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Implications of parton-model concepts for large-transverse-momentum production of hadrons

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We study the implications of the parton-parton scattering picture for large-transverse-momentum hadronic events. Topics discussed are single and double inclusive cross sections, two-jet cross sections, multiplicities, quantum numbers, and contributions to σ_{tot} and $d\sigma/dt|_{el}$.

INTRODUCTION

From a theoretical standpoint an essential difficulty one must face in the development of a complete treatment of strong interaction physics is the composite nature of hadrons. Thus one is barred from directly applying the methods of quantum field theory which work so well in quantum electrodynamics. However, recent, and by now well known, experimental results for electromagnetic and weak inclusive processes[1] seem to have revived the possibility that elementary fields may play a role in hadron physics. To explain these new data the "parton" picture of hadrons has been developed.[2-4] One treats the hadron as a bound system of elementary particles (partons) which are assumed to be described by elementary fields and which seem to behave as if they were "almost free" in certain kinematic regions. Thus the relatively large size and scaling behavior of these semileptonic inclusive cross sections can be understood in terms of absorption of the weak currents by a structureless spin-$\frac{1}{2}$ parton, which is never far from the mass shell.

It is, of course, true that this does not yield directly a simple theoretical picture. In fact no complete field theory of partons yet exists. Moreover, there are some indications that some of the partons should be quarks and hence are not to be observed as asymptotic states, at least at present energies. There are also alternative explanations of the scaling results which avoid the explicit introduction of elementary fields.[5] However, the possibility of the validity of parton ideas is so appealing, and the present state of theoretical understanding is so unclear, that it is extremely important to search for other indications of the parton's existence.

In the following paper we shall discuss the possibilities of directly observing parton-parton scattering effects in purely hadronic reactions. Since the appropriate experimental data are appearing at this time, it is important to have clearly in mind what one can expect to observe as the result of the "existence" of partons.[6] Particularly, we wish to clarify what assumptions are necessary to get which results.[7]

The paper will proceed as follows: In Sec. I we review the parton model and its general role in hadron-hadron scattering; in Sec. II we discuss contributions to single-particle inclusive cross sections; in Sec. III we discuss the expected structure of doubly inclusive processes and the related concept of a two-jet cross section; Sec. IV is concerned with the question of multiplicities; Sec. V involves a discussion of what can be expected with respect to quantum numbers in simple models where the partons are quarks; in Sec. VI we discuss the questions of elastic and total cross sections; and finally, in the conclusion, we shall briefly summarize and present a few general concluding remarks. The Appendix reviews how the various cross section formulas are obtained. The reader who is not interested in the details may proceed directly from Sec. I to the conclusion.

I. PARTON-PARTON SCATTERING AND THE PARTON MODEL

An exciting possibility which arises when one considers the role of partons in hadronic reactions is the opportunity to observe directly parton-parton interactions. In the semileptonic case one measures directly the properties of, for example, electron-single-parton scattering and only indirectly the properties of parton-parton dynamics, i.e., that they are consistent with scaling at present energies. This is also to some extent true of parton interchange descriptions of hadronic reactions when one derives relations between hadronic cross sections and electromagnetic form factors.[8] We shall here concentrate on studying parton-parton scattering directly, bearing in mind the possibility that the parton concept may be valid but yet direct parton-parton scattering may never play a dominant role in hadronic reactions.

The essence of a simple treatment of parton-parton effects in hadron-hadron scattering is the assumption that under some kinematic conditions two partons from different hadrons and with large rapidity difference can suffer a "hard" scattering process,[4,9,10] which is effectively both elastic and incoherent. By this we mean that the scattering occurs as if independent from the other partons present in the original hadrons and that the basic process is just two partons go to two partons. The final partons subsequently evolve into hadrons but in a fashion which is largely independent of the other partons except for the constraint that only hadron quantum numbers be observed in the final state. As in deep-inelastic lepton-induced processes we shall assume that it is valid to deal directly with probabilities instead of amplitudes. The remaining and presumably dominant part of strong interactions is assumed to involve interactions between partons of small rapidity difference, i.e., the "wee" partons.[2,4] This is a complicated question which we shall not treat here. We shall concentrate on the small perturbation due to "hard" collisions of partons which offer some possibility of simple description.

With the above assumptions, the kinematics for "hard" parton-parton collisions are simple and many features of the scattering process are determined by these assumptions alone. Further assumptions about the specific form of the parton-parton scattering cross section will lead to more specific predictions. Within this framework the other information required in order to calculate cross sections is presumed to be obtainable, at least approximately, from lepton-hadron processes. This includes the distribution of partons within the hadrons and the distribution of hadrons in the decay of the parton. These distributions are clearly outside the realm of what is presently calculable from first principles. The approximation alluded to above results from the possibility of initial- and final-state interactions present in hadron-hadron and not in lepton-hadron scattering which may affect the parton distribution functions. However, within the general concept here being discussed, where the bulk of strong interactions involves only partons which are nearby each other in phase space and interactions involving large rapidity gaps are rare, the distributions of the hard partons should be unchanged from the lepton case. In fact we are essentially doing perturbation theory in hard effects, so that we consider first-order scattering between zeroth-order distributions of the hard partons.

As has already been pointed out in the literature,[8-10] it seems that the best candidate for where the effects mentioned above may be present is in the inclusive production of hadrons at large transverse momentum.[11,12] This hope is raised by the idea that some sort of impulse approximation will justify the above assumptions about isolated parton-parton scattering when there is a large transfer of momentum. In the $ep \to e + X$ case, one considered the electron as scattering at a large angle off an individual parton. Since, to order α, the scattering occurs via the exchange of a single large-q^2 photon, it is easy to imagine that in the limit $q^2 \to \infty$, $\nu \to \infty$ the scattering occurs in a time which is short compared to the characteristic time of the strong interactions. This serves to motivate treating the parton as free momentarily and leads to the usual scaling results. For parton-parton scattering the situation is much more complicated. If large-angle scattering occurs via the exchange of a single elementary gluon, either vector or scalar, the impulse approximation is again justifiable. This is in fact the situation which is best studied. However, one finds, as expected, that to fit the magnitude of the observed cross sections the effective gluon-parton coupling α_{eff} should be of order unity. One is now faced with the problem of including multiple gluon exchange which is of the same order as single exchange for large q^2 when $\alpha_{\text{eff}} \approx 1$. Keeping a few more gluons will in fact have little effect on the basic form of the cross section, but one is somewhat embarrassed about the motivation for the impulse approximation.[13] In the limit that we eikonalize the isolated parton-parton cross section with gluon exchange,[14] unwanted form factors appear, but by now the assumption of isolated elastic parton-parton scattering is certainly unjustified. This last problem of the appearance of form factors for the parton is, of course, also present in the case of lepton-induced reactions.[15] We shall return to this question briefly in the conclusion.

In summary, it is important to keep in mind that at the present stage of our understanding of parton models the picture being discussed here is somewhat more difficult to motivate than the usual application to lepton-induced processes which is itself somewhat poorly defined. However, the extension to purely hadronic reactions does seem reasonable in its general form, and the successes in the lepton case make a comparison with the new hadronic data of extreme interest. In what follows we shall simply assume that the impulse approximation, and hence, the picture discussed above is appropriate.

Before proceeding to the details of the parton-model calculations we shall briefly preview some of the results so that the reader, if so inclined, may proceed directly to the conclusion. We shall see that the essential signal for a parton-parton scattering event is a two-jet structure for the large-transverse-momentum hadrons in the final state. In the c.m. system (ISR) we also expect two jets of hadrons along the beam lines, whereas in the lab system (NAL) we expect one jet along the beam and a cloud of hadrons at rest. Henceforth, when we refer to two-jet structure, we mean for the large-transverse-momentum hadrons. If one observes, for example, a jet on one side of the beam but only a poorly defined "fan" of particles on the other, the event is at best one parton scattering off several other partons and at worst bears no relation to parton effects. Secondly, the sort of events of interest here correspond to the two jets forming a common plane with the incident beam direction to a very good accuracy (± 300 MeV/c). The jets need not, however, be "back to back" in the hadron c.m. system. If there is no indication of such structure in the data (the inverse seems true at present), one may simply ignore what follows.

Another result which should be straightforward to check experimentally but which involves more specific assumptions is the form of the single-particle inclusive cross section. The form

$$\frac{d\sigma}{d^3p/E} \sim \frac{1}{s^2} f(p_\perp/\sqrt{s}\,,\theta) \quad (s\to\infty,\ p_\perp/\sqrt{s}\ \text{fixed}) \tag{1.1}$$

results from the assumption of single-vector-gluon exchange between spin-$\frac{1}{2}$ partons. This specific choice reflects current theoretical bias plus the twin virtues of simplicity and consistency with the assumed impulse approximation. The reader is reminded that it is not a necessary result in the general parton picture, but we use it as an illustrative example of typical results expected in parton models.

A simple way to remember the form in Eq. (1.1) is to note that it would also result from assuming that either parton-parton scattering or the hadronic process itself should exhibit true scaling behavior. Then Eq. (1.1) follows from dimensional analysis (in the vector-gluon case masses are ignored and the couplings are dimensionless, which would also be true for scalar gluons).

We also find a strong correlation between the $x_\perp = 2p_\perp/\sqrt{s}$ of the fastest particle in one large $p_\perp$ jet and the mean multiplicity in the other jet. The multiplicity in the same jet is found to be generally much smaller. By identifying partons with quarks we find that in pp reactions the ratios π^+/π^- and K^+/K^0 should be larger than 2 for $x_\perp \gtrsim 0.5$, whereas p/π and $\bar{K}/K$ should be small. Finally, we discuss the possibility of a component of σ_{tot} which grows as $(\ln s)^2$, at least for present energies.

The reader who has not been motivated to seek further details may now proceed to the Conclusion.

II. CONTRIBUTIONS TO SINGLE-PARTICLE INCLUSIVE REACTIONS

In order to illustrate how the parton picture introduced above can be applied to calculations of hadronic cross sections we will first study the familiar example of the single-particle inclusive cross section. This will suffice to introduce and define all the various quantities of interest and even allow a comparison with data. However, as we shall see in the following sections, the picture is actually most naturally suited to the study of two-particle (or two-jet) inclusive cross sections.

To proceed we define the usual distribution functions. For simplicity we assume, for now, the existence of only one variety of parton (and one type of gluon) and two types of hadrons, baryons, and mesons (pions). This eliminates various summations which can be reinstated in an obvious way as indicated in Sec. VI. The probability that a hadron contains a parton with fraction x of its momentum is given by

$$P(x)dx = F(x)\frac{dx}{x}\,, \tag{2.1}$$

where we assume that $F(x)$ for spin-$\frac{1}{2}$ partons in the proton can be inferred from inelastic e-p data. Note that in principle Eq. (2.1) should also contain a distribution in the transverse direction $[(dx/x)d^2k_\perp F(x,k_\perp^2)]$. We have, for simplicity, approximated this distribution as a δ function and done the $k_\perp^2$ integration. Including a more realistic but still narrow distribution (e. g., $e^{-6k_\perp}$ as suggested by π distributions) would serve only to smear our results slightly. Our conclusions would remain essentially the same.[16] Our equations would, of course, be more complicated but in an obvious fashion. We shall maintain this simplification throughout.

The corresponding distribution for a parton materializing into a cloud of hadrons containing one with momentum fraction y is given by

$$P(y)dy=\frac{G(y)}{y}\,dy\,. \tag{2.2}$$

Note that we have already assumed a dy/y distribution for the soft hadrons [if $G(0)\neq 0$] as is characteristic of parton models. The behavior of $G(y)$ for y approaching unity is not obvious, although in principle it can be measured in the process $lp\to l+h+X$, where h is in the so-called current fragmentation region. With the idea in mind that $G(y)$ can eventually be measured elsewhere and then applied to hadronic reactions, we shall make an assumption which leads to a definite form for $G(y)$ which can then be tested. We assume that the behavior near "threshold," $y\approx 1$, of the probability that a parton is one hard hadron (+wee hadrons) is very similar to the behavior of the probability that the same hadron is one hard parton (+wee partons).[2,4] We shall in fact make the assumption that $G(y)\approx F(y)$ for all nonwee y so that, for example, $G_\pi(y)\sim F_\pi(y)\sim 1-y$, whereas for a baryon (proton) $G_B(y)\sim F_B(y)\sim \nu W_2(y)$. Such an assumption is not unreasonable since we expect the same configuration to give the same behavior whether we are projecting from partons onto hadrons or hadrons onto partons. This simple relation between F and G seems to be borne out in specific model calculations.[17]

With this assumption we have specified the behavior for finite y which is of interest for the calculations of cross sections; however, we have left unresolved the question of the behavior as $y\to 0$, which will be important when we discuss multiplicities.

Note that as a result of the above assumption baryons will be damped more rapidily with increasing $x_\perp$ than mesons. This is quite different from the results of the first paper in Ref. 12, for example, where it is suggested that large-$p_\perp$ baryon production may be relatively larger, compared to meson production, than at small $p_\perp$. Heavy-meson production is presumably not damped here.

With these distributions one can now define the invariant cross section for the process $a+b\to c+X$ as depicted in Fig. 1. Deferring the details to the Appendix, the result is

$$E_c\frac{d\sigma}{d^3p_c}\sim\frac{4}{\pi x_\perp^{\,2}}\int\!\!\int dx_1dx_2F(x_1)F(x_2)G\left[\frac{x_\perp(1+\eta)}{2x_1\tan(\frac{1}{2}\theta)}\right]\frac{\eta}{(1+\eta)^2}\frac{d\sigma}{d\hat{t}}(\hat{s},\hat{t})\quad (s\to\infty,\ p_\perp/\sqrt{s},\ \theta\text{ fixed}), \tag{2.3}$$

where $\eta=(x_1/x_2)\tan^2(\frac{1}{2}\theta)$, $x_\perp=2p_{\perp c}/\sqrt{s}$, θ is the c.m. angle of the particle c, and $d\sigma/d\hat{t}$ is the invariant differential elastic cross section for parton-parton scattering. The appropriate parton variables in terms of the hadron variables are

$$\hat{s}=x_1x_2s\,,$$
$$\hat{t}=-x_1x_2s\eta/(1+\eta)\,,$$
$$\hat{u}=-x_1x_2s/(1+\eta)\,,$$

where the parton masses have been ignored, i.e., $\hat{s}+\hat{t}+\hat{u}\approx 0$.

It is useful to recall at this point that the simple form of Eq. (2.3) results directly from our assumptions about the independent, incoherent nature of the parton-parton interaction. The general factorization and kinematic properties are clearly independent of a specific form for $d\sigma/d\hat{t}$. Of course interesting cross sections result only if $d\sigma/d\hat{t}$ is not a rapidly falling function of $\hat{s}$ and $\hat{t}$. Furthermore, simple scaling laws $[E\,d\sigma/d^3p\sim s^{-n}f(p_\perp/\sqrt{s})]$ result from simple forms for $d\sigma/d\hat{t}$ [i.e., $d\sigma/d\hat{t}\sim\hat{s}^{-n}f(x_1,x_2,\theta)$], not solely from the parton-model assumptions. Equation (2.3) also suggests how to define our impulse approximation. We may choose to assume that Eq. (2.3) is valid for $|t/s|$ fixed as $s\to\infty$, although the early onset of scaling at SLAC might suggest validity for $|t|>|t_0|$, where $|t_0|$ is possibly a few GeV^2.

In Fig. 2 we give the form of $d\sigma/d\hat{t}$ for various interesting choices of partons and gluons where $\alpha_{\rm eff}$ is the effective parton-gluon coupling constant. For comparable distributions (and even those favoring gluon-gluon effects) and couplings, the largest cross-section results for spin-$\frac{1}{2}$ partons with

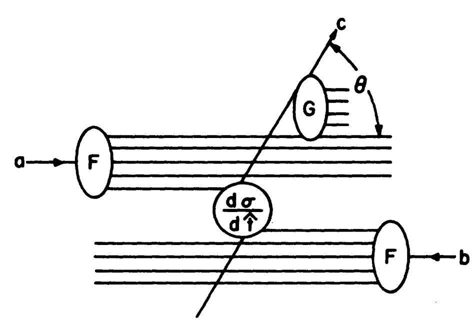

FIG. 1. Pictorial representation of parton-parton scattering contribution to the single-particle inclusive cross section.

vector-gluon exchange. This is illustrated in Fig. 3. Since we are primarily interested in proton-proton processes, we take the F distribution for spin-$\frac{1}{2}$ partons in protons from data on νW_2, i.e.,

$$F_p(x) \propto 3.8(1-x)^3 + 14.9(1-x)^4 = 17.7(1-x)^5 .$$

We also indicate the result for $F \propto (1-x)^3$. The $G(y)$ was taken proportional to $1-y$ as assumed appropriate for the production of a π.[10] The distribution forms for the gluons are indicated. In each case the distributions were normalized so that the hadron's momentum is evenly divided between partons and gluons. Note that the assumed relationship between $F_p(x)$ and $\nu W_2(x)$ does not include the normalization unless assumptions are made about the partons' charges, which are included in νW_2 (in fact even the x dependence is not obvious unless the various partons appear symmetrically). Evidently, the various results are quite similar except for magnitude. As mentioned above, we shall henceforth use the largest for calculations, i.e., vector exchange on spin-$\frac{1}{2}$ partons. Note that we have illustrated both the case where the $\hat{t}$ and $\hat{u}$ channel exchanges are added incoherently and added coherently. The major effect occurs when $\theta \approx \frac{1}{2}\pi$ and $\langle \eta \rangle \sim 1$ where the coherent case is larger by approximately a factor 18/10. In the calculations that follow we have chosen to use the incoherent cross sections although the choice was treated as arbitrary. We shall return to this point again in Sec. V.

