PHYSICAL REVIEW D VOLUME 9, NUMBER 7 1 APRIL 1974

Pion electromagnetic form factor—data analysis and asymptotic behavior

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(Received 15 February 1973; revised manuscript received 20 September 1973)

Existing data on the pion form factor are analyzed with a view to suggesting a method of verifying theoretical predictions about its asymptotic behavior. A Broadhurst-type analysis gives a falloff faster than $(\ln t)^c/t^{(p+1)/2}$ for $t\to\infty$, where p is the exponent of the threshold factor of the structure function νW_2. Most of the dynamical models predict an asymptotic decrease $(\ln t)^{2m}/t^n$ with m and n as integers. For data analysis a G/D method is developed and parametrized, the D function posessing the analyticity property due to the two-body elastic cut and the G function possessing the inelastic cut and the ρ-ω interference term. The logarithms occur naturally in a conformally mapped parabolic variable which is used to parametrize the G function. An analysis of the available data yields the result that although a $1/t^2$ or $(\ln t)^2/t^2$ type behavior is not ruled out, the best fit is given for $c\geq 0$ and $p=5$, suggesting a $1/t^3$ behavior. We also obtain the results (i) $r_\pi=(0.70\pm0.01)$ fm for the pion's electromagnetic radius and (ii) $\xi=(0.0094^{+0.006}_{-0.001})\times\exp i(101^\circ{}^{+28^\circ}_{-39^\circ})$ for the ρ-ω interference amplitude and phase.

I. INTRODUCTION

It is conjectured that if the electromagnetic form factor of any particle falls off more rapidly than a dipole form it is composite in nature, whereas a single-pole-like behavior could imply that the particle is elementary. Since the pion is the least massive of the strongly interacting particles, the asymptotic behavior of the pion form factor is an important problem in particle physics. Following Cooper and Pagels[1] and Broadhurst[2] we have deduced an exact inequality for the asymptotic bound of the pion form factor. We have also considered predictions from various theoretical models for elementary and composite objects. It is the aim of the present paper to suggest a method of studying the asymptotic behavior by analyzing the experimental data in a scheme of parametrization specially tailored for the purpose.

In a parallel work[3] on the proton form factor a similar scheme of parametrization had been undertaken incorporating (i) analyticity in t, (ii) correct behavior near elastic threshold, (iii) the lowest inelastic branch point, and also (iv) leading to an asymptotic behavior $(\ln t)^{2m}/t^n$ with m and n being integers. Acceptable fits to the spacelike data were obtained for $m=2$ and $n=2$ or 3. The data alone could not discriminate between these two fits. The fit with $n=3$, however, gave a better ρ signal in the timelike region on extrapolation. Our analysis definitely indicated a falloff faster than the dipole and at the same time reproduced the Frascati datum point at $t=4.41$ GeV^2.

In the pion case the situation is quite different and rather opposite. Extensive data are available in the timelike region. There are no reliable spacelike data since the analysis of pion electroproduction can not unambiguously determine the pion form factor. Even in the timelike data there are discrepancies between different experimental groups. There are also measurable effects due to the ρ-ω interference. The problem is to extract meaningful results, if any, from the data as they exist at present. In case of the proton[3] the formula which fitted the spacelike data yielded a ρ signal on extrapolation to the timelike region. It may be possible, then, to use the timelike data for the pion to extrapolate to the spacelike region. In addition, the use of the existing spacelike data, even though unreliable, will provide a distinct corridor for extrapolation. In the literature we have not found any attempt to fit both the timelike and the spacelike data with the same formula. The extensively quoted formula of Gounaris and Sakurai,[4] the more recent formula due to Lyth,[5] and the fit proposed by Benaksas[6] *et al.* yield unusually high χ^2 for the spacelike region (see Table III, below). Our formula is distinctly better and yields the positive result that a falloff faster than the dipole cannot be ruled out for the pion.

We plan the paper as follows: In Sec. II we make a brief review of the existing experimental data and some existing phenomenological fits. In Sec. III we discuss the derivation of the asymptotic behavior. In Sec. IV we propose a G/D method suitable for verifying the asymptotic behavior. In Sec. V we discuss our results.

II. EXPERIMENTAL DATA AND PHENOMENOLOGICAL FITS

The existing data on the pion form factor can be classified into two groups: (a) data in the timelike region ($t>0$) and (b) data in the spacelike region ($t<0$) where t is the square of the four-momentum transfered to the photon. Experimental data for $t>0$ have been obtained from electron-positron colliding beam experiments [Fig. 1(a)]. The earliest measurements of Auslander *et al.*[7] suggests the mass and the width of the ρ meson to be $m_\rho = 764 \pm 11$ MeV and $\Gamma_\rho = 93 \pm 15$ MeV. Orsay results[8] indicate a peak almost 50% higher than the Novosibirsk[7] results with $m_\rho = 777.5$ MeV and a much higher width. Recent results of Augustin *et al.*[9] give indications of ρ-ω interference instead of a single ρ resonance although the accuracy of these data has been questioned.[6] The latest reports by Lefrançois[10] and Benaksas *et al.*,[6] however, confirm the existence of ρ-ω interference. Data reported by the groups of experimentalists mentioned above cover only the range $0.33 \leq t \leq 1$ (GeV^2). Experimental data for higher values of t, but with $t \leq 4.41\ \mathrm{GeV}^2$, have been reported from Novosibirsk[11] and Frascati.[12]

