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Rising cross sections and scaling

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A parametrization for hadronic total cross sections, which incorporates a logarithmically rising term, is proposed. The new parameters present in it are restricted, using a scaling assumption for q^2 in the timelike region. The resulting expression contains only one parameter in addition to the usual Regge parameters. In the energy range between 10 and 400 GeV/c this model gives very good fits for $\pi^{\pm}p$, $K^{\pm}p$, pp, and $\bar{p}p$ total cross sections. Above 400 GeV/c, the agreement with recent data on the pp total cross section is only qualitative. The model also predicts the vanishing of the ratio of real to imaginary parts of the pp forward elastic amplitude at 360 GeV/c in agreement with experimental data, the increase of this ratio to a maximum value, and a vanishing asymptotic limit.

I. INTRODUCTION

A possible increase of hadronic total cross sections has often been considered during the last three years. Several models have been constructed to accommodate it. Among them, we can quote for example, the quantum-electrodynamics model of Cheng and Wu[1] and the absorbed multiperipheral model of Finkelstein and Zachariasen.[2] It is also known that several modifications of the Regge-pole model lead to rising cross sections. Such is the case for various pole-plus-cut(s) models and for the pole-plus-dipole model.[3] The motivation for those works can be traced back to the fact that total cross sections for laboratory momenta between 10 and 65 GeV/c are observed either to flatten out more rapidly than is predicted by the Regge-pole model or are seen to increase slowly. The various modifications of the naive Regge picture are largely arbitrary and contain at least one additional parameter for each reaction. Therefore, even if the ensuing fits are good, they are not very compelling from a theoretical point of view. Recently, several groups[4] have measured the pp total cross section and the real over the imaginary part of the forward pp amplitude α_{pp} at CERN ISR (intersecting storage rings) energies.

The total cross section is seen to increase substantially and α_{pp} is compatible with zero for energies in the neighborhood of 400 GeV. Moreover, they have shown that their data can be fitted by the expression

$$\sigma_{\text{tot}}(pp) = 38.5 + 0.5(\ln \tfrac{1}{137} s)^2 . \tag{1}$$

This remarkable result raises several important questions: Does the increase of $\sigma_{\text{tot}}(K^+p)$ observed between 10 and 65 GeV/c come from a $\ln^2(s/s_0)$ term? Do other total cross sections increase in a similar way? If we introduce a term $\sigma'[\ln(s/s_0)]^2$ in the expressions of total cross sections, how can be interpret it? How can we fix the scale s_0 and the new parameter σ'? In this paper, we present a model which in first approximation gives for total cross sections the expression

$$\sigma_{\text{tot}} = \sigma_{\text{Regge}} + C\frac{\mu^2}{(1+\mu^2/M^2)^2}\ln^2\left(\frac{s}{\mu^2}\right) , \tag{2}$$

with

$$\sigma_{\text{Regge}} = a + \frac{b}{(P_{\text{lab}})^{1/2}} . \tag{2$'$}$$

In (2), μ is the mass of the projectile and M an effective mass chosen to be the proton mass. C is a dimensionless universal constant. In (2′), a and b are the effective Regge couplings for Pomeron and Regge exchanges.

The work develops as follows: We make a few qualitative considerations on the meaning of the $[\ln(s/s_0)]^2$ and explain the additive assumption made in (2). Assuming that "timelike scaling" is valid for strong amplitudes and making a few approximations, we show how to fix the scale of s and determine σ' up to a dimensionless constant independent of the particles involved in the reaction. The fit of (2) to experimental cross sections is done and $\sigma_{\text{tot}}(pp)$ is seen to increase in qualitative agreement with the data of Amaldi *et al.*[4] From the pp dispersion relation, we calculate the high-energy behavior of α_{pp} and find that it goes through zero at about 350 GeV. Finally, several comments relating to the present work are made.

II. INTUITIVE CONSIDERATIONS ON THE PHENOMENOLOGICAL PARAMETRIZATION OF HADRONIC TOTAL CROSS SECTIONS

In this section, we attempt to explain intuitively the phenomenological parametrization of total cross sections we have chosen, i.e.,

$$\sigma_{\text{tot}} = \sigma_{\text{Regge}} + \sigma' \ln^2\left(\frac{s}{s_0}\right) . \tag{3}$$

It involves two assumptions:

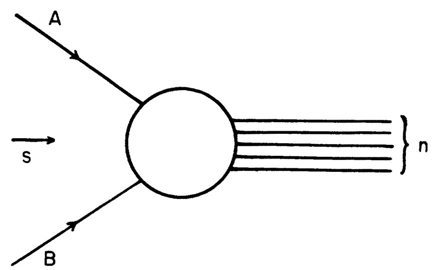

FIG. 1. The n-particle production amplitude.

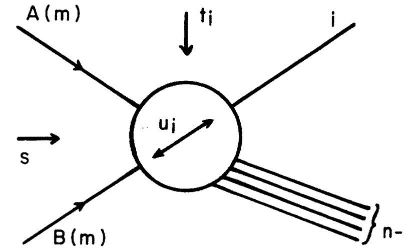

FIG. 2. The n-particle production amplitude when a particular particle i is picked out.

(a) The extra term adds to the usual cross section;

(b) as a function of energy, it behaves as the square of a logarithm.

A. Discussion of assumption (a)

We consider σ_{tot} as the distribution for the inclusive reaction $A+B\to$anything. It can be written

$$\sigma_{tot}=\frac{1}{s}\sum_{n\geq 2}^{\infty}\int d\Omega_n |A_{2\to n}|^2, \tag{4}$$

where $A_{2\to n}$ is the production amplitude for n particles and is drawn in Fig. 1. For simplicity, we suppose that there are only particles of one kind.

Let us concentrate on one particle among the n final-state particles and call it i. Then, the same process can be represented as in Fig. 2 and it is clear that the $n-1$ particles left can be kinematically defined as a pseudoparticle i^*. For particle i we can define in the usual way the Mandelstan variables t_i and u_i, as well as the Feynman variable

$$x_i=\frac{2p_{iL}}{\sqrt{s}}, \tag{5}$$

where p_{iL} is its longitudinal momentum in the center-of-mass system.

