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[16]Many of these experiments have fairly small changes in the confidence level of only approximately a factor of 2. However, this change is such as to take them, for example, from a confidence level of 12% to only a few percent. The decrease to the few-percent level is the justification for the statement that it does not work well for these data sets.
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PHYSICAL REVIEW D VOLUME 9, NUMBER 1 1 JANUARY 1974

Description of the triple-Regge region with a very large triple-Pomeron coupling and a bare Pomeron*

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(Received 3 August 1973)

A description of data in the triple-Regge region up to CERN ISR energies is proposed. The description utilizes a very large PPP (triple-Pomeron) coupling g_P and assumes that the "bare" or "unrenormalized" Pomeron controls the region near $x = 1$. The "bare" Pomeron intercept is $\hat{\alpha}_0 = 0.85$. One secondary term is employed to extend the description to $x = 0.7$. This term is chosen to be of the form $\pi\pi P$. We use a modified ABFST (Amati-Bertocchi-Fubini-Stanghellini-Tonin) model to roughly constrain g_P and $\hat{\alpha}_0$. The number of unconstrained parameters otherwise is one. Other phenomenological applications utilizing the bare Pomeron as the definition of diffraction at intermediate energies are discussed, and the total cross section with and without diffraction is discussed in the context of the ABFST model.

I. INTRODUCTION

The nature of diffractive scattering is one of the central problems of high-energy physics. A number of authors have proposed that a possible handle on this problem lies in the magnitude of the triple-Pomeron coupling at zero momentum transfer, $g_{PPP}(0)$. It has usually been assumed that $g_{PPP}(0)$ is either zero or else very small, and the data may well be consistent with this assumption, though we shall show this is not necessary and perhaps not desirable at current energies. We shall adopt the generalized two-component schemes of Chew[1] or Ter-Martirosyan,[2] but *we shall insist that the transition into the "asymptotic" region controlled by the true Pomeron is describable in terms of a pole in an auxiliary "unrenormalized" partial-wave amplitude* $\hat{A}_l(t)$.[3] This pole $l = \hat{\alpha}_P(t)$ is termed the "bare" or "unrenormalized" Pomeron, and is taken here as a convenient way of summarizing (and/or defining) the nature of diffraction scattering at intermediate energies, where the multiparticle amplitudes have neighboring subenergies almost entirely in the resonance

region. The behavior of the elastic amplitude is controlled at high s by the "renormalized" Pomeron, which occurs as a singularity (or set of singularities) in the usual positive-signatured t-channel partial-wave amplitude $A_l(t)$ at $l=\alpha_P(t)$.

It should be emphasized that what is being proposed is not an energy-dependent pole but rather poles in different functions controlling the behavior of the elastic amplitude in different regions in s. The renormalization effect is due to the possible presence of neighboring subenergies in multiparticle amplitudes lying above the resonance region. At high energies, say those at the CERN Intersecting Storage Rings (ISR), these diffractive subenergies are at intermediate values on the average, with most neighboring subenergies still lying in the resonance region.

At this stage, to avoid conflict with the known behavior of σ_{tot}, one has two choices:

(i) $\hat{\alpha}_0$ is very close to unity and the renormalization effect characterized by $g_{PPP}(0)$ is small, though enough to account for the rise in σ_{tot}.[4]

(ii) $\hat{\alpha}_0$ is substantially below unity but $g_{PPP}(0)$ is large enough to produce $\alpha_0 \approx 1$. The rise in σ_{tot} at ISR energies can come either from the renormalization alone or along with negative PP cuts.[5]

The first choice is the one preferred by several authors, while we shall, in this work, assume the second alternative. We feel the reasons for the attractiveness of this choice (aside from the fact that it provides an alternative description of the data) are several. First, it would be most surprising if a resonance multiperipheral model, which is the conventional mechanism for producing $\hat{\alpha}_0$, were so close to the unitarity limit. Indeed, realistic strong-coupling multiperipheral models, such as the ABFST model,[6] have difficulty producing $\hat{\alpha}_0$ above 0.5 unless strong off-shell modifications are employed. If one utilizes an off-shell prescription suggested by ABFST-model calculations[7] and calculations of certain exclusive multibody processes,[8] and further increases the resonance couplings G^2 by about 50% to account for extra exchange mechanisms, cross graphs, 3π resonances, etc., one obtains $\hat{\alpha}_0 = 0.85$, still far below unity. This value is large enough, however, so that if one calculates the renormalization effect in the same ABFST model, one can obtain $\alpha_0 \approx 0.98$–0.99 with $g_{PPP}(0) \approx 1$–3 GeV^{-1}, the exact value being dependent on the value of $\sigma_{\text{tot}}^{\pi\pi}$ (see Sec. III). We regard this as sufficient motivation to choose $\hat{\alpha}_0 = 0.85$ and $g_{PPP}(0) = 2$ GeV^{-1}.