The pion form factor in the spacelike region has been extracted from experiments[13-17] of an entirely different nature, namely the electroproduction of pions. Akerlof *et al.*[13] and Mistretta *et al.*[14] have extracted pion form factor from the electroproduction cross section by using the fixed-t dispersion theory of Zagury[18] and Adler.[19] Their results in the low-$|t|$ region can be reasonably fitted by a formula of the type $F_\pi(t) = 1/(1 - t/m^2)$, with $m = 0.6$ GeV. More accurate data covering larger values of $|t|$ have been recently reported by Brown *et al.*[17] by using the dispersion theory of Berends.[20] Although a single-pole fit to their data are not ruled out, a better fit is obtained by a dipole-like form factor: $F_\pi(t) = 1/(1 - t/m^2)^2$, with $m^2 = 1.312$ GeV^2. However the analysis of pion electroproduction cannot unambiguously determine the pion form factor. This is partly due to the presence of unknown subtraction constants in the amplitude which satisfies a subtracted dispersion relation. Recently Kellett and Verzegnassi[21] have made a useful review of the problem.

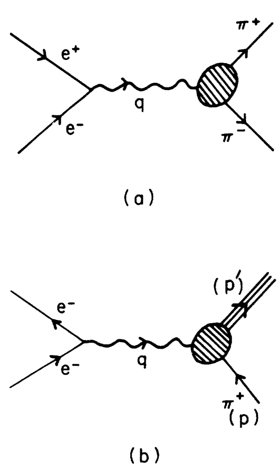

FIG. 1. (a) One-photon-exchange diagram for the process $e^+e^- \to \pi^+\pi^-$. (b) One-photon-exchange diagram for inelastic electron-meson scattering.

Several attempts have been made to construct an analytic formula for the pion form factor. The most familiar method for construction utilizes the dispersion-relation technique suggested by Chew and Mandelstam[22] and parametrizes the $J = I = 1$ $\pi\pi$-scattering partial-wave amplitude by an N/D method. Such a procedure was at first adopted by Frazer and Fulco[23] and subsequently refined by Gounaris and Sakurai[4] using the hypothesis of vector-meson dominance. Although the formula of Gounaris and Sakurai[4] (GS) can explain the earliest Novosibirsk results very well, the extrapolated curve falls much lower than the high-energy data in the timelike region and somewhat higher than the data in the spacelike region. For clarity the χ^2 value by the GS formula has been reported in Table III. The hard-pion current-algebra method[24] which introduces a zero into the imaginary part of the form factor also yields similar results. A phenomenological parametrization procedure which introduces a zero into the numerator has been proposed by Vaughn,[25] Kanazawa and Haruyama,[26] and Lyth.[27] Many authors[28] have studied the application of the Veneziano model to the pion form factor while others[29] have studied the approach of the GS type. But all the formulas suffer, more or less, from the same type of difficulties as the GS formula. These formulas could not account for the recently observed ρ-ω interference effects. Recently a practical way of parametrization including inelasticity and the interference effects has been suggested by Lyth[5] and a similar fit has been used in the data analysis by Benaksas[6] *et al.* This formula ignores the analyticity properties of the pion form factor for the ρ-ω interference term and the global fit is also not good, giving a high χ^2 (see Table III). In a subsequent section we shall suggest a formula which possesses the correct analyticity properties and gives a much better fit to the data.

III. ASYMPTOTIC BEHAVIOR

The knowledge of the asymptotic behavior of the pion form factor is also an essential ingredient in our scheme of parametrization. It is therefore necessary to know the theoretical predictions of different models in this regard. In the case of the proton Broadhurst[2] has obtained an exact bound on the form factor using the sidewise dispersion method for inelastic electron-proton scattering initiated by Cooper and Pagels.[1] Calculations of the Broadhurst[2] type were repeated by us for inelastic electron-meson scattering, Fig. 1(b). A Drell-Yan-West[30,31] relation is obtained for the meson and the bound on the pion form factor turns out to be

$$\lim_{Q^2\to\infty} |F_\pi(Q^2)| \leq (\ln Q^2)^c/(Q^2)^{(p+1)/2} , \qquad (1)$$

where c is any arbitrarily positive constant including zero and p is a constant related to the threshold behavior of the inelastic mesonic structure function[32,33,34]

$$\lim_{Q^2\to\infty} \nu W_2 = F_2(\omega) = (\omega - 1)^p . \qquad (2)$$

In (1) Q^2 is related to t by the relation $t = q^2 = -Q^2$.

We briefly summarize below our findings about the asymptotic behavior by taking different models for the elementary and the composite pion.

1. Elementary pion-gluon emission model

Here we consider the triangle graph with point vertices given in Fig. 2(a). The intermediate state consists of two spin-zero objects: a bare pion and a gluon. Thus the vertex in Fig. 2(a) can be written as

$$\Gamma_\mu = \langle p' | j_\mu(0) | p \rangle \sim \int \frac{(p'+p-2k)_\mu d^4k}{[(p-k)^2 - m_\pi{}^2][(p'-k)^2 - m_\pi{}^2][k^2 - m_x{}^2]} , \qquad (3)$$