When s becomes large,[5] we can write

$$p_{iT}^2\simeq\frac{t_i u_i}{s}, \tag{6}$$

where p_{iT} is the transverse momentum. Therefore, small t_i or u_i means small p_{iT}. Intuitively, this result is almost obvious.

Of course, this kinematical separation can be done for every particle.

We divide the phase-space integration in (4) into two parts. To this end, we define for any fixed n

(i) F_n as the total n-particle phase-space domain;

(ii) $D_n(p_{iT})$ as that part of F_n for which

$$p_{iT}^2<a\ll s \tag{7}$$

for every particle i (i.e., $i=1$ to n); in Eq. (7), the constant a is assumed to be of the order of a few $(\mathrm{GeV}/c)^2$;

(iii) $\bar{D}_n(p_{iT})$ as the complementary of D_n in F_n.

With these definitions, we can write

$$\sigma_{tot}=\frac{1}{s}\sum_{n\geq 2}\int_{D_n(p_{iT})} d\Omega_n |A_{2\to n}|^2 +\frac{1}{s}\sum_{n\geq 2}\int_{\bar{D}_n(p_{iT})} d\Omega_n |A_{2\to n}|^2 . \tag{8}$$

The first term includes contributions from small-momentum-transfer (forward or backward) processes only. In the second term, *one particle at least has a large momentum transfer*. Of course, we are aware that this separation is somewhat arbitrary insofar as dynamically there is certainly no clear-cut separation between the two types of events. Also the present stage of our knowledge of the underlying dynamics does not allow us to define precisely the value of the constant a.

Next, we examine the physical content of each term in (8) when s is large.

The first term receives contributions from elastic scattering, quasi-two-body reactions of diffractive and nondiffractive nature, and also from multiparticle processes of a peripheral nature. All these processes can be theoretically described by the Regge model suitably modified by inclusion of absorptive corrections. The range of the interaction seems to be roughly independent of their nature and to be about 1 fm. If elementary particles have structures, they are not seen because the subunits act coherently. Therefore, this term is to be identified with σ_{Regge} [see Eq. (2)] and we expect it to behave roughly as a constant at infinity.

The second term certainly receives very little contributions from both elastic and quasi-two-body scattering. Indeed, model calculations[6,7] show that at fixed angle

$$\frac{d\sigma(pp\to pp)}{dt}\simeq s^{-n}F(\cos\theta), \tag{9}$$

with $n\sim 10$, and available data look compatible with such a behavior. This term also contains the contribution of inclusive processes of the type

$$A+B\to C+\text{anything}, \quad (10)$$

where particle C has a big transverse momentum. From recent experiments done at the ISR, we know that they are much more frequent than expected on the basis of small-p_T data. Moreover, it is quite possible that their cross sections do increase with s.[8] A recent model calculation[7] which assumes pointlike interaction between partons shows that at 90° the inclusive cross section for

$$pp\to\pi^0+\text{anything} \quad (11)$$

is given by

$$E\frac{d\sigma}{d^3p}=\frac{as^2}{{p_{\pi_0 T}}^8} . \quad (12)$$

We must also add that recently Krisch[9] has shown that a geometrical picture of the interaction at high p_T would lead for the inclusive cross section to a Gaussian-like behavior in disagreement with experiment. This indicates the essential incoherence of large-p_T processes. We can conclude that this term is much smaller than the first one at conventional accelerator energies but *it should increase with s.*

B. Discussion of assumption (b)

How fast does the second term of (8) increase with s? As long as we do not have a detailed dynamical picutre of the large-p_T processes, it is impossible to answer this question in precise terms. Several qualitative arguments can be put forward to generate a $\ln^2(s/s_0)$ increase of total cross sections. They are much too naive and can even be wrong. One of them is based on potential scattering and asserts that if the coupling constant behaves as

$$g=g_0\left(\frac{E}{E_0}\right)^\alpha , \quad (13)$$

where E is the energy and α a real positive number and if

$$\sigma_{\text{tot}}\propto R^2 , \quad (14)$$

where R is the effective radius for the particles,

$$\sigma_{\text{tot}}\propto\frac{\alpha^2}{\mu}\ln^2\frac{E}{E_0} . \quad (15)$$

In Eq. (15), μ^{-1} gives the range of the interactions.[10]

Apart from the difficulty in understanding the assumption (13), the expression (14) for σ_{tot} suggests a geometrical picture of the interaction, so that it is probably irrelevant to the actual problem.

The following argument is more suggestive. We start from the one-particle inclusive distribution $F(x,\vec{p}_T,s)$. From the relation

$$\int F(x,\vec{p}_T,s)\frac{d^3p}{p_0}=\langle n\rangle\sigma_{\text{tot}} , \quad (16)$$

we require

$$\int F(x,\vec{p}_T,s)\frac{d^3p}{p_0}\sim\ln^3\left(\frac{s}{s_0}\right), \quad (17)$$

since we know from experimental data that

$$\langle n\rangle\sim\ln\left(\frac{s}{s_0}\right) . \quad (18)$$

Assuming Feynman scaling, for very high s the left-hand side of (17) can easily be written

$$\int_{1/\sqrt{s}}^{1}\frac{dx}{x}\int F(0,\vec{p}_T)d^2p_T\sim\ln^3 s . \quad (19)$$

In Eq. (19), all energies are measured in units of 1 GeV. Therefore, we find

$$\int F(0,\vec{p}_T)\,d^2p_T\sim\ln^2 s. \quad (20)$$

Such a relation is impossible unless some violation of scaling is introduced in F. A simple way to do that is to assume

$$F(0,\vec{p}_T)\propto e^{-ap_T^2} , \quad (21)$$

where a is a certain function of s. Inserting Eq. (21) into Eq. (20), we get immediately

$$a=\frac{a_0}{\ln^2 s} , \quad (22)$$

where a_0 is a constant. Equation (22) gives

$$\langle p_T\rangle\sim c\ln s. \quad (23)$$

Though a Gaussian form for the transverse-momentum distribution can only be chosen for small p_T (i.e., $p_T\lesssim 1$ GeV/c),[11] the above argument is interesting because it shows that a close relation between the s dependence of the transverse-momentum inclusive distributions and the growth of total cross sections may exist. Experimental data are as yet unable to tell whether such a broadening of the small-p_T distribution exists or not.

