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Analysis of $\bar{p}p$ elastic scattering using two coherent exponentials

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Published data from many groups on $\bar{p}p$ elastic scattering at twenty-eight laboratory momenta have been analyzed with a parametrization corresponding to two coherent interfering exponentials. The data from almost all of these experiments can be adequately represented by this parametrization. In most of the experiments the data are such that the fit does not uniquely determine all five parameters with reasonable errors. In fits to at least eight experiments the parameters are well determined. Using these parameters as guides, attempts are made to reduce the total number of parameters and still fit all the data.

I. INTRODUCTION

In studies of $\bar{p}p$ elastic scattering at incident laboratory momenta of 1.11, 1.33, and 1.52 GeV/c, Kalbfleisch *et al.*[1] used two coherent interfering exponentials to parametrize the dependence of the differential scattering cross section on t, the square of the four-momentum transfer. Later experiments at 2.33 (Ref. 2) and 2.85 GeV/c (Ref. 3) have also been successfully represented over a large t range with this parametrization as well. An additional experiment[4] at 2.32 GeV/c has also been successfully analyzed[2] using this formula. This led to speculation that this form might represent, at least qualitatively, the data over a large range of center-of-mass energy as well as a large range in t. The change in the qualitative appearance of the data as the laboratory momentum varies from about 1 GeV/c to about 16 GeV/c, the range over which data on this reaction are available, might then be understood as a decrease in the amplitude of the second exponential term relative to the first, or as a change in the amplitude combined with a change in the phase between the two terms.

In order to test this idea, the data summary on this reaction from the Particle Data Group[5] was used to select a beginning series of experiments to fit. The initial selection criteria were quite simple. Each experiment had to cover the t range of the diffraction peak, the first minimum corresponding to $-t\approx 0.35$ (GeV/c)^2, and the region out to $-t\approx 0.6$ (GeV/c)^2, where the second maximum occurs. The most useful data sets also go beyond the second maximum to show a secondary decrease in the differential cross section. To this group of data we added the 2.32-GeV/c data of the Oxford-Argonne collaboration,[4] the data of our own experiments at 2.33 and 2.85 GeV/c, and the data of the Argonne EMS (effective mass spectrometer) group at 3.0, 3.65, 5.0, and 6.0 GeV/c.[6,7] This yielded a data sample of 28 energies for which the differential cross section has been measured and the above criteria satisfied. These 28 energies and the references[1-14] are summarized in Table I.

It has also been necessary to limit the t range of the data used at any energy. If the measured values correspond to t values that go beyond the second-peak structure, a maximum-t cutoff on the data has been used. If the experiment has reported a cross section at very low t values where detection efficiencies may be causing an apparent decrease in the differential cross section, then a minimum-t-value cutoff on the data has been applied. In order to completely specify what has been done, the t range of each data sample used is also shown in Table I.

II. THE FIVE-PARAMETER FITS

Two coherent interfering exponentials correspond to the equation

TABLE I. Specifications of the $\bar{p}p$ elastic-scattering data sets discussed.

p_{lab} (GeV/c)	$-t_{min}$ [(GeV/c)^2]	$-t_{max}$ [(GeV/c)^2]	Reference and source	Confidence level[a] (percent)
1.11	0.015	0.695	Kalbfleisch *et al.*, Ref. 1[b]	0.7
1.23	0.035	0.780	Bacon *et al.*, Ref. 8[b]	16.1
1.30	0.035	1.08	Bacon *et al.*, Ref. 8[b]	46.0
1.33	0.025	0.695	Kalbfleisch *et al.*, Ref. 1[b]	1.2
1.36	0.035	1.08	Bacon *et al.*, Ref. 8[b]	2.3
1.43	0.035	1.08	Bacon *et al.*, Ref. 8[b]	17.7
1.44	0.170	0.90	Berryhill *et al.*, Ref. 9[b]	5.1
1.51	0.0475	1.0715	Parker *et al.*, Ref. 10[b]	39.4
1.65	0.054	1.117	Parker *et al.*, Ref. 10[b]	8.4
1.73	0.097	1.227	Daum *et al.*, Ref. 11[b]	1.7
1.80	0.0615	1.352	Parker *et al.*, Ref. 10[b]	65.6
1.95	0.069	1.472	Parker *et al.*, Ref. 10[b]	6.3
2.15	0.0795	1.532	Parker *et al.*, Ref. 10[b]	61.7
2.32	0.0441	1.469	Allison *et al.*, Ref. 4	19.7
2.33	0.0425	1.50	Crawley *et al.*, Ref. 2	42.5
2.45	0.0945	1.5815	Parker *et al.*, Ref. 10[b]	39.7
2.60	0.103	1.473	Parker *et al.*, Ref. 10[b]	80.8
2.69	0.045	1.6135	Domingo *et al.*, Ref. 12[b]	46.6
2.75	0.111	1.5555	Parker *et al.*, Ref. 10[b]	21.8
2.85	0.0425	1.75	Crawley *et al.*, Ref. 3	39.3
2.90	0.1185	1.50	Parker *et al.*, Ref. 10[b]	51.8
3.00	0.035	1.35	Ambats *et al.*, Ref. 6[c]	7.5
3.65	0.035	1.45	Ambats *et al.*, Ref. 6[c]	87.5
5.00	0.035	1.55	Ambats *et al.*, Ref. 6[c]	30.8
5.70	0.022	1.54	Böckmann, Ref. 13[b]	85.2
6.00	0.045	1.55	Ambats *et al.*, Ref. 6[c]	50.4
8.00	0.046	0.86	Birnbaum *et al.*, Ref. 14[b]	0.003
16.00	0.111	1.18	Birnbaum *et al.*, Ref. 14[b]	82.5

[a] Confidence level of the fit to Eq. (1).
[b] Source of data used: Particle Data Group, Ref. 5.
[c] Source of data used: A. B. Wicklund, private communication.

$$\frac{d\sigma}{dt}=\left(\frac{d\sigma}{dt}\right)_0\left|\frac{e^{b_1t/2}+|A|e^{i\varphi}e^{b_2t/2}}{1+|A|e^{i\varphi}}\right|^2. \qquad (1)$$

In this representation $(d\sigma/dt)_0$ is the differential cross section at $t=0$, b_1 and b_2 are the first and second slopes, respectively, $|A|$ is the relative amplitude between the two terms, and φ is the phase angle between these terms.

This equation of course may be susceptible to numerical analysis problems most easily exemplified by examination of what happens if $A\rightarrow 1$, $\varphi\rightarrow 180°$, and $b_2\rightarrow b_1$. Under these conditions the equation becomes undefined, approaching the value zero over zero. When this occurs the four variables b_1, b_2, $|A|$, and φ become highly correlated and as a result the parameters become poorly determined. These conditions are nearly satisfied, particularly at low incident momenta. The intercept $(d\sigma/dt)_0$ is not highly correlated with the other variables. We are able to determine all five parameters with uncertainties that correspond typically to less than 25% of the parameter value for only eight of the data sets. These are the experiments at incident laboratory momenta of 1.80, 2.15, 2.32, 2.33, 2.60, 2.69, 2.85, and 3.65 GeV/c.

