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Some rare meson-decay modes in a relativistic quark model*

D. Parashar† and P. N. Dobson, Jr.
Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822
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A version of the relativistic quark model with phenomenological form factors and prescriptions developed largely by Mitra and coworkers is here applied to the calculation of some rare meson-decay rates. First, vector-meson dominance is incorporated into the model and electromagnetic decay rates are computed for pseudoscalar and vector mesons. Then some three-body decays of the $\eta'(958)$, $A_1(1070)$, and $E(1422)$ are considered. In general, reasonably good agreement with the available experimental evidence is obtained, without the introduction of any new parameters. It is found that the best over-all consistency is obtained if the $\eta'(958)$ is assumed to be a pure SU(3) singlet, and the $E(1422)$ is taken to be an octet state with $J^P = 1^+$.

I. INTRODUCTION

Since its inception, a nonrelativistic quark model of hadrons[1] has been sucessfully employed for the construction of strong and electromagnetic couplings, based on the notion of a single quark transition with the emission of a meson, regarded as a radiation quantum. For transitions from a particular supermultiplet, the coupling structure will involve two terms: one proportional to the momentum of the emitted meson (the direct term) and the other proportional to the recoil momentum of the active quark (the recoil term), thus introducing two sets of parameters and making quantitative predictions rather difficult. A parameterization better suited for confrontation of the predictions with experiment can be achieved by suitably combining the direct and recoil terms in the form factors.[2] In this way, a single parameter can describe the entire supermultiplet transition. These ideas have been extended to a relativistic formulation via a standard relativistic boosting of the SU(6)×O(3) couplings.[3] Symmetry breaking is achieved through the masses by multiplying the couplings by suitable form factors.[4,5]

In Sec. II, we apply the model to the electromagnetic decays of the pseudoscalar and vector mesons. The quark model form for VVP coupling is used to determine the effective $\gamma\gamma P$ and $V\gamma P$ couplings, using Schwinger's technique of partial symmetry and the idea of vector-meson dominance of the electromagnetic interactions of hadrons.[6] All the electromagnetic vertices are expressible in this way in terms of the $\rho\pi\pi$ coupling constant. The results are in reasonably good agreement with the experimental evidence available, and favor the assignment of η and η' to pure octet and singlet, respectively. This is consistent with the conclusion reached by application of the model directly to strong decays of $L=1$ mesons into vector and pseudoscalar mesons.[5] (L here is the internal orbital angular momentum of the quark-antiquark state and is 0 for the P and V nonets.)

In Sec. III, we take a further look at the η and η' assignments. A technique is proposed for calculating the $\eta'(958)\rightarrow\eta\pi\pi$ decay width in the context of the relativistic quark model. This involves the assumption that the decay proceeds via an intermediate state consisting of $\sigma\eta$, where σ represents the broad enhancement in the $I=0$ s-wave $\pi\pi$ system at about the ρ mass. In the model we are using, this is taken to be an $L=1$ scalar meson, mixed with the $S^*(1070)$ with an ideal mixing angle (so that the σ contains only nonstrange quarks).[5] The calculated widths for both ideal mixing and a pure singlet-octet assignment are consistent with experimental upper bounds. A further application of the idea of using a σ intermediate state is made to the $A_1(1070)\rightarrow 3\pi$ decay. The result, when combined with previous predictions of this model for $A_1\rightarrow\rho\pi$, is not in disagreement with the experimental evidence.

In Sec. IV, the decays ot the $E(1422)$ meson are discussed in order to further clarify the role of the $\eta'(958)$ as a member of the pseudoscalar nonet. The possibility that the E should be assigned this role in the present model is completely ruled out by experiment. We find that the best assignment is $J^P=1^+$, with SU(3) content nearly pure octet. A small singlet admixture provides good agreement with experimental values for the decay modes.

II. ELECTROMAGNETIC DECAYS OF PSEUDOSCALAR AND VECTOR MESONS

In this section, we study the decays $(\pi^0, \eta, \eta')\rightarrow 2\gamma$ and $(\rho, \omega, \phi)\rightarrow(\pi^0, \eta)\gamma$. The couplings needed are of the type $\gamma\gamma P$ and $V\gamma P$. These are constructed from the basic VVP coupling by replacing the vector-meson fields successively by their electromagnetic equivalents in the format of Schwinger's partial symmetry and the vector-dominance model.[4,6] For this purpose, we use the relativistic form of the coupling:

$$\frac{i}{m_\omega} g_{\omega\rho\pi}\epsilon_{\lambda\mu\beta\nu}(\partial_\lambda \omega_\mu \rho_\beta^a)(\partial_\nu \pi_a)\,, \tag{1}$$

where the $\omega\rho\pi$ coupling constant in the context of this model is given by