C. Conclusion

Throughout this section we have tried to show that several mechanisms can be invoked to generate cross sections increasing[12] as $[\ln(s/s_0)]^2$. No doubt several other mechanisms do exist. In the present status of experimental data there is a large arbitrariness in that respect. We have made clear that the phenomenological expression (2) for σ_{tot} is reasonable, independent of any detailed dynamical assumptions to generate it. If the increase of total cross sections is due to the big-p_T processes and if these display the existence of a hadronic substructure, one cannot consider formula (2) as an *asymptotic expression* for σ_{tot} since sooner or later a new kind of physics comes into play. On the contrary, if the expression (2) for σ_{tot} is generated through the small-p_T processes, it should hold up to the highest energies presently accessible.

III. DETERMINATION OF THE PARAMETERS σ' AND s_0

As we saw, it is reasonable to assume for σ_{tot} the phenomenological expression

$$\sigma_{tot}=\sigma_{Regge}+\sigma'\ln^2\left(\frac{s}{s_0}\right). \tag{24}$$

The problem we want to solve is the determination of σ' and s_0. This can be done within very plausible assumptions:

(i) The off-shell forward amplitude shown in Fig. 3 is an analytic function of q^2; and

(ii) the off-shell Compton amplitudes have the same structure as strong ones and still satisfy scaling for both timelike and spacelike q^2.

Let us consider the implications of these assumptions on σ' and s_0. Since we will consider on-mass-shell strong amplitudes, we restrict ourselves to the timelike q^2 region and put, as usual,

$$\omega=\frac{q^2}{2M\nu},\quad \nu=\frac{p\cdot q}{M}=\frac{s-M^2-q^2}{2M}. \tag{25}$$

Then

$$\sigma_{L,T}(q^2,\nu)=\sigma^{(1)}_{L,T}(q^2,\nu)+C_{L,T}(q^2,\nu)\ln^2\frac{1}{\omega}, \tag{26}$$

where $\sigma^{(1)}_{L,T}(q^2,\nu)$ gives the Regge term for large ν and q^2 fixed and scales in the Bjorken limit[13]:

$$\nu\sigma^{(1)}_{L,T}(q^2,\nu)\xrightarrow[\substack{q^2,\nu\to\infty\\ \omega\ \text{fixed}}]{}\tilde{\sigma}(\omega). \tag{27}$$

On the other hand, $C_{L,T}(q^2,\nu)$ must satisfy several conditions:

$$\text{(a)}\ C_{L,T}(q^2,\nu)\xrightarrow[\substack{q^2\to 0\\ \nu\ \text{fixed}}]{}0, \tag{28}$$

since for the real photon, there is no infrared divergence in the total cross section;

$$\text{(b)}\ C_{L,T}(q^2,\nu)\xrightarrow[\substack{\nu\to\infty\\ q^2\ \text{fixed}}]{}f(q^2) \tag{29}$$

in order to recover Eq. (24); and

$$\text{(c)}\ \nu C_{L,T}(q^2,\nu)\xrightarrow[\substack{q^2,\nu\to\infty\\ \omega\ \text{fixed}}]{}\tilde{C}(\omega) \tag{30}$$

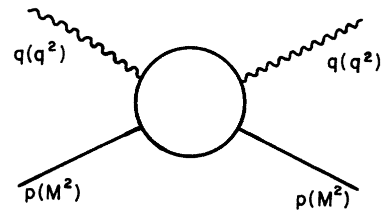

FIG. 3. The off-shell forward elastic amplitude $q(q^2)+p(M^2)\rightarrow q(q^2)+p(M^2)$.

in order to satisfy scaling. It is easily seen that the most general function satisfying these conditions may be written as follows:

$$C(q^2,\nu)=\frac{1}{q^2}\left(\frac{dq^2}{1+dq^2}\right)^f\left[\beta_0(q^2)+\sum_n\beta_n(q^2)\left(\frac{q^2}{\nu}\right)^{b_n}\right], \tag{31}$$

with

$$f>1,\quad b_n\geq 1,$$

$$\lim_{q^2\to 0}\beta_0(q^2)=\beta\neq 0,$$

$$\lim_{q^2\to 0}\beta_n(q^2)(q^2)^{b_n}\ \text{finite},$$

$$\lim_{q^2\to\infty}\beta_0(q^2)=\gamma\neq 0,$$

$$\lim_{q^2\to\infty}\beta_n(q^2)=\gamma_n\ \text{finite}.$$

In order not to introduce too many parameters, we put

$$\beta_0(q^2)=\beta, \tag{32}$$

and since we will be interested only in a region where $q^2/\nu\ll 1$, we neglect $\sum_n\beta_n(q^2)(q^2/\nu)^{b_n}$. Moreover, in order to avoid cuts in the q^2 plane, we set $f=2$ so that

$$C(q^2,\nu)=\frac{\beta}{q^2}\left(\frac{dq^2}{1+dq^2}\right)^2. \tag{33}$$

Up to now, d is a parameter which is only restricted by the condition $d>0$ in order to avoid a pole in $\sigma(q^2,\nu)$. However, since d has the dimension of an inverse mass squared and since the proton mass is the only other dimensioned parameter, it is natural to choose

$$d=1/M^2. \tag{34}$$

Using assumption (ii), we get from (33) and (34) the expressions for σ' and s_0. The expression for σ_{tot} for the physical hadronic processes is obtained by putting $q^2=\mu^2$, where μ is the projectile mass.