Referring again to Fig. 3, we point out that, although for the present purposes gluon-gluon scattering seems negligible, even with fairly generous distributions, we do not wish to rule out the possibility that gluons may play an important role in some region of phase space.

Although we shall concentrate on gluon exchange between spin-$\frac{1}{2}$ partons, it is useful to keep in mind two other types of interactions which lead to quite different forms for the inclusive cross section. First is the inclusion of a four-Fermi contact-type interaction.[18] This leads to $d\sigma/d\hat{t} \sim$ const (which would be indistinguishable from a heavy-gluon theory at finite $\hat{t}$, $M_g{}^2 \gg |\hat{t}| \gg |t_0|$), and

$$\frac{d\sigma}{d^3p/E} \sim s^0 f(p_\perp/\sqrt{s}) .$$

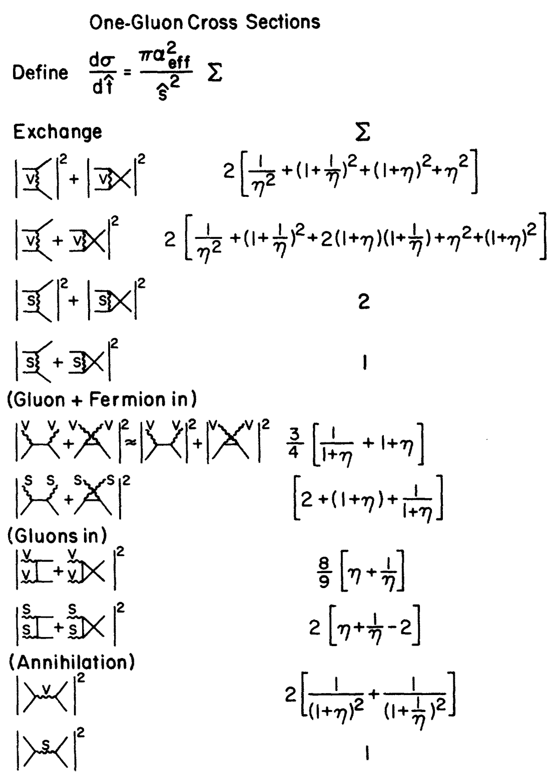

FIG. 2. Form of $d\sigma/d\hat{t}$ for various single exchanges. α_{eff} is the effective parton-gluon coupling constant.

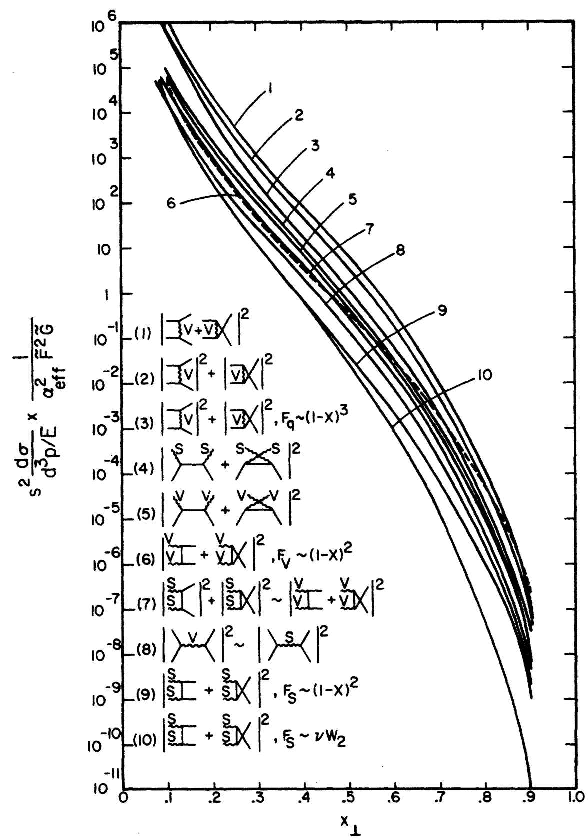

FIG. 3. Comparison of the relative sizes of contributions to the single-particle inclusive cross section for various choices of partons and exchanges (see Table I). Unless otherwise noted, $F \sim \nu W_2$, $G \sim (1-x)$.

Also one might consider a scalar-scalar event leading to

$$\frac{d\sigma}{d^3p/E} \sim s^{-4} f(p_\perp/\sqrt{s})$$

if scalar partons are a predominant constituent.

In this case one arrives at results similar to the parton interchange picture.[8]

The most striking feature of Eq. (2.3) with single-gluon exchange is, in fact, the $1/s^2$ behavior at fixed $x_\perp = 2p/\sqrt{s}$, e.g., with a vector gluon:

$$E_c \frac{d\sigma}{d^3p_c} \sim \frac{1}{s^2}\left(\frac{8\alpha_{\rm eff}{}^2}{x_\perp{}^2}\right)\int\int \frac{dx_1}{x_1{}^2}\frac{dx_2{}^2}{x_2{}^2} F(x_1)F(x_2)G\left(\frac{x_\perp(1+\eta)}{2x_1\tan(\frac{1}{2}\theta)}\right)\frac{\eta}{(1+\eta)^2}\left[(1+\eta)^2+\eta^2+1/\eta^2+(1+1/\eta)^2\right]. \qquad (2.4)$$

This behavior obtains more generally whenever the impulse approximation is appropriate and when the theory has dimensionless couplings as in scalar or vector gluons on fermion partons. In this case $d\sigma/d\hat{t} \sim (1/s^2) f(x_1, x_2, \theta)$ is guaranteed by dimensional analysis in the limit in which the masses can be ignored. The existence of this type of behavior in the data, although not strictly mandatory to establish the validity of parton models in general, is a clean check of the type of simple parton-gluon model we have in mind here. Although the forms mentioned in the previous paragraph are interesting, we do not consider them to be in the "mainstream" of parton ideas.

Before proceeding to compare the one-gluon-exchange formula with experiment let us briefly review what it is that we can hope to learn. By assumption we have taken $F(x) \propto \nu W_2(x)$ and $G_\pi \propto 1-x$. This fixes everything but the normalization $\bar{F} = \int dx\, F(x)$ and $\bar{G} = \int dx\, G(x)$, which give the total fraction of the proton momentum carried by the parton in question and the fraction of the parton momentum carried by the hadron studied. As mentioned, these numbers are not directly available from lepton induced reactions due to the appearance of parton charges. Hence, by comparing the gluon formula with data, we obtain a value for $\bar{F}^2\bar{G}\alpha_{\rm eff}{}^2$.[19] Comparison with the recent $pp \to \pi^0 + X$ ISR data[20] at $2p_\perp/\sqrt{s} \sim 0.2$ yields a value $\bar{F}^2\bar{G}\alpha_{\rm eff}{}^2 \approx 3\times 10^{-3}$. To arrive at an estimate of $\alpha_{\rm eff}{}^2$ we take $\bar{F} \sim \frac{1}{2}$ (momentum of hadron equally divided between spin-$\frac{1}{2}$ partons and gluons) and $\bar{G}_{\pi^0} \sim \frac{1}{6}$ as if all varieties of π's were equally

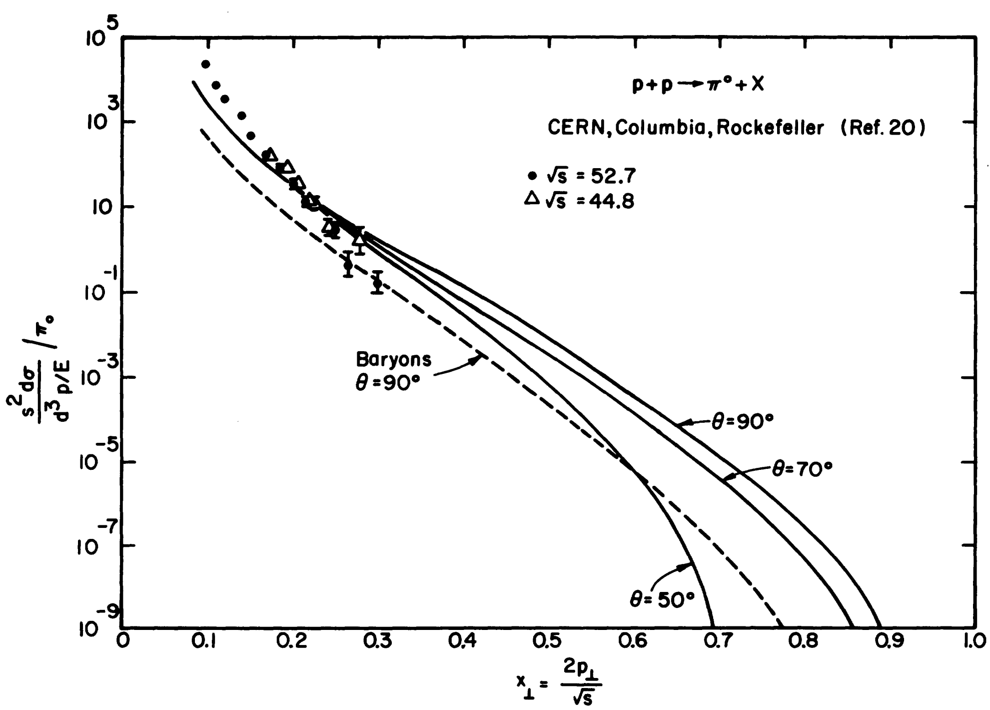

FIG. 4. One-vector-gluon-exchange form of single-particle inclusive cross section with $\alpha_{\rm eff} \approx 0.3$, $\bar{F} = \bar{G} = \frac{1}{2}$ compared to data of Ref. 20 for π^0 production (solid line). Baryon production is also indicated (dashed line).

likely and the primary final states were divided as $(p+\bar{p}):K:\pi \sim 2:3:5$, as suggested by recent data.[21] This gives $\alpha_{\text{eff}} \sim (0.072)^{1/2} \sim 0.3$. This general magnitude for α_{eff} seems to be an inescapable result for vector-gluon exchange (other exchanges give larger values), but the specific value depends clearly on model-dependent assumptions.

In Fig. 4 we see that the general shape of the one-gluon cross section is in quite reasonable agreement with the data and is at least encouraging.[22] From the indicated variations of the curves with θ, it is clear that a steeper curve results when data are summed over a finite solid angle. This will presumably be an important effect when one truly attempts to fit the data. The present data, in fact, seem to require even slightly steeper behavior than is achievable this way, indicating a possible need for more sophisticated distribution functions.

Also shown in Fig. 4 is the type of cross section which we would predict for baryon production at large $p_\perp$. We used $G_B(y) \propto \nu W_2(y)$ and took $\tilde{G}_B = \tilde{G}_{\pi^0}$ (the numbers above give $\tilde{G}_p = \frac{6}{5}\tilde{G}_{\pi^0}$). The interesting result is that, as expected for our assumptions, the ratio of π production to inelastic baryon production should increase as $x_\perp \to 1$ following from a similar behavior for $G_\pi(y)/G_B(y)$ as $y \to 1$. We have ignored all complications due to decays of the originally produced hadrons, etc.

Using this specific example as a model it is interesting to calculate what parton parameters are probed for various kinematic regions. In Figs. 5 and 6 we show the mean values of x ($\langle x_1 \rangle \approx \langle x_2 \rangle \approx \langle x \rangle$) and y ($y = p_{\text{hadron}}/p_{\text{parton}}$) measured

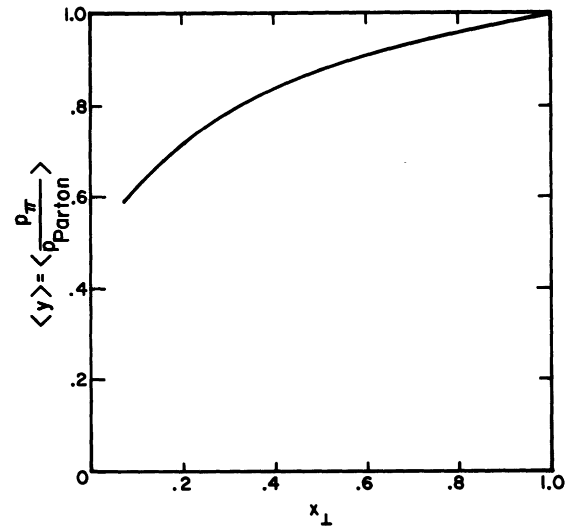

FIG. 6. Average fraction y of the scattered parton's transverse momentum carried by the observed π as a function of the $x_\perp$ of the $\pi, \theta = 90°$.

for various values of $x_{\perp c}$ at $\theta \simeq \frac{1}{2}\pi$. For the present range of data, $x_\perp \lesssim 0.3$, we see that we are looking at quite-small-x partons in the initial state as is expected. We can only hope to observe large-x partons when we are very near the edge of phase space ($x_\perp \approx 1$). We should be able to probe large y much more easily. Note that although $\langle x_1 \rangle = \langle x_2 \rangle = x$ at $\theta \approx \frac{1}{2}\pi$ as discussed above, the dispersion $\langle (x_1 - x_2)^2 \rangle$ is quite large (Fig. 7), which will be quite important in Sec. III. Unlike the value of α_{eff}, these results for $\langle x \rangle, \langle y \rangle$ are fairly independent of specific details of the precise form of

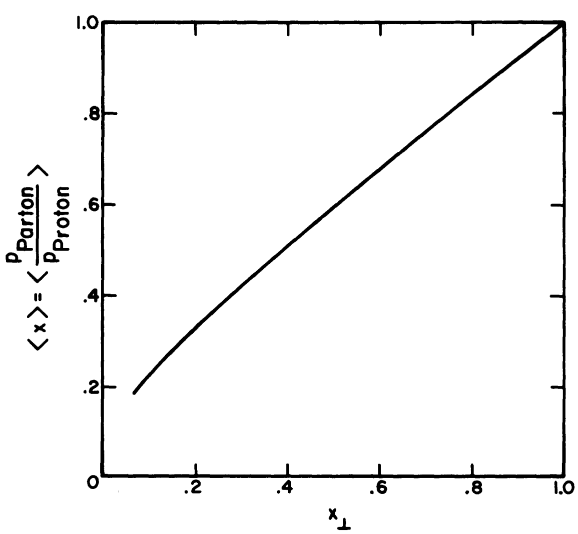

FIG. 5. Average x of scattered parton in the original proton as a function of $x_\perp$ of the observed $\pi, \theta = 90°$.

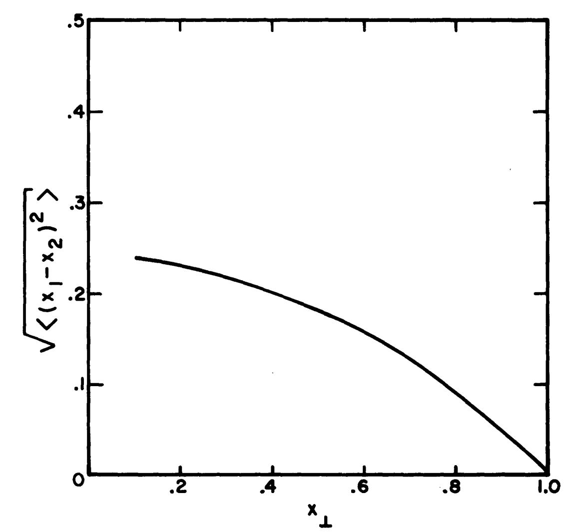

FIG. 7. Average dispersion $[(x_1 - x_2)^2]^{1/2}$ between the momentum fractions carried by the two scattering partons as a function $x_\perp$ of the observed $\pi, \theta = 90°$.

the distributions.