Before considering the triple-Regge data, we wish to remark on several other phenomenological points that would be relevant to this model of diffraction scattering. First, it seems to us that the use of the diffractive "bare pole" $\hat{\alpha}_P$ in the auxiliary amplitude $\hat{A}_l(t)$ may facilitate intermediate-energy phenomenology greatly. It is conceivable, for example, that the failure of conventional absorption models at $p_{\text{lab}} = 5$–20 GeV/c could be attributed to the use of the wrong diffraction component, rather than the wrong prescription for generating cuts. Indeed, as Kane has emphasized,[9] the major modification for such a scheme must be the introduction of a large real component for the diffractive term at $t \neq 0$. Having $\hat{\alpha}_0 \approx 0.85$ with some finite value of α' defining the intermediate-energy diffractive term accomplishes just this feature in a direct and simple way.

Secondly, it is not at all clear theoretically whether or not the bare pole $\hat{\alpha}_0$ must be supplemented with a secondary P' term. It is certainly consistent to imagine that a P' exists, but that the resonance multiperipheral model building up $\hat{\alpha}_0$ is not a sufficiently accurate representation of reality to produce it.[10] If this were the case, one would add in the P' by hand. This means that the bare pole can be quite *distinct* from a simple "average" of trajectories at 1 and $\frac{1}{2}$. We anticipate that any intermediate-energy 2-body phenomenology may in fact involve a P' pole in addition to the bare Pomeron. This does not require that this P' enter into triple-Regge fits, of course, since the $P'P'P$ coupling could be small. Another possibility could be that the P' (f) is generated by a schizophrenic-Pomeron mechanism[1] but that intermediate-energy 2-body phenomenology will require a P'' (f') pole.

Finally, our scheme implies a large amount of "diffraction" in σ_{tot} at high energies due to the importance of large-rapidity-gap-separation events. This leads to a more complicated separation of "diffractive" scattering in multiparticle events than has been proposed in the simple "low-prong diffraction plus Poisson multiperipheral" model separation.[11] It is helpful in this regard to keep in mind that simple strong-coupling ladder models do *not* produce a Poisson distribution in any variable[12] (this happens only when weak-coupling formulas are used at the large coupling strengths required in hadron physics).

In the next section we describe the triple-Regge fits, while in Sec. III the ABFST-model calculation of g_P and $\hat{\alpha}_0$ is discussed.

II. TRIPLE-REGGE PHENOMENOLOGY

The triple-Regge region is characterized by the normalized subenergies $s/M^2 = (1-x)^{-1} \gg 1$ and $M^2/s_0 \gg 1$. It should be noted that even at $s = 1000$ GeV^2, both these quantities lie approximately in the region 5–100 if $0.9 < x < 0.99$. If we define the intermediate-energy region for the purposes of triple-Regge phenomenology as encompassing

roughly these values, as emphasized in the Introduction, the *bare pole* $\hat{\alpha}_0$ must be employed for the diffractive component in this region near $x=1$. For x substantially away from unity, M^2 can become large, and in this region the renormalized $\hat{P}\hat{P}P$ coupling should be used. However, in this region this term is small anyway. Similarly, for $x>0.99$ the coupling $PP\hat{P}$ should be used. To avoid cluttering up the parametrization we will use $\hat{\alpha}_0$ throughout.