For high s we obtain

$$\sigma_{tot}=\sigma_{Regge}+B\,\frac{\mu^2}{(1+\mu^2/M^2)^2}\ln^2\left(\frac{s}{\mu^2}\right), \tag{35}$$

where

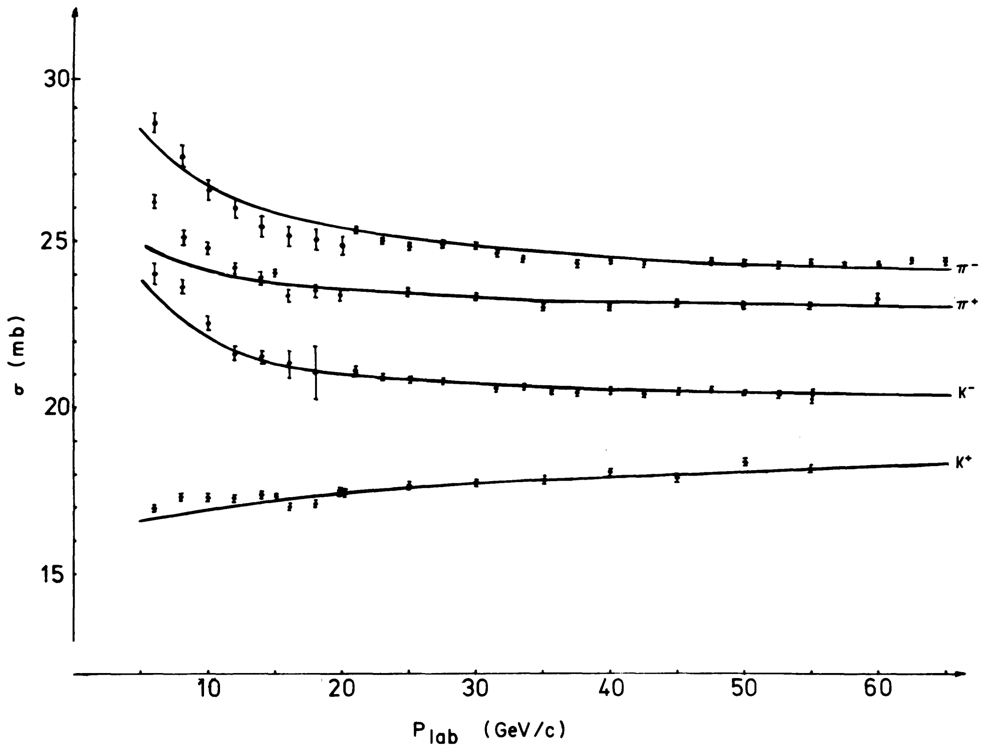

FIG. 4. $\pi^{\pm}p$ and $K^{\pm}p$ total cross sections between 10 and 65 GeV/c. The curves are the results of our fits, while experimental points are taken from Ref. 14.

$$B=\frac{\beta}{M^4} \tag{36}$$

is the only unknown parameter left. It can only depend on the target and is therefore the same for all reactions involving the proton, i.e., for πN, KN, NN, ...

β has no dimension and is probably related to the coupling constant of the pointlike interaction between partons. In Eq. (35), $\sigma_{\rm Regge}$ is the high-ν limit of $\sigma^{(1)}(q^2, \nu)$. It may be asked, as we did for σ', why we do not use our assumptions in order to get a mass dependence of $\sigma_{\rm Regge}$. In fact, this term has certainly no universal dependence on q^2. The example of $\sigma_L^{(1)}$ and $\sigma_T^{(1)}$ well illustrates this fact. Indeed,

$$\lim_{q^2\to 0} \sigma_L^{(1)}=0, \quad \lim_{q^2\to 0} \sigma_T^{(1)}\neq 0. \tag{37}$$

Before we close this section, let us remark that Eq. (35) could be obtained without any recourse to Compton amplitudes if we replace assumption (ii) by the following:

(ii′) The strong off-shell forward amplitudes satisfy scaling:

$$\lim_{\substack{q^2,\,\nu\to\infty\\ \omega\ \text{fixed}}} \nu\sigma_{\rm tot}(q^2, \nu)=f(\omega) \tag{38}$$

and they remain finite when $q^2 \to 0$.

IV. FIT TO TOTAL CROSS SECTIONS

Let us now write Eq. (35) as

$$\sigma_{\rm tot}=\sigma_{\rm Regge}+\frac{C\mu^2}{(1+\mu^2/M^2)^2}\log_{10}{}^2\left(\frac{2M}{\mu^2}\,p_{\rm lab}\right) \tag{39}$$

and fit total cross sections for πN, KN, and NN scattering processes with this formula. To this end, we must introduce an explicit expression for $\sigma_{\rm Regge}$. It is known that, if we take

$$\sigma_{\rm tot}=\sigma_{\rm Regge}=a+\frac{b}{(p_{\rm lab})^{1/2}}\ , \tag{40}$$

it is not possible to get a good fit to all total cross sections between 10 and 65 GeV/c. In accordance with what has been said in Sec. II, we adopt the

TABLE I. Values of parameters used to fit total cross sections.

	a (mb)	b (GeV$^{1/2}$ mb)	$\frac{C\mu^2}{(1+\mu^2/M^2)^2}$ (mb)	$\frac{2M}{\mu^2}$ (GeV^{-1})
K^+p	15.7	0	0.36	7.66
K^-p	15.7	16.1	0.36	7.66
π^+p	21.6	6.5	0.045	95.7
π^-p	21.6	14.5	0.045	95.7
pp	35.0	13.5	0.53	1.07
$\bar{p}p$	35.0	53.0	0.53	1.07