It should be emphasized, however, that this does not mean that the data at the remaining momenta cannot be represented adequately by Eq. (1). In order to illustrate how well Eq. (1) is able to represent the data, the calculated distribution

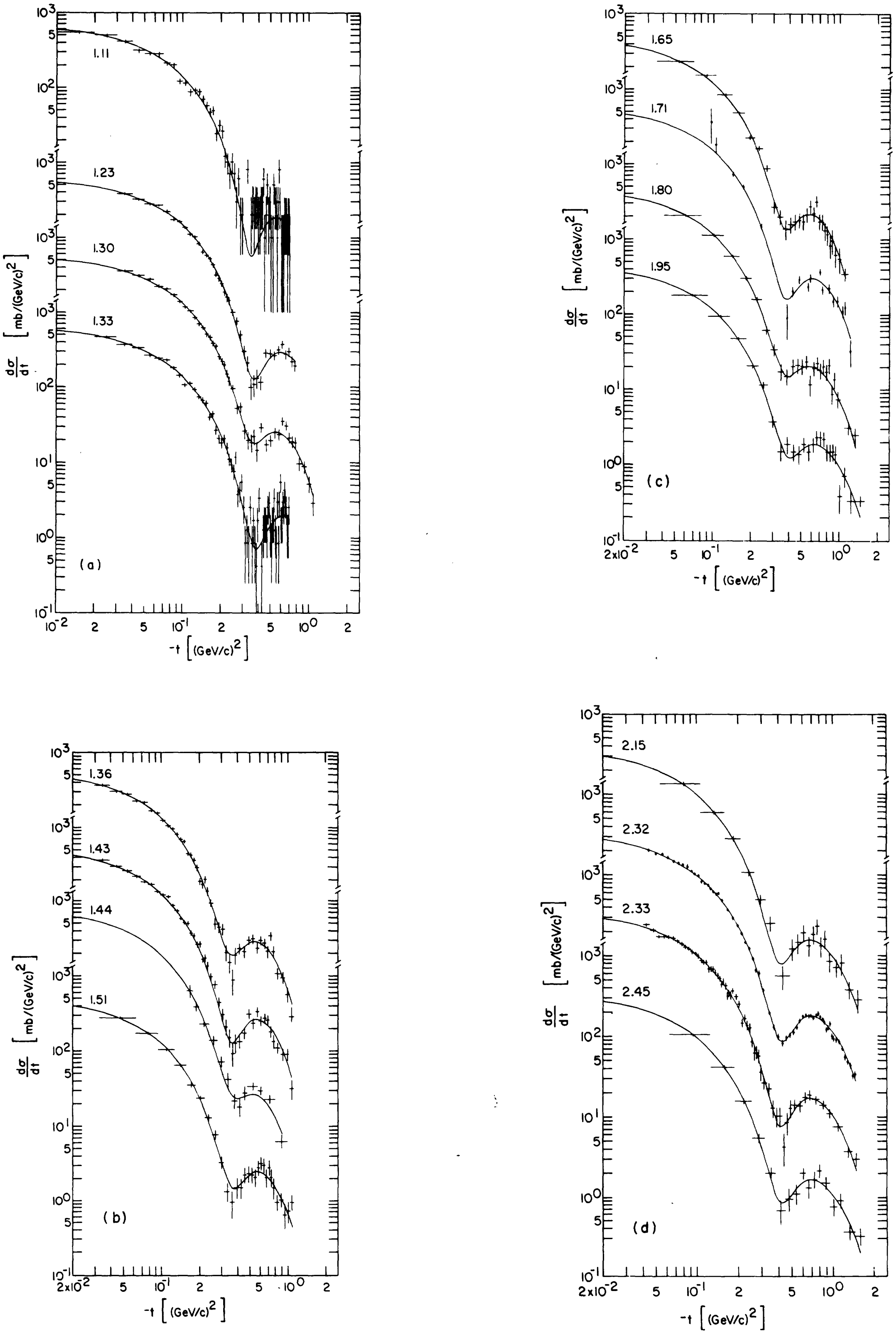
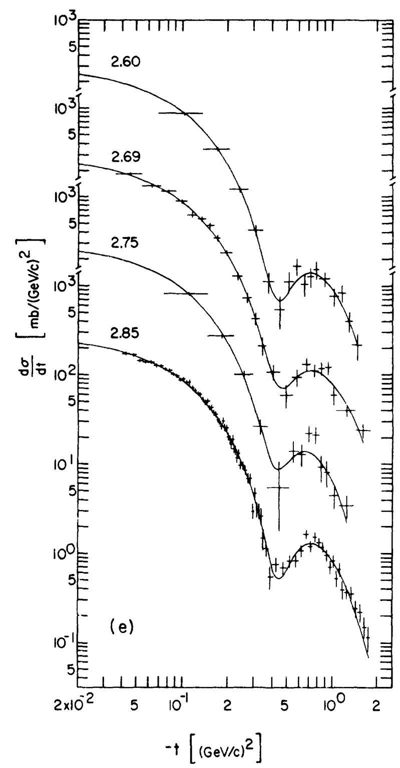
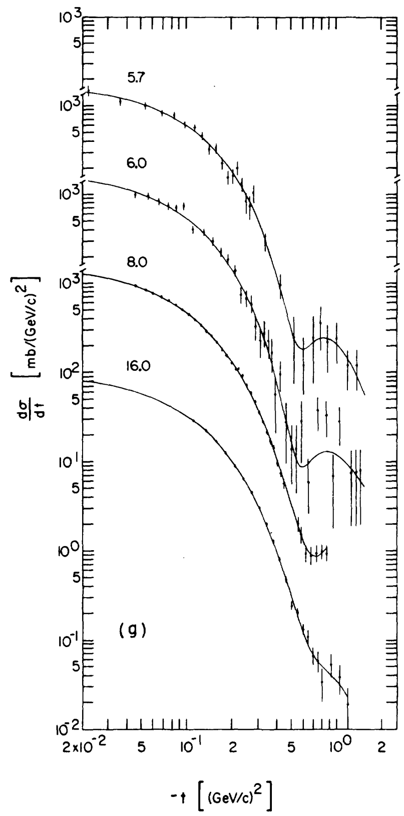


FIG. 1. *(Continued on following page.)*





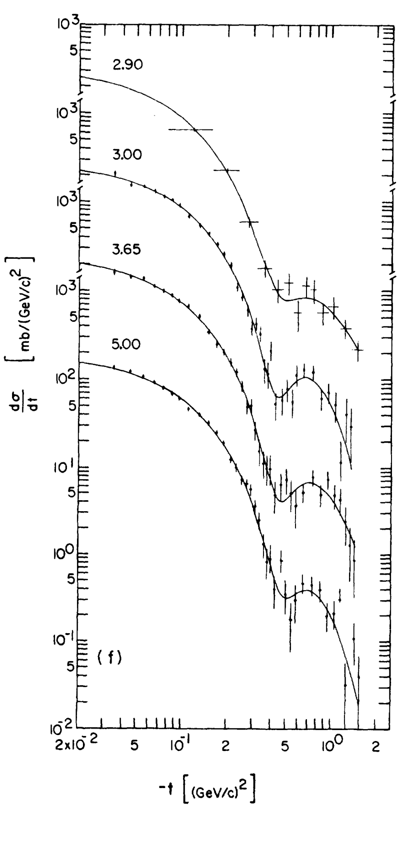


FIG. 1. $\bar{p}p$ elastic differential cross section corresponding to the 28 data samples used in the analysis. Smooth curves are best fits to the respective data sample obtained by adjusting all five parameters of Eq. (1). The curve labels are P_{lab} in GeV/c.

using the best values of the five parameters is shown superimposed upon the experimental data in Fig. 1. The typical set of data at these momenta is well represented but, except for the eight experiments mentioned above, four of the five parameters are so poorly determined that they are of little use in understanding the energy dependence of the parameters. The confidence level of the fit for each set of data is listed in Table I. These confidence levels show that of the 28 data sets which we attempted to fit, not all can be well represented by this form. The quality of the fit is poor for at least five sets of data. The 8.0-GeV/c data have a confidence level of about $3\times10^{-3}\%$. The 1.11, 1.33, 1.36, and 1.73 GeV/c data have confidence levels between 0.5 and 2.5%. Twenty of the 28 data sets, however, have very acceptable fits.