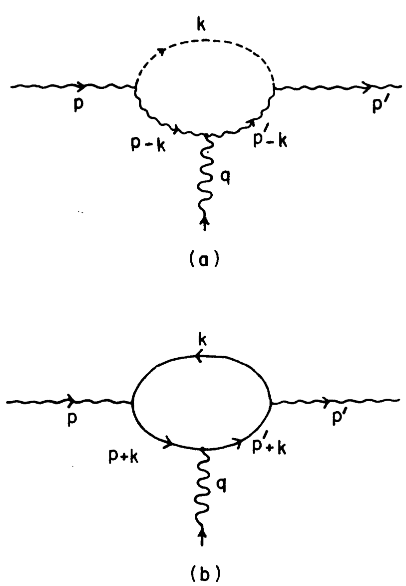

FIG. 2. (a) Elementary pion-gluon emission model for the pion. (b) Elementary $\bar{N}N$ mode of the pion.

where m_x is the mass of the gluon. Using the Feynman method of integration equation, (3) gives

$$\Gamma_\mu \sim (p'+p)_\mu \int_0^1 y\,dy \int_0^1 \frac{dx(1-y)}{cx^2 + bx + a} , \qquad (4)$$

where

$$c = Q^2 y^2 = -b ,$$
$$a = -m_\pi{}^2 y^2 - m_x{}^2 (1-y) . \qquad (5)$$

In obtaining the result (4) from (3) we have used Lorentz invariance and current conservation. Now integrating over x and noting that for $Q^2 \to \infty$, the region $0 < y \leq m_x/Q$ gives the major contribution to the integral, we obtain the leading terms in Γ_μ as

$$\Gamma_\mu \underset{Q^2\to\infty}{\sim} (p'+p)_\mu \frac{1}{Q} \int_0^1 \frac{dy}{(Q^2y^2 + 4m_x{}^2)^{1/2}} \times \ln\left[\frac{Qy - (Q^2y^2 + 4m_x{}^2)^{1/2}}{Qy + (Q^2y^2+4m_x{}^2)^{1/2}}\right]$$
$$= (p'+p)_\mu (\ln Q^2)^2/Q^2 . \qquad (6)$$

Thus the prediction of this model is that

$$|F_\pi(t)| \underset{t\to\infty}{\sim} (\ln t)^2/t . \qquad (7)$$

We observe that this asymptotic behavior is obtained by saturating the asymptotic bound of the Broadhurst type given in formula (1) with the values of $c = 2$ and $p = 1$.

2. Elementary $N\bar{N}$ mode

We consider the $N\bar{N}$ mode of the elementary pion as shown in Fig. 2(b), where all the vertices are point vertices. The vertex function can be written as

$$\Gamma_\mu \sim \int d^4k \,\mathrm{Tr}\left(\frac{1}{\not{k} - m}\gamma_5 \frac{1}{\not{p}' + \not{k} - m}\gamma_\mu \frac{1}{\not{p} + \not{k} - m}\gamma_5\right) , \qquad (8)$$

where m is the mass of the nucleon. Using Lorentz invariance and current conservation we reduce it to a logarithmically divergent integral

$$\Gamma_\mu \sim (p'+p)_\mu \int \frac{d^4k}{[(p'+k)^2 - m^2][(p+k)^2 - m^2]} . \qquad (9)$$

The infinity is usually absorbed as a charge renormalization. Carrying out the necessary renormalization one obtains

$$\Gamma_\mu^R \underset{Q^2\to\infty}{\sim} (p'+p)_\mu \ln(Q^2/m^2) \tag{10}$$

so that $|F_\pi(t)|$ goes to infinity as $\ln t$ in this model.

3. Composite $N\bar{N}$ mode

One can treat the pion as a bound state of the nucleon-antinucleon pair $N\bar{N}$ as suggested by the Fermi-Yang[35] model. This situation, illustrated in Fig. 3(a), has been considered by several authors.[36] Without solving the Bethe-Salpeter equation one can use a deuteron-type wave function to arrive at the same result as in Ref. 36. From Fig. 3(a) we have

$$\Gamma_\mu = \langle p'|j_\mu(0)|p\rangle$$

$$\sim \int d^4k \frac{\Gamma^*(m_\pi{}^2, k^2, (k+p')^2)}{(k^2-m^2)[(p'+k)^2-m^2]} \frac{\Gamma(m_\pi{}^2, k^2, (k+p)^2)}{(p+k)^2-m^2}$$

$$\times \mathrm{Tr}[\gamma_5(\not{k}+m)\gamma_5(\not{p}+\not{k}+m)\gamma_\mu(\not{p}'+\not{k}+m)] \;. \tag{11}$$

The function $\Gamma(p'^2, p^2, (p'+k)^2)$ describes the vertex connecting a pion of momentum p' to its constituent particles of momenta $-k$ and $p'+k$. We recall that the wave function of the deuteron, which is a bound state of the two nucleons, has the asymptotic behavior

$$\psi(r) \underset{r\to\infty}{\sim} e^{-\alpha r} \tag{12}$$

and its Fourier transform gives the form factor

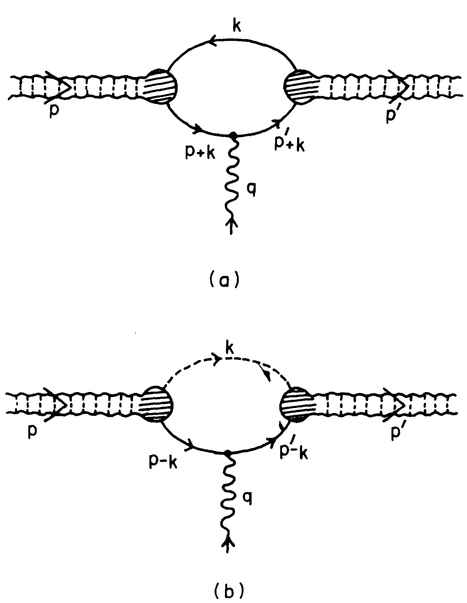

FIG. 3. (a) Vertex diagram for the $\bar{N}N$ bound state of the pion. (b) Vertex diagram for the pion-gluon bound state of the pion.