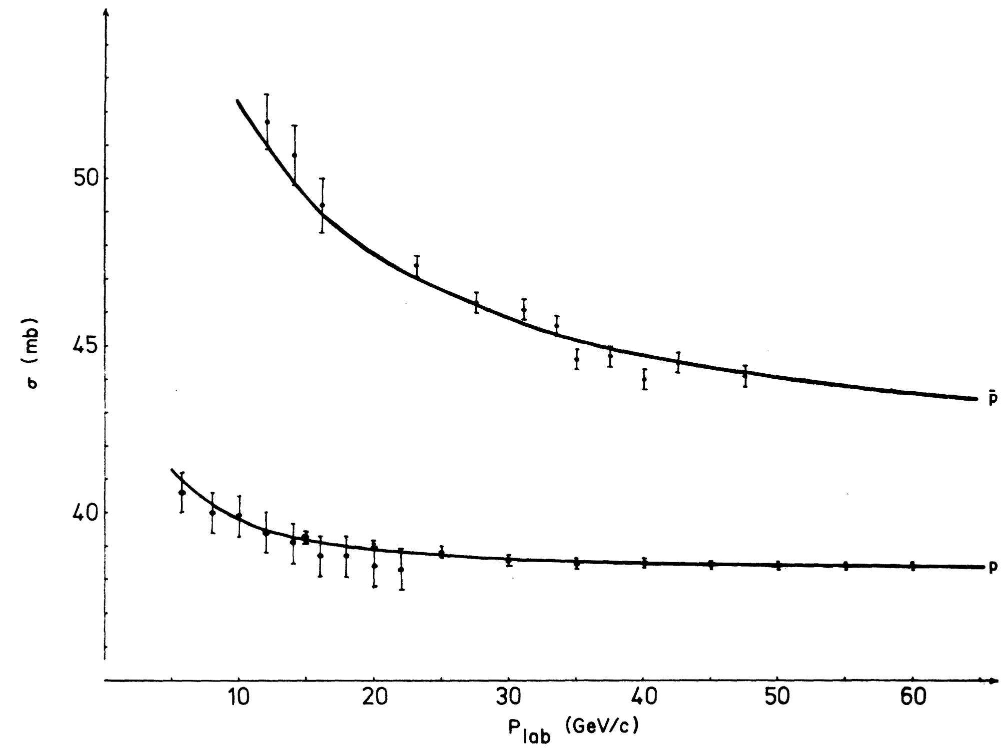

FIG. 5. pp and $\bar{p}p$ total cross sections between 10 and 65 GeV/c. The curves are the results of our fits, while experimental points are taken from Ref. 14.

point of view that once the logarithmic term has been added, the simplest Regge expression (40) already should be enough to get a reasonable fit in the above energy range. *In other words, we think that the observed increase of* $\sigma_{tot}(K^+p)$ *above 10 GeV/c is due mainly to the presence of the logarithmic term and not to the Regge cut(s) which could be introduced.*

This is an hypothesis worth testing. It explains the way the fit was done. The constant C in (35) has been fixed from the K^+p total-cross-section data, putting $b=0$, and it turns out to be

$$C=0.45 \text{ mb GeV}^{-2}. \tag{41}$$

This value of C corresponds from Eq. (36) to a β value of

$$\beta \simeq 0.95\,. \tag{42}$$

With this value of C all other cross sections have been fitted within the same energy range requiring the validity of the Pomeranchuk theorem so as to make the number of free parameters as small as possible. The results of the fits are shown in Figs. 4 and 5 (see Ref. 14), while the values of the various parameters are given in Table I.

Let us notice the weakness of the coefficient of the $\log_{10}^2$ term for $\pi^\pm p$. It is due to the smallness of the pion mass and is related to the vanishing of the $\log_{10}^2$ term for real photons in order to avoid infrared divergences. We will also remark that a and b are, within 10%, approximately the same values as those obtained from fits with the Regge term alone.

From the curves, we see that the contribution of the $\log_{10}^2$ term is hidden by ordinary Regge-trajectory exchange when it is important (as in $\bar{p}p$), while it flattens the curves more quickly than a naive Regge-pole model would do, as in ($\pi^\pm p, pp, K^-p$). In all cases, our model is able to reproduce the experimental data surprisingly well.

These curves have been extrapolated to Batavia (NAL) and ISR energies. For πN and KN, our predictions are plotted in Fig. 6. For pp, the extrapolated curve is compared to the data at these energies.[4,15] The result is shown in Fig. 7. For momenta less than 400 GeV/c, we get a very good agreement. Above 400 GeV/c, the experimental data show a steeper increase of $\sigma_{tot}(pp)$. The origin of this discrepancy is difficult to understand because it seems that our logarithmic term is of the right size to fit data up to 400 GeV/c. In order to see this clearly, we have arbitrarily modified the coefficient $C\mu^2/(1+\mu^2/M^2)^2$ in order to obtain a qualitative agreement between 400 and 2000 GeV/c. We found

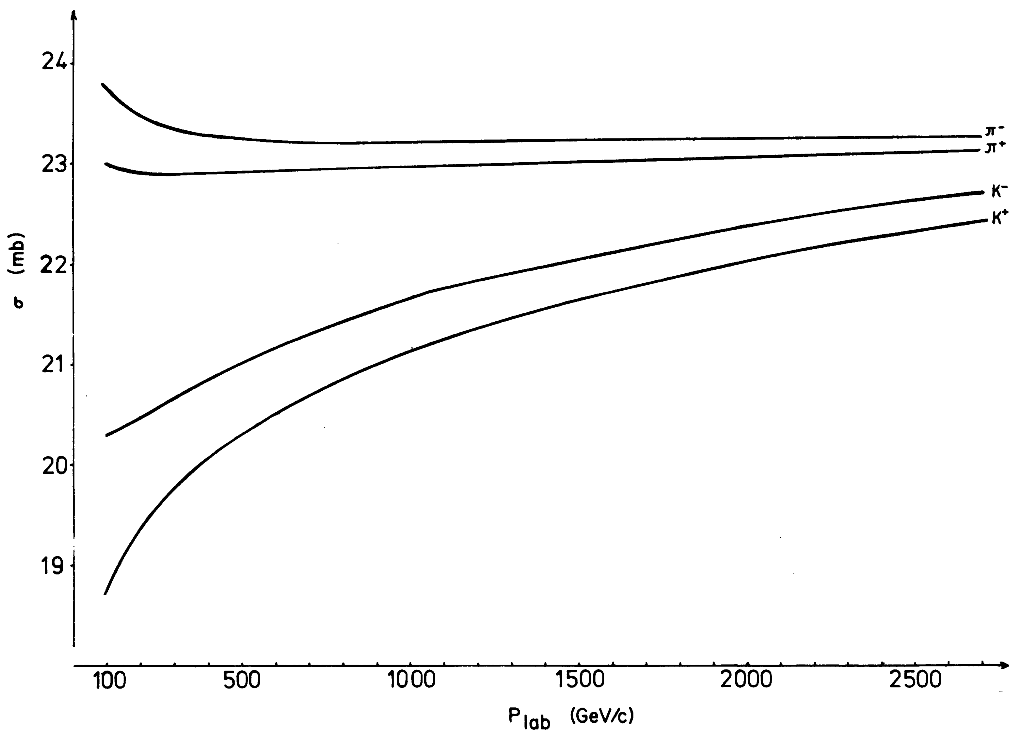

FIG. 6. Extrapolation of our formulas for $\pi^{\pm}p$ and $K^{\pm}p$ cross sections from 100 to 2700 GeV/c.