The values of the parameters which result from fitting Eq. (1) to the data of the eight experiments

TABLE II. Five-parameter fits to Eq. (1) for the eight data sets which had well-determined parameters.

P_{lab} (GeV/c)	$(d\sigma/dt)_0$ [mb/(GeV/c)^2]	b_1 [(GeV/c)$^{-2}$]	b_2 [(GeV/c)$^{-2}$]	φ (deg)	$\lvert A\rvert$
1.80	495.8 ± 19.1	11.20 ± 0.81	4.56 ± 0.57	161.9 ± 4.9	0.305 ± 0.073
2.15	392.0 ± 22.0	11.16 ± 0.73	3.09 ± 0.43	160.6 ± 4.7	0.206 ± 0.045
2.32	356.2 ± 5.0	10.04 ± 0.20	3.44 ± 0.13	163.8 ± 1.1	0.269 ± 0.017
2.33	375.6 ± 9.3	10.21 ± 0.46	3.57 ± 0.35	164.9 ± 2.7	0.268 ± 0.042
2.60	314.8 ± 21.1	9.62 ± 0.97	3.53 ± 0.66	164.7 ± 5.5	0.281 ± 0.092
2.69	308.8 ± 11.6	10.64 ± 0.56	2.77 ± 0.36	156.7 ± 4.7	0.188 ± 0.035
2.85	292.7 ± 5.5	9.24 ± 0.38	3.83 ± 0.30	168.0 ± 2.1	0.319 ± 0.045
3.65	262.3 ± 9.6	10.88 ± 0.71	3.27 ± 0.63	159.4 ± 5.7	0.194 ± 0.056

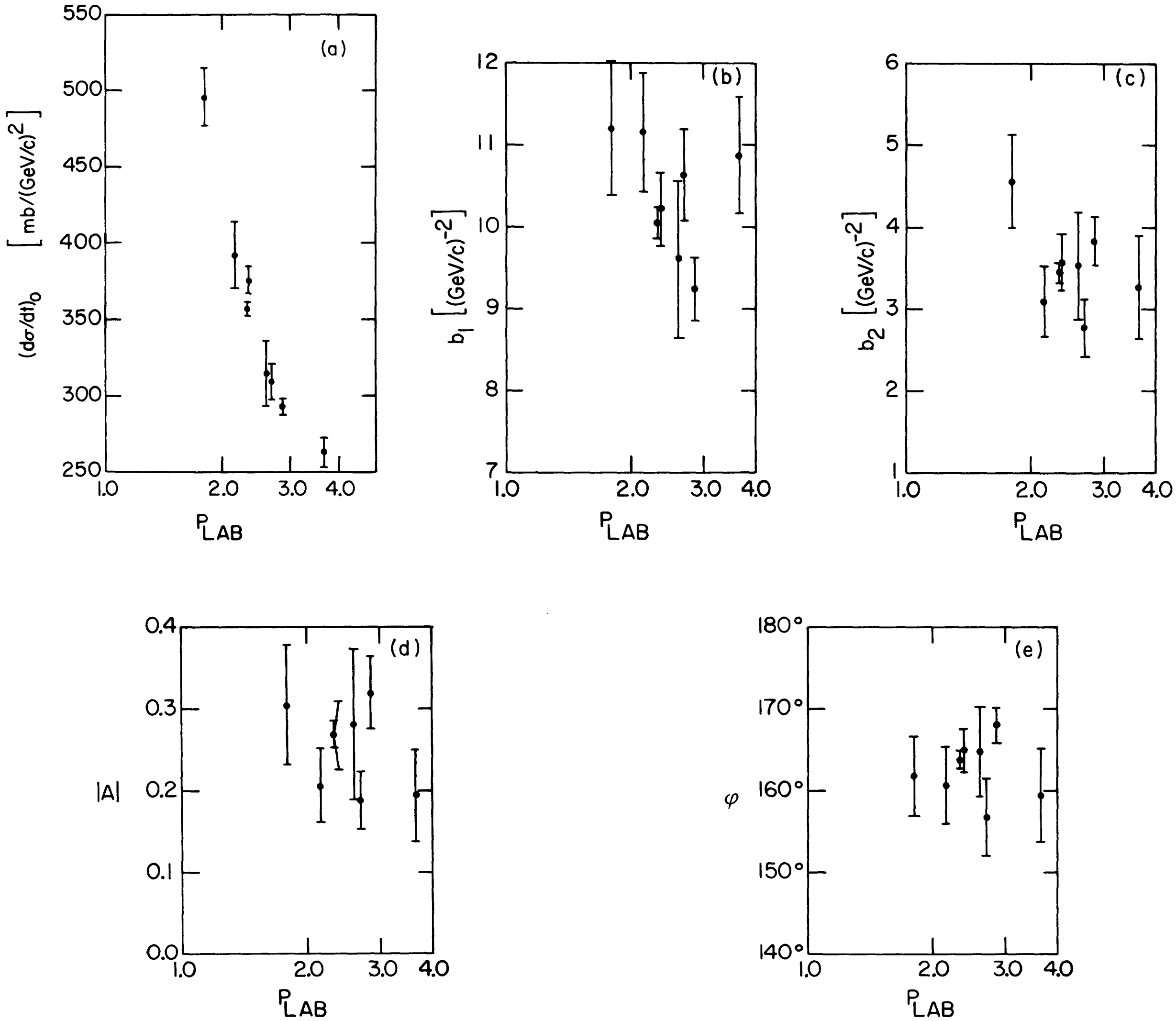


FIG. 2. Equation (1) parameter values for the eight data samples which yield well-determined parameters when fitted with all five parameters allowed to vary. (a) Intercept $(d\sigma/dt)_0$ versus laboratory beam momentum P_{lab} (GeV/c). (b) Slope b_1 versus P_{lab}. (c) Slope b_2 versus P_{lab}. (d) Relative amplitude $|A|$ versus P_{lab}. (e) Relative phase angle φ versus P_{lab}.

TABLE III. Four-parameter fits to Eq. (1) with b_1 fixed at the constant value 10 $(\text{GeV}/c)^{-2}$.