Although the reader is reminded that a direct test of these parton models is the s dependence at fixed $x_\perp$, he is faced at present with only a limited $p_\perp$ range. Hence fixed $p_\perp$ behavior is most easily checked. If we assume that the $p_\perp$ dependence in the current range is $\propto p_\perp^{-8}$ at fixed s, we expect $d\sigma/d^3p/E$ to behave as s^{+2} for gluon exchange and s^4 for the contact interaction at fixed $p_\perp$ such that $2p_\perp/\sqrt{s} > 0.2$, for example. Parton interchange, nonparton pictures, and scalar-parton theories tend to give s^0 results, i.e., approximatic hadronic scaling at all $p_\perp$ except very near the kinematic boundary. Hence one can hope to distinguish these possibilities in the near future.

It is important to note that although one uses simple power-behaviored functions for $F(x)$ and $G(y)$ the resulting convolution in Eq. (2.3) yields forms for the function $f(x_\perp, \theta)$ in Eq. (2.4) which are in general fairly complicated. Simple power behavior such as the $p_\perp^{-8}$ form mentioned above can be approximately true only over a finite range in $x_\perp$, so care should be taken when using such a simple approximation. For $x_\perp \gtrsim 0.6$ the cross section is much more rapidly cut off. For $x_\perp \lesssim 0.1$ the result approaches its asymptotic behavior for small $x_\perp$, which for vector-gluon (VG) exchange is

$$f_{\rm VG}(x_\perp, \theta) \underset{x_\perp \to 0}{\sim} \frac{1}{x_\perp^4} \ln \frac{1}{x_\perp}, \tag{2.5a}$$

$$\frac{d\sigma}{d^3p/E^{\rm VG}} \underset{s\to\infty;\, p_\perp \text{ fixed}}{\sim} \frac{\text{const}}{p_\perp^4} \ln \frac{\sqrt{s}}{2p_\perp}. \tag{2.5b}$$

Thus the expected discrimination between various models is feasible only for finite $x_\perp$. Further, in the limit $s \to \infty$, $p_\perp$ fixed, the vector-gluon model is well behaved except for the presence of the logarithmic term. This is, of course, a serious question in principle. One may interpret this result either as an indication that the model is only good for $x_\perp \geq$ const rather than $x_\perp \geq 2p_{\perp\min}/\sqrt{s}$ or as an indication that at fixed $p_\perp$ the inclusive cross section will asymptotically rise as $\ln\sqrt{s}$. This behavior is not unrelated to the result of a rising $\sigma_{\rm tot}$ contribution to be found in Sec. VI.

III. DOUBLY-INCLUSIVE CROSS SECTIONS

As mentioned in the Introduction, a truly distinctive feature of the present picture of large-transverse-momentum events is the prediction that all such events proceed via the production of two jets of hadrons in the transverse direction on opposite sides of the beam direction. This is the direct result of the idea that the initial step is the large-angle scattering of two partons which subsequently evolve into jets of hadrons. This general kinematic picture is thus inherent to the simple parton-parton scattering picture independent of the details of $d\sigma/d\hat{t}$.

Although the details of how the partons evolve into hadrons are unclear at present,[23] they must be such as to ensure the nonobservation of isolated partons at current energies. Simple models suggest average multiplicities which grow logarithmically with the parton's momentum (as we shall see in Sec. IV) and distributions of hadrons in the jet which are well collimated around the original direction with mean transverse momentum of order 300 MeV. This ensures a uniform distribution in momentum space along the original parton direction and no isolated partons, at least in momentum space.

This two-jet picture (one on each side of the beam plus some distribution along the beam) is to be contrasted with other pictures[8,12] wherein at least one of the jets is a single hadron and the distribution on the other side of the beam is not clearly jetlike. The over-all multiplicity in these alternative pictures is expected to be lower than in the two-jet model described here.[24]

Since data are now becoming available from the ISR on the doubly-differential cross section where one particle is observed on each side of the beam, it is of great interest to study the contribution from the present model to the situation where these particles come from the two jets. Using the distributions introduced previously and the kinematics of Fig. 8 we find (again details are to be found in the Appendix; for simplicity we shall no longer explicitly write the implied limit)

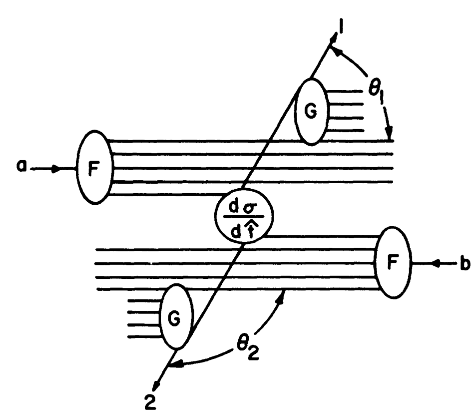

FIG. 8. Pictorial representation of parton-parton scattering contribution to the double-particle cross section where one is from each jet.

$$\frac{d^6\sigma}{(d^3p_1/E_1)\,(d^3p_2/E_2)} \sim \frac{16}{\pi s x_{\perp 1}{}^2 x_{\perp 2}{}^2}\int \frac{dx\, xF(x)F(x\tan(\frac{1}{2}\theta_1)\tan(\frac{1}{2}\theta_2))}{[\cot(\frac{1}{2}\theta_1)+\cot(\frac{1}{2}\theta_2)]^2}$$

$$\times G\left(\frac{x_{\perp 1}}{2x}[\cot(\tfrac{1}{2}\theta_2)+\cot(\tfrac{1}{2}\theta_1)]\right) G\left(\frac{x_{\perp 2}}{2x}[\cot(\tfrac{1}{2}\theta_2)+\cot(\tfrac{1}{2}\theta_1)]\right)\left(\frac{d\sigma}{d\hat{t}}\right)\delta(\phi_1-\phi_2+\pi)\,. \tag{3.1}$$

These variables are defined analogously to before, i.e., $x_\perp = 2p_\perp/\sqrt{s}$, the θ's are in the hadron c.m. system (see Fig. 8), $x_1 = x$, $x_2 = x\,\tan(\frac{1}{2}\theta_1)\tan(\frac{1}{2}\theta_2)$, and

$$\hat{t} = -x_1{}^2 s\tan(\tfrac{1}{2}\theta_1)/[\cot(\tfrac{1}{2}\theta_1)+\cot(\tfrac{1}{2}\theta_2)]\ .$$

The forms for $d\sigma/d\hat{t}$ in Fig. 2 can again be used where now $\eta = \tan(\frac{1}{2}\theta_1)\cot(\frac{1}{2}\theta_2)$. Note that the exact correlation in ϕ results from the simplifying assumption of zero-width transverse distributions. In Fig. 9 we indicate typical values for this cross section using the one-vector-gluon-exchange cross section as an illustrative example. The cross section in this case takes the form

$$\frac{d\sigma}{(d^3p_1/E_1)\,(d^3p_2/E_2)} \sim \frac{32\alpha_{\text{eff}}{}^2}{s^3 x_{\perp 1}{}^2 x_{\perp 2}{}^2}\int \frac{dx}{x^3}F(x)F(x\tan(\tfrac{1}{2}\theta_1)\tan(\tfrac{1}{2}\theta_2))$$

$$\times G\left(\frac{x_{\perp 1}}{2x}[\cot(\tfrac{1}{2}\theta_1)+\cot(\tfrac{1}{2}\theta_2)]\right) G\left(\frac{x_{\perp 2}}{2x}[\cot(\tfrac{1}{2}\theta_1)+\cot(\tfrac{1}{2}\theta_2)]\right)$$

$$\times\frac{\cot^2(\frac{1}{2}\theta_2)}{(1+\eta)^2}\left(\eta^2+(1+\eta)^2+\frac{1}{\eta^2}+(1+\eta)^2\right)\delta(\phi_1-\phi_2+\pi)\,. \tag{3.2}$$

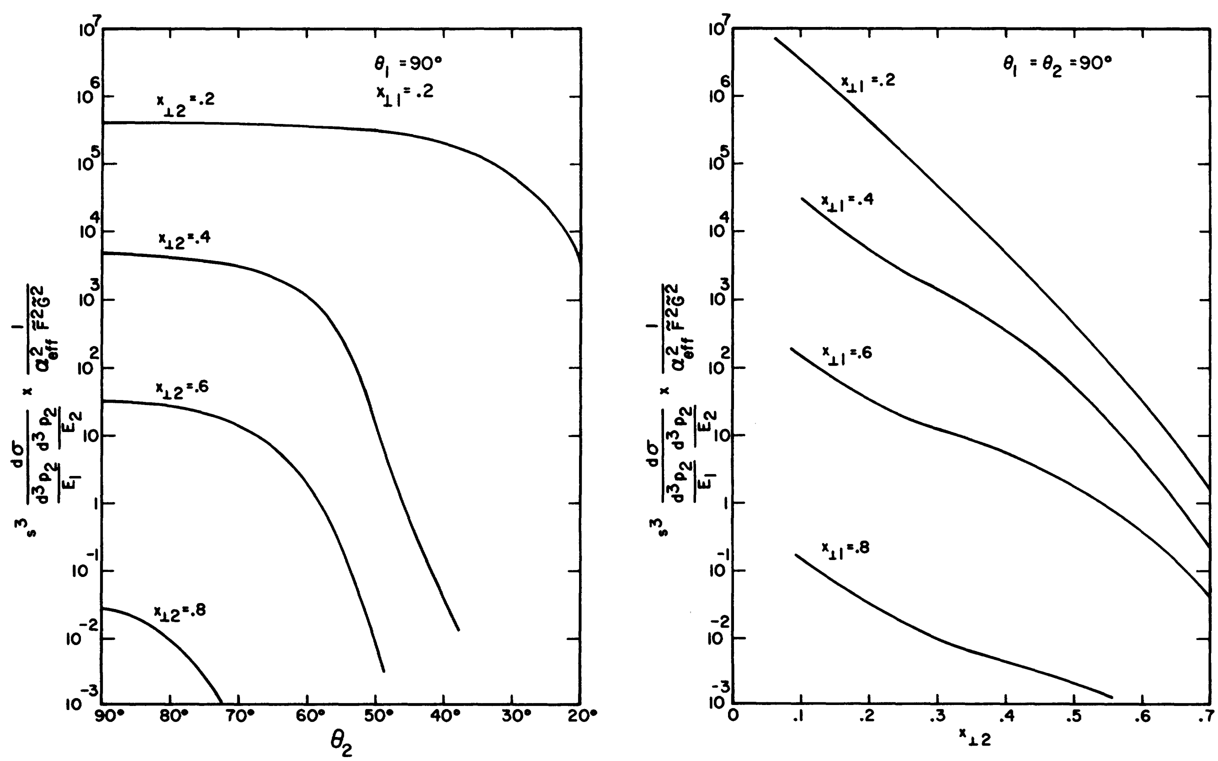

FIG. 9. Double-particle cross section from one-gluon exchange with distribution functions described in the text: (a) x_1, x_2, θ fixed, varying θ_2; (b) θ_1, θ_2, x_1 fixed, varying x_2.

The normalizations correspond to those given in Sec. II. Note especially the important but experimentally unfortunate feature that for relatively small ($\lesssim 0.5$) values of $x_{\perp 1,2}$, where the cross section is relatively sizable, the correlation between θ_1 and θ_2 is fairly weak. This results from the large value of $[\langle (x_1 - x_2)^2 \rangle]^{1/2}$ mentioned in Sec. II. Although the two jets appear "back to back" in the parton-parton c.m. system, this system is typically moving in the over-all hadron c.m. system. Hence the distinctive feature of "back to back" jets will not appear event by event except for large $x_\perp$'s. It will occur on the average, but the angular width of the averaged second jet will be quite large and not characteristic of a single jet. Of course good measurements of these cross sections will be extremely important in verifying whether this parton picture has any validity for hadronic reactions.

If one, in fact, does observe clean two-jet events in the data, so as to verify the picture discussed here, one is led to consider measuring the two-jet cross section itself.[25] Unlike the two-hadron cross section, the differential two-jet cross section totally specifies the kinematics of the parton-parton scattering, and so one can study independently the distribution functions and the cross section $d\sigma/d\hat{t}$. For example, if we take the independent variables to be $y_\perp = 2p_{\perp \text{jet}}/\sqrt{s}$, the common jet transverse momentum, and the angles of the two jets in the c.m. system, θ_1 and θ_2 measured from the same axial direction, we find (see Appendix)

$$\frac{d\sigma}{dy_\perp d\Omega_1 d\Omega_2} \sim \frac{sy_\perp}{4\pi}\frac{F(x_1)F(x_2)}{\sin^2\theta_1 \sin^2\theta_2}\frac{d\sigma}{d\hat{t}}(\hat{s},\hat{t}) \times \delta(\phi_1 - \phi_2 + \pi), \tag{3.3}$$

where

$$x_1 = \tfrac{1}{2}y_\perp[\cot(\tfrac{1}{2}\theta_1) + \cot(\tfrac{1}{2}\theta_2)],$$

$$x_2 = \tfrac{1}{2}y_\perp[\tan(\tfrac{1}{2}\theta_1) + \tan(\tfrac{1}{2}\theta_2)],$$

$$\hat{s} = x_1 x_2 s,$$

and

$$\hat{t} = -\tfrac{1}{4}sy_\perp^2[1 + \tan(\tfrac{1}{2}\theta_1)\cot(\tfrac{1}{2}\theta_2)].$$

Again we can use Fig. 2 with $\eta = \tan(\frac{1}{2}\theta_1)\cot(\frac{1}{2}\theta_2)$. Thus by varying the independent variables s, $y_\perp$, θ_1, and θ_2 one can study independently the dependence on x_1, x_2, $\hat{s}$, $\hat{t}$ and check explicitly the forms of $F(x)$ and $d\sigma/d\hat{t}$. Note that one can rewrite Eq. (3.3) in terms of the standard invariant form

$$\frac{d\sigma}{(d^3p_1/E_1)\,(d^3p_2/E_2)}$$

as long as the jet energy is well defined. This is a nontrivial problem since in any measurement of a jet there is an ambiguity about which slow hadrons should be counted as members of the jet. This leads to a finite error (say ~ 4 GeV/c) in both $p_{\perp \text{jet}}$ and E_{jet} and an error of order $E_{\text{jet}}\,m$ in $M_{\text{jet}}{}^2$, which complicates the situation but does not detract from its essential interest. For $E_{\text{jet}} \sim p_{\text{jet}}$ we have

$$\frac{d\sigma_{\text{jet}}}{(d^3p_1/E_1)\,(d^3p_2/E_2)} \sim \frac{1}{\pi}F(x_1)F(x_2)\frac{d\sigma}{d\hat{t}}(\hat{s},\hat{t}) \times \delta^2(\vec{p}_{\perp 1} + \vec{p}_{\perp 2}). \tag{3.4}$$

For completeness we also give here the single-jet cross section

$$\frac{d\sigma_{\text{jet}}}{d^3p/E} \sim \frac{2}{\pi}\int \frac{dx_1}{2x_1 - y_\perp \cot(\frac{1}{2}\theta)} \times F(x_1)F\left(\frac{x_1\tan^2(\frac{1}{2}\theta)}{\eta}\right)\frac{d\sigma}{d\hat{t}}, \tag{3.5}$$

where the integration is over

$$\frac{y_{\perp \text{jet}}}{2 - y_{\perp \text{jet}}\tan(\frac{1}{2}\theta)} \le x_1 \le 1,$$

$$\eta = [2x_1\tan(\tfrac{1}{2}\theta) - y_{\perp \text{jet}}]/y_{\perp \text{jet}},$$

$$x_2 = x_1\tan^2(\tfrac{1}{2}\theta)/\eta,$$

and

$$\hat{t} = -\tfrac{1}{2}x_1 s y_{\perp \text{jet}}\tan(\tfrac{1}{2}\theta).$$

Again the forms of Fig. 2 can be applied in a direct fashion.

IV. MULTIPLICITIES

The question of multiplicities of hadrons in large-transverse-momentum purely hadronic events is very interesting since logarithmic multiplicities are essentially built into parton models at step one in order to ensure that partons do not appear as isolated particles as was discussed at the beginning of Sec. III. This is particularly important if one wants to identify partons with quarks. As was mentioned above, other models tend to predict quite different multiplicity distributions so that multiplicity measurements should prove extremely useful in determining how the world actually behaves.