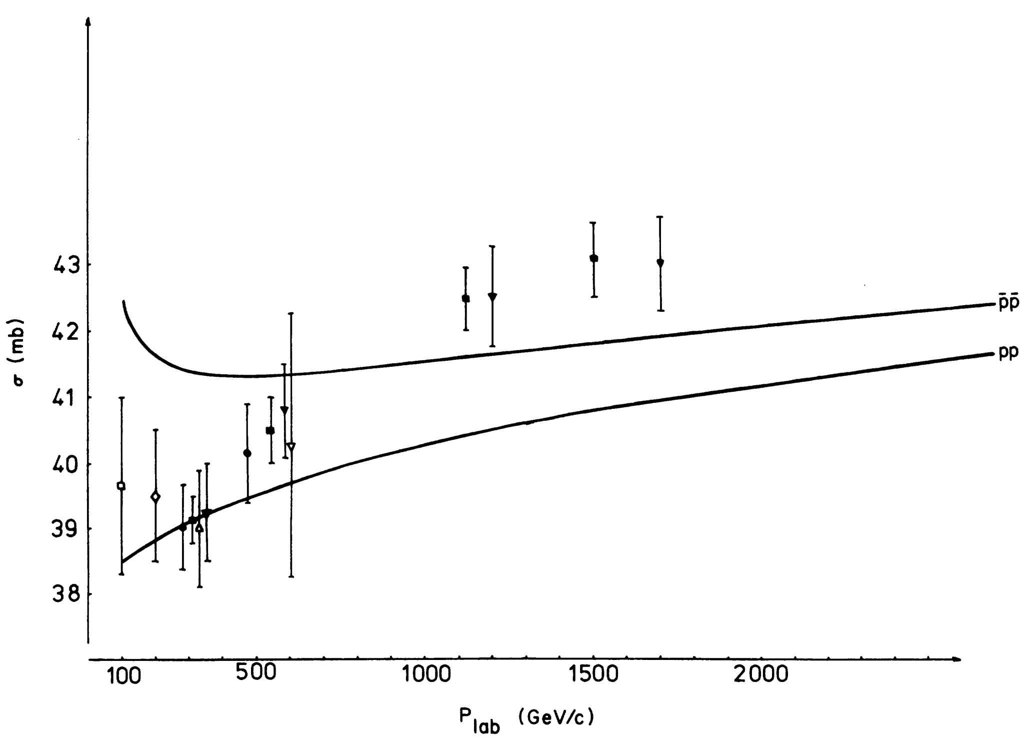

FIG. 7. Extrapolation of our formulas for pp and $\bar{p}p$ cross sections from 100 to 2700 GeV/c. Experimental points for $\sigma_{tot}(pp)$ are taken from the following works: □, Chapman *et al.* (Ref. 15); ◇, Charlton *et al.* (Ref. 15); △, Dao *et al.* (Ref. 15); ▽, Holder *et al.* (Ref. 15); ■●, Amaldi *et al.* (Ref. 4); ▼, Amendolia *et al.* (Ref. 4).

$$\frac{C\mu^2}{(1+\mu^2/M^2)^2} \simeq 0.7 \text{ mb}. \tag{43}$$

However, with this new value,

$$\sigma_{\text{tot}}(pp) \simeq 38.9 \text{ mb} \tag{44}$$

at 60 GeV instead of

$$\sigma_{\text{tot}}^{\text{exp}}(pp) \simeq 38.4 \pm 0.1 \text{ mb}. \tag{45}$$

We see that the value (44) is definitely too high. We can reduce it by reducing the Regge contribution. For instance, a reduction of the b parameter would do the job but then we would have difficulty in fitting $\sigma_{\text{tot}}(pp)$ correctly between 5 and 20 GeV.

In conclusion, the discrepancy may have one of the following origins:

(1) There are systematic errors in the data at the highest energies. This possibility looks improbable since in the two experiments of Amaldi *et al.*[4] and Amendolia *et al.*[4] systematic errors are different, but nevertheless their results agree.

(2) The Regge parametrization we have chosen is too naive, i.e., Regge cuts do play a significant role. In the parametrization chosen above their effect is already partially taken into account by breaking exchange degeneracy of the f and ω Regge residues; therefore the introduction of further effects looks quite arbitrary.

(3) The observed increase of $\sigma_{\text{tot}}(pp)$ inside the ISR energy range is partly due to nonasymptotic phenomena. This possibility is strongly suggested by the fact that our fit starts to disagree with the data only above 400 GeV/c. This fact finds a simple explanation if both small-p_T and large-p_T events do contribute to the rise of total cross sections, the contribution of the large-p_T events becoming significant above 400 GeV/c. A model based on this idea is presently being investigated.

V. THE REAL PART OF THE pp AMPLITUDE

Using forward dispersion relations and our parametrization for total cross sections, we can of course give asymptotic estimates of the real parts of the forward amplitudes. In the 100-GeV/c region, we only have data for pp scattering. Though the errors are big, these data suggest that

$$\alpha_{pp} = \frac{\text{Re}A_{pp}}{\text{Im}A_{pp}} \tag{46}$$

probably vanishes somewhere between 200 and 400 GeV/c.

In this section, we present the calculation of this quantity. Apart from minor normalization differences we follow the notations of Ref. 16. The dispersion relations for

$$A_\pm(E) = \tfrac{1}{2}(A_{pp} \pm A_{p\bar{p}}) \tag{47}$$

are

$$\text{Re}A_+(E) = \text{Re}A_+(M) + 2\pi M f^2 \frac{(E^2-M^2)}{(E_B{}^2-M^2)} \frac{2E_B}{(E_B{}^2-E^2)} + \frac{2M}{\pi}(E^2-M^2)\int_M^{+\infty}\frac{dE'}{p'}\sigma_+(E')\left(\frac{1}{E'-E}+\frac{1}{E'+E}\right)$$
$$-\frac{1}{\pi}\int_U \frac{E'dE'\text{Im}A_{p\bar{p}}(E')}{E'^2-M^2}, \tag{48}$$

$$\text{Re}A_-(E) = 4\pi f^2 \frac{ME}{E_B{}^2-E^2} + \frac{4M^3}{\pi E}\int_M^{+\infty}\frac{dE'}{p'}\sigma_-(E') + \frac{4ME}{\pi}\left(1-\frac{M^2}{E^2}\right)\int_M^{+\infty}\frac{E'^2}{E'^2-E^2}\sigma_-(E')\frac{dE'}{p'}. \tag{49}$$

In these relations

$$\sigma_\pm(E) = \tfrac{1}{2}(\sigma_{pp} \pm \sigma_{p\bar{p}}), \tag{50}$$

E is the laboratory energy of the incident proton, $f^2 = 0.08$, $E_B = (\mu^2 - 2M^2)/2M$, μ is the pion mass, M is the proton mass, and U is the unphysical cut.