P_{lab} (GeV/c)	$(d\sigma/dt)_0$ [mb/(GeV/c)^2]	b_2 [(GeV/c)$^{-2}$]	$\lvert A\rvert$	φ (deg)	χ^2/ν	Confidence level (%)
1.43	569.3 ± 9.2	5.91 ± 0.18	0.486 ± 0.019	170.5 ± 0.6	1.24	14
1.44	873.5 ± 138.4	7.53 ± 0.66	0.655 ± 0.080	173.0 ± 2.4	1.77	7
1.51	531.0 ± 11.8	5.87 ± 0.26	0.484 ± 0.026	170.8 ± 0.7	1.03	42
1.65	507.4 ± 10.8	5.80 ± 0.24	0.468 ± 0.024	168.7 ± 0.8	1.48	6
1.73	604.8 ± 44.1	4.86 ± 0.30	0.392 ± 0.026	168.1 ± 1.9	1.86	3
1.80	476.5 ± 10.9	5.44 ± 0.22	0.441 ± 0.022	168.5 ± 0.9	0.89	61
1.95	445.1 ± 11.4	4.54 ± 0.22	0.355 ± 0.020	166.9 ± 1.1	1.61	4
2.15	366.9 ± 12.3	3.76 ± 0.21	0.292 ± 0.018	166.6 ± 1.7	0.92	55
2.32	355.3 ± 2.9	3.47 ± 0.05	0.272 ± 0.004	164.1 ± 0.4	1.14	22
2.33	372.4 ± 5.9	3.73 ± 0.13	0.288 ± 0.011	166.1 ± 0.9	1.01	46
2.45	343.3 ± 11.5	3.43 ± 0.21	0.269 ± 0.017	164.0 ± 1.6	0.99	46
2.60	322.1 ± 11.3	3.30 ± 0.24	0.249 ± 0.018	162.6 ± 1.8	0.59	85
2.69	299.2 ± 7.1	3.17 ± 0.18	0.233 ± 0.014	161.4 ± 1.7	1.00	46
2.75	339.3 ± 17.4	4.29 ± 0.42	0.324 ± 0.037	162.0 ± 2.3	1.29	23
2.85	301.9 ± 3.6	3.29 ± 0.10	0.243 ± 0.008	163.7 ± 0.8	1.09	29
2.90	276.6 ± 14.2	3.34 ± 0.35	0.235 ± 0.026	159.2 ± 2.8	1.21	28
3.00	296.0 ± 6.4	4.27 ± 0.29	0.306 ± 0.023	163.7 ± 1.2	1.45	5
3.65	254.2 ± 6.1	4.02 ± 0.31	0.276 ± 0.023	165.3 ± 1.6	0.74	86
5.00	202.2 ± 5.0	4.59 ± 0.34	0.289 ± 0.029	161.9 ± 1.6	1.09	33
5.70	181.9 ± 7.5	2.78 ± 0.60	0.142 ± 0.035	153.7 ± 5.2	0.68	88
6.00	177.5 ± 7.4	4.25 ± 0.69	0.222 ± 0.046	165.7 ± 3.7	1.02	44
16.00	97.5 ± 1.6	4.65 ± 0.52	0.203 ± 0.034	145.6 ± 1.4	0.75	76

(1.80, 2.15, 2.32, 2.33, 2.60, 2.69, 2.85, and 3.65 GeV/c) are listed in Table II and shown in Figs. 2(a) through 2(e). These eight were selected for presentation as was mentioned before, because in each case the parameters have typical uncertainties of ≤ 25%. It should also be noted that the confidence level is 20% or greater for the fits to these data sets.

A good test of the validity of Eq. (1) would be to have better experimental data at many energies from which all five parameters could be accurately determined. In the absence of data from which this can be done, it seems worthwhile to try some educated guesses in an attempt to reduce the number of free parameters. In a four-parameter representation the correlations might be greatly reduced and the other experiments would be able to provide at least an indication of how valid the reduced parametrization is.

We have no *a priori* knowledge about the energy dependence of most of these parameters. Thus we are forced to try to find those parameters or combinations of parameters which might be independent of energy. The known variation of the total cross section for $\bar{p}p$ interactions, coupled with the optical theorem, eliminates $(d\sigma/dt)_0$ as a possible constant. Our assumption, based on examination of the data from 1.1 to 16 GeV/c, that a major part of the qualitative change may be due to a change in $|A|$ makes this a second unlikely candidate for energy independence. The other variables b_1, b_2, and φ might be energy-independent, at least over a large range of incident momenta. For the phase angle, φ, this is essentially what Kalbfleisch *et al.*[1] reported, although over a much smaller range of incident momenta.

Assuming a constant value for the first slope, b_1, is a way of questioning the usual assumption of antishrinkage of the $\bar{p}p$ diffraction peak. If an energy-independent slope works down to low momenta, then the validity of the concept of antishrinkage in this reaction would be questionable. From the results shown in Fig. 2(b) we conclude that a constant value of 10 for b_1 is marginally consistent with all eight experiments.

In an attempt to provide some indication of the type of models which could be tried in connection with these data, we have tried to fit the data with the ratio b_2/b_1 fixed. Any model where the second slope is generated by iteration of two interactions