Now the normalization for the $pp\to pX$ inclusive cross section, in mb/GeV^2, is

$$s\frac{d^2\sigma}{dtdM^2}=\pi E\frac{d^3\sigma}{dp^3}$$
$$=\frac{s_0(0.389)^{1/2}}{16\pi s}\sum_{ij}\beta_i^2(t)\tilde{\beta}_j(0)g_{iij}(t)\times(s/M^2)^{2\alpha_i(t)}(M^2/s_0)^{\alpha_j(0)}, \tag{2.1}$$

with $M^2/s=1-x$ and $t=-[p_T^2+m_p^2(1-x)^2]/x$. For $pp\to pX$ scattering, the pp elastic scattering data yield[13]

$$\beta_P^2(t)\cong 100e^{4t}\ \text{GeV}^{-2},$$
$$\tilde{\beta}_P(0)\cong(40\ \text{mb})^{1/2}.$$

We shall take, in accordance with the above discussion,

$$\hat{\alpha}_P(t)=\hat{\alpha}_0+0.2t\quad(\hat{\alpha}_0=0.85),$$
$$g_P=g_{PPP}(t)=2\ \text{GeV}^{-1}\quad(\text{independent of } t).$$

This value of g_P is extremely large (seven times bigger than Capella's value,[14] for example), and is phenomenologically possible only because $\hat{\alpha}_0=0.85$ is a fair distance from unity.

We have found that a suitable and economical parametrization for the region away from $x=1$ is describable in terms of a single $(\pi\pi P)$ term.[15] We do not deny the existence of other terms; we find only that they are not necessary in this context. It should be noted that the $\pi\pi P$ term scales in a rather complicated way in principle because of the difference between $\hat{\alpha}_0$ and α_0, which is important at large M^2. At $x=0.8$, M^2 does not equal 100 until past NAL (National Accelerator Laboratory) energies; up to NAL energies the $\pi\pi P$ term scales just like the PPP term ($\sim s^{-0.15}$). Past these energies, the $\pi\pi P$ term would scale like a constant, however. We discuss this further below.

Our parametrization for the pion trajectory is $\alpha_\pi(t)=t$. This has been done for simplicity in spite of the fact that absorption due to $\hat{P}\otimes\pi$ cuts undoubtedly decreases the effective pion slope. We then obtain the following parametrization for the inclusive cross section in the triple-Regge region:

$$\left(s\frac{d^2\sigma}{dtdM^2}\right)_{pp\to pX}=[16.1e^{4t}(1-x)^{-\hat{\alpha}_0-0.4t}+200(1-x)^{\hat{\alpha}_0-2t}]\left(\frac{s}{s_0}\right)^{\hat{\alpha}_0-1}\ \text{mb/GeV}^2. \tag{2.2}$$

The only unconstrained parameter is the $\pi\pi P$ normalization. For simplicity it has been taken independent of t. The graphs of the $pp\to pX$ data[16] and model predictions up to $s=1000$ GeV^2 are shown in Figs. 1–4(a). The numbers in parentheses give the ratio of the PPP to the $\pi\pi P$ term. It is seen that the $s^{-0.15}$ drop in the PPP region is well obeyed up to $s=1000$, though some deviation is seen in the $\pi\pi P$ region. As mentioned above, the scaling behavior of the $\pi\pi P$ term is perhaps more complicated than the parametrization we have used, which predicts a 10% drop in the invariant cross section over the ISR range. Figure 5 shows the results at $s=1995$ GeV^2 and $p_T^2=0.49$ GeV^2. The model prediction is somewhat low. Even if scaling is restored by using α_0 in place of $\hat{\alpha}_0$ for $x<0.8$ above NAL energies, the results are similar, however. The reason for this is that the x shape changes to offset the increase due to restored scaling. A real test of the model at the top of the ISR range awaits the $x>0.9$, small-p_T data.

It should be mentioned that our simple parametrization works well only out to $t=-0.5$ GeV^2 near $x=1$. Below this value of t, predictions systematically too low are obtained.

The reaction $p\pi\to pX$ is describable in this model in a very simple way. Since the $(M^2)^{\alpha_j(0)}$ is $j=P$ in both terms, we may find the cross section for this reaction by the simple factorization procedure

$$\left(\frac{d^2\sigma}{dtdM^2}\right)_{p\pi\to pX}=\left(\frac{\sigma_{\text{tot}}^{\pi\pi}}{\sigma_{\text{tot}}^{NN}}\right)^{1/2}\left(\frac{d^2\sigma}{dtdM^2}\right)_{pp\to pX}. \tag{2.3}$$

The graph in Fig. 4(b) shows the results assuming $\sigma_{\text{tot}}^{\pi\pi}/\sigma_{\text{tot}}^{NN}=16/40$. The couplings were taken at $t=-0.2$ GeV^2 and the data are averaged over a t cut of $(-0.17,-0.34)$. The model seems consistent with these data.

