D. Lee, Phys. Rev. D 6, 1110 (1972); M. S. Chanowitz and S. D. Drell, Phys. Rev. Lett. 30, 807 (1973). We especially wish to call attention to the work of K. Matumoto [Prog. Theor. Phys. 47, 1795 (1972)], which we inadvertently over-

looked in our previous paper.

[5]Aspects of this problem are now under investigation by P. Pesic (unpublished).

[6]Chanowitz and Drell, Ref. 4.

[7]These curves are taken from K. Stanfield, C. Canizares, W. Faissler, and F. Pipkin, Phys. Rev. C 3, 1448 (1971).

[8]See, for instance, J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969), and references therein.

[9]K. Johnson, Phys. Rev. D 6, 1101 (1972).

[10]R. F. Dashen and M. Gell-Mann, Phys. Rev. Lett. 17, 340 (1966); H. J. Melosh, Ph.D. thesis Caltech (unpublished); F J. Gilman and M. Kugler, Phys. Rev. Lett. 30, 518 (1973); H. J. Melosh, Phys. Rev. D 9, 1095 (1974).

[11]D. Flamm and J. Sanchez, Nuovo Cimento Lett. 6, 129 (1973).

[12]For an expression of this point of view, see "The Straton Model—Relativistic Structure Theory of Baryons and Mesons" in the 1966 Summer Physics Colloquium of the Peking Symposium (available only in unpublished report form in English), p. 403.

[13]In renormalizable theories, with spin but no momentum cutoffs and therefore without superconvergent behavior when treated with an order-by-order iterative perturbation expansion, scaling is violated by logarithmic powers $\ln Q^2/M^2$ for $Q^2/M^2 \gg 1$. These factors are introduced by the fact that massive states $\approx Q^2$ are important in the Feynman-loop integrations.

[14]As a numerical example, based on the work of P. Pesic (Ref. 5), for a spin-zero bound state of spin-zero constituents, each of mass m, interacting via massive scalar gluons of mass $M_G \sim 10m$, the coupling as defined in Eq. (3.1) is determined to be approximately $f \sim \frac{1}{4}$.

[15]See comment by K. Gottfried, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, New York, 1972), p. 144.

[16]E. D. Bloom, in International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973 (unpublished).

[17]E. M. Riordan, thesis, MIT, 1973 (unpublished).

[18]E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. 25, 1140 (1970); Phys. Rev. D 4, 2901 (1971). This explanation in terms of ω' as the scaling variable can also account for the data discussed in Riordan's thesis (see Ref. 16) with no indicated deviation from scaling behavior.

[19]P. N. Kirk *et al.*, Phys. Rev. D 8, 63 (1973).

[20]T. Massam and A. Zichichi, Nuovo Cimento Lett. 1, 387 (1969).

[21]We thank Dr. Steve Stein for performing this analysis.

[22]M. Chanowitz and R. Pettorino (unpublished).

[23]Possible effects of virtual gluon exchange in proton-proton scattering are discussed by A. Casher, S. Nussinov, and L. Susskind, Phys. Lett. 44B, 511 (1973).

[24]V. Z. Jankus, Phys. Rev. 102, 1586 (1956).

[25]Reference 2 discusses this for scalar gluons; the vector-gluon case has been studied by Ahmet Gokalp (unpublished).

[26]In Ref. 2 these corrections were reduced by powers of M^2/Q^2 since no large mass M_G was introduced there.

[27]S. L. Adler, Phys. Rev. 143, 1144 (1966).

[28]Y. Frishman, Ref. 3.

[29]K. Wilson, Phys. Rev. 179, 1499 (1969).

[30]Here we follow the pedagogical development of S. L. Adler and R. F. Dashen, *Current Algebras* (Benjamin, New York, 1968).

[31]See Chap. 4 of Ref. 30 for the precise definition.

[32]More precisely, the integral is cut off by a Gaussian factor, $\exp(-\nu^2/\nu_{\max}^2)$.

[33]The integration limits $\pm Q^2/2M\nu_{\max}$ are shorthand for a Gaussian factor $\exp\{-[(Q^2/2M\nu_{\max})/\alpha]^2\}$.

[34]Here we follow the derivation given by H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone*, 1971 Coral Gables Conference on Fundamental Interactions at High Energies, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971).

[35]The over-all factor 2 is due to the vector plus axial-vector contributions.

[36]Notice that if instead we assume Regge behavior, i.e., $A(\alpha) \sim \alpha^{-1/2}$, then when the sum rule is truncated at some $\omega_{\max}$, the missing contribution is $\sim 1/(\omega_{\max})^{1/2}$. Thus at $\omega_{\max} = 5$ we would expect almost a 50% mismatch.

[37]The formulas in Sec. III of the last of Refs. 1 can be integrated directly to obtain these results. We have also been informed that R. L. Jaffe and H. Quinn have explicitly confirmed the Adler sum rule in second-order perturbation theory in a model with an elementary spin-$\frac{1}{2}$ target and a vector gluon (private communication).

[38]A. Litke *et al.*, Phys. Rev. Lett. 30, 1189 (1973).

[39]E. J. Moniz, Phys. Rev. 184, 1154 (1969).