$$g_{\omega\rho\pi}{}^2/4\pi = (4m_\omega/m_\pi)^2 f_q{}^2/4\pi$$

and similarly,

$$g_{\rho\pi\pi}{}^2/4\pi = (2m_\rho/m_\pi)^2 f_q{}^2/4\pi\,.$$

Here, f_q is the coupling strength of the basic $\bar{q}qP$ coupling[4] and is

$$f_q{}^2/4\pi = 0.03\,,$$

and so

$$g_{\rho\pi\pi}{}^2/4\pi = 3.6\,.$$

Noting that $m_\rho \cong m_\omega$, we may use these relations to rewrite Eq. (1) in the form

$$(g_{\rho\pi\pi}/m_\rho)[{}^\star\omega_{\mu\nu}\rho_a^\nu + {}^\star\rho_a^{\mu\nu}\omega_\nu](\partial_\mu \pi^a)\,, \tag{2}$$

where ${}^\star\omega_{\mu\nu}$, etc., is given by

$${}^\star\omega_{\mu\nu} = \epsilon_{\mu\nu\lambda\sigma}\partial_\lambda \omega_\sigma\,.$$

To obtain the necessary electromagnetic couplings, we make the following transformations in accordance with the vector-dominance model:

$$\rho_\mu \to (e/g_{\rho\pi\pi})A_\mu \text{ and } \omega_\mu \to \tfrac{1}{3}(e/g_{\rho\pi\pi})A_\mu\,.$$

For example, we find the $\omega\gamma\pi^0$ coupling to be

$$-(e/m_\rho)\,{}^\star\omega_{\mu\nu}F^{\mu\nu}\pi^0 \tag{3}$$

and the $\pi^0\gamma\gamma$ coupling to be

$$-\tfrac{1}{3}(e^2/m_\rho g_{\rho\pi\pi})\,{}^\star F_{\mu\nu}F^{\mu\nu}\pi^0\,. \tag{4}$$

The other $V\gamma P$ and $\gamma\gamma P$ couplings are constructed in exactly the same way from the basic VVP couplings of the model. Of particular interest are the 2γ decays of η and η', determined from the $\rho\rho P$ coupling:

$$2C(g_{\rho\pi\pi}/m_\rho)(m_\pi/m_\eta)^{1/2}\,{}^\star\rho^{\mu\nu}\rho_\nu(\partial_\mu' P), \tag{5}$$

where C is an SU(3) factor equal to $(\frac{1}{3})^{1/2}$ for a pure octet isosinglet state, and $(\frac{2}{3})^{1/2}$ for a pure singlet. Vector-meson dominance gives us the $\gamma\gamma P$ coupling

$$2C(e^2/m_\rho g_{\rho\pi\pi})(m_\pi/m_\eta)^{1/2}\,{}^\star F^{\mu\nu}F_{\mu\nu}P, \tag{6}$$

where the P can be either η or η'. The physical η and η' are presumably mixtures of octet and singlet states. We have calculated the decays for the cases where η' is a pure singlet, and where it has no strange quark content (ideal mixing angle).

The results of the calculations of the electromagnetic decays are given in Table I, along with the available experimental values.[7] The agreement is fairly good, and it should be noted that $\Gamma(\omega\to\gamma\pi^0)$ and $\Gamma(\pi^0\to\gamma\gamma)$ are the same as obtained by Schwinger,[6] and compare very well with experiment. All the 2γ modes of pseudoscalar mesons agree rather well with the calculations[8] of Matsuda and Oneda, Pietschmann and Thirring, and Dar and Weisskopf. Our result for $\Gamma(\omega\to\eta\gamma)$ suggests that the pure singlet assignment for the η' is favored in this model over the ideal mixing angle.

III. THREE-BODY DECAYS AND THE SU(3) ASSIGNMENT OF THE $\eta'(958)$

We can further investigate the nature of the η' meson by applying the model directly to its strong decay modes. The $\eta'(958)$ decays predominantly into $\rho^0\gamma$, which was treated in Sec. II, and into $\eta\pi\pi$. The latter is a three-body decay mode and must be regarded as proceeding through a two-body intermediate state in order to be treated easily in the present model. We will assume that the two pions in the final state arise from an internal line assigned to a scalar meson with a mass close to that of the ρ. The experimental evidence at present does not seem to favor an actual resonance, but there does seem to be a broad enhancement in this channel, and such a state, which we will call the σ, has already been discussed in the context of this model.[5] The σ is assumed to be an $L=1$ meson, belonging to a nonet that includes the

TABLE I. Electromagnetic decays of vector and pseudoscalar mesons.