$$\Gamma(|\vec{q}|) \sim \frac{1}{|\vec{q}|^2+\alpha^2} \;. \tag{13}$$

This can be generalized to have the relativistically invariant form $(-q^2+\alpha^2)^{-1}$. In analogy to the deuteron form factor we can represent

$$\Gamma(m_\pi{}^2, k^2, (k+p')^2) \sim \frac{1}{(k+\frac{1}{2}p')^2-K^2} \tag{14}$$

and a similar function for the $\Gamma(m_\pi{}^2, k^2, (k+p)^2)$, where K is a constant. Evaluating the integrals one obtains the asymptotic behavior

$$|F_\pi(t)| \underset{t\to\infty}{\sim} (\ln t)^2/t^2 \tag{15}$$

which saturates the general asymptotic bound (1) with $c=2$ and $p=3$.

4. Composite pion-gluon model

The asymptotic behavior of the pion form factor can also be calculated by considering the pion as a bound state of a bare pion and a scalar gluon as shown in Fig. 3(b). This type of situation has been studied extensively in the Drell-Lee bound-state model[37] of the nucleon. Following our ansatz for the bound-state vertex described above, the vertex function for large Q^2 is written as

$$\Gamma_\mu \sim \int d^4k (p'+p+2k)_\mu$$

$$\times \frac{1}{[(k-\eta p)^2-K^2][(k-\eta p')-K^2][k^2-m_x{}^2]}$$

$$\times \frac{1}{[(p+k)^2-m_\pi{}^2][(p'+k)^2-m_\pi{}^2]} \;, \tag{16}$$

where $\eta = \mu/m_\pi$, with $\mu = m_x m_\pi/(m_x+m_\pi)$, the reduced mass of the pion-gluon system. After some laborious calculations we obtain the asymptotic behavior

$$|F_\pi(t)| \underset{t\to\infty}{\sim} (\ln t)^2/t^3 \;. \tag{17}$$

This important result is obtained as the extremum of Eq. (1) with $c=2$ and $p=5$. The results derived in this section are summarized in Table I. We see that the most general type of asymptotic behavior of the type $(\ln t)^{2m}/t^n$, with m and n as integers, can be obtained by considering a suitable model of the pion. The parametrization with the help of a parabolic variable suggested in the next section also exhibits such behavior.

IV. FORM-FACTOR PARAMETRIZATION

It is well known that $F_\pi(t)$ is analytic in the cut t plane with a right-hand cut at $4m_\pi{}^2 \leq t \leq \infty$. It has the phase δ_1^1 of the $I=J=1$ $\pi\pi$ partial-wave amplitude (modulo π) in the region $4m_\pi{}^2 \leq t \leq 16m_\pi{}^2$.

TABLE I. Asymptotic behavior of the pion form factor from different models.

	Asymptotic behavior	
Models	Elementary	Composite
1. Scalar gluon	$(\ln t)^2/t$	$(\ln t)^2/t^3$
2. $N\bar{N}$ mode	$\ln t$	$(\ln t)^2/t^2$

The most convenient way of parametrizing the pion form factor is to write an N/D representation such that the numerator contains the zeros and the denominator contains the poles in addition to satisfying the analyticity properties. It has been shown by Bjorken and Drell[38] and by Chew[39] that one can parametrize the pion form factor in the form

$$F_\pi(t) = G(t)D(0)/D(t) , \tag{18}$$

where $D(t)$ is the denominator function for the $J=I=1$ $\pi\pi$ scattering partial-wave amplitude containing the contribution from the elastic cut alone. $G(t)$ is an unknown function which can contain the inelastic branch points. Gounaris and Sakurai[4] have taken the form

$$F_\pi(t) = D(0)/D(t) , \tag{19}$$

where $D(t)$ satisfies an effective-range formula of the Chew-Mandelstam[22] type and is expressed as

$$D(t) = a + bt + h(t) , \tag{20}$$

with

$$h(t) = \frac{2}{\pi}\frac{k^3}{\sqrt{t}}\ln\left[\left(\frac{t}{4m_\pi^2}-1\right)^{1/2} + \left(\frac{t}{4m_\pi^2}\right)^{1/2}\right] - ik^3/\sqrt{t} \tag{21}$$

and

$$k = (\tfrac{1}{4}t - m_\pi^2)^{1/2} . \tag{22}$$

The fact that the D function includes the threshold structure of the p-wave $\pi\pi$ scattering can be verified by observing that $h(t)$ satisfies a twice-subtracted dispersion relation

$$h(t) = \frac{1}{\pi}\left(-m_\pi^2 + \frac{t}{3} - t^2\int_{4m_\pi^2}^{\infty}\frac{(\frac{1}{4}t' - m_\pi^2)^{3/2}dt'}{t'^{5/2}(t'-t)}\right) . \tag{23}$$