Finally we mention that a crucial test of the model would involve the $t\approx 0$ region. For x near unity the fixed-M^2 distribution in t should not turn over. Preliminary 200-GeV/c NAL data are evidently in some disagreement on this point. Bubble chamber experiments for both $pp\to pX$[17(a)] and $\pi^-p\to pX$[17(b)] show no turnover, but a $pp\to pX$ counter experiment[17(c)] does show some turnover. We can fit the data of Ref. 17(a), but evidently some

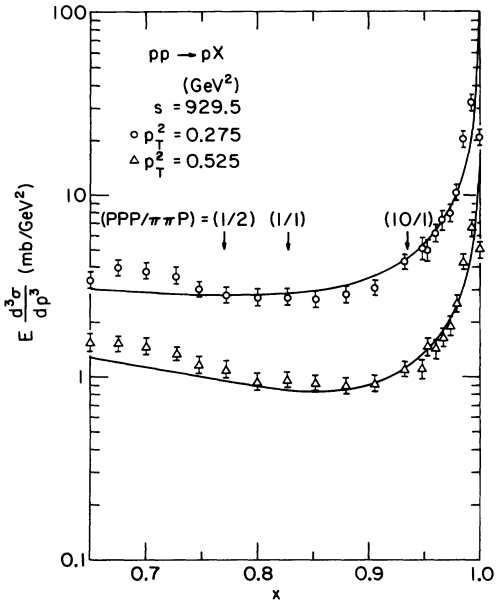

FIG. 1. ISR data for $pp \to pX$ at $s = 929.5$ GeV^2 with $p_T^2 = 0.275$ and 0.525 GeV^2. The numbers in parentheses are the ratios of the PPP term to the $\pi\pi P$ term.

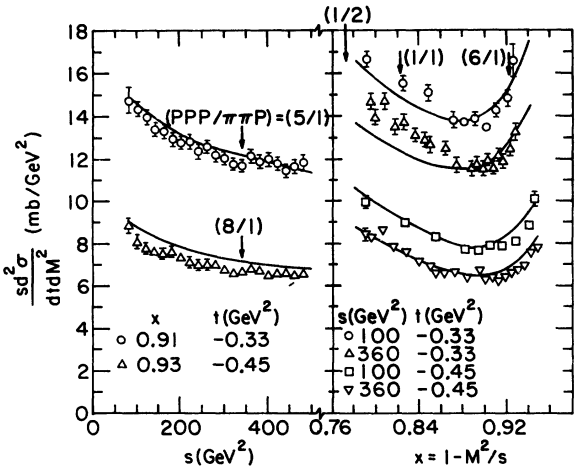

FIG. 3. NAL data for $pp \to pX$ at $s = 100$–480 GeV^2 and $t = -0.33$, -0.45 GeV^2.

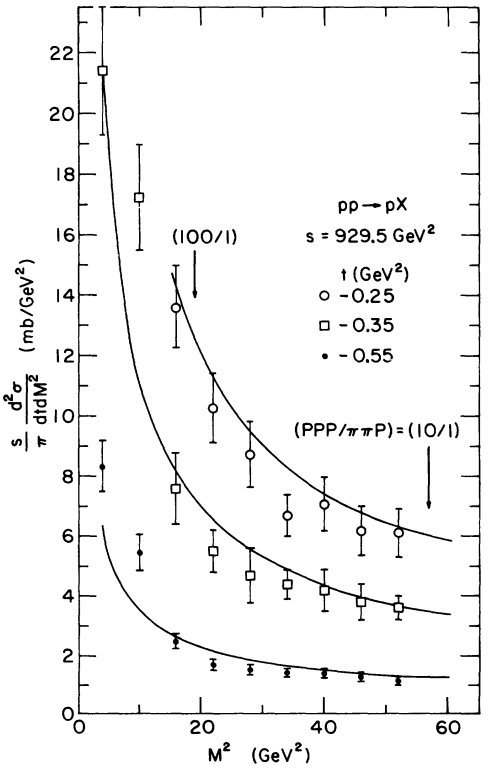

FIG. 2. ISR data for $pp \to pX$ at $s = 929.5$ GeV^2 and small M^2 in the PPP region.