In the present context one can see how logarithmic multiplicities arise by considering Eq. (2.2). The $G(y)\,dy/y$ distribution (y is the fraction of the parton's momentum carried by the hadron) assumed for hadrons within a jet yields naturally such behavior if $G(0) \neq 0$. The distribution does not vanish for small momentum in models with finite-momentum ("wee" as $p_{\text{parton}} \to \infty$) hadrons (and partons), although such a zero is not uncommon in specific field-theory calculations.[26]

If we consider an event where the scattered parton carries momentum defined by

$$y_\perp = \frac{2p_{\perp\,\text{parton}}}{\sqrt{s}} \equiv 2p_{\perp\text{jet}}/\sqrt{s}$$

in the direction θ, where

$$y_\perp = 2\left(\frac{\cot(\frac{1}{2}\theta)}{x_1} + \frac{\tan(\frac{1}{2}\theta)}{x_2}\right)^{-1} ,$$

we may ask for the average multiplicity of hadrons of type h with transverse momentum $x_\perp = 2p_{\perp h}/\sqrt{s}$ greater than some minimum value $x_{\perp\min} = 2p_{\perp\min}/\sqrt{s}$ (where $p_{\perp\min}$ defines the range of validity of the present model and may either be a constant or proportional to $\sqrt{s}$). This seems a natural question both experimentally and within the context of our parton model. We can evaluate this average multiplicity in the limit of small $x_{\perp\min}$ by using the relation ($y = x_\perp/y_\perp$)

$$\langle n_h(y_\perp, x_{\perp\min})\rangle_{\text{jet}} = \int_{x_{\perp\min}}^{y_\perp} \frac{dx_\perp}{x_\perp}\, G^h(x_\perp/y_\perp)$$

$$\approx G^h(0)\ln(y_\perp/x_{\perp\min}) + O(1), \quad (4.1)$$

where the second line is valid for $x_{\perp\min}/y_\perp \ll 1$. Thus it obtains for a fixed $p_{\perp\min}$ as $s\to\infty$ or for a fixed $x_{\perp\min} \ll y_\perp$. Treating the behavior of G as $y \to 0$ as an experimental question we may then define a new parameter $d_h = G^h(0)$ which characterizes the logarithmic multiplicity of hadrons of type h in a jet (note that d_h will in general also have a superscript, d_h^i, to specify that we started with a parton of type i). If one assumes that $G(y)$ describes the evolution of a parton essentially in isolation from any other partons present (an attitude not without some theoretical difficulty, see Ref. 23), then one expects the constants d_h to specify the multiplicities observed in all parton-instigated processes,[27,28] e.g., e^+e^- annihilation, ep inelastic scattering, and large-transverse-momentum hadronic processes. On the other hand, if the other partons present participate in an essential way, one can expect only to correlate the general logarithmic behavior (which is presumably related only to the fact that partons do not become free).

For the present purpose we shall just treat d_h as a parameter and proceed to calculate the expected distribution of hadrons. From the above we have in the limit $s\to\infty$, $y_{\perp\,\text{fixed}}$,

$$\langle n_h(x_{\perp\min}, y_\perp)\rangle_{\text{jet}}$$

$$\sim d_h \ln(y_\perp/x_{\perp\min}) \;(x_{\perp\min}\text{ fixed}) \quad (4.2a)$$

$$= d_h\left(\ln\frac{\sqrt{s}}{2p_{\perp\min}} + \ln y_\perp\right) \;(p_{\perp\min}\text{ fixed}) \quad (4.2b)$$

$$= d_h \ln\frac{p_{\perp\text{jet}}}{p_{\perp\min}} \;(p_{\perp\min}\text{ fixed}), \quad (4.2c)$$

where we have not kept the next term, which is an unknown constant.

Note that our choice to discuss the multiplicity of hadrons with $x_\perp > x_{\perp\min}$ ($p_\perp > p_{\perp\min}$) is of essential importance in order to obtain the simple result of Eqs. (4.2). It has the virtue of having no explicit θ dependence and is invariant under z boosts. Hence it is appropriate as given both at NAL and the ISR. If one looked at $|\vec{p}| > |\vec{p}_{\min}|$, sin θ dependence would appear in an obvious way. Equation (4.2a) is valid when either $x_{\perp\min}$ or $p_{\perp\min}$ is held fixed as $s\to\infty$. To use Eqs. (4.2b) and (4.2c) one must require the validity of the present model explicity for fixed $p_{\perp\min}$. The validity of this assumption is not at all clear.

It is clear that the most straightforward procedure to test the behavior indicated in Eqs. (4.2) is

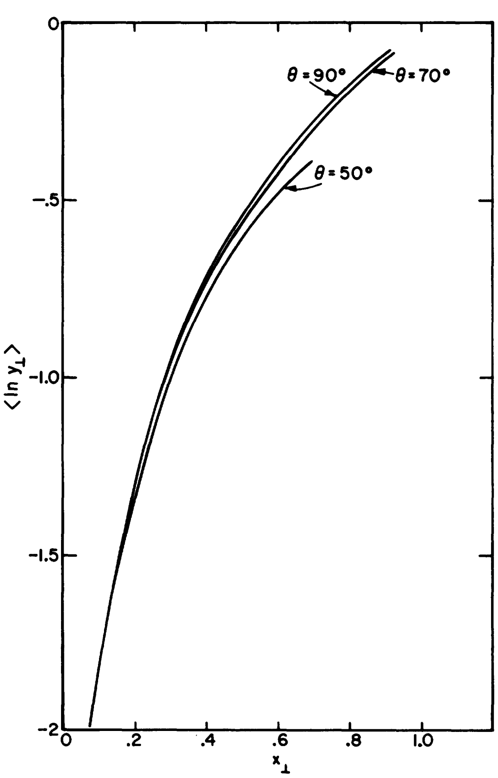

FIG. 10. Average value of $\ln y_\perp$, where $y_\perp = 2p_{\perp\,\text{parton}}/\sqrt{s}$, as a function of $x_\perp$ of the observed π.

to measure both the total momentum of a jet and its multiplicity. This will presumably be done eventually but for the present we are interested primarily in situations where the momentum of only one hadron is measured. (Note that we must require at least one high-transverse-momentum hadron in order to focus on the kind of event for which the present analysis is applicable.) What we need is the value of $\langle n(y_\perp)\rangle_{\text{jet}}$ averaged over those events (values of $y_\perp$) which yield a hadron of momentum $x_\perp$. Using Eq. (4.2) we calculate $\langle \ln y_\perp \rangle$ as a function of $x_\perp$ and θ (hadron) which is shown in Fig. 10. Although it is not generally true that the mean multiplicity averaged over some $y_\perp$ distribution should equal the mean multiplicity at the average $y_\perp$ it is approximately true in our case, i.e., $\langle\langle n(y_\perp)\rangle\rangle \approx \langle n(\langle y_\perp\rangle)\rangle$ (compare Figs. 6 and 10). This analysis applies to events where the parton evolves "freely." In particular we expect it to apply to the hadron jet which appears on the opposite side of the beam from the hadron whose momentum is measured. Thus we expect, for an observed hadron with transverse momentum $x_\perp$, the mean multiplicity in the other jet to be

$$\langle n_h(x_{\perp\min}, x_\perp)\rangle_{\text{jet opposite}} \sim d_h\langle \ln(y_\perp/x_{\perp\min})\rangle \sim d_h\left(\ln\frac{1}{x_{\perp\min}} + \ln\langle y_\perp\rangle\right). \qquad (4.3)$$

As an illustration we use the results of Fig. 10 to show a typical variation of this correlation effect with $d_h = 0.8$. There is an over-all $x_\perp$- independent constant which at this level is unknown and which we have chosen to allow comparison with the data of Ref. 20 as shown in Fig. 11(a). This constant reflects not only theoretical effects but presumably also experimental effects such as angular acceptance. Recall that the jet opposite is confined to a smaller and smaller angular region near 90° as $x_\perp$ approaches 1. This will serve to enhance the observed correlation between $x_\perp$ and $\langle n\rangle_{\text{jet opposite}}$ in an experiment with fixed angular acceptance around 90°.

The above analysis presumably does not apply to the jet from which we selected the hadron with momentum fraction $x_\perp$. To see this note that the kinematic structure of such a jet cannot, in general, correspond to the type of event which leads to the multiplicity of Eqs. (4.2). As an illustrative

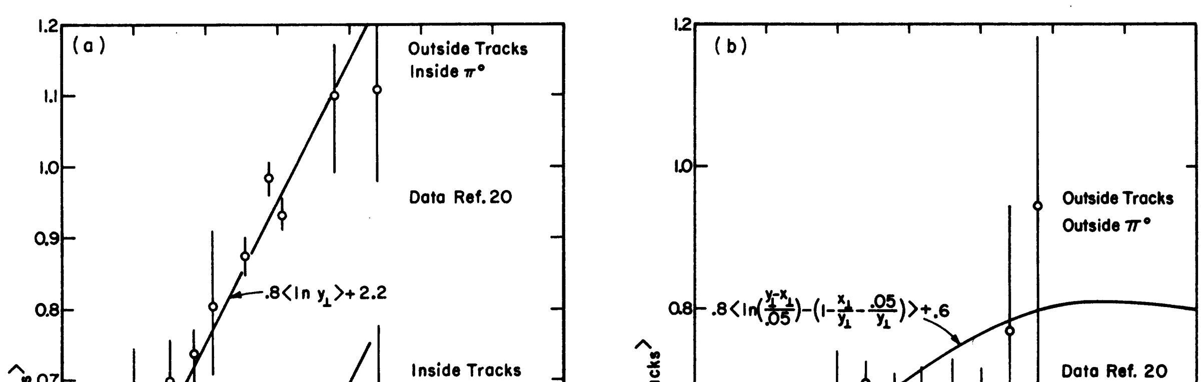

FIG. 11. Variation of the expected mean multiplicity of hadrons in a jet as a function of $x_\perp$ of the observed π. Shown are (a) the case for the jet opposite the observed π, and (b) the jet in which the π is observed. Data from Ref. 20 are shown for comparison. The constant terms are fitted.

example consider an average multiperipheral type of event (see Fig. 12) for a parton with $y_\perp$. The fastest hadron has $x_\perp = (1-\alpha)y_\perp$, the next fastest $x_\perp = \alpha(1-\alpha)y_\perp$ and so on. If this is an average event, then after $\langle n\rangle \sim d_h \ln y_\perp/x_{\perp\min}$ steps we have $\alpha^{\langle n\rangle}(1-\alpha)y_\perp \sim x_{\perp\min}$. This gives that α is approximately e^{-1/d_h}. So for d_h of order unity, as we expect, the fastest hadron should have $x_\perp/y_\perp \lesssim 0.6$. From Fig. 5 we see that this can be satisfied only for small $x_\perp$. As we increase $x_\perp$ we expect a configuration where most of the parton's momentum is carried by one hadron and there is little momentum left for the other hadrons. In order to have any explicit calculations of the multiplicity in the same jet as the observed hadron one needs the form of the two-particle distribution function

$$\frac{dx_1}{x_1}\frac{dx_2}{x_2}\,\overline{G}(x_1, x_2)$$

which describes the "decay" of a parton into a hadron of momentum fraction x_1, one of a fraction x_2 and anything. We find that for reasonable forms of $\overline{G}(x_1, x_2)$, i.e., $\overline{G}$ behaving analogously to G[e.g., $\overline{G} \sim \theta(1-x_1-x_2)(1-x_1)(1-x_2)$], the mean multiplicity assumes the form (the 1 accounts for the observed hadron)

$$\langle n_h(x_{\perp\min}, x_\perp\rangle_{\text{jet on same side}} \sim \overline{d}_h\left\langle \ln \frac{y_\perp - x_\perp}{x_{\perp\min}}\right\rangle + 1 + \left\langle F\left(1-\frac{x_\perp}{y_\perp}, \frac{x_{\perp\min}}{y_\perp}\right)\right\rangle. \tag{4.4}$$

The indicated average is over events with one hadron with momentum fraction $x_\perp$ and the function F is generally more slowly varying than the logarithm but also vanishes as $y_\perp - x_\perp$ approaches $x_{\perp\min}$. In general one might expect that $\overline{d}_h$ is a totally new constant. However, if we require that we recover Eqs. (4.2) for the "average" event when $x_\perp = (1-\alpha)y_\perp \cong (1-e^{-/d_h})y_\perp$, we find that $\overline{d}_h = d_h$ as happens in very simple models. In explicit calculations we find that for $\overline{d}_h \sim 1$ the associated, same side, mean multiplicity will increase slightly ($\Delta\langle n\rangle \lesssim 0.3$) as $x_\perp$ is varied from 0.1 to 0.3. As $x_\perp$ is further increased the mean multiplicity will decrease back to one, the observed particle. In Fig. 11(b) we compare this behavior to the data of Ref. 20 using the example $\overline{G} \sim (1-x_1) \times (1-x_2)$, $x_{\perp\min} = 0.05$, $d_h = 0.8$ and allowing an additive constant to normalize to the data. Although these details are not to be considered rigorous, the general picture of the associated, mean multiplicity in the same jet as the observed hadron showing weak positive correlation for small $x_\perp$ and then weak negative correlation for larger $x_\perp$, is expected to be a general feature of the parton model.

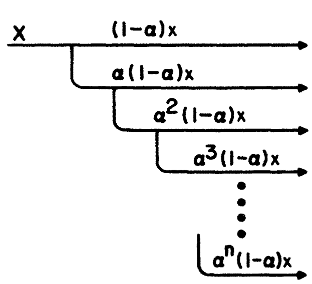

FIG. 12. Expected average momentum distribution in typical "multiperipheral-" type event.

In summary we expect in general to find hadron jets whose multiplicities (of hadrons with $p_\perp > p_{\perp\min}$) vary as the logarithm of the jet momentum, as in Eqs. (4.2). For events where the large transverse momentum ($x_\perp$) of one hadron is observed, we expect a strong positive correlation with the multiplicity of the other jet as expressed by Eq. (4.3) and Figs. 10 and 11(a). For example in the range $0.1 < x_\perp < 0.2$, for $d_h = 0.8$, $\langle n\rangle$ should vary by about 0.4. Naive calculations suggest that the mean multiplicity in the same jet as the observed hadron will behave as suggested in Eq. (4.4). In the range $0.1 < x_\perp < 0.2$, for $d_h = 0.8$, it is expected to increase by only about 0.15 and will in fact decrease as $x_\perp$ becomes greater than about 0.3 (where almost all the jet momentum is carried by this single hadron) as indicated in Fig. 11(b). Both these features seem in reasonable agreement with current experimental results.[20] More detailed tests should prove most informative. In all cases the mean multiplicity behaves as $\ln\sqrt{s}$ as $s \to \infty$ for $x_\perp$ and $p_{\perp\min}$ fixed, if the model is valid there. Finally we emphasize that the results of this section are generally independent of detailed assumptions about $d\sigma/d\hat{t}$, $F(x)$, or $G(y)$ except that $G(y)$ be finite for small y. The type of behavior discussed is inherent in any simple parton model.

V. QUANTUM NUMBERS

Having discussed the kinematics of parton-parton scattering effects, we should like now to discuss what can be said about the quantum numbers of the produced large-transverse-momentum hadrons in the parton picture.[29] The first feature which is characteristic of naive parton models is that the hadron spectrum at large $p_\perp$ is predicted to be very similar to the hadron spectrum observed in deep-inelastic ep scattering in the "photon fragmentation" region since both processes are pictured as being the result of a parton materializing into hadrons. For a more detailed account of what is expected in the leptonic case the reader is referred to the literature.[30] The only important change from the present situation is that in the electromagnetic cases the probability that a specific parton participates is weighted by a charge squared. In the hadronic case the scattering is assumed to be the same for all partons. We remind the reader that the predictions given below apply to the limit $p_\perp/\sqrt{s}$ fixed as $s\to\infty$.

In order to obtain more specific results, via more specific assumptions, we must reinstate the subscripts which we dropped for simplicity at the beginning. In particular, for the single-particle inclusive cross section, $a+b\to c+x$, we make the following replacement:

$$FF\frac{d\sigma}{d\hat{t}}G\to\sum_{ij}F_i^aF_j^b\left[\frac{d\sigma_{ij}}{d\hat{t}}(\hat{s},\hat{t})G_i^c+\frac{d\sigma_{ij}}{d\hat{u}}(\hat{s},\hat{u})G_j^c+\delta_{ij}\sigma_{\text{int}}G_i^c\right]\ . \tag{5.1}$$

The superscripts label hadrons and the subscripts label partons. We have included the possibility, as discussed earlier, of interference terms in the scattering of identical partons (see Fig. 2). In what follows we shall continue to assume the predominance of effects due to the spin-$\frac{1}{2}$ partons, which we shall identify with quarks. It is clear, however, that gluon terms can easily be included in the summations of Eq. (5.1). If the gluons are neutral SU(2) singlets, their contributions may be easily excluded by studying appropriate differences of various charge-state reactions.[31] For simplicity in the present discussion we shall disregard this possibility. It should be noted, however, that such a neutral contribution may be detectable as more complete data becomes available. Its general effect should be to reduce the ratios discussed below.