Instead of Eq. (20) for the D function we write a general expression

$$D(t) = L(t) + h(t) + m_\pi^2/\pi , \tag{24}$$

where $L(t)$ is a polynomial in t which should be rapidly convergent in the whole t plane without cuts in the first sheet. In the effective-range formula of the GS type the large-t behavior of $D(t)$ comes from $h(t)$ which behaves as $t\ln t$. The presence of this term will vitiate the asymptotic form $(\ln t)^{2m}/t^n$ for the case when $n=1$. Thus only when $L(t)$ is a polynomial not higher than the first degree ($n=1$) can we write in place of $h(t)$ a subtracted form

$$\begin{aligned} H(t) &= h(t) - \frac{2}{\pi}\frac{(\frac{1}{4}t - m^2)^{3/2}}{\sqrt{t}}\ln\left[\left(\frac{t}{4m^2}-1\right)^{1/2} + \left(\frac{t}{4m^2}\right)^{1/2}\right] + i\frac{(\frac{1}{4}t - m^2)^{3/2}}{\sqrt{t}} - \frac{m^2}{\pi} \\ &= \frac{1}{\pi}\left(-m_\pi^2 + t^2\int_{4m^2}^{\infty}\frac{(\frac{1}{4}t' - m^2)^{3/2}}{t'^{5/2}(t'-t)}dt' - t^2\int_{4m_\pi^2}^{\infty}\frac{(\frac{1}{4}t' - m_\pi^2)^{3/2}dt'}{t'^{5/2}(t'-t)}\right) , \end{aligned} \tag{25}$$

m being the nucleon mass. This function does not disturb the correct p-wave threshold property of the D function but makes the form-factor parametrization purposeful by removing the dominant kinematical factor[40] in the asymptotic region for $n=1$. With modifications suggested by (25) (for $n=1$ only) we represent the pion form factor as

$$F_\pi(t) = \frac{G(t)}{D(t)} = \frac{G(t)}{\sum_n a_n t^n + h(t) + m_\pi^2/\pi} , \tag{26}$$

where the numerator function should grow as the powers of $(\ln t)^2$ in the limit of large t and contain the nearest inelastic branch point. A way of parametrizing $G(t)$ has been proposed[25,26,29] in terms of the zeros of the amplitude

$$G(t) = 1 - \lambda t , \tag{27}$$

a form deduced[26] from hard-pion current algebra with $\lambda = 0.042\ \text{GeV}^{-2}$.

To meet our ansatz and ideas regarding extrapolation with an emphasis on large-$|t|$ behavior, $G(t)$ must be a convergent power-series expansion[41,42,43] in a suitably chosen conformally mapped variable where the inelastic cut opens out to form the boundary of the plane of analyticity. To take into account the prominent ρ-ω interference feature we write $G(t)$ as the sum of two terms

$$G(t) = G_\rho(t) + G_\omega(t) , \tag{28}$$

where $G_\rho(t)$ has the desired analyticity and contains the inelastic branch point at $t=16m_\pi^2$. $G_\omega(t)$ contains the inelastic branch point starting from $t=9m_\pi^2$ and a Breit-Wigner form for the ω-resonance denominator. Using the ideas of analytic approximation theory[41,42,43] the cuts are mapped into parabolas such that the region of analyticity coincides with the region of a polynomial expansion. The variable is

$$Z_{t_c}(t)=[\sinh^{-1}(-t/t_c)^{1/2}]^2 \qquad (29)$$

which maps the cut extending from $t=t_c$ to ∞ onto the branches of a parabola in the Z_{t_c} plane with origin as the focus and the entire region of analyticity in the t plane being mapped onto the interior of the parabola. The physical region of the e-π scattering is then mapped onto the right-hand half of the real axis of the Z_{t_c} plane. Further the variable has the very useful and desirable feature that $|Z_{t_c}|$ tends to $(\ln t)^2$ for large $|t|$. Thus the variable is potentially useful for the study of the asymptotic behavior of the pion form factor and it has been employed to good effect for studying the nucleon form factor.[3] In the absence of the ω term, the form factor is

$$F_\pi^\rho(t)=\frac{G_\rho(t)}{D(t)} \qquad (30)$$

and the interference term is

$$F_\pi^\omega(t)=\frac{G_\omega(t)}{D(t)} \qquad (31)$$

such that

$$F_\pi(t)=F_\pi^\rho(t)+F_\pi^\omega(t)\ . \qquad (32)$$

In the ρ region this term should be smooth even though $\mathrm{Re}D(m_\rho^2)=0$. Also we impose the condition that $F_\pi^\omega(0)=G_\omega(0)=0$. Keeping these and the analyticity properties in mind we write

$$G_\omega(t)=\frac{f_1Z_{3\pi}(t)+f_2Z_{4\pi}(t)}{m_\omega^2-t-i\Gamma_\omega m_\omega k_{3\pi}^3/k_\omega^3}\ , \qquad (33)$$

where $Z_{3\pi}$ is the parabolic variable with $t_c=(3m_\pi)^2$, $Z_{4\pi}$ is the variable with $t_c=16m_\pi^2$, f_1 and f_2 are unknown parameters, and

$$k_{3\pi}=(\tfrac{1}{9}t-m_\pi^2)^{1/2}\ ,$$
$$k_\omega=(\tfrac{1}{9}m_\omega^2-m_\pi^2)^{1/2}\ . \qquad (34)$$

A denominator of the modified Breit-Wigner form similar to that used in (33) has also been used by Roos and Pišút.[40] We approximate $G_\rho(t)=\sum_n g_n Z_{4\pi}{}^n$ and the polynomial in $D(t)$ as $L(t)=\sum_n a_n t^n$. These forms of the G function and the L function were put into (32) and the world data on the pion form factor were used to search for the best set of parameters. The mass and the width of the ω meson were taken from tables and the amplitude and the "phase" of $F_\pi^\omega(t)$ was determined from the fit to the form-factor data. The excellent ω peak with a smooth tail at the ρ mass which almost approaches zero after a few MeV away from the ω mass shows that our choice for the $G_\omega(t)$ is correct.