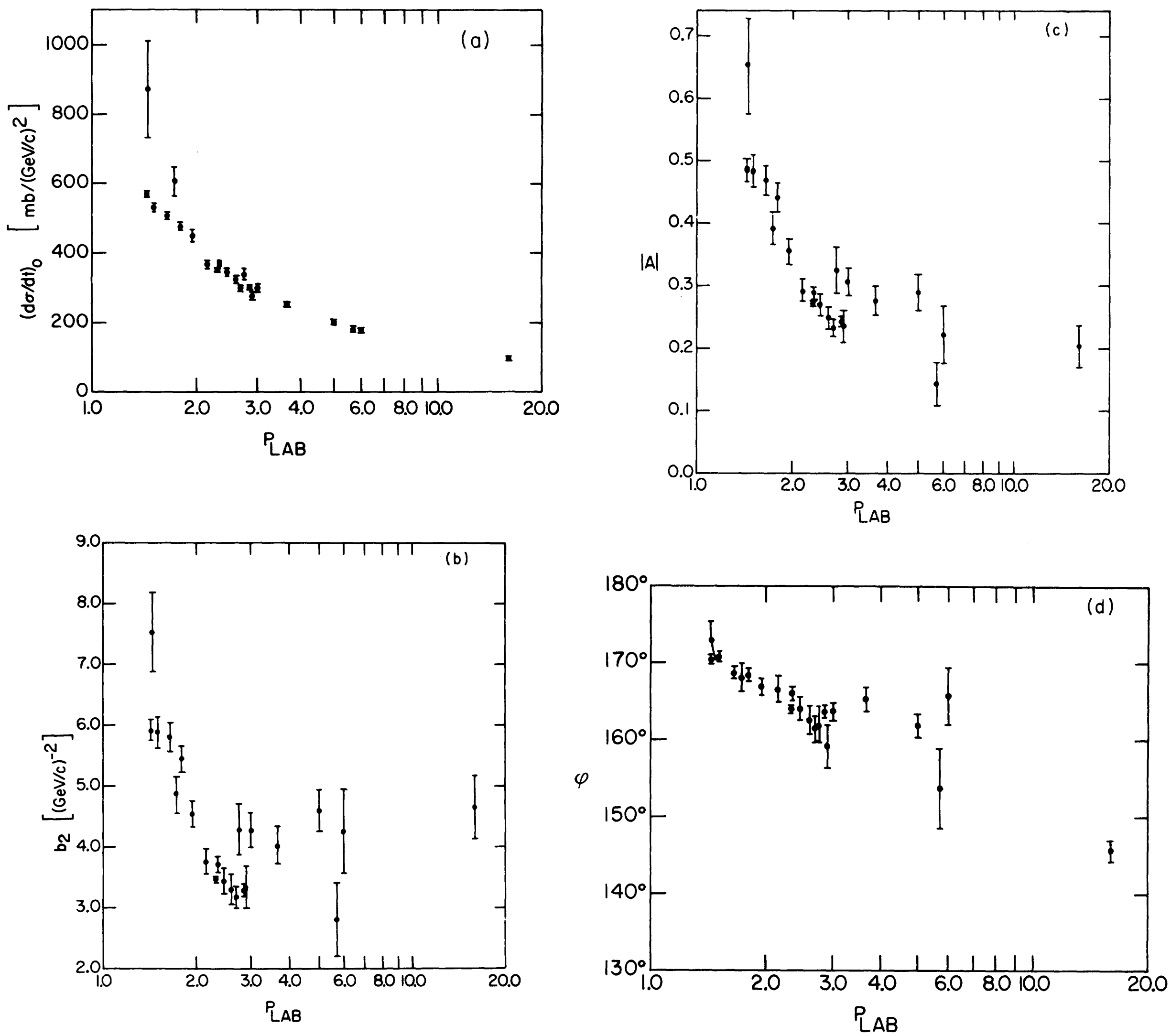


FIG. 3. Equation (1) parameter values corresponding to the four-parameter fit in which b_1 is fixed at the value of 10. Only the parameter values for the 22 well-fitted data samples are plotted. (a) Intercept $(d\sigma/dt)_0$ versus laboratory beam momentum P_{lab} (GeV/c). (b) Slope b_2 versus P_{lab}. (c) Relative amplitude $|A|$ versus P_{lab}. (d) Relative phase angle φ versus P_{lab}.

will lead to at least approximate energy independence of this ratio, provided that b_1 itself is relatively independent of energy. This is essentially valid whether the model is expressed as a multiple-scattering model, an absorption model, or a double-Regge-exchange model. The eight experiments have a ratio b_2/b_1 which is typically in the range 0.3 to 0.4, but most of the experiments have uncertainties in this ratio of approximately 20%. Thus the ratio might arbitrarily be chosen such that $b_1 = 2b_2$ or $b_1 = 3b_2$. Both ratios were attempted but only $b_1 = 3b_2$ can be said to work successfully.

If a guess is to be made for a constant value of φ, then, as can be seen from Fig. 2(e), a value in the range 160° to 165° would be reasonable. We have tried both of these values and the results are quite similar. We will present in detail only the 160° results. The results of each of these four-parameter fits will be discussed below.

In order to simplify later sections of this analysis it is worth stating that the 8.0 GeV/c data of Birnbaum *et al.* do not have a reasonable χ^2 in any attempt to fit these data, either with all five parameters or with any reduced set of four parameters. In fact we attempted to fit this data set in the t range $0.046 < -t < 0.287$ (GeV/c)^2 to a simple exponential and the confidence level was less than

TABLE IV. Four-parameter fits to Eq. (1) with the constraint $b_2=\frac{1}{3}b_1$.

P_{lab} (GeV/c)	$(d\sigma/dt)_0$ [mb/(GeV/c)^2]	b_1 [(GeV/c)$^{-2}$]	$\|A\|$	φ (deg)	χ^2/ν	Confidence level (%)
1.80	510.7 ± 14.4	12.01 ± 0.21	0.240 ± 0.004	156.8 ± 1.6	0.87	64
1.95	466.5 ± 14.7	11.17 ± 0.22	0.246 ± 0.005	160.3 ± 1.7	1.47	8
2.15	378.9 ± 15.7	10.50 ± 0.22	0.253 ± 0.005	164.2 ± 1.9	0.86	63
2.32	358.0 ± 3.7	10.14 ± 0.05	0.261 ± 0.001	163.3 ± 0.4	1.14	22
2.33	377.7 ± 7.2	10.37 ± 0.13	0.254 ± 0.003	164.1 ± 1.1	1.01	46
2.45	346.8 ± 15.1	10.13 ± 0.23	0.260 ± 0.005	163.4 ± 1.7	0.98	46
2.60	320.2 ± 14.8	9.94 ± 0.25	0.252 ± 0.006	162.9 ± 1.9	0.58	86
2.69	299.5 ± 8.7	9.90 ± 0.18	0.247 ± 0.004	162.1 ± 1.8	1.03	42
2.75	351.5 ± 24.0	10.86 ± 0.43	0.240 ± 0.007	155.7 ± 3.5	1.41	18
2.85	299.5 ± 4.4	9.90 ± 0.10	0.248 ± 0.003	164.3 ± 1.0	1.08	31
2.90	286.8 ± 20.3	10.23 ± 0.38	0.232 ± 0.008	158.1 ± 3.1	1.18	30
3.00	299.3 ± 7.6	10.64 ± 0.22	0.227 ± 0.006	159.1 ± 2.0	1.55	3
3.65	259.9 ± 7.5	10.60 ± 0.25	0.219 ± 0.006	161.4 ± 2.5	0.70	90
5.00	207.0 ± 6.1	10.93 ± 0.27	0.183 ± 0.007	153.5 ± 3.2	1.16	24
5.70	179.3 ± 9.1	9.60 ± 0.41	0.180 ± 0.012	158.0 ± 6.1	0.68	88
6.00	182.7 ± 9.3	10.69 ± 0.45	0.153 ± 0.013	160.3 ± 6.3	0.98	49
16.00	99.7 ± 2.2	10.63 ± 0.23	0.115 ± 0.008	133.2 ± 4.6	0.67	85

10^{-2}%. These data will not be referred to again. This is due to the inability to fit these data with any of these parametrizations.

III. FITS ASSUMING A CONSTANT b_1

For the eight well-determined sets of parameters in the five-parameter fit, the value of b_1 varies between 11.2 and 9.2. We have selected a constant value of 10 for b_1 and refit all the data to Eq. (1) with the other four parameters free to vary.

Below an incident beam momentum of 1.4 GeV/c three of the five data sets have confidence levels of less than 2%, and cannot be described as being well fitted with these parameters.[15] Of the 22 data sets considered above 1.4 GeV/c, only fits to five data sets have confidence levels of less than 10%, and none of the fits to these 22 data sets have a confidence level less than 3%. For the data above 1.4 GeV/c, the results of these fits are listed in Table III, and they are plotted in Figs. 3(a) through 3(d).

The result seems to us to be quite interesting. It has become an established part of the "Regge folklore" that $\bar{p}p$ elastic scattering antishrinks. However, all the data of these 22 experiments above 1.4 GeV/c can be adequately reproduced with a fixed value of the leading slope in Eq. (1). If any credence at all can be attributed to these fits, then a serious question must be raised relative to any appreciable antishrinkage of the diffraction peak in $\bar{p}p$ elastic scattering at laboratory momenta greater than 1.4 GeV/c.