If we identify the partons with quarks we can proceed to make predictions. If we consider the production of π's, isospin and charge-conjugation invariance tell us that these are just three independent G functions[2]

$$G_u^{\pi^+}=G_d^{\pi^-}=G_{\bar{u}}^{\pi^-}=G_{\bar{d}}^{\pi^+}\ , \tag{5.2a}$$

$$G_d^{\pi^+}=G_u^{\pi^-}=G_{\bar{d}}^{\pi^-}=G_{\bar{u}}^{\pi^+}\ , \tag{5.2b}$$

$$G_s^{\pi^+}=G_s^{\pi^-}=G_{\bar{s}}^{\pi^+}=G_{\bar{s}}^{\pi^-}=G_s^{\pi^0}=G_{\bar{s}}^{\pi^0}\ , \tag{5.2c}$$

$$G_u^{\pi^0}=G_{\bar{u}}^{\pi^0}=G_d^{\pi^0}=G_{\bar{d}}^{\pi^0}=\tfrac{1}{2}(G_u^{\pi^+}+G_u^{\pi^-})\ . \tag{5.2d}$$

In order to further simplify the situation and obtain directly simple ratios between the production probabilities of various particles, we shall make another assumption which can eventually be eliminated by more complete data and explicit calculations as discussed shortly. Recall (see Fig. 5) that for $x_\perp\gtrsim0.3$ we are probing on the average the behavior of nonwee quarks ($x\gtrsim0.4$) in the initial hadrons. Thus it is reasonable to expect that we are observing primarily the valence quarks (those which give the quantum numbers of the specific hadron). This is consistent with current neutrino data which suggest that few antiquarks are present in baryons (qqq) at nonwee momentum fraction.[32] Hence we expect little contribution from the $q\bar{q}$ sea at finite x compared to the valence-quark contribution. More explicitly, if we define two distributions, $V(x)$ for the valence quarks and $C(x)$ for the $q\bar{q}$ sea, we expect the quantity $\epsilon(x)=C(x)/[V(x)+C(x)]$ to be much less than one for $x\geqslant0.3$, for example. In the proton case the usual identification yields

$$F_u^p=2V(x)+C(x)\ ,$$

$$F_d^p=V(x)+C(x)\ ,$$

and

$$\begin{aligned}F_{\bar{u}}^p/F_d^p&\simeq F_{\bar{d}}^p/F_d^p\\&\simeq F_s^p/F_d^p\\&\simeq F_{\bar{s}}^p/F_d^p\\&\simeq\epsilon(x).\end{aligned}$$

Thus for an $x_\perp$ such that $\epsilon(\langle x\rangle)$ is small we can drop these last four contributions from the sum in (5.1). In principle all these ratios are experimentally accessible so that when the data are available these contributions can be explicitly included. For now we shall assume that $\langle\epsilon(x)\rangle$, averaged over x's appropriate to $x_\perp\gtrsim0.3$, for example, is small in order to obtain simple limiting results. Eventually we will do a sample calculation with an assummed form of $\epsilon(x)$ in order to see how things change.

For the reaction $p+p\to\pi+x$ we have the form ($s\to\infty$, $x_\perp=2p_{\perp\pi}/\sqrt{s}$ fixed)

$$\begin{aligned}E_\pi\frac{d\sigma}{d^3p_\pi}\sim\frac{4}{\pi x_\perp{}^2}\int\!\!\int dx_1dx_2\frac{\eta}{(1+\eta)^2}\Bigg[&F_u^p(x_1)F_u^p(x_2)\left(\frac{d\sigma_{uu}}{d\hat{t}}+\frac{d\sigma_{uu}}{d\hat{u}}+\sigma_{\text{int}}uu\right)G_u^\pi\\&+F_u^p(x_1)F_d^p(x_2)\left(\frac{d\sigma_{ud}}{d\hat{t}}G_u^\pi+\frac{d\sigma_{ud}}{d\hat{u}}G_d^\pi\right)+F_d^p(x_1)F_u^p(x_2)\left(\frac{d\sigma_{du}}{d\hat{t}}G_d^\pi+\frac{d\sigma_{du}}{d\hat{u}}G_u^\pi\right)\\&+F_d^p(x_1)F_d^p(x_2)\left(\frac{d\sigma_{dd}}{d\hat{t}}+\frac{d\sigma_{dd}}{d\hat{u}}+\sigma_{\text{int}}dd\right)G_d^\pi\Bigg]+O(\langle\epsilon(x)\rangle)\ .\end{aligned} \tag{5.3}$$

[Unless stated, the following relations will also be considered to be correct to order $\langle\epsilon(x)\rangle$.] Assuming an isosinglet exchange for the parton-scattering process so that $(d\sigma/d\hat{t})uu=(d\sigma/d\hat{t})dd=(d\sigma/d\hat{t})ud$, etc., and taking $F_u^p(x)=\beta(x)F_d^p(x)=\beta(x)F^p(x)$ (for example $\beta=2$ in the simple quark model, which is consistent with current data for x near 0.5), the following ratios apply to the process $p+p\to\pi+x$, where the π's have large transverse momentum:

$$\frac{\langle\pi^+ +\pi^-\rangle}{2\langle\pi^0\rangle}=\frac{\langle\frac{1}{2}(G_u^{\pi^+}+G_u^{\pi^-})[(\beta^2+\beta)(d\sigma/d\hat{t}+d\sigma/d\hat{u})+\beta^2\sigma_{\rm int}]+\frac{1}{2}(G_d^{\pi^+}+G_d^{\pi^-})[(\beta+1)(d\sigma/d\hat{t}+d\sigma/d\hat{u})+\sigma_{\rm int}]\rangle}{\langle G_u^{\pi^0}[(\beta^2+\beta)(d\sigma/d\hat{t}+d\sigma/d\hat{u})+\beta^2\sigma_{\rm int}]+G_d^{\pi^0}[(\beta+1)(d\sigma/d\hat{t}+d\sigma/d\hat{u})+\sigma_{\rm int}]\rangle}$$

$$=1, \qquad (5.4a)$$

$$\frac{\langle\pi^+\rangle}{\langle\pi^-\rangle}=\frac{\langle G_u^{\pi^+}[(\beta^2+\beta)(d\sigma/d\hat{t}+d\sigma/d\hat{u})+\beta^2\sigma_{\rm int}]+G_d^{\pi^+}[(\beta+1)(d\sigma/d\hat{t}+d\sigma/d\hat{u})+\sigma_{\rm int}]\rangle}{\langle G_u^{\pi^-}[(\beta^2+\beta)(d\sigma/d\hat{t}+d\sigma/d\hat{u})+\beta^2\sigma_{\rm int}]+G_d^{\pi^-}[(\beta+1)(d\sigma/d\hat{t}+d\sigma/d\hat{u})+\sigma_{\rm int}]\rangle}, \qquad (5.4b)$$

where the bracket $\langle\ \rangle$ represents the integrals over x_1 and x_2 with weight function $F^p(x_1)F^p(x_2)\times\eta/(1+\eta)^2$. The result (5.4a) follows directly from Eq. (5.2), i.e., isospin and charge-conjugation invariance plus the assumption that produced η and η' mesons do not decay into π's before reaching the detectors. If this latter condition is not fulfilled, the π's from the decays of the η and η' should result in a ratio $\langle\pi^+ +\pi^-\rangle/2\langle\pi^0\rangle\sim0.9$, i.e., π^0 production is enhanced by about 10% assuming that the relative production of π^0, η, and η' is given by U(3). Note that Eq. (5.4a) did not require extra assumptions about the F's and G's. The results of Refs. 20 and 21 seem in reasonable agreement with this prediction.

To obtain a value for the ratio π^+/π^- we need to assume a relationship between $G_u^{\pi^+}$ $(=G_d^{\pi^-})$ and $G_d^{\pi^+}$ $(=G_u^{\pi^-})$. In light of our earlier assumptions that the structure of the G's is similar to that of the F's and that for nonwee x we see essentially only the valence quarks in the F's, the natural assumption is that the leading hadron in a jet has as one of its valence quarks the original scattered quark. Thus with the usual valence-quark assignments we have $G_u^{\pi^+}=G_d^{\pi^-}\gg G_d^{\pi^+}=G_u^{\pi^-}$, i.e., $G_d^{\pi^+}/G_u^{\pi^+}\simeq G_u^{\pi^-}/G_u^{\pi^+}\simeq\epsilon(x)$. Again these ratios are in principle directly measureable in lepton-induced reactions. In the limit that we drop $G_d^{\pi^+}$ and $G_u^{\pi^-}$ $(\langle\epsilon(x)\rangle\ll1)$ we see that Eq. (5.4b) assumes the very simple form $(G_u^{\pi^+}=G_d^{\pi^-}$, define $\sigma\equiv d\sigma/d\hat{t}+d\sigma/d\hat{u})$

$$\frac{\langle\pi^+\rangle}{\langle\pi^-\rangle}\simeq\frac{\langle\beta[\sigma(\beta+1)+\beta\sigma_{\rm int}]\rangle}{\langle\sigma(\beta+1)+\sigma_{\rm int}\rangle} \qquad (5.5a)$$

$$\simeq2\left(\frac{\langle3\sigma+2\sigma_{\rm int}\rangle}{\langle3\sigma+\sigma_{\rm int}\rangle}\right), \qquad (5.5b)$$

where in (5.5b) we have used the usual quark-model result $\langle\beta\rangle=2$. In Eq. (5.5) the integral implied by the brackets includes G^π in the weight function. Clearly in the absence of interference terms in the cross section we have $\langle\pi^+\rangle/\langle\pi^-\rangle=\langle\beta\rangle\sim2$ independent of the specific scattering process. Hence the ratio $\langle\pi^+\rangle/\langle\pi^-\rangle$ essentially measures the ratio of u quarks to d quarks in the proton at least in the limit $\langle\epsilon\rangle\ll1$. Using single-vector-gluon exchange as an example and keeping the interference terms we find that the quantity in the brackets $\{\ \}$ in Eq. (5.5) has the value 1.2 at $\theta\sim90°$ $(\sigma\sim\frac{5}{4}\sigma_{\rm int})$ and falls to 1.1 at $\theta\sim40°$ or 140° (it will eventually go to unity at very small angles). Hence even with this complication $\langle\pi^+\rangle/\langle\pi^-\rangle$ essentially measures β for all θ's and all $x_\perp$ where the above approximations are valid. Note that unlike the ratio $\langle\pi^+ +\pi^-\rangle/\langle\pi^0\rangle$, Eq. (5.5) depends explicitly on assumptions about the F's and G's. In particular if $\beta=F_u^p/F_d^p$ exceeds 2 as $x\to1$, as suggested by a ratio $F_2^{ep}(x)/F_2^{en}(x)\to4$ as $x\to1$, the ratio $\langle\pi^+\rangle/\langle\pi^-\rangle$ should behave accordingly. Again, if the effects of the η and η' decays are not properly treated, a simple U(3) analysis suggests the above ratio may be as much as 30% lower in the data $(2.4\to1.6)$.

Turning now to the production of K's, again ignoring strange quarks in the initial state $(F_{u,d}^p\gg F_{s,\bar{s}}^p)$, we must in general consider the four independent production functions

$$G_u^{K^+}=G_d^{K^0}=G_{\bar{u}}^{K^-}=G_{\bar{d}}^{\bar{K}^0}, \qquad (5.6a)$$

$$G_d^{K^+}=G_u^{K^0}=G_{\bar{d}}^{K^-}=G_{\bar{u}}^{\bar{K}^0}, \qquad (5.6b)$$

$$G_u^{K^-}=G_d^{\bar{K}^0}=G_{\bar{u}}^{K^+}=G_{\bar{d}}^{K^0}, \qquad (5.6c)$$

$$G_d^{K^-}=G_u^{\bar{K}^0}=G_{\bar{d}}^{K^+}=G_{\bar{u}}^{K^0}. \qquad (5.6d)$$

For the various ratios in the process $p+p\to K+X$, where the K has large transverse momentum and c.m. angle near 90°, we have, using Eqs. (5.6),

$$\frac{\langle K^+\rangle}{\langle K^-\rangle}=\frac{\langle[\sigma(\beta^2+\beta)+\beta^2\sigma_{\rm int}]G_u^{K^+}+[\sigma(\beta+1)+\sigma_{\rm int}]G_d^{K^+}\rangle}{\langle[\sigma(\beta^2+\beta)+\beta^2\sigma_{\rm int}]G_u^{K^-}+[\sigma(\beta+1)+\sigma_{\rm int}]G_d^{K^-}\rangle}, \qquad (5.7a)$$

$$\frac{\langle K^0\rangle}{\langle\bar{K}^0\rangle}=\frac{\langle[\sigma(\beta^2+\beta)+\beta^2\sigma_{\rm int}]G_u^{K^0}+[\sigma(\beta+1)+\sigma_{\rm int}]G_d^{K^0}\rangle}{\langle[\sigma(\beta^2+\beta)+\beta^2\sigma_{\rm int}]G_u^{\bar{K}^0}+[\sigma(\beta+1)+\sigma_{\rm int}]G_d^{\bar{K}^0}\rangle}, \qquad (5.7b)$$

$$\frac{\langle K^+\rangle}{\langle K^0\rangle}=\frac{\langle[\sigma(\beta^2+\beta)+\beta^2\sigma_{\rm int}]G_u^{K^+}+[\sigma(\beta+1)+\sigma_{\rm int}]G_d^{K^+}\rangle}{\langle[\sigma(\beta^2+\beta)+\beta^2\sigma_{\rm int}]G_u^{K^0}+[\sigma(\beta+1)+\sigma_{\rm int}]G_d^{K^0}\rangle}. \qquad (5.7c)$$

If we again assume that the leading meson at large $p_\perp$ has the scattered quark as a valence quark, we have $G_u^{K^+}=G_d^{K^0}\gg G_u^{K^-}, G_d^{K^-}, G_u^{\bar{K}^0}, G_d^{\bar{K}^0}, G_d^{K^+}, G_u^{K^0}$. Again we can assume that all small ratios are parametrized by $\epsilon(x)$. In this case we find

$$\frac{\langle K^+\rangle}{\langle K^0\rangle}=\frac{\langle \pi^+\rangle}{\langle \pi^-\rangle}\sim 2 \text{ to } 2.4 \qquad (5.8)$$

with all the comments which applied for the pion ratio applying again here. In the limit that all the G^K except the ones in Eq. (5.6a) are small, we expect $\bar{K}/K\to 0$ [i.e., $\bar{K}/K\sim O(\langle\epsilon(x)\rangle)$]. In fact this ratio [Eqs. (5.7a) and (5.7b)] is a direct measure of the validity of our assumptions that (1) the nonwee quarks in the initial state (e.g., the proton) are valence quarks, and (2) leading $p_\perp$ hadrons have the scattered quark as a valence quark. Of course we are also assuming that we are at sufficiently large $p_\perp$ that the entire picture being described here is valid.