V. RESULTS AND DISCUSSION

We have collected 65 data points on the pion form factor from the literature.[6-17] Out of these 22 are in the spacelike region in the interval $0\geq t\geq -1.18\ \mathrm{GeV}^2$ and the rest are in timelike region in the interval $0.33\leq t\leq 4.41\ \mathrm{GeV}^2$. The data reported in Ref. 3 by Augustin *et al.* show a wide discrepancy from the measurements of the other experimental groups in the same range of t. As has been pointed out by Benaksas *et al.*[6] one of the reasons for such a discrepancy is the overestimation of the nuclear absorption correction (namely 22%). This systematical error decreases the cross section by 8%. Thus we have used the form-factor data of Ref. 3 reduced by 8%. The data for higher values of t are from the $\mu\pi$ and the BCF groups[12] and are much higher than given by a dipole fit. This has led to the speculation[12] that the form factor vanishes much more slowly than $1/t$ as $|t|\to\infty$.

Total χ^2 values were obtained for different values of m and n. In Table II we give nine such fits with $m=0, 1, 2$ and $n=1, 2, 3$ corresponding to $c=0, 2, 4$ and $p=1, 3, 5$ in the inequality (1). The mass and the width of the ω meson were taken from the Particle Data Tables to be $m_\omega=784$ MeV and $\Gamma_\omega=10$ MeV. We find that our formula gives the best fit for the case $c\geq 0$ and $p=5$, which implies a $1/t^3$ behavior for the pion form factor. For this fit, we have $(4+2)$ unknown parameters [with the normalization[44] $F_\pi(0)=1$], the number of degrees of freedom (NDF) is 59, giving a χ^2/NDF

TABLE II. Total χ^2 values for different c and p values where $|F_\pi(t)| \underset{t\to\infty}{\sim} (\ln t)^c/t^{(p+1)/2}$ with the formula (32) and for 65 data points. The first quantity inside the parenthesis represents the contribution to the χ^2 from the timelike data whereas the second term is the contribution from the spacelike data alone.

p \ c	0	2	4
1	156.34 (110.07+46.27)	95.13 (46.01+49.12)	86.81 (45.11+41.70)
3	87.98 (52.47+35.51)	83.40 (46.30+37.10)	83.38 (45.81+37.57)
5	79.36 (44.77+34.59)	79.35 (45.08+34.27)	79.34 (44.87+34.47)

$=1.32$. This is definitely a good fit in view of the uncertainties and discrepancies discussed above. The first term inside the bracket in Table II shows the χ^2 value for the timelike data points whereas the second one in it represents the χ^2 for the spacelike ones. We are aware that the fits giving the $1/t^2$ or $(\ln t)^2/t^2$ behavior cannot be ignored as χ^2/NDF is not very much different (1.4) from the above-mentioned value. Although the asymptotic behavior of the type $(\ln t)^4/t$ seems to be likely ($\chi^2=86.8$), the curve passes far below the timelike data available at large t and behaves like a ρ tail. One of the important features in our fit is that we can still account for the large $+t$ data even with asymptotic behaviors faster than the single-pole fit. For definiteness we write the formula and the parameters with errors for the $c=0$, $p=5$ fit

$$F_\pi^\rho(t)=\frac{a_0}{a_0+a_1t+a_2t^2+a_3t^3+h(t)+m_\pi{}^2/\pi}\,, \tag{35}$$

$$F_\pi^\omega(t)=\frac{f_1Z_{3\pi}+f_2Z_{4\pi}}{(a_0+a_1t+a_2t^2+a_3t^3+h(t)+m_\pi{}^2/\pi)(m_\omega{}^2-t-i\Gamma_\omega m_\omega k_{3\pi}{}^3/k_\omega{}^3)}\,, \tag{36}$$

with

$$\begin{aligned}
a_0&=(0.365\pm0.006)\,,\\
a_1&=-(0.785\pm0.012)\ \mathrm{GeV}^{-2}\,,\\
a_2&=(0.109\pm0.009)\ \mathrm{GeV}^{-4}\,,\\
a_3&=-(0.010\pm0.003)\ \mathrm{GeV}^{-6}\,,\\
f_1&=(0.042\pm0.007)\times10^{-2}\ \mathrm{GeV}^2\,,\\
f_2&=-(0.057\pm0.009)\times10^{-2}\ \mathrm{GeV}^2\,,
\end{aligned} \tag{37}$$

where the errors have been computed by the usual error-matrix method. The fit to the form-factor data with the above formula is shown in Fig. 4. The kink due to the ρ-ω interference is sufficiently prominent. Figure 5 represents the real and the imaginary parts of the form factor. It is worth mentioning that the form taken for parametrization saturates the bound of Eq. (1) and to study the possible asymptotic behavior of the form $(\ln t)^{2m}/t^n$ for $F_\pi(t)$ there are $m+n+1$ free parameters for