To obtain the dispersion relation for $\text{Re}A_-(E)$ [Eq. (49)], we have eliminated the unphysical-cut contribution using the fact that in our parametrization

$$\sigma_-(E) \sim O(1/\sqrt{E}). \tag{51}$$

Since we want only to know α_{pp} for $E > 20$ GeV, we have calculated the first few terms of the asymptotic developments of $\text{Re}A_\pm(E)$. The calculational details can be found in the Appendix and in Ref. 17. We get, using (A9), (A10), and (A11),

$$\text{Re}A_+ \sim \alpha E \ln E + \beta E + \gamma\sqrt{E}, \tag{52}$$

$$\text{Re}A_- \sim O(1/\sqrt{E}). \tag{53}$$

From Table I, we obtain for the parameters α, β, and γ the values

$$\alpha = 0.56, \quad \beta = 0.036, \quad \gamma = -61.9. \tag{54}$$

When $E = 100$ GeV, we can see that

$$\frac{\text{Re}A_-}{\text{Re}A_+} \sim 1\%; \tag{55}$$

therefore, we set

$$\text{Re}A_- = 0. \tag{56}$$

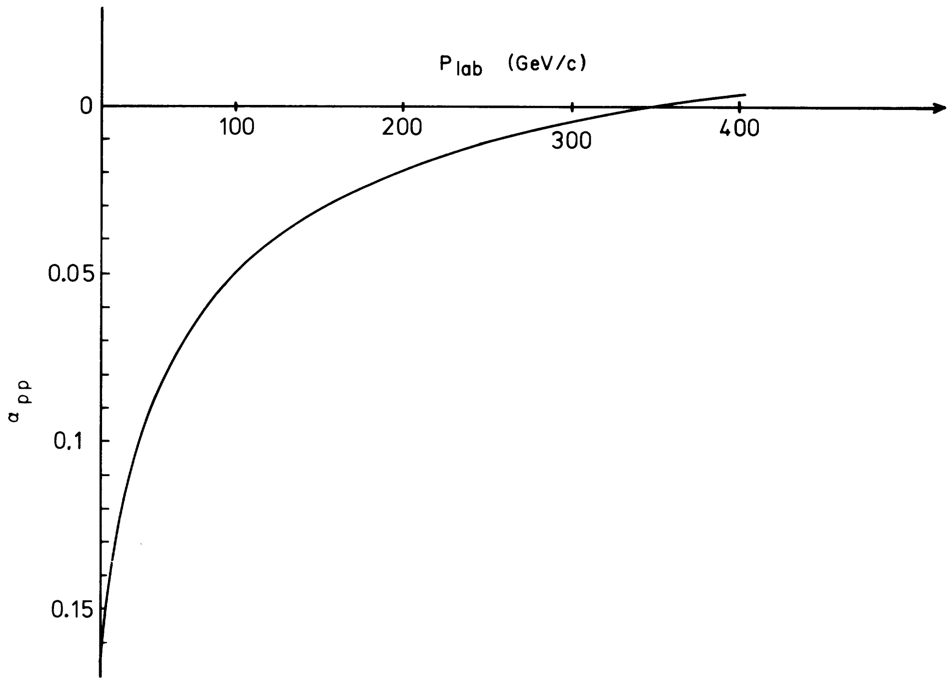

FIG. 8. Our prediction for $\alpha_{pp}=\mathrm{Re}A_{pp}/\mathrm{Im}A_{pp}$.

It follows that

$$\alpha_{pp}(E)\approx\alpha_+(E)\approx\alpha_{p\bar{p}}(E). \quad (57)$$

From (A11) and the dispersion relation (48), we see immediately that

$$\lim_{E\to+\infty}\frac{\alpha_{pp}(E)}{(\pi/\ln E)}=1\,. \quad (58)$$

When the energy is large enough, α_{pp} becomes positive, reaches a maximum, and tends to zero as $\pi/\ln E$.

Using (48), (52), and the optical theorem, we have computed α_{pp}. The resulting curve is given in Fig. 8. We see that

$$\alpha_{pp}=0 \quad (59)$$

for $E=360$ GeV. When $E>360$ GeV, α_{pp} increases very slowly and it attains its maximum value

$$\alpha_{pp}=+0.08 \quad (60)$$

when

$$E\simeq 2.4\times 10^8\ \mathrm{GeV}.$$

VI. CONCLUSIONS AND FURTHER REMARKS

We have introduced a model for total cross sections which rise as fast as they are allowed to by the Froissart bound. Making use of a scaling assumption in the timelike region, we have been able to fix the scale inside the logarithmic term and to determine its relative strength. The resulting expression has several convenient features:

(i) It includes the Regge contribution quite naturally.

(ii) It incorporates the Pomeranchuk theorem.

(iii) It gives a correction to the Regge contribution which becomes negligible for small energies.

(iv) It is able to fit beautifully all total cross sections from 10 to 400 GeV/c.

(v) It predicts that α_{pp} should be zero at 360 GeV/c.

However, everything is not clear. We have no simple picture of the relation between the increase with energy of the large-p_T processes and the appearance of the logarithmic term in the total cross section. The discrepancy between our $\sigma_{\mathrm{tot}}(pp)$ curve and the experimental data above 400 GeV/c cannot be explained convincingly. As we have noticed in Sec. IV, our curve is very sensitive to the value of β. It is determined from the fit to $\sigma_{\mathrm{tot}}(K^+p)$ assuming *exact* strong exchange degeneracy for the ω and f trajectories. This is certainly an oversimplification. If we break it slightly, we shall get a higher value of β and consequently a better fit to the highest-energy data.