Since at any finite $x_\perp$ and finite s, the experimental value of $\langle K^-\rangle/\langle K^+\rangle$ will be nonzero, it is important to estimate how big the ratio can be just from quark-sea effects, i.e., from the fact that $\epsilon(x)$ is actually only expected to vanish as $x\to 1$. Then if $\langle K^-\rangle/\langle K^+\rangle$ is not approaching this value as s is increased at fixed $x_\perp$, we can feel fairly certain that our simple quark picture is invalid. In the absence of more complete data from lepton-induced reactions, we shall assume a form for the sea distribution utilizing the functions $V(x)$ and $C(x)$ introduced earlier [recall $F^p(x)=3V^p(x)+6C^p(x)$]. As an illustrative example we use the forms suggested in Ref. 10 which satisfactorily describe the ep data. Normalizing $\int F^p(x)dx$ to unity (we will only look at ratios), we have $V^p(x)\sim 1.75\sqrt{x}(1-x)^3$ and $C^p(x)\sim 0.35(1-x)^{7/2}$. Taking a similar form for $G^\pi\sim 2V^\pi+6C^\pi$ with $(1-x)^3\to 1-x$ and the over-all ratio of V and C contributions maintained, we use $V^\pi(x)\sim 1.17\sqrt{x}(1-x)$ and $C^\pi(x)\sim 0.16(1-x)^{3/2}$. This leads to the results illustrated in Fig. 13 which indicate fairly sizable deviations (20–30%) in $\langle\pi^-\rangle/\langle\pi^+\rangle$ from its limiting value for $x_\perp$ less than 0.6 due to the $q\bar{q}$ sea contribution. Likewise $\langle K^-\rangle/\langle K^+\rangle$ is less than 10% only for $x_\perp$ very near unity. Although the specific numbers depend on our choices of V and C, presumably the general magnitude is indicative of any reasonable quark model which has a background sea. Larger deviations (as are present in low-energy data and even at the ISR for $p_\perp\lesssim 3$ GeV/c) are expected to vanish in the $x_\perp$ fixed, $s\to\infty$ limit if the present model is valid (the limit should be approached from above). The measurement of $\langle K^-\rangle/$

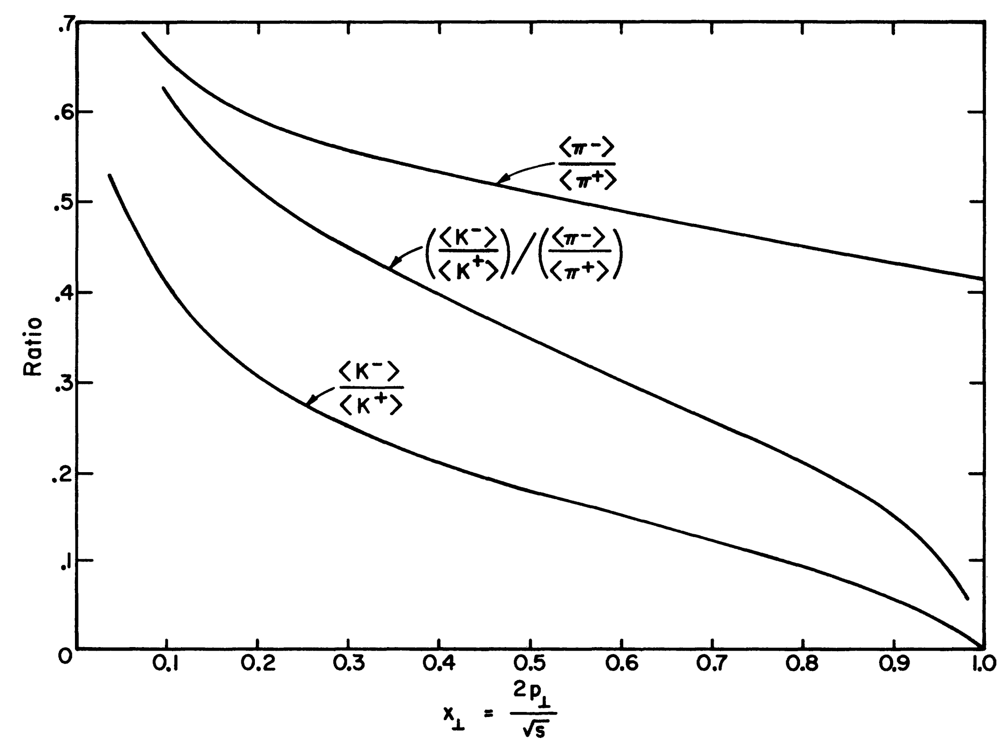

FIG. 13. Ratios of limiting ($s\to\infty$, $x_\perp$, θ fixed) mean charged multiplicities including the effect of an assumed $q\bar{q}$-sea contribution.

$\langle K^+\rangle$ is particularly interesting, as mentioned above, but the situation is obscured at present by the fact that even at small $p_\perp$ the experimental value for the ratio does not match the predictions of simple (Pomeranchukon dominated) models, i.e., $\langle K^-\rangle \approx \langle K^+\rangle$ is not presently true for $p_\perp \lesssim 1$ GeV/c.

If we now make the much stronger assumption of SU(3) symmetry for the G functions, we obtain the result that $G_u^{\pi^+} = G_u^{K^+}$ and hence

$$\frac{\langle \pi^+\rangle}{\langle K^+\rangle} = \frac{\langle \pi^-\rangle}{\langle K^0\rangle} = 1. \tag{5.9}$$

However, SU(3) is badly broken at small transverse momentum and within the present picture it is difficult to predict how much and how rapidly SU(3) symmetry improves (if at all) as we increase the transverse momentum. Hence the value of Eq. (5.9) as a usable prediction is rather limited. At the same time measurements of $\langle K^+\rangle/\langle \pi^+\rangle$ as a function of $p_\perp$ will have important implications for the dynamics of SU(3) breaking, at least within the present framework.[33]

Returning to Eq. (5.3) a similar analysis for pn scattering yields identical results for the ratios $(\pi^+ + \pi^-)/2\pi^0$ and $\bar{K}/K$, but a value 1 for $\pi^+/\pi^- = K^+/K^0$ for $\theta \approx 90°$. By the same methods one can also analyze the various πN interactions. All of these results for the limiting production of mesons at large $p_\perp$ for one-vector-gluon exchange at $\theta = 90°$ are summarized in Table I. Deviations such as illustrated in Fig. 13 due to the $q\bar{q}$ sea are expected throughout.

Finally we recall that our earlier assumptions about the $G^B(y)$ for the production of baryons behaving like νW_2 serves to suppress baryon production at large $x_\perp$ compared to mesons. It should be noted, however, that the ratio of protons to pions, for example, in the range $0.1 \leqslant x_\perp \leqslant 0.3$ (where some data exist) depends sensitively on the ratio of $G^p(y)/G^\pi(y)$ in a similar range for y. In the absence of complete knowledge of these functions the best statement that can be made is that this ratio for hadronic large-$p_\perp$ reactions should be similar to what is observed in inelastic ep and e^+e^- annihilation reactions. For our choice of normalizations we find $\langle \pi^+\rangle/\langle p\rangle \sim 2$ to 5 for $0.1 \leqslant x_\perp \leqslant 0.3$. Also by similar arguments to those for $\bar{K}$ production, the absence of $\bar{q}$'s in the initial state [to $O(\epsilon)$] will further suppress the production of antibaryons in pp reactions.

Note that within the context of our incoherent scattering picture one would expect little correlation of quantum numbers between the two transverse jets. However, if we include the interference terms in the scattering of two identical quarks, correlations can arise (just as $\langle \pi^+\rangle/\langle \pi^-\rangle$ is found to be 2.4 instead of 2). A naive analysis

TABLE I. Naive limiting charged multiplicity ratios ($s \to \infty$, x nonwee, $\langle\epsilon\rangle \ll 1$).[a]

Process	$\frac{\langle\pi^+ + \pi\rangle}{2\langle\pi^0\rangle}$	$\frac{\langle\pi^+\rangle}{\langle\pi^-\rangle} \sim \frac{\langle K^+\rangle}{\langle K^0\rangle}$	$\frac{\langle K^+\rangle}{\langle K^-\rangle}$	$\frac{\langle K^0\rangle}{\langle \bar{K}^0\rangle}$	$\frac{\langle K^+ + K^0\rangle}{\langle K^- + K^0\rangle}$
pp	1	2 (2.4)[b]	$\gg 1$	$\gg 1$	$\gg 1$
pn	1	1	$\gg 1$	$\gg 1$	$\gg 1$
$\pi^+ p$	1	5 (6.6)	$\gg 1$	$\frac{2}{3}$	3 (4.7)
πn	1	$\frac{1}{5}\left(\frac{1}{6.6}\right)$	$\frac{2}{3}$	$\gg 1$	3 (4.7)
$\pi^- p$	1	$\frac{1}{2}\left(\frac{1}{2.4}\right)$	$\frac{4}{3}$	$\gg 1$	3 (3.5)
$\pi^+ n$	1	2 (2.4)	$\gg 1$	$\frac{4}{3}$	3 (3.5)

[a] In general the ratios which are not unity will be much larger (or smaller if <1) if $F_u^p/F_d^p \gg 2$ for $x \to 1$.

[b] The numbers in parentheses indicate the effect of interference terms in $d\sigma/d\hat{t}$ for one-vector-gluon exchange.

keeping only valence effects and ignoring all the complications discussed above suggests that if we observe a leading π^+ in one jet near $\theta = 90°$ (i.e., a u quark was scattered out), then the ratio of π^+ to π^- on the other side is enhanced as

$$\left(\frac{\langle\pi^+\rangle}{\langle\pi^-\rangle}\right)_{\pi^+ \text{ opposite jet}} \approx \left\langle \frac{F_u^p}{F_d^p}\left(\frac{\sigma + \sigma_{\text{int}}}{\sigma}\right)\right\rangle \tag{5.10a}$$

$$\approx 2(1.8) = 3.6\ . \tag{5.10b}$$

(We have again taken specific numbers from the vector-gluon case for $\theta \approx 90°$, $\sigma \approx \frac{5}{4}\sigma_{\text{int}}$.) This result is to be compared with the value (2.4) obtained for the case of no measurement of the opposite jet. An identical result holds for the observation of a K^+ in the other jet and the ratio $\langle K^+\rangle/\langle K^0\rangle$. Hence the inclusion of interference terms in the parton-parton cross section (a point which is not totally unambiguous theoretically) can in fact lead to strong correlations. This should prove to be an interesting experimental question.

Let us now briefly review our results. The first and overriding assumption is that in the limit $x_\perp$ fixed, $s \to \infty$ the parton picture described here has some validity. Having accepted this we identified spin-$\frac{1}{2}$ partons with quarks and assumed that the quarks (as opposed to neutral gluons) are the dominant participants from the initial state in large-transverse-momentum events. Within this framework we are then able to make statements about the average multiplicities of various species of hadrons at fixed $x_\perp$ (we are implicitly assuming that for any given $x_\perp$ we are, on the average, looking at the largest $p_\perp$ hadron present, which is true in explicit calculations). In particular the ratios of of such multiplicities are relatively independent of the details ($d\sigma/d\hat{t}$) of parton-parton scattering but depend on the distributions and quantum numbers.

With the assumption of isospin and charge-conjugation invariance (plus partons with isospin less than or equal to $\frac{1}{2}$ as with quarks) we find $\langle \pi^+ + \pi^- \rangle / 2\langle \pi^0 \rangle = 1$. Then we identified nonwee quarks in the initial state with valence quarks and assumed that the most energetic hadron in a given jet in the final state has the scattered quark as a valence quark. This led to the simple limiting ratios shown in Table I. For example $\langle \pi^+ \rangle / \langle \pi^- \rangle$ measures essentially the number of u quarks to the number of d quarks in the initial hadrons (if care is taken with events where an η or η' decays into the observed π).

These results are valid within the naive quark model in the limit that the $q\bar{q}$ sea does not contribute. Although the contribution from the $q\bar{q}$ sea is in principle calculable from lepton-induced-reaction data, we are at present only able to estimate this effect as illustrated in Fig. 13. If the quark model as described here is valid, the data should approach these curves from above as $s \to \infty$, $x_\perp$ fixed. The ratio $\langle K^- \rangle / \langle K^+ \rangle$ is particularly sensitive both to the contribution of the $q\bar{q}$ sea and deviations from the quark picture.

We note in passing that contributions from neutral gluons in the initial state which scatter to form hadrons will contribute to $\langle \pi^+ \rangle / \langle \pi^- \rangle \sim \langle K^+ \rangle / \langle K^- \rangle \sim 1$. We anticipate that this contribution is small but in any case it can be eliminated by considering, for example, the ratio of the differences $(pp \to K^+ + x) - (pn \to K^+ + x)$ and $(pp \to K^- + x) - (pn \to K^- + x)$, which should be free of SU(2) singlet gluon effects in the initial states.

Then we pointed out that the assumption of SU(3) symmetry leads to $\langle \pi^+ \rangle \sim \langle K^+ \rangle$, $\langle \pi^- \rangle \sim \langle K^0 \rangle$, etc. The theoretical validity of the assumption is an open question but its experimental validity as a function of $p_\perp$ should be most informative. Finally we reminded the reader that our assumption about the direct relation between F's and G's $[F^B \sim G^B \sim (1-y)^3]$ leads to the suppression, relative to mesons, of baryon production at large $x_\perp$, although the ratio at small $x_\perp$ depends in detail on the functions $G^B(y)$ and $G^\pi(y)$. Antibaryon production is further suppressed in pp reactions by the lack of antiquarks in the initial state except in the $q\bar{q}$ sea.

In general, for specific charge states in $pp \to (\ ) + X$, we expect $\langle \pi^+ \rangle \gtrsim \langle K^+ \rangle > \langle p \rangle$ and $\langle \pi^- \rangle \gg \langle K^- \rangle > \langle \bar{p} \rangle$, where $\langle \pi^+ \rangle \gtrsim 2\langle \pi^- \rangle$, $\langle K^+ \rangle \gg \langle K^- \rangle$, and $\langle p \rangle \gg \langle \bar{p} \rangle$. Except for the discussion of η and $\eta' \to \pi$'s, we have systematically ignored the complication due to the decay of initially produced, higher-mass mesons.[29] The general effect will presumably be to obscure the simple results given here via the production of pairs $(\pi^+\pi^-, K\bar{K})$ in the decay. A similar effect will result if gluon scattering plays a sizable role, particularly for $\langle \bar{K} \rangle / \langle K \rangle$ and $\langle \bar{p} \rangle / \langle p \rangle$.

Finally we note that the results of this section follow largely from quark quantum numbers and assumptions about the behavior of the F and G distribution functions, not parton-parton dynamics. A more direct test of the basic quark-quark scattering picture utilizing quantum numbers and multiplicities, and tainted by a minimum of assumptions, is obtained by checking the relationship between the spectra observed in hadronic reactions and those in lepton-induced reactions as given by the model. The only problem here is the contribution of gluon scattering.

VI. CONTRIBUTIONS TO $d\sigma/dt|_{\text{elastic}}$ AND σ_{tot}

In the preceding sections we have considered the effect in inclusive processes of assuming that there exists an interaction between partons which does not vanish as the rapidity difference of the two partons becomes very large. Further, we assumed that this scattering occurred essentially in isolation from the other partons whenever the momentum exchanged is large enough. By assuming that this process in fact should explain the observed large-$p_\perp$ events in inclusive reactions we arrived at an estimate of the size of the parton-parton cross section. Since we may expect this process to contribute also to the elastic amplitude and likewise to the total cross section, one must determine that this does not yield contradictory results.

In the elastic case one expects a direct contribution from the hard scattering of two partons which then reassociate with the other partons to form outgoing protons, for example. In the usual picture this occurs with reasonable probability only when the incoming protons each consist of one parton with x near unity and the others near $x = 0$.[2,22,34] This picture is essentially identical to the one which appears in the calculation of electromagnetic form factors (we here do not differentiate between the two form factors of the proton) and leads to the Drell-Yan-West[35] relationship between the electromagnetic form factor of a hadron and the parton distribution function $F(x)$ for that hadron. We are thus led to insert the electromagnetic form factor $F(t)$ for the vertex which appears in the parton-parton scattering contribution to elastic hadron-hadron scattering (see Fig. 14). This leads to the following formula for Fig. 14.

$$\left.\frac{d\sigma}{dt}\right|_{\text{elastic hadron from elastic parton}} \sim F^4(t) \left.\frac{d\sigma}{d\hat{t}}\right|_{\hat{t}=t}, \qquad (6.1)$$

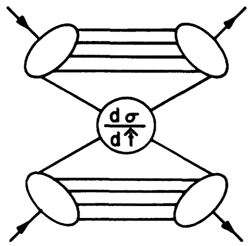

FIG. 14. Pictorial representation of parton-parton scattering contribution to elastic scattering.

where, as before, $d\sigma/d\hat{t}$ is the elastic parton-parton cross section and we have suppressed the implied summation over the various partons [by identifying $F(t)$ with the electromagnetic form factor we have excluded contributions from neutral partons, e.g., gluons]. This is presumed to be a valid description of the parton-parton contribution whenever $|t|>|t_{\min}|$ although there is no assurance that this is the dominant contribution. For the example of elastic proton-proton scattering it is assumed that the appropriate form of $F(t)$ is $(1-t/M_V^2)^{-2}$ where $M_V^2\sim 0.7$ GeV^2, i.e., we chose the normalization $F(0)=1$. This choice is not without some ambiguity although it seems reasonable if we consider $F(t)$ is measuring the probability that the proton "puts itself back together" after the hard parton-parton collision. Using this normalization and the single vector gluon $d\sigma/d\hat{t}$ with $\alpha_{\text{eff}}\sim 0.5$, we find an elastic cross-section contribution well below the observed values for large t (see Fig. 15). We conclude that for exclusive processes parton-parton scattering is a small effect and that some process involving a more collective interaction of the partons is the dominant feature. Such a result is not surprising and we consider that it is not in contradiction with our naive picture.