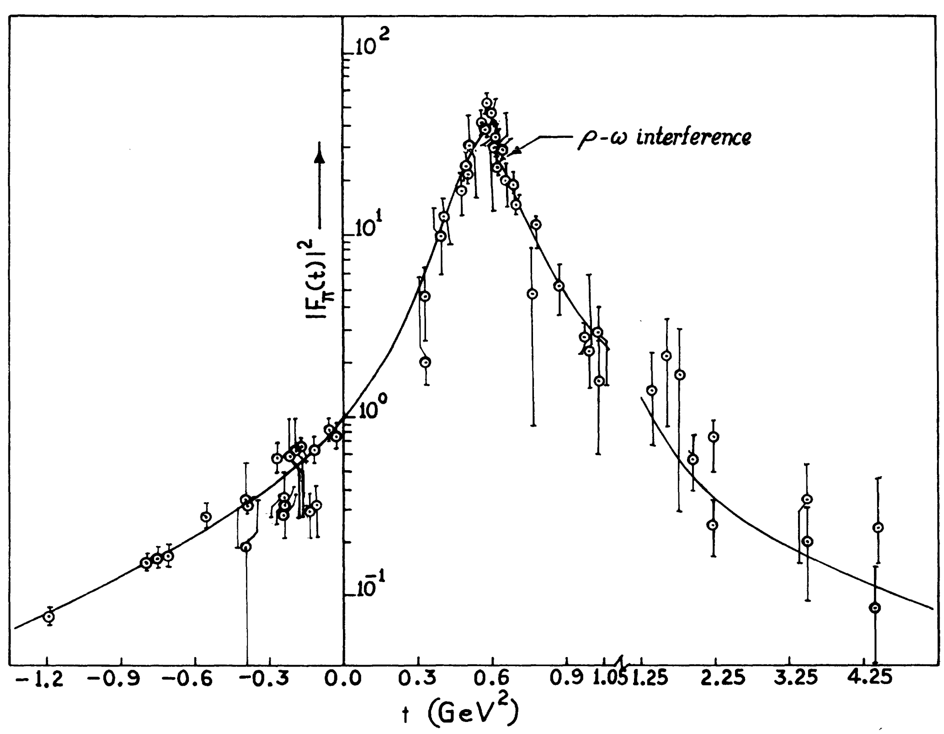

FIG. 4. Fit to the pion form-factor data. The solid line shows the fit for the case $c=0$, $p=5$. The kink appears due to the ρ-ω interference. The data points were taken from Refs. 6 to 17.

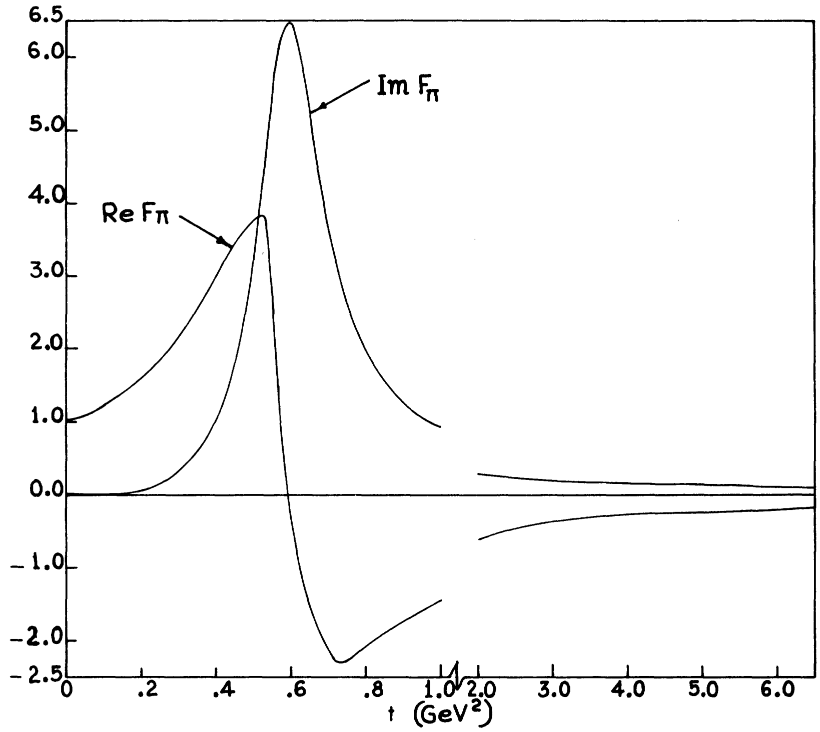

FIG. 5. Real and imaginary parts of the pion form factor corresponding to the fit of Fig. 4.

$F_\pi^\rho(t)$ and in addition $F_\pi^\omega(t)$ introduces two more parameters.[44]

It is of interest to compare our fit with the fits obtained in earlier analyses. The χ^2 values with different fits are given in Table III. One immediately finds that our formula is a superior one in both the timelike and the spacelike regions taken separately and together.

Further to illustrate the ρ and ω contributions (35) and (36) more clearly we have plotted $|F_\pi^\rho(t)|^2$ and $|F_\pi^\omega(t)|^2$ in Fig. 6. We find that the ρ resonance occurs at the mass 765 MeV and has the width $\Gamma_\rho = 144.6$ MeV. Since the parameters in this case are determined from a fit consistent with the analyticity and the experimental data they may be taken as more reliable.