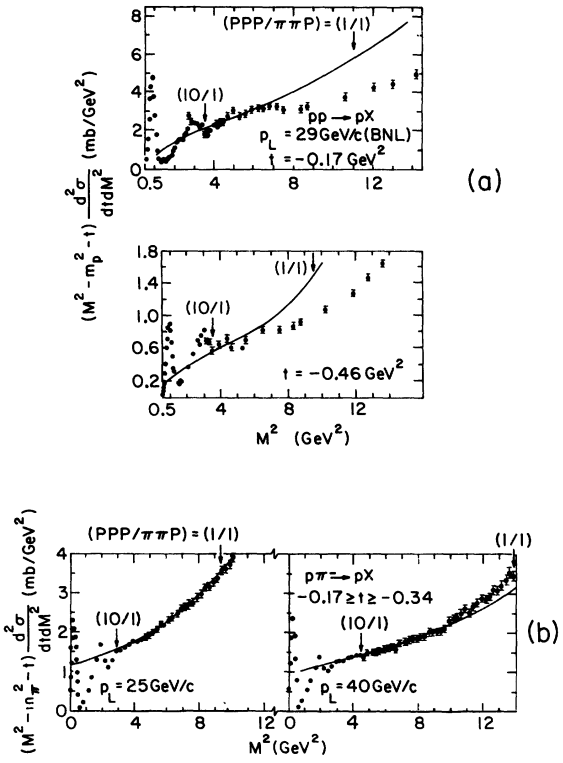

FIG. 4. (a) BNL data for $pp \to pX$ at $t = -0.17$, -0.46 GeV^2; (b) CERN-IHEP data for $p\pi^- \to pX$ for $-0.34 < t < -0.17$ GeV^2. The curves are the model evaluated at $t = -0.2$ GeV^2, and factorization is employed using $\sigma_{tot}^{\pi\pi}/\sigma_{tot}^{NN} = 16/40$.

judgment must be reserved.

Another test, this time of the $\pi\pi P$ term, is provided by the inclusive reaction $pn \to pX$. Preliminary NAL data[17(d)] show this cross section to be on the order of that predicted by our parametrization.

III. A MODEL CALCULATION OF $g_{PPP}(0)$ AND $\hat{\alpha}_0$

In this section we wish to discuss the parameters $g_P = g_{PPP}(0)$ and $\hat{\alpha}_0$ within the context of a modified ABFST model.[6] This model assumes that, neglecting absorptive corrections, the multiparticle amplitudes $T_{AB\to n}$ are multiperipheral in the (off-shell) $\pi\pi$ elastic amplitude T_{22} with appropriate end-chain connections made to particles A and B. We shall for simplicity take $A = B = \pi$. Unitarity for T_{22} then produces a highly nonlinear equation for T_{22} itself. The proposal is that representing $T_{22}(s,0) \sim \hat{\beta}(-s/s_0)^{\hat{\alpha}_0}$ in the intermediate region and $T_{22}(s,0) \sim \beta(-s/s_0)^{\alpha_0}$ asymptotically is a reasonably accurate solution of the nonlinear equation. Figure 6 shows a pictorial representation of this statement. The solid curve is a computer evaluation of the inelastic part of the total cross section by integrating its partial-wave projection over the Sommerfeld-Watson contour [see Eq. (3.5)]. The nearly straight lines are the approximations $\hat{\beta}(s/s_0)^{\hat{\alpha}_0 - 1}$ and $\beta(s/s_0)^{\alpha_0 - 1}$ with $\hat{\alpha}_0 = 0.86$ and $\alpha_0 = 0.992$, with the appropriate residues. The dashed curve is the inelastic resonance production cross section, i.e., that part of the cross section without any diffraction [Eq. (3.1)]. It is seen that over the intermediate-energy region bracketed by the arrows, the *shape* of the exact curve is more closely followed by the *bare* Pomeron, while the renormalized Pomeron pole is more accurate at high energies. We are proposing that this feature be abstracted

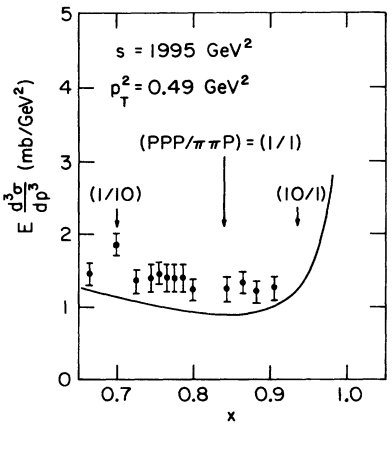

FIG. 5. ISR data for $pp \to pX$ at $s = 1995$ GeV^2 and $p_T^2 = 0.49$ GeV^2.