We make a short detour here to point out the amusing result that the essential behavior of Eq. (5.1) (at least for $\theta\approx 90°$) can be obtained via Eq. (3.1) and the "correspondence principle" of Bjorken and Kogut.[36] The essential idea is that exclusive process $pp\to pp$ should be related to the inclusive process $pp\to pp+X$ if we integrate over the region very close to the edge of phase space. Here we choose to define "close" in terms of the parameter M_V as generally used in the derivation of the Drell-Yan-West relation. Changing variables in Eq. (3.1) from $p_1, p_2, \theta_1, \theta_2$ $(\theta_1\approx\pi-\theta_2\approx\theta)$ to t, y, z, w, where

$$y = x\tan(\tfrac{1}{2}\theta_1)\tan(\tfrac{1}{2}\theta_2),$$

$$z = \frac{x_{\perp 1}}{2x}[\cot(\tfrac{1}{2}\theta_1)+\cot(\tfrac{1}{2}\theta_2)],$$

and

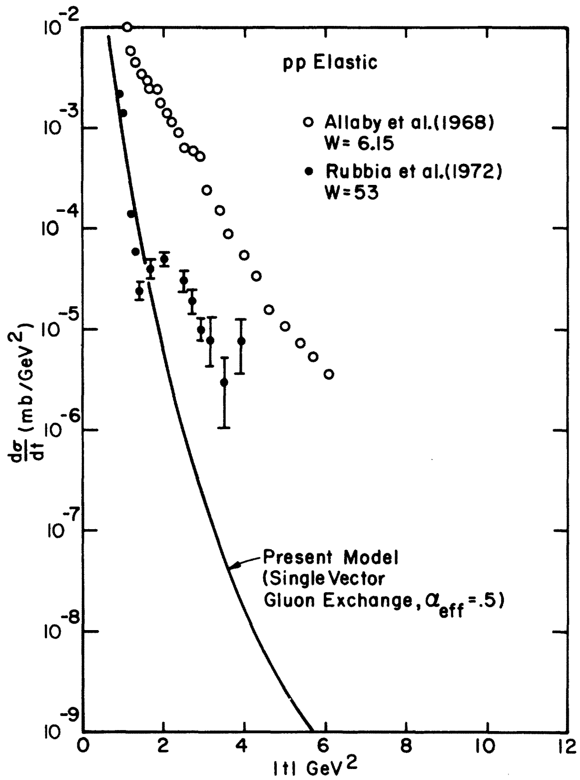

FIG. 15. Comparison of parton-parton scattering contribution to elastic scattering with data. Normalization is defined in the text.

$$w = \frac{x_{\perp 2}}{2x}[\cot(\tfrac{1}{2}\theta_1)+\cot(\tfrac{1}{2}\theta_2)],$$

we have in the limit x, y, z, w all approach unity, $x_{\perp 1}\approx x_{\perp 2}\approx\sin\theta$ and

$$\frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\approx\frac{s}{8}\,d\phi_1 d\phi_2 dt\,dy dz dw\,\sin^2\theta. \qquad (6.2)$$

Substituting this into Eq. (3.1) we find

$$\left.\frac{d\sigma}{dt}\right|_{\text{el}}\sim\int\!\!\int\!\!\int\!\!\int_{1-(M_V/\sqrt{-t})}^{1} dx dv dz dw \times F(x)F(y)G(z)G(w)\left.\frac{d\sigma}{d\hat{t}}\right|_{\hat{t}\sim t}. \qquad (6.3a)$$

Now we take $F\approx G\approx C(1-x)^3$ for $x\to 1$, and we find

$$\left.\frac{d\sigma}{dt}\right|_{\text{el}}\sim\left(\frac{C}{4}\right)^4\left(\frac{M_V}{\sqrt{-t}}\right)^{16}\frac{d\sigma}{d\hat{t}}. \qquad (6.3b)$$

For large t this is the same behavior as Eq. (6.1). Again the over-all normalization is ambiguous, essentially for the same reasons as in the form-factor case, i.e., how are C and M_V chosen?

As was pointed out by Susskind and collaborators,[9] before either effect was observed, it is natural within the parton model to have both an "abundance" of large-transverse-momentum events and a term in σ_{tot} which rises as $\ln^2 s$. The subsequent observation of both effects must be considered as at least encouraging for the parton picture. The first effect we discussed above. We will point out now how the second effect arises within the framework we have developed. The essential assumption is that there exists a fixed minimum momentum transfer $|\hat{t}_{min}|$, such that for $|\hat{t}| > |\hat{t}_{min}|$ our previous analysis is valid. In this case

$$\int_{-s/2}^{\hat{t}_{min}} d\hat{t}\,(d\sigma/d\hat{t})$$

may have a term independent of s, e.g., for the one-gluon case the result is

$$2\pi\alpha_{eff}^2\left(\frac{1}{-\hat{t}_{min}} - \frac{2}{s}\right),$$

assuming that M_{VG} can be ignored. Since the number of partons which can participate is determined by $\frac{1}{2}\hat{s} = (x_1 x_2/2)s > |\hat{t}_{min}|$, i.e., $p_{parton} > (\frac{1}{2}|\hat{t}_{min}|)^{1/2}$, this number grows with s. For the sort of models discussed here the number of eligible partons grows as $\ln s$ in each hadron (we eventually allow wee partons to participate). Assuming no shielding effects over a finite range in s, this leads to a $\ln^2 s$ contribution to σ_{tot}. More explicitly we have

$$\Delta\sigma_{tot} = \int_{2|\hat{t}_{min}|/s}^{1} \frac{dx_1}{x_1} F(x_1) \int_{2|\hat{t}_{min}|/x_1 s}^{1} \frac{dx_2}{x_2} F(x_2) \times \int_{-\hat{s}/2}^{\hat{t}_{min}} \frac{d\sigma}{d\hat{t}}\, d\hat{t} \tag{6.4a}$$

$$\approx \frac{2\pi\alpha_{eff}^2}{|\hat{t}_{min}|}\left(\ln\frac{s}{|\hat{t}_{min}|}\right)^2 F^2(0) + O\left(\ln\frac{s}{|\hat{t}_{min}|}\right). \tag{6.4b}$$

An equal contribution is obtained by integrating over $d\sigma/d\hat{u}$. Clearly if $|\hat{t}_{min}| \sim x_{\perp min}^2 s/2$ instead of a constant, no interesting effect would arise, although our large $p_\perp$ analysis would survive for $x_\perp > x_{\perp min}$. If, as an optimistic example, we take $\alpha_{eff} \sim 0.3$, $F(0) \sim 1$, and $|\hat{t}_{min}| \sim 2$ GeV^2, we obtain a value for $\Delta\sigma_{tot} \sim 0.2 \ln^2 s$ (mb) which is at least suggestive of the rise now observed at the ISR[37] although there are clearly uncertainties of factors of two or more in the present analysis. This is not expected to be the true asymptotic behavior since shielding will eventually set in. In any case more complete experimental studies of both large $p_\perp$ effects and a rising σ_{tot} should help test the parton picture via the suggested correlation. As noted earlier, similar assumptions yield a single-particle cross section which asymptotically rises as $\ln(\sqrt{s}/p_\perp)$ as fixed $p_\perp$.

Note that only these last results $[\sigma_{tot} \sim \ln^2 s$, $E(d\sigma/d^3p) \sim \ln s]$ plus the multiplicity formulas Eqs. (6.2b) and (6.2c) require the assumption that the parton-parton scattering model be valid for $p_\perp \geq p_{\perp min}$ instead of just $x_\perp \geq x_{\perp min}$. Since it is difficult to theoretically decide which criterion is appropriate, it will be interesting to see what the data say about which predictions are correct. For example, if the cross sections of Sec. II are valid at fixed $x_\perp$ but the data do not rise as $\ln s$ at fixed $p_\perp$, the region of validity of the model will be clearly defined.

CONCLUSION

In this final section we shall attempt to review our results and put them into perspective. Our basic assumptions are that hadrons have a representation in terms of partons, which are treated essentially as elementary particles, and that under certain kinematic conditions these partons can interact independently and incoherently from the other partons present in the initial hadrons. This led directly to simple cross-section formulas for various hadronic inclusive processes which are of interest if the partons have interactions with a long range in rapidity. In particular, we attempted to motivate the application of parton concepts to large-transverse-momentum hadronic events by analogy with deep-inelastic leptonic processes, where an impulse approximation is presumably valid and the model seems successful. This required not only the usual caveats about ignoring standard perturbation-theory results but also an explicit assumption as to the validity of the impulse approximation via one-gluon exchange, for example. The quantities which appear in these cross sections are related to measurements made in lepton-induced processes, and to the basic parton-parton interaction. Thus the exploration of large-transverse-momentum hadronic events may serve not only to confirm (or disprove) parton-model ideas but also to measure directly parton-parton scattering.

The essential features of inelastic, large-transverse-momentum events which are characteristic of simple parton-parton scattering models are the two-jet, coplanar kinematic structure and the logarithmic multiplicities in the jets.

The general form of the single-particle inclusive cross section is, ignoring quantum numbers for now ($s \to \infty$, $p_\perp/\sqrt{s}$, θ fixed, see Fig. 1),

$$E\frac{d\sigma}{d^3p} \sim \frac{4}{\pi x_\perp^2}\int dx_1 dx_2 F(x_1)F(x_2)\frac{\eta}{(1+\eta)^2}\frac{d\sigma}{d\hat{t}}(\hat{s},\hat{t}) \times G\left(\frac{x_\perp(1+\eta)}{2x_1\tan(\frac{1}{2}\theta)}\right), \tag{7.1}$$

where $\eta=(x_1/x_2)\tan^2(\frac{1}{2}\theta)$, θ is the c.m. scattering angle, and $x_\perp=2p_\perp/\sqrt{s}$. For simple forms of the parton-parton elastic cross section $d\sigma/d\hat{t}$ one finds

$$E\frac{d\sigma}{d^3p}\sim s^{-n}F(x_\perp,\theta)\quad(s\to\infty;\ x_\perp,\theta\text{ fixed}).\tag{7.2}$$

The case $n=2$ follows either from gluon models with fermion partons or from assuming that naive scaling, i.e., dimensional analysis, is appropriate. The latter picture has at least the virtue of phenomenological simplicity. Other possibilities include $n=0$ for free-fermion interactions and $n=4$ for a purely scalar model. The specific example of one-vector-gluon exchange yields (t- and u-channel exchanges added incoherently)

$$E\frac{d\sigma}{d^3p}\sim\frac{8\alpha_{\text{eff}}{}^2}{s^2x_\perp{}^2}\int\frac{dx_1}{x_1^2}\frac{dx_2}{x_2^2}F(x_1)F(x_2)G\left(\frac{x_\perp(1+\eta)}{2x_1\tan(\frac{1}{2}\theta)}\right)\frac{\eta}{(1+\eta)^2}\left[(1+\eta)^2+\eta^2+\frac{1}{\eta^2}+\left(1+\frac{1}{\eta}\right)^2\right],\tag{7.3}$$

which gives reasonable agreement with current data[20] for $\alpha_{\text{eff}}\sim0.3$. Although the general shape of the cross section is independent of specific assumptions, this particular value of α_{eff} depends on our choice of $d\sigma/d\hat{t}$ and normalization of the distributions F and G.

We also exhibited the 2-hadron and 2-jet cross sections:

$hh\to h_1h_2+X$ (one hadron in each jet),

$$\frac{d\sigma}{(d^3p_1/E_1)(d^3p_2/E_2)}\sim\frac{16}{\pi s x_{\perp1}{}^2x_{\perp2}{}^2}\int dx\,x\frac{F(x)F(x\tan(\frac{1}{2}\theta_1)\tan(\frac{1}{2}\theta_2))}{[\cot(\frac{1}{2}\theta_1)+\cot(\frac{1}{2}\theta_2)]^2}\times G_1\left\{\frac{x_{\perp1}}{2x}[\cot(\tfrac{1}{2}\theta_1)+\cot(\tfrac{1}{2}\theta_2)]\right\}G_2\left\{\frac{x_{\perp2}}{2x}[\cot(\tfrac{1}{2}\theta_1)+\cot(\tfrac{1}{2}\theta_2)]\right\}\times\frac{d\sigma}{d\hat{t}}\,\delta(\phi_1-\phi_2+\pi),\tag{7.4}$$

$hh\to\text{jet}_1+\text{jet}_2+X$,

$$\frac{d\sigma_{\text{jet}}}{(d^3p_1/E_1)(d^3p_2/E_2)}\sim\frac{1}{\pi}F(x_1)F(x_2)\frac{d\sigma}{d\hat{t}}(\hat{s},\hat{t})\delta^2(\vec{p}_{\perp1}+\vec{p}_{\perp2}).\tag{7.5}$$

In the last case

$$x_1=(y_\perp/2)[\cot(\tfrac{1}{2}\theta_1)+\cot(\tfrac{1}{2}\theta_2)],$$

$$x_2=(y_\perp/2)[\tan(\tfrac{1}{2}\theta_1)+\tan(\tfrac{1}{2}\theta_2)],$$

$$y_\perp=2p_{\perp\text{jet}}/\sqrt{s}\,,\quad\hat{s}=x_1x_2s\,,$$

and

$$\hat{t}=(-s/4)y_\perp{}^2[1+\tan(\tfrac{1}{2}\theta_1)\cot(\tfrac{1}{2}\theta_2)].$$

Thus, with sufficiently clever experiments, one may either confirm or dismiss the simple parton-scattering picture as applied to hadronic reactions. By individually varying x_1 and x_2 one can check if the observed $F(x)$ conforms to leptonic results. If the results are affirmative, one may interpret the $\hat{s}$ and $\hat{t}$ dependence as exhibiting the behavior of parton-parton scattering.

We discussed at some length the question of the mean multiplicities of large-transverse-momentum hadrons ($x_\perp>x_{\perp\min}$) in the jets. The characteristic behavior of current naive parton models is a mean multiplicity which varies as the logarithm of the total jet momentum. However, if we select events by requiring one hadron to have a certain $x_{\perp c}$, the structure of the jet in which that hadron is observed will in general not correspond to that which yields the logarithmic mean multiplicity. Thus, although we can infer from the observed $x_{\perp c}$, the average momentum of the parton from which it came ($\langle p_\perp\rangle_{\text{parton}}$), the mean multiplicity of that jet will not behave as $\ln\langle p_\perp\rangle_{\text{parton}}$. In fact for increasing $x_{\perp c}\gtrsim0.4$, s fixed, we expect the multiplicity of hadrons, $x_\perp>x_{\perp\min}$, to approach unity. The other transverse jet should satisfy the naive analysis and look very similar to what is observed in the lepton-induced case. It should be noted that all these results are independent of specific details of $d\sigma/d\hat{t}$ and the distribution function $G(y)$ except that $G(y)\neq0$.

We also looked into the question of what quantum numbers we expected for the leading transverse-momentum particles. This point was approached by identifying partons with quarks. This leads directly to the result $\langle\pi^++\pi^-\rangle/2\langle\pi^0\rangle\approx1$, where $\langle\ \rangle$ means average multiplicity at some finite $x_\perp$, i.e., we measure one (presumably leading) π per event and average over many events. If we assume that for $x_\perp\gtrsim0.3$, we see only valence quarks in the initial state and that the leading transverse meson has the scattered quark as a valence quark, we find for $pp\to$ meson $+X$, $\langle\pi^+\rangle/\langle\pi^-\rangle=\langle K^+\rangle/\langle K^0\rangle=2$ (2.4 if we include interference terms in the scattering of identical quarks in the vector-gluon example) which just measures the ratio of u quarks to d quarks in the proton. Under the same conditions we have $\langle \bar{K} \rangle / \langle K \rangle \ll 1$ and $\langle \bar{B} \rangle / \langle B \rangle \ll 1$. SU(3) symmetry would give $\langle K^+ \rangle = \langle \pi^+ \rangle$ etc. We noted how the inclusion of $q\bar{q}$ sea effects introduce $x_\perp$ dependence in the above ratios. We pointed out that the decay of higher-mass resonances, e.g., η and η', into π's could tend to obscure our results if such events were not properly treated. Other points were the correlation between jets due to the enhancement of identical quark scattering if interference terms are kept and the suppression of baryons at large $x_\perp$ relative to mesons due to our assumption that $G^B(y) \sim \nu W_2(y) \sim (1-y)^3$ as $y \to 1$. Although these results are in general independent of details of parton-parton dynamics, they involve some fairly specific assumptions about the behavior of the G's, particularly the last result concerning the ratio of baryons produced at large $p_\perp$ compared to mesons. A definitive test of the basic quark-quark picture, with a minimum of assumptions, will involve a careful test of the predicted relation between the quantum number and multiplicity structure of large-transverse-momentum hadron jets and the corresponding distributions measured in lepton-induced reactions. In this case the only remaining major assumption is the absence of gluon-scattering contributions.

In Sec. VI we discussed contributions to $d\sigma/dt|_{\text{elastic}}$ and σ_{tot} due to parton-parton scattering effects. In the former case we expect the contribution is negligible although the normalization is not unambiguous. In the latter case, the assumption of a finite $\hat{t}_{\min}$ where parton-parton scattering is important leads to a $\ln^2(s/|\hat{t}_{\min}|)$ term in σ_{tot} which could be of the same order of magnitude as the observed rise. Under the similar assumptions the single-particle cross section at fixed $p_\perp$ will asymptotically behave as $\ln(\sqrt{s}/p_\perp)f(\theta)$. These last specific results depend on the assumption of a one-gluon-exchange form for $d\sigma/d\hat{t}$.

The general conclusion is that naive parton ideas plus reasonable specific assumptions yield some fairly definite predictions for large-transverse-momentum inelastic hadronic processes. These are at present in general agreement with data,[20,21] and the outlook for clean tests in the near future at NAL and the ISR is very good. We have then a very good chance of soon determining the viability of parton ideas as the basis of an understanding of hadron physics.