IV. FITS ASSUMING A FIXED RATIO OF b_1 AND b_2

Various attempts to reduce the parameters of Eq. (1) from five to four seem to be able to reduce the correlations so that reasonably well-determined parameters result. Instead of attempting to fix b_2 to a constant, we chose to try to fix the ratio of b_2/b_1. We made two complete attempts to fix this ratio; one attempt set $b_2=\frac{1}{2}b_1$ and the other attempt set $b_2=\frac{1}{3}b_1$. When we compared the fits with $b_2=\frac{1}{2}b_1$ and those with $b_2=\frac{1}{3}b_1$, the confidence levels for many experiments were better by factors of 2 to 4 for $b_2=\frac{1}{3}b_1$ than for those with $b_2=\frac{1}{2}b_1$. Most significantly the 2.32-GeV/c experiment[4] had a confidence level of 22% for $b_2=\frac{1}{3}b_1$, and of 5×10^{-2}% for $b_2=\frac{1}{2}b_1$. Thus, of these two attempts, only the attempt with $b_2=\frac{1}{3}b_1$ can reasonably represent any extensive range of incident momenta of this reaction.

For incident laboratory momenta below 1.8 GeV/c, the existing experimental data cannot in general be well represented by Eq. (1) with $b_2=\frac{1}{3}b_1$. Of the ten sets of data below 1.8 GeV/c, only fits to three sets of data (1.23, 1.30, and 1.51 GeV/c) have confidence levels greater than 10%. Five experiments (1.11, 1.36, 1.43, 1.44, and 1.65

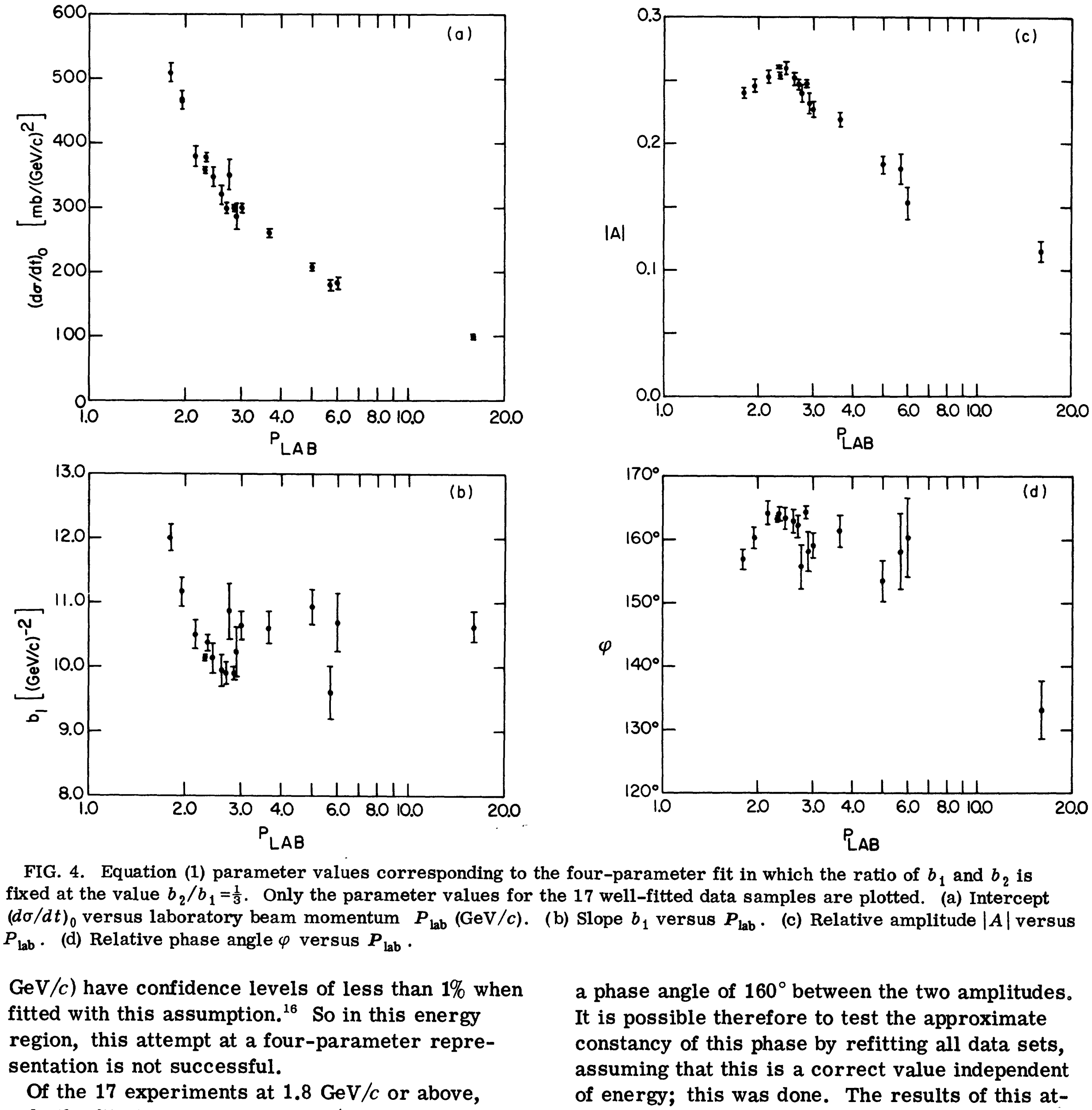


FIG. 4. Equation (1) parameter values corresponding to the four-parameter fit in which the ratio of b_1 and b_2 is fixed at the value $b_2/b_1=\frac{1}{3}$. Only the parameter values for the 17 well-fitted data samples are plotted. (a) Intercept $(d\sigma/dt)_0$ versus laboratory beam momentum P_{lab} (GeV/c). (b) Slope b_1 versus P_{lab}. (c) Relative amplitude $|A|$ versus P_{lab}. (d) Relative phase angle φ versus P_{lab}.

GeV/c) have confidence levels of less than 1% when fitted with this assumption.[16] So in this energy region, this attempt at a four-parameter representation is not successful.

Of the 17 experiments at 1.8 GeV/c or above, only the fits to two experiments (1.95 and 3.00 GeV/c) have confidence levels less than 10%. For this energy region, the attempt to use Eq. (1) with the constraint $b_2=\frac{1}{3}b_1$ is generally successful at reproducing the data. For these 17 experiments the results of this fit are listed in Table IV and they are plotted in Figs. 4(a) through 4(d).

V. FITS ASSUMING A CONSTANT PHASE OF 160°

The eight experiments with incident momenta between 1.8 and 3.65 GeV/c, from which all five parameters can be determined with reasonable accuracy, are all at least marginally consistent with a phase angle of 160° between the two amplitudes. It is possible therefore to test the approximate constancy of this phase by refitting all data sets, assuming that this is a correct value independent of energy; this was done. The results of this attempt can be summarized by stating that below 1.8 GeV/c this does not work well, but it does seem to work reasonably well above 1.8 GeV/c.