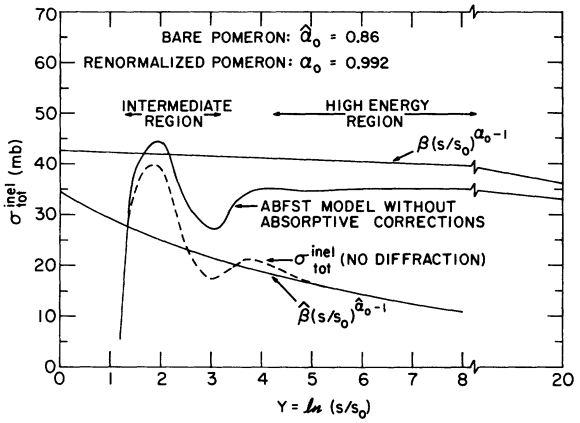

FIG. 6. Results of the modified ABFST model for $\sigma_{\text{tot}}^{\text{inel}}$ including resonance and diffractive kernels. The exact result is the oscillating curve, and the two relatively straight lines are the bare and renormalized Pomeron pole approximations. The dashed line is $\sigma_{\text{tot}}^{\text{inel}}$ without diffraction. The intermediate-energy region is bracketed by the arrows.

from the model. We stress that corrections to the model producing the P' even at intermediate energies may be required for 2-body phenomenology, and that absorptive corrections to T_{2n} leading to negative PP cuts would probably be needed to produce the rise in the total cross section seen at the ISR.[5]

The solution of the ABFST equation when the subenergies s_i are all in resonance regions (corresponding to the "intermediate energy" region in s) can be found by separable-kernel techniques.[18] If one employs the off-shell prescription proposed in Ref. 7, a prescription compatible with Monte-Carlo calculations of certain exclusive multibody cross sections,[8] it can be shown that for $\pi\pi$ scattering

$$\sigma_{\text{tot}}^{\text{inel}} \,|_{\text{no diffraction}} \cong \frac{1}{3s} \int_{c-i\infty}^{c+i\infty} \frac{dl}{2\pi i} \left(\frac{s}{m_0^2}\right)^l \frac{N_l^R}{1 - C_l^R}, \tag{3.1}$$

where

$$N_l^R = 16\pi^3 (l+1) \frac{\Gamma(2l)}{\Gamma^2(l)} (C_l^R)^2 \tag{3.2}$$

and

$$C_l^R = \frac{G^2}{l+1} \frac{\Gamma^2(l)}{\Gamma(2l)}. \tag{3.3}$$

Here $G^2 \cong 0.8$ is the experimental $\pi\pi$-resonance coupling crossed to $I_t = 0$; we take the resonance mass $m_0^2 = 1$ GeV^2. The quantity

$$\hat{A}_l = N_l^R (1 - C_l^R)^{-1} \tag{3.4}$$

is the auxiliary unrenormalized partial-wave amplitude, and $\hat{\alpha}_0$ is found as a solution of the equa-

tion $C^R_{\hat{\alpha}_0}=1$. If we increase G^2 to 1.4 to atone for the neglect of the effects of crossed graphs, 3-pion resonances, kaon production, etc., then we find that $\hat{\alpha}_0=0.85$.