We close with a short comment about a possible implication of the present work for inelastic leptonic processes. If the elementary gluon-parton coupling α_{eff} is of order 0.5 or less as suggested here, it is not impossible that scale-breaking factors of the form suggested by naive perturbation theory calculations,[15] i.e., $(q^2/M^2)^{\alpha/\pi}$, could have escaped detection at SLAC. Such effects, if present, should, however, be observable at NAL.

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APPENDIX

In the following we shall briefly indicate how the various formulas in the text can be arrived at from the parton picture. All are understood to apply in the limit $s \to \infty$, and $x_\perp$, θ fixed.

In terms of the distribution functions defined in Sec. II, we have for the general parton-parton contribution to the single-particle cross section (see Fig. 1).

$$d\sigma = \frac{dx_1}{x_1} F(x_1) \frac{dx_2}{x_2} F(x_2) \frac{d\sigma}{d\hat{t}} (\hat{s}, \hat{t})\, d\hat{t}\, G(y)\, dy/y \,. \tag{A1}$$

To arrive at the single-particle inclusive cross section we need to define $\hat{t}$ and y in terms of the observed hadron's momentum. We have

$$y = \frac{p_\perp}{\sqrt{s}} \left[\frac{\tan(\frac{1}{2}\theta)}{x_2} + \frac{\cot(\frac{1}{2}\theta)}{x_1} \right], \tag{A2a}$$

$$\hat{t} = -x_1 s \tan(\tfrac{1}{2}\theta) \Big/ \left[\frac{\tan(\frac{1}{2}\theta)}{x_2} + \frac{\cot(\frac{1}{2}\theta)}{x_1} \right] \tag{A2b}$$

which yields

$$dy d\hat{t} = \frac{\sqrt{s}}{\sin(\frac{1}{2}\theta)\cos(\frac{1}{2}\theta)\left[\frac{\tan(\frac{1}{2}\theta)}{x_2} + \frac{\cot(\frac{1}{2}\theta)}{x_1} \right]} \, dp_\perp \, d\theta \,. \tag{A3}$$

Then using

$$dp_\perp \, d\theta = \frac{\sin\theta}{2\pi p_\perp} d^3p/E \,, \tag{A4}$$

we have

$$\frac{d\sigma}{d^3p/E} \sim \int \frac{dx_1}{x_1} F(x_1) \frac{dx_2}{x_2} F(x_2) \frac{d\sigma}{d\hat{t}} G\left(\frac{p_\perp}{\sqrt{s}} \left[\frac{\tan(\frac{1}{2}\theta)}{x_2} + \frac{\cot(\frac{1}{2}\theta)}{x_1} \right] \right) \frac{(\sqrt{s})^2}{\pi (p_\perp)^2 \left[\frac{\tan(\frac{1}{2}\theta)}{x_2} + \frac{\cot(\frac{1}{2}\theta)}{x_1} \right]^2} \,, \tag{A5a}$$

$$\frac{d\sigma}{d^3p/E} \sim \frac{4}{\pi x_\perp^2} \iint dx_1\, dx_2\, F(x_1)\, F(x_2)\, \frac{d\sigma}{d\hat{t}}\, G\left(\frac{x_\perp}{2}\, \frac{(1+\eta)}{x_1 \tan(\frac{1}{2}\theta)}\right) \frac{\eta}{(1+\eta)^2}\,, \tag{A5b}$$

where $x_\perp = 2p_\perp/\sqrt{s}$ and $\eta = (x_1/x_2)\tan^2(\frac{1}{2}\theta)$. Recall that we have simplified our analysis by ignoring the transverse distribution implicit in F and G, i.e., took it to be a δ-function distribution and did the $d^2k_\perp$ integral trivially.

The reader may now use his favorite parton cross section to calculate the hadronic result. The single-vector-gluon result follows directly from the $d\sigma/d\hat{t}$ given in Fig. 2. The limits of integration result from requiring the arguments of the various distributions to be bounded by unity. This yields $Tx_1 x_\perp/(2x_1 - Cx_\perp) \leq x_2 \leq 1$ and $Cx_\perp/(2 - Tx_\perp) \leq x_1 \leq 1$, where $T = \tan(\frac{1}{2}\theta)$ and $C = \cot(\frac{1}{2}\theta)$.

For two hadrons in the final state we have (see Fig. 8)

$$d\sigma = \frac{dx_1}{x_1}\, F(x_1)\, \frac{dx_2}{x_2}\, F(x_2)\, \frac{d\sigma}{d\hat{t}}\, d\hat{t}\, \frac{dx}{x}\, G(x)\, \frac{dy}{y}\, G(y)\,. \tag{A6}$$

In this case the Jacobian is

$$\begin{aligned} dx_2\,dx\,dy\,d\hat{t} &= 2x_1 \frac{\tan(\frac{1}{2}\theta_1)\tan(\frac{1}{2}\theta_2)}{\sin\theta_1 \sin\theta_2}\, dp_{\perp 1}\, dp_{\perp 2}\, d\theta_1\, d\theta_2 \\ &= \frac{2x_1 \tan(\frac{1}{2}\theta_1)\tan(\frac{1}{2}\theta_2)}{p_{\perp 1}\, p_{\perp 2}\, 2\pi}\, \frac{d^3p_1}{E_1}\, \frac{d^3p_2}{E_2}\, \delta(\phi_1 - \phi_2 + \pi)\,. \end{aligned} \tag{A7}$$

Defining $x_{\perp 1} = 2p_{\perp 1}/\sqrt{s}$, $x_{\perp 2} = 2p_{\perp 2}/\sqrt{s}$, and $\eta = \tan(\frac{1}{2}\theta_1)\cot(\frac{1}{2}\theta_2)$, we arrive at

$$\begin{aligned} E_1 E_2 \frac{d\sigma}{(d^3p_1)(d^3p_2)} &\sim \frac{16}{\pi s\, x_{\perp 1}^2\, x_{\perp 2}^2} \int \frac{dx_1 x_1\, F(x_1)\, F(x_1 \tan(\frac{1}{2}\theta_1)\tan(\frac{1}{2}\theta_2))}{(1+\eta)^2} \tan^2(\tfrac{1}{2}\theta_1) \\ &\quad \times \frac{d\sigma}{d\hat{t}}\, G_1\left(\frac{x_{\perp 1}}{2x_1}\, \frac{1+\eta}{\tan(\frac{1}{2}\theta_1)}\right) G_2\left(\frac{x_{\perp 2}}{2x_1}\, \frac{1+\eta}{\tan(\frac{1}{2}\theta_1)}\right) \delta(\phi_1 - \phi_2 + \pi)\,. \end{aligned} \tag{A8}$$

We note for completeness that $\hat{t} = -x_1^2 s \tan^2(\frac{1}{2}\theta_1)/(1+\eta)$ and x_2 in Eq. (A6) is given by $x_2 = x_1 \tan(\frac{1}{2}\theta_1) \times \tan(\frac{1}{2}\theta_2)$. The limit of the integration is given by

$$\left[x_{\perp 1}\left(\frac{1+\eta}{2\tan(\frac{1}{2}\theta_1)}\right), x_{\perp 2}\left(\frac{1+\eta}{2\tan(\frac{1}{2}\theta_1)}\right)\right]_{\max} \leq x_1 \leq 1\,.$$

Finally, for the two-jet cross section, we consider the case where no G functions are present and define the cross section in terms of the outgoing parton momentum:

$$d\sigma \sim \frac{F(x_1)}{x_1}\, dx_1\, \frac{F(x_2)}{x_2}\, dx_2\, \frac{d\sigma}{d\hat{t}}\, d\hat{t}\,. \tag{A9}$$

Using $y_\perp = 2p_{\perp\text{jet}}/\sqrt{s}$, $\cos\theta_1$, $\cos\theta_2$ (see Fig. 8) as the independent variables we have

$$dx_1\, dx_2\, d\hat{t} = x_1 x_2 \frac{s y_\perp}{4\pi}\, \frac{dy_\perp\, d\cos\theta_1\, d\cos\theta_2}{\sin^2\theta_1 \sin^2\theta_2}\, d\phi_1\, d\phi_2 \times \delta(\phi_1 - \phi_2 + \pi)\,. \tag{A10}$$

This gives

$$\frac{d\sigma}{dy_\perp\, d\Omega_1\, d\Omega_2} \sim \frac{s y_\perp}{4\pi}\, \frac{F(x_1)\, F(x_2)}{\sin^2\theta_1 \sin^2\theta_2}\, \frac{d\sigma}{d\hat{t}}\, \delta(\phi_1 - \phi_2 + \pi), \tag{A11}$$

with

$$x_1 = (\tfrac{1}{2}y_\perp)[\cot(\tfrac{1}{2}\theta_1) + \cot(\tfrac{1}{2}\theta_2)]\,,$$

$$x_2 = (\tfrac{1}{2}y_\perp)[\tan(\tfrac{1}{2}\theta_1) + \tan(\tfrac{1}{2}\theta_2)]\,,$$

$$\hat{t} = -(\tfrac{1}{4}s)\, y_\perp^2 [1 + \tan(\tfrac{1}{2}\theta_1)\cot(\tfrac{1}{2}\theta_2)]\,,$$

and

$$\eta = \tan(\tfrac{1}{2}\theta_1)\cot(\tfrac{1}{2}\theta_2)\,.$$

Using

$$\begin{aligned} y_\perp\, dy_\perp\, d\cos\theta_1\, d\cos\theta_2 &= \frac{p_{\perp 1}\, dp_{\perp 1}\, p_{\perp 2}\, dp_{\perp 2}}{s/4}\, \delta(p_{\perp 1}^2 - p_{\perp 2}^2) \\ &\quad \times \frac{d\Omega_1\, d\Omega_2}{2\pi}\, \delta(\phi_1 - \phi_2 + \pi) \\ &= \frac{\sin^2\theta_1 \sin^2\theta_2}{\pi s/2}\, \delta^2(\vec{p}_{1\perp} + \vec{p}_{2\perp}) \\ &\quad \times \frac{d^3p_1}{E_1}\, \frac{d^3p_2}{E_2}\,, \end{aligned} \tag{A12}$$

where we have ignored masses in E, we arrive at

$$\frac{d\sigma_{\text{jet}}}{(d^3p_1/E_1)(d^3p_2/E_2)} \sim \frac{1}{\pi}\, F(x_1)\, F(x_2) \frac{d\sigma}{d\hat{t}}\, \delta^2(\vec{p}_{1\perp} + \vec{p}_{2\perp})\,. \tag{A13}$$

Finally, for the single-jet cross section we again start with Eq. (A9) and use $y_\perp = 2p_{\perp\text{jet}}/\sqrt{s}$. With

$$\hat{t} = -\frac{x_1 s y_\perp}{2} \tan(\tfrac{1}{2}\theta)$$

and

$$x_2 = \tan(\tfrac{1}{2}\theta) \Big/ \left[\frac{2}{y_\perp} - \frac{\cot(\frac{1}{2}\theta)}{x_1}\right],$$

the appropriate change of variables is

$$dx_2\, d\hat{t} = \frac{2x_1^2\, y_\perp \tan(\frac{1}{2}\theta)}{\pi[2x_1 - y_\perp \cot(\frac{1}{2}\theta)]^2}\, d^3p/E\ . \qquad \text{(A14)}$$

So with

$$\eta = [2\tan(\tfrac{1}{2}\theta)\, x_1 - y_\perp]/y_\perp$$

we have

$$\frac{d\sigma_{\text{jet}}}{d^3p/E} \sim \frac{2}{\pi} \int \frac{dx_1}{2x_1 - \cot(\frac{1}{2}\theta)\, y_\perp} F(x_1) \times F\left(x_1 \frac{\tan^2(\frac{1}{2}\theta)}{\eta}\right) \frac{d\sigma}{d\hat{t}}, \qquad \text{(A15)}$$

with integration limits

$$\frac{y_\perp \cot(\frac{1}{2}\theta)}{2 - y_\perp \tan(\frac{1}{2}\theta)} \leqslant x_1 \leqslant 1.$$

Again the reader is invited to use his favorite cross section for the partons. For completeness we write out the example of vector-gluon exchange without interference terms as was used for most of the calculations.

One particle:

$$\frac{d\sigma}{d^3p/E} \sim \frac{8\alpha_{\text{eff}}^2}{s^2 x_\perp^2} \int \frac{dx_1}{x_1^2} \int \frac{dx_2}{x_2^2} F(x_1) F(x_2)\, G\left(\frac{x_\perp(1+\eta)}{2x_1\tan(\frac{1}{2}\theta)}\right) \frac{\eta}{(1+\eta)^2}\left[(1+\eta)^2 + \eta^2 + \frac{1}{\eta^2} + \left(1+\frac{1}{\eta}\right)^2\right]. \qquad \text{(A16)}$$

Two particles (one in each jet):

$$E_1 E_2 \frac{d\sigma}{(d^3p_1)(d^3p_2)} \sim \frac{32\alpha_{\text{eff}}^2}{s^3 x_{\perp 1}^2 x_{\perp 2}^2} \int \frac{dx\, F(x)\, F(x\tan(\frac{1}{2}\theta_1)\tan(\frac{1}{2}\theta_2))}{x^3(1+\eta)^2\tan^2(\frac{1}{2}\theta_2)} \times G\left(\frac{x_{\perp 1}(1+\eta)}{2x\tan(\frac{1}{2}\theta_1)}\right) G\left(\frac{x_{\perp 2}(1+\eta)}{2x\tan(\frac{1}{2}\theta_1)}\right)\left[(1+\eta)^2 + \eta^2 + \frac{1}{\eta^2} + \left(1+\frac{1}{\eta}\right)^2\right]\delta(\phi_1 - \phi_2 + \pi)\ . \qquad \text{(A17)}$$

Two jets:

$$E_1 E_2 \frac{d\sigma}{(d^3p_1)(d^3p_2)} \sim \frac{2\alpha_{\text{eff}}^2}{s^2} \frac{F(x_1)}{x_1^2} \frac{F(x_2)}{x_2^2}\, \delta^2(\vec{p}_{\perp 1} + \vec{p}_{\perp 2})\left[\eta^2 + (1+\eta)^2 + \frac{1}{\eta^2} + \left(1+\frac{1}{\eta}\right)^2\right]. \qquad \text{(A18)}$$

One jet:

$$\frac{d\sigma}{d^3p/E} \sim \frac{4\alpha_{\text{eff}}^2}{y_\perp^2 s^2} \int \frac{dx_1[2x_1\tan(\frac{1}{2}\theta) - y_\perp]}{x_1^4 \tan^3(\frac{1}{2}\theta)} F(x_1)\, F\left(x_1 \frac{\tan^2(\frac{1}{2}\theta)}{\eta}\right)\left[(1+\eta)^2 + \eta^2 + \frac{1}{\eta^2} + \left(1+\frac{1}{\eta}\right)^2\right]$$

$$\sim \frac{4\alpha_{\text{eff}}^2}{y_\perp s^2 \tan^3(\frac{1}{2}\theta)} \int \frac{dx_1}{x_1^4}\, \eta\, F(x_1)\, F\left(x_1 \frac{\tan^2(\frac{1}{2}\theta)}{\eta}\right)\left[(1+\eta)^2 + \eta^2 + \frac{1}{\eta^2} + \left(1+\frac{1}{\eta}\right)^2\right]. \qquad \text{(A19)}$$

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[1]See, e.g., G. Miller *et al.*, Phys. Rev. D 5, 528 (1972).

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[3]J. D. Bjorken and E. A. Paschós, Phys. Rev. 184, 1975 (1969); Phys. Rev. D 1, 3151 (1970).

[4]J. Kogut and L. Susskind, Phys. Rep. 8C, 75 (1973).

[5]For a recent summary see Y. Frishman, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 119.

[6]By existence we do not mean to imply necessarily the presence of partons as asymptotic states, but existence only in the sense that partons are an efficient and valid method to describe hadron dynamics in specific kinematic regions.

[7]For the sake of clarity and completeness we have included in this paper several results which have also appeared elsewhere in the literature. We have attempted to indicate the original or independent sources of such results. While the other detailed results represent the original work of the authors, we recognize the possibility that, owing to the recent popularity of this topic, some of these results may have been developed independently by others. We apologize to anyone whose work has been inadvertently left unreferenced. In particular, we reference, once and for all, the notebooks of R. P. Feynman and J. D. Bjorken.

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[12]For alternative descriptions see D. Amati, L. Caneschi, and M. Testa, Phys. Lett. 43B, 186 (1973); Minh Duong-Van and P. Carruthers, Phys. Rev. Lett. 31, 133 (1973).
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[17]For a review see S. D. Drell, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (Ref. 5), Vol. 2, p. 8.
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