In particular the four-parameter fits with φ fixed at 160° for eight of the ten experiments below 1.8 GeV/c have confidence levels of less than 10%.[16] Only the experiments at 1.30 and 1.51 GeV/c are reasonably fitted with this assumption. Our conclusion, based on this fit, the five-parameter fits, and the other four-parameter fits is that the phase begins at nearly 180° at low momenta and decreases with increasing energy until in the region above 1.8-GeV/c incident momentum it has become a constant or a slowly varying function

TABLE V. Four-parameter fits to Eq. (1) with φ fixed at the constant value 160°.

P_{lab} (GeV/c)	$(d\sigma/dt)_0$ [mb/(GeV/c)^2]	b_1 [(GeV/c)$^{-2}$]	b_2 [(GeV/c)$^{-2}$]	$\|A\|$	χ^2/ν	Confidence level (%)
1.80	501.1±13.9	11.51±0.21	4.36±0.20	0.280±0.018	0.82	70
1.95	467.8±15.0	11.24±0.24	3.66±0.22	0.240±0.019	1.46	8
2.15	393.5±18.5	11.24±0.35	3.04±0.23	0.202±0.021	0.81	68
2.32	367.4±4.0	10.66±0.07	3.06±0.05	0.222±0.004	1.35	4
2.33	385.5±7.7	10.93±0.16	3.09±0.15	0.212±0.013	1.06	36
2.45	358.5±16.8	10.70±0.30	3.04±0.21	0.218±0.019	0.98	47
2.60	327.5±16.0	10.35±0.31	3.08±0.25	0.222±0.023	0.63	82
2.69	304.0±9.2	10.27±0.23	3.00±0.21	0.214±0.019	0.97	50
2.75	341.2±21.9	10.24±0.35	4.10±0.39	0.298±0.039	1.32	22
2.85	304.9±4.7	10.39±0.12	3.02±0.11	0.212±0.010	1.21	11
2.90	284.8±20.2	10.14±0.41	3.32±0.37	0.229±0.035	1.21	28
3.00	298.3±7.3	10.46±0.18	3.84±0.30	0.254±0.024	1.52	3
3.65	261.6±7.3	10.81±0.25	3.32±0.36	0.199±0.027	0.70	90
5.00	203.0±5.6	10.20±0.20	4.40±0.35	0.263±0.032	1.10	32
5.70	178.3±8.6	9.42±0.43	3.38±0.73	0.199±0.065	0.68	88
6.00	184.1±9.1	11.05±0.56	2.71±0.97	0.107±0.048	0.97	52
16.00	96.5±2.0	9.16±0.12	6.11±0.34	0.411±0.030	0.85	64

of energy with a value of approximately 160°.

Of the 17 experiments above 1.8 GeV/c only the fits to two experiments (1.95 and 3.0 GeV/c) have confidence levels of less than 10%. For these 17 experiments the parameters that result from the fit with φ fixed at 160° are listed in Table V, and they are plotted in Figs. 5(a) through 5(d).

VI. CONCLUSIONS

Our main conclusion from this study of much of the available $\bar{p}p$ elastic-scattering data can be summarized as a "plea for admission of ignorance"; that is, the values of $(d\sigma/dt)_0$ and the slope of the diffraction peak are not as well known as has been thought. The secondary conclusions that follow from this are the need for better experimental data and the need for theoretical guidance on the correct way to interpret the data and to extrapolate the data to $t=0$. We will try to outline below the reasons for these conclusions.

The important information to be gleaned from studies of elastic scattering is usually contained in the energy dependence of the parameters. For example, one can combine a knowledge of $(d\sigma/dt)_0$ and the total cross section to extract the ratio of the real to imaginary amplitudes. This ratio as a function of energy can yield some real insight into or test of our theoretical understanding of the interaction. But this process requires as a starting point a reliable value of $(d\sigma/dt)_0$ at a particular energy. At 1.51 GeV/c Parker *et al.*[10] used a limited t range and a simple exponential and estimated $(d\sigma/dt)_0$ to be 620±25 mb/(GeV/c)^2. For the same data, in the fit using Eq. (1) with b_1 fixed at 10, we would estimate $(d\sigma/dt)_0$ to be 531±12 mb/(GeV/c)^2 with a confidence level for the fit of 42%; with all five parameters free we would estimate this same quantity to be 522±17 mb/(GeV/c)^2 with a confidence level for the fit of 39%. An additional example is the 1.8-GeV/c data. Parker *et al.*,[10] using a limited t range and a simple exponential, estimate $(d\sigma/dt)_0$ to be 551±22 mb/(GeV/c)^2. For the same data, using Eq. (1) with five free parameters, we would estimate this intercept to be 496±19 mb/(GeV/c)^2 with a confidence level for the fit of 66%. If we fix b_1 equal to 10 and refit the same data, we would estimate this quantity to be 477±11 mb/(GeV/c)^2; the confidence level for the fit is 61%. The appropriate question would seem to be: What is $(d\sigma/dt)_0$ at either of these energies? Because the calculation of the ratio of the real to the imaginary amplitudes requires the subtraction of the optical point (determined using the measured total cross section) from the estimated value of $(d\sigma/dt)_0$ (using only elastic-scattering data), the ratio of the real to the imaginary amplitudes is sensitive to even fairly small changes in $(d\sigma/dt)_0$. If $(d\sigma/dt)_0$ is uncertain by ~20% due to differences in the parametrizations used to extrapolate the measured data, then there is an extremely large uncertainty in the ratio of the real to the imaginary amplitudes. These examples are not unique and in our opinion exemplify one aspect of the lack of knowledge presently existing