Now that the behavior of the total cross section is known in the intermediate region, the *high-s* behavior is found by inserting the intermediate-energy behavior $\hat{\beta}e^{\gamma t_i}(-s_i/s_0)^{\hat{\alpha}_P(t_i)}V_{\rm off}{}^2$ into $T_{22}(s_i,t_i)$ whenever s_i is in this region. It can then be shown that the total $\pi\pi$ cross section has its inelastic part given by

$$\sigma_{\rm tot}^{\rm inel}(s)\cong\frac{1}{3s}\int_{c-i\infty}^{c+i\infty}\frac{dl}{2\pi i}\left(\frac{s}{m_0{}^2}\right)^l\frac{N_l}{1-C_l^R-C_l^H}, \tag{3.5}$$

where

$$N_l=16\pi^3(l+1)\left\{\left[\frac{\Gamma(2\hat{\alpha}_0)}{\Gamma(l)\Gamma(2\hat{\alpha}_0-l)}\right]^{1/2}C_l^H+\left[\frac{\Gamma(2l)}{\Gamma^2(l)}\right]^{1/2}C_l^R\right\}^2 \tag{3.6}$$

and

$$C_l^H=3\left(\frac{\hat{\beta}}{16\pi^2 s_0}\right)^2\frac{(m_0{}^2/s_0)^l}{2\alpha' s_0(l+1)}\frac{\Gamma(l)\Gamma(2\hat{\alpha}_0-l)}{\Gamma(2\hat{\alpha}_0)}\times\left(\frac{s_{\rm min}}{s_0}\right)^{\hat{\alpha}_c-l}e^x E_1(x). \tag{3.7}$$

Here

$$x=(l-\hat{\alpha}_c)\left[\frac{\gamma}{\alpha'}+\ln\left(\frac{s_{\rm min}}{s_0}\right)\right] \tag{3.8}$$

and

$$\hat{\alpha}_c=2\hat{\alpha}_0-1. \tag{3.9}$$

With the values $\hat{\alpha}_0=0.85$, $\alpha'=0.2$ GeV^{-2}, $\gamma=2$ GeV^{-2}, and the intermediate-region threshold $s_{\rm min}=4$ GeV^2, we obtain the renormalized Pomeron position at $\alpha_0=0.98$. Raising $\hat{\alpha}_0$ to 0.86 produces $\alpha_0=0.99$.

One should note that the factor $(s_{\rm min}/m_0{}^2)^{-l}$ is a threshold factor for diffraction production. The approximate equality of Eqs. (3.5) and (3.1) in the intermediate-s region can be seen by expanding the denominator in Eq. (3.5) and using Cauchy's theorem to bend the contour to the right. This might lead one to think that the renormalized pole would be as good a representation of the cross section as the bare pole in the intermediate-energy region, but this is not necessarily so. The reason is that the mechanism for diffraction in most subenergies is not available at intermediate energies, so that if the renormalized pole is used, many secondary terms coming from (probably complex) zeros of the renormalized D_l function must *also* be used. All this is equivalently characterized by the simple prescription of using the bare pole.

We shall now calculate g_P in this model. To do this, we first use the prescription of Ref. 19 and write the renormalized D_l function as

$$D_l=1-C_l^R-C_l^H\approx\left(\frac{\partial C_l^R}{\partial l}\right)_{\hat{\alpha}_0}\left(l-\hat{\alpha}_0-\eta\ln\frac{1}{l-\hat{\alpha}_c}\right) \tag{3.10}$$

up to a term regular in l at $l=\hat{\alpha}_c$ from C_l^H. The connection of η with g_P is found by

$$\eta=\frac{g_P{}^2}{32\pi\alpha'}. \tag{3.11}$$

Equivalently we may find g_P by the formula obtained by coupling three (bare) Pomerons together[19,20]:

$$g_P=\left(\frac{\hat{\beta}}{0.389}\right)^{3/2}\frac{3(m_0{}^2)^{2-2\hat{\alpha}_0}}{16\pi^3(\hat{\alpha}_0+1)}\int_0^\infty\frac{dy\,y^{\hat{\alpha}_0+1}V_{\rm off}{}^2(\hat{y})V_{\rm off}(y)}{(y+m_\pi{}^2)^{3-\hat{\alpha}_0}(1+\hat{y}/m_0{}^2)^{2\hat{\alpha}_0+2}(1+y/m_0{}^2)^{\hat{\alpha}_0+1}}, \tag{3.12}$$

where $\hat{y}=\frac{1}{2}(y+m_\pi{}^2)$.