From our fit the ρ-ω interference amplitude comes out to be[45,46]

$$\xi = (0.009^{+0.006}_{-0.001}) \exp[i(101^\circ {}^{+28^\circ}_{-39^\circ})] \,. \qquad (38)$$

Table IV gives the comparison of our result with those obtained from photoproduction and colliding-beam data. It is to be noted that our phase is t-dependent.[45] The value of the pion's charge radius obtained from the fit is

$$r_\pi = (0.709 \pm 0.011) \text{ fm} \,, \qquad (39)$$

as compared to the vector-dominance value $r_{\pi\text{VDM}} = 0.63$ fm. Thus we obtain $r_\pi^2/r_{\pi\text{VDM}}^2 = 1.25$. Recent Serpukhov-UCLA[47] measurements on $e\pi$ scattering suggest that $r_\pi^2/r_{\pi\text{VDM}}^2 = 2.02 \pm 0.58$. All the formulas on the pion form factor suggested so far

TABLE III. Comparison of our formula with other phenomenological fits.

References	Total χ^2	χ^2 from the timelike data	χ^2 from the spacelike data	χ^2/NDF
Gounaris and Sakurai (Ref. 4)	398.70	237.80	160.90	6.1
Benaksas *et al.* (Ref. 6)	182.70	56.10	126.60	3.0
This analysis	79.36	34.59	44.77	1.32

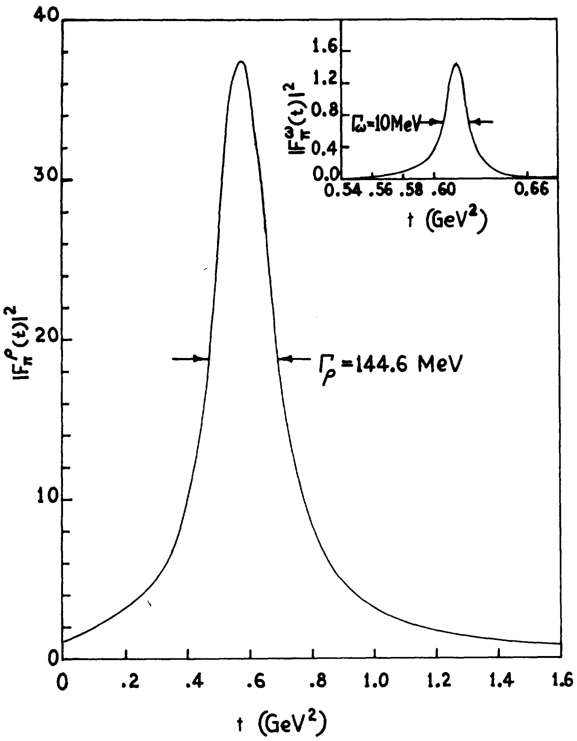

FIG. 6. Contributions of the ρ and ω terms to the pion form factor corresponding to the fit of Fig. 4 and the formulas (35), (36), and (37).

gives a radius nearly equal to the vector-meson-dominance value. In fact it has been demonstrated by Levin, Mathur, and Okubo[48] that the pion's charge radius must be smaller than the naive vector-dominance value unless a peaked phase[49] δ_1^1 is assumed between $4m_\pi^2 \leq t \leq 16m_\pi^2$ which is contradictory to the result obtained with the Chew-Low extrapolation method. Our formula does not suffer from the flaw of a peaked phase but gives a ratio 25% higher than the vector-dominance value.

TABLE IV. Results on the ρ-ω interference amplitude obtained by different experimental groups and this analysis. We follow the notations of Lemke and Sachs.[45]

Parameter	Measured value	References
$\xi(\gamma, 2\pi)$	$(0.009\,97 \pm 0.008)e^{i(104^\circ \pm 5^\circ)}$	Ref. 46(d)
	$(0.014 \pm 0.016)e^{i(96^\circ \pm 15^\circ)}$	Ref. 46(e)
	$(0.0106 \pm 0.0012)e^{i(96^\circ \pm 15^\circ)}$	Ref. 46(b)
$\xi(\gamma, 2e)$	$(0.106 \pm 0.022)e^{i(41^\circ \pm 20^\circ)}$	Ref. 46(a)
	$(0.142^{+0.039}_{-0.032})\exp[i(100^\circ{}^{+38^\circ}_{-30^\circ})]$	Ref. 46(c)
$\xi(2e, 2\pi)$	$(0.016 \pm 0.004)e^{i(95^\circ \pm 15^\circ)}$	Ref. 9
	$(0.010 \pm 0.003)e^{i(85.7^\circ \pm 15.3^\circ)}$	Ref. 6,10
	$(0.0094^{+0.006}_{-0.001})\exp[i(101^\circ{}^{+28^\circ}_{-39^\circ})]$	This analysis

VI. CONCLUSION

As mentioned earlier, a fall-off of the form factor faster than dipole cannot be ruled out. A more positive statement is possible only if unambiguous data are available in the spacelike region at a large values of $|t|$. The pion being a very light object, the *asymptotic* region is likely to set in at a much smaller value of $|t|$ $(t<0)$ than for the proton. We suggest that experiments be performed to find $F_\pi(t)$ at least in the range $-4m_\pi^2 \leq t < 0$.

We are also aware that the main body of data lies in the ρ-ω interference region. This has introduced two more parameters into our theory, but has not affected our main aim of suggesting a way of determining the asymptotic behavior and showing that this may yield meaningful results.

ACKNOWLEDGMENT

One of us (M. K. P.) wishes to thank Dr. D. J. Broadhurst for some useful communication. The calculations were performed by the IBM1130 Computer of the Utkal University and we are thankful to the personnel of the computer for all possible help in computation.

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