The scaling assumption we have made must be further investigated and may well turn out to be untrue. A very specific property of the model is that it predicts a much slower rise for $\sigma_{\mathrm{tot}}(\pi^\pm p)$ than for $\sigma_{\mathrm{tot}}(pp)$. For instance, at 200 GeV/c

$\sigma_{tot}(\pi^- p)$ is *smaller* than at 65 GeV/c by about 0.5 mb, while $\sigma_{tot}(pp)$ is *bigger* at 200 GeV/c than at 65 GeV/c by about 0.5 mb. A measurement of $\sigma_{tot}(\pi N)$ in the 100 GeV/c range is therefore a crucial test for it.

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APPENDIX

In many cases, the principal-value integral

$$\phi(x) = \text{P.V.} \int_0^\infty \frac{f(x')}{x'-x}\, dx' \tag{A1}$$

can be written as a Cauchy-integral representation. This is always the case in dispersion theory. Therefore, to calculate (A1), we do not need to make an integration. We have only to guess a complex function $F(Z)$ such that

(i) $F(Z)$ is holomorphic in the Z plane cut along the positive real axis;

(ii) $F(Z) = F^*(Z^*)$;

(iii) $F(Z) \to 0$ when $|Z| \to \infty$.

For infinitely oscillating functions, condition (iii) can be violated along the positive real axis. Since we do not need to consider this case here, we refer the reader to the specialized literature and to Ref. 17.

The function $F(Z)$ determines the integral since

$$\lim_{\epsilon \to 0} \text{Re} F(Z)|_{Z=x+i0} = \phi(x)\,. \tag{A2}$$

Following this procedure, we can easily calculate the P.V. integrals we need here. For $x > 0$ and $\alpha \in [0, 1]$ we find that

$$\text{P.V.} \int_0^\infty \frac{dx'}{x'^\alpha} \frac{1}{x'-x} = \pi \cot\pi\alpha \frac{1}{x^\alpha}\,, \tag{A3}$$

$$\text{P.V.} \int_0^\infty \frac{\ln x'}{x'^\alpha} \frac{1}{x'-x}\, dx' = \pi \cot\pi\alpha \frac{\ln x}{x^\alpha} + \frac{\pi^2}{\sin^2\pi\alpha} \frac{1}{x^\alpha}\,, \tag{A4}$$

$$\text{P.V.} \int_0^\infty \frac{\ln^2 x'}{x'^\alpha} \frac{1}{x'-x}\, dx' = \pi \cot\pi\alpha \frac{\ln^2 x}{x^\alpha} + \frac{2\pi^2}{\sin^2\pi\alpha} \frac{\ln x}{x^\alpha} + 2\pi^3 \frac{\cot\pi\alpha}{\sin^2\pi\alpha} \frac{1}{x^\alpha}\,. \tag{A5}$$

To determine the dominant terms of the asymptotic expansion of $\text{Re}F_\pm(E)$ we have to determine the dominant contributions of the dispersion integrals over $\sigma_\pm(E)$. We show how this can be done in the case of $\text{Re}F_+(E)$. The case of $\text{Re}F_-(E)$ can be treated similarly. The dispersion integral is

$$\int_M^{+\infty} \frac{E'}{(E'^2 - M^2)^{1/2}} \frac{\sigma_+(E')}{E'^2 - E^2}\, dE'\,, \tag{A6}$$

where $E \gg M$. We neglect terms of $(1/E^2)$; therefore, (A6) can be replaced by the integral

$$\int_0^{+\infty} dE' \frac{\sigma_+(E')}{E'^2 - E^2}\,, \tag{A7}$$

with

$$\sigma_+(E) = a_+ + \frac{b_+}{\sqrt{E}} + c_+ (\ln\lambda E)^2\,, \tag{A8}$$

where a_+, b_+, c_+, and λ are the parameters defined in the text. The constant a_+ gives no contribution to (A7). The b_+/E term gives [using Eq. (A3)]

$$b_+ \int_0^\infty \frac{1}{\sqrt{E'}} \frac{dE'}{E'^2 - E^2} = -\frac{b_+\pi}{2} \frac{1}{E\sqrt{E}}\,. \tag{A9}$$

The third term $c_+(\ln\lambda E)^2$ can be decomposed into three parts:

$$P_1 = c_+ \ln^2 E\,,$$

$$P_2 = 2c_+ \ln\lambda \ln E\,,$$

$$P_3 = c_+ \ln^2\lambda\,.$$

P_3 gives no contribution. P_2 gives [using Eq. (A4)]

$$2c_+ \ln\lambda \int_0^\infty \frac{\ln E'}{E'^2 - E^2}\, dE' = \tfrac{1}{2}\pi^2 c_+ (\ln\lambda) \frac{1}{E}\,. \tag{A10}$$

P_1 gives [using Eq. (A5)]

$$c_+ \int_0^\infty \frac{\ln^2 E'}{E'^2 - E^2}\, dE' = \frac{c_+\pi^2}{2} \frac{\ln E}{E}\,. \tag{A11}$$

Notice that to get (A9), (A10) and (A11), we have made the substitutions $x = E^2$, $x' = E'^2$. The complex function $F(Z)$ which solves (A11) is given in the x plane by

$$F(Z) = i\frac{\ln^2(-Z)}{\sqrt{Z}} + i\frac{\pi^2}{\sqrt{E}}\,, \tag{A12}$$

where the arguments of the logarithm and of the square-root function have been chosen in such a way that for $Z = x + i0$, $x \geq 0$,

$$\ln(-Z) = \ln x - i\pi\,, \quad \sqrt{Z} = \sqrt{x}\,. \tag{A13}$$

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$$P_{iT}^2 = \frac{tu}{s} - \frac{1}{s}(m^2 - M_{i*}^2)(m^2 - m_i^2) - \frac{m^2(M_{i*}^2 - m_i^2)^2}{s^2},$$

where M_{i*} is the mass of the pseudoparticle. The last two terms are of $O(1)$. Therefore, they do not alter the validity of the subsequent argument.

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