Our dynamical off-shell $\pi\pi$-resonance vertex is[7,8]

$$V_{\rm off}(y)=1+y/m_0{}^2. \tag{3.13}$$

A reasonable approximation to Eq. (3.12) using this off-shell prescription is

$$g_P\approx\left(\frac{\hat{\beta}}{0.389}\right)^{3/2}\frac{3m_0{}^2}{16\pi^3}\frac{\Gamma^2(\hat{\alpha}_0)}{(\hat{\alpha}_0+1)\Gamma(2\hat{\alpha}_0)}\ {\rm GeV}^{-1}. \tag{3.14}$$

If on the other hand we set $V_{\rm off}=1$, we get

$$g_P|_{V_{\rm off}=1}\approx\left(\frac{\hat{\beta}}{0.389}\right)^{3/2}\frac{3m_0{}^2}{16\pi^3}\frac{\Gamma(\hat{\alpha}_0)\Gamma(\hat{\alpha}_0+1)}{\Gamma(2\hat{\alpha}_0+2)}\ {\rm GeV}^{-1}. \tag{3.15}$$

Comparison of Eq. (3.3) with Eq. (3.14) shows a close relationship between g_P and the trace of the resonance kernel; a similar analogy holds for Eq. (3.15).

At this stage, one has the choice of utilizing Eqs. (3.10) and (3.11) to calculate g_P. This is equivalent to using the model prediction for $\hat{\beta}$ (35 mb), which is too large. This yields, with $\hat{\alpha}_0 = 0.85$,

$$g_P \approx 3 \text{ GeV}^{-1}. \tag{3.16}$$

If on the other hand one calculates g_P using Eq. (3.12) with $\hat{\beta} = 22$ mb (giving $\sigma_{\text{tot}}^{\pi\pi} = 14$ mb at $s = 20$ GeV^2, a typical intermediate value), one obtains with $\hat{\alpha}_0 \equiv 0.85$

$$g_P \cong 1.3 \text{ GeV}^{-1}. \tag{3.17}$$

The model prediction for g_P is thus ambiguous, but the value $g_P = 2$ GeV^{-1} used in our triple-Regge phenomenology in the previous section is compatible with these numbers.

These values for g_P are much larger than those obtained with the dynamical off-shell factor V_{off} removed. With $\hat{\alpha}_0 = 1$ and $\hat{\beta} = 14$ mb, for example, we obtain $g_P \cong 0.15$ GeV^{-1}. This value, obtained by Sorensen,[20] is consistent with triple-Regge fits at the ISR,[14] but is inconsistent insofar as setting $V_{\text{off}} = 1$ leads to $\hat{\alpha}_0 < 0.5$ in the ABFST model.[1]

Finally, it is interesting that the relation (3.11) is satisfied with

$$\eta = 1 - \hat{\alpha}_0 \tag{3.18}$$

rather than with $\eta = 1 - \alpha_0$.[21] We have shown that this is compatible with the amount of renormalization needed to get the renormalized Pomeron very near unity in this model. It is not clear to us whether or not Eq. (3.18) is interpretable in the context of t-channel unitarity arguments for Reggeons.

IV. DISCUSSION

We have leaned heavily on the concept of the "bare Pomeron" as a *physical* object describing the leading term in the total cross section at intermediate energies and the leading term in the triple-Regge region when s/M^2 and M^2/s_0 are at comparable intermediate values. The $t = 0$ triple-Pomeron coupling g_P thus obtained is very large; it is the coupling between three bare Pomerons. We do not know what the $t = 0$ coupling is between three renormalized Pomerons; in the context of this model that quantity would have relevance only at extremely high energies when s/M^2 and M^2/s_0 were both very large. It could be that this coupling (call it g_P) is very small or even zero.

We showed that, within the context of a modified ABFST model, our value of g_P is consistent with the amount of renormalization needed to produce the renormalized Pomeron $\alpha_0 \approx 0.99$. This model was also capable of producing the bare Pomeron required. With absorptive corrections, it can produce locally rising cross sections. We regard all this as providing a strong plausibility argument for taking the bare Pomeron as a serious dynamical concept.

Further phenomenological analysis utilizing the bare Pomeron as the diffractive component at intermediate s is of course required. As mentioned in the Introduction, it may be that the built-in real part of the bare Pomeron may actually facilitate the description of these data. Further work along these lines is in progress.

ACKNOWLEDGMENTS

I would like to thank D. Colley for informing me of the results of his NAL data. I would also like to thank D. Snider for stimulating conversations, and the theory group at the University of Washington for its generous hospitality during which time this work was completed.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

†Summer visitor at the University of Washington, Seattle, Washington 98195.

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