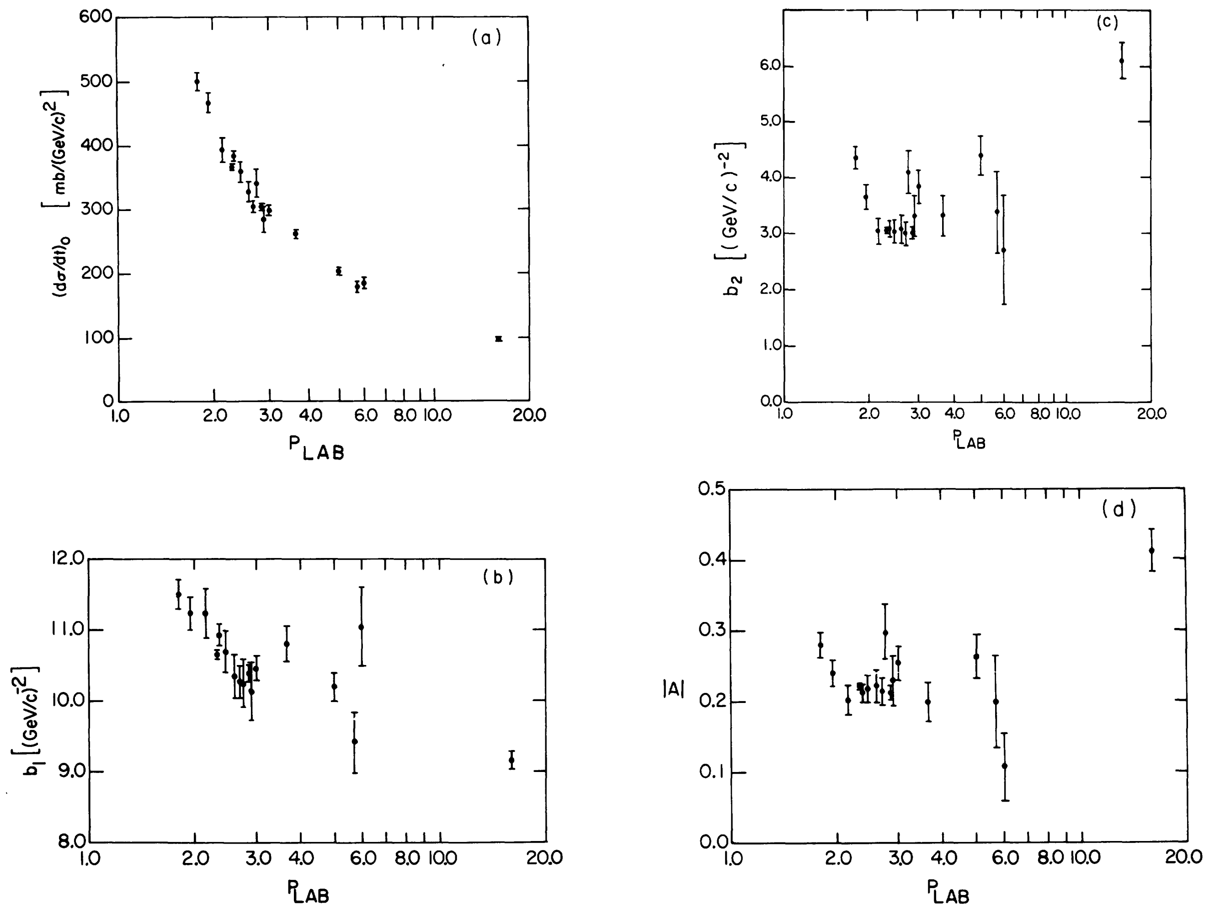


FIG. 5. Equation (1) parameter values corresponding to the four-parameter fit in which the phase angle φ is fixed at the value 160°. Only the parameter values for the 17 well-fitted data samples are plotted. (a) Intercept $(d\sigma/dt)_0$ versus laboratory beam momentum P_{lab} (GeV/c). (b) Slope b_1 versus P_{lab}. (c) Slope b_2 versus P_{lab}. (d) Relative amplitude $|A|$ versus P_{lab}.

about the interpretation of $\bar{p}p$ elastic scattering.

Another important part of the information which can be extracted from studies of elastic $\bar{p}p$ scattering is the energy dependence of the slope in the diffraction region. In various models, particularly Regge-pole models, this can be used to study the properties of the interaction. Again a starting point is a reliable estimate of the slope at each energy. At 1.51 GeV/c Parker *et al.*[10] used a simple exponential to estimate the slope to be 16.3 ± 0.4 (GeV/c)$^{-2}$. In our four- and five-parameter fits, b_1 (Ref. 17) is variously estimated between 8.0 and 12.0 (GeV/c)$^{-2}$. At 1.8 GeV/c Parker *et al.*[10] estimated the slope to be 15.7 ± 0.4 (GeV/c)$^{-2}$ using a simple exponential. Using all five parameters in Eq. (1) we estimate b_1 to be 11.2 ± 0.8 (GeV/c)$^{-2}$, with a confidence level for the fit of 66%. Our four-parameter fits to these data have b_1 values between 10.4 and 11.5 (GeV/c)$^{-2}$ with uncertainties which typically are ± 0.2 (GeV/c)$^{-2}$. Given this lack of knowledge of what the slope in the diffraction region is at each energy, it is very difficult to conclude anything about the energy dependence.

A compelling argument that we lack knowledge of the slope and its energy dependence comes from the attempt to fit all of these data with a constant slope. In the past there have been many discussions of the antishrinkage of the diffraction slope in $\bar{p}p$ elastic scattering. However, it is possible to fit the data between incident momenta of 1.4 and 16 GeV/c using Eq. (1) with the slope b_1 fixed 10.0 and to have a reasonable confidence level for each experiment.

If it is desirable to test the applicability of functions such as Eq. (1) against the data, significantly better data will be needed at a wider range of energies. Any alternative method of deriving reasonable estimates of $(d\sigma/dt)_0$ and of the slope based on existing data would require a functional form of the t dependence with acceptable theoretical justification.

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[15]It is worth pointing out that the five data sets below 1.4 GeV/c can be fitted with b_1 fixed at 10 as well as they can with b_1 free to vary. The confidence levels for the fits with b_1 fixed at 10 for these five data sets are 0.8%, 18.7%, 50.4%, 0.6%, and 1.2%. These confidence levels are to be compared with the best-fit values from Table I of 0.7%, 16.1%, 46.1%, 1.2%, and 2.3%, respectively. A careful examination shows that the relatively poor confidence levels for three of the five of these data sets is due to large χ^2 contributions from a limited number of data points and not due to an inability of the function to reproduce the t dependence of the data.
[16]Many of these experiments have fairly small changes in the confidence level of only approximately a factor of 2. However, this change is such as to take them, for example, from a confidence level of 12% to only a few percent. The decrease to the few-percent level is the justification for the statement that it does not work well for these data sets.
[17]For the purposes of this discussion we are comparing b_1, the slope of the leading term in this exponential expansion, with the usual determination of the slope using only a single exponential; this is, of course, only an approximate identification.

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Description of the triple-Regge region with a very large triple-Pomeron coupling and a bare Pomeron*

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A description of data in the triple-Regge region up to CERN ISR energies is proposed. The description utilizes a very large PPP (triple-Pomeron) coupling g_P and assumes that the "bare" or "unrenormalized" Pomeron controls the region near $x = 1$. The "bare" Pomeron intercept is $\hat{\alpha}_0 = 0.85$. One secondary term is employed to extend the description to $x = 0.7$. This term is chosen to be of the form $\pi\pi P$. We use a modified ABFST (Amati-Bertocchi-Fubini-Stanghellini-Tonin) model to roughly constrain g_P and $\hat{\alpha}_0$. The number of unconstrained parameters otherwise is one. Other phenomenological applications utilizing the bare Pomeron as the definition of diffraction at intermediate energies are discussed, and the total cross section with and without diffraction is discussed in the context of the ABFST model.

I. INTRODUCTION

The nature of diffractive scattering is one of the central problems of high-energy physics. A number of authors have proposed that a possible handle on this problem lies in the magnitude of the triple-Pomeron coupling at zero momentum transfer, $g_{PPP}(0)$. It has usually been assumed that $g_{PPP}(0)$ is either zero or else very small, and the data may well be consistent with this assumption, though we shall show this is not necessary and perhaps not desirable at current energies. We shall adopt the generalized two-component schemes of Chew[1] or Ter-Martirosyan,[2] but *we shall insist that the transition into the "asymptotic" region controlled by the true Pomeron is describable in terms of a pole in an auxiliary "unrenormalized" partial-wave amplitude* $\hat{A}_l(t)$.[3] This pole $l = \hat{\alpha}_P(t)$ is termed the "bare" or "unrenormalized" Pomeron, and is taken here as a convenient way of summarizing (and/or defining) the nature of diffraction scattering at intermediate energies, where the multiparticle amplitudes have neighboring subenergies almost entirely in the resonance