Decay process	Calculated width (MeV)	Experimental width (MeV)
$\pi^0\to\gamma\gamma$	7.4×10^{-6}	$(7.8\pm0.9)\times10^{-6}$
$\eta\to\gamma\gamma$	0.28×10^{-3}: $\eta(\underline{8})$ 0.64×10^{-3}: η(ideal)	$(1.0\pm0.22)\times10^{-3}$
$\eta'\to\gamma\gamma$	1.7×10^{-3}: $\eta'(\underline{1})$ 0.6×10^{-3}: η'(ideal)	$<8\times10^{-3}$
$\eta'\to\rho^0\gamma$	0.20: $\eta'(\underline{1})$ 0.15: η'(ideal)	<1.2
$\omega\to\pi^0\gamma$	0.96	(0.9 ± 0.05)
$\omega\to\eta\gamma$	0.13: $\eta(\underline{8})$ 0.44: η(ideal)	0.15
$\rho\to\eta\gamma$	0.04: $\eta(\underline{8})$ 0.12: η(ideal)	...
$\phi\to\eta\gamma$	8×10^{-3}: $\eta(\underline{8})$ 7.7×10^{-2}: η(ideal)	(0.11 ± 0.09)
$\rho^0\to\pi^0\gamma$	0.22	<0.23

$S^*(1070)$, and mixed with the latter so that it contains no strange quarks.

In accordance with the prescriptions of the model, the coupling of an $L=1$ scalar meson (S) to two pseudoscalar mesons (P_1, P_2) is written

$$-(\mu^2/\sqrt{3})SP_1P_2,$$

where μ is the mass of the "radiation quantum," taken to be the heavier of the pseudoscalar mesons emitted for phenomenological reasons. This must be multiplied by an appropriate SU(3) factor and a phenomenological form factor, which for this transition takes the form

$$g_1(2M_s/\mu^2)(\mu/m_\pi)^{1/2},$$

with M_s the mass of the scalar meson, and g_1 determined from $A_2 \to \rho\pi$ to be

$$g_1^2/4\pi = 0.08.$$

Thus, the $\sigma\pi\pi$ vertex is represented in the model by

$$-C_1 g_1(2m_\sigma/\sqrt{3})(\sigma\pi\pi), \tag{7}$$

where C_1 is the relevant SU(3) factor, and is $2\sqrt{3}$ for the ideal-mixing-angle assumption. This gives for the decay width of the σ the value $\Gamma(\sigma\to\pi\pi)=224$ MeV, which is consistent with our assumption of a very broad enhancement in the $I=0$, s-wave $\pi\pi$ state.

In a similar fashion, we find the coupling at the $\eta'\eta\sigma$ vertex to have the form

$$-C_2 g_1(2m_{\eta'}/\sqrt{3})(m_\eta/m_\pi)^{1/2}(\eta'\eta\sigma). \tag{8}$$

Now the $\sigma\eta$ intermediate state has the σ very far off the mass shell, so it is not appropriate to treat this as a simple cascade decay. Instead, we use the formalism of Kumar[9] for the three-pion final state, using an invariant matrix element dominated by the σ pole with vertices given by Eqs. (7) and (8). The σ pole is taken at a complex mass, with real part m_ρ and imaginary part $i\Gamma/2$. The resulting expression for the width can be written in the form

$$\Gamma(\eta'\to\eta\pi\pi)=\frac{(C_1C_2)^2}{9\pi}\left(\frac{g_1^2}{4\pi}\right)^2(m_\eta m_\sigma^2/m_\pi m_{\eta'})I, \tag{9}$$

where the phase space integral I in Eq. (9) is explicitly given by

$$I=\int_{4m_\pi^2}^{(m_{\eta'}-m_\eta)^2}\frac{ds}{S|m_\sigma^2-S|^2}\lambda^{1/2}(S,m_\pi^2,m_\pi^2)\times\lambda^{1/2}(S,m_{\eta'}^2,m_\eta^2) \tag{10}$$

and

$$\lambda(x,y,z)=x^2+y^2+z^2-2xy-2yz-2xz.$$

In Eq. (10), m_σ refers to the complex mass of the pole position. If we make the assumption that η is a pure octet state, and η' a pure singlet, we obtain $\Gamma(\eta'\to\eta\pi\pi)$ about 0.44 MeV, while the assumption that the η' has no strange quarks gives about 0.15 MeV. Both these values are below the experimental upper limit of about 1.7 MeV.

To test the over-all consistency of this approach, the model was also applied to the 3π decay of the $A_1(1070)$. This state is also taken to be an $L=1$ excitation, like the σ, and there are similar doubts about its interpretation as a resonance. Nonetheless, the enhancement appears to occur mainly in the $\rho\pi$ mode, so the hope is that the present model gives a relatively small fraction of $\sigma\pi$. For the $A_1\sigma\pi$ vertex, we must use the coupling between two $L=1$ states (one with $J=1$ and one with $J=0$) and a pseudoscalar meson, which is[5]

$$(1/\sqrt{6})A_1^\mu\sigma(\partial_\mu\pi).$$

This must be multiplied by an appropriate SU(3) factor and the form factor

$$(2m_{A_1}/m_\pi)g_0,$$

where

$$g_0^2/4\pi = f_q^2/4\pi = 0.03.$$

We find that the decay width is $\Gamma(A_1\to\sigma\pi)=9$ MeV, which is reasonably small compared with the model's prediction[5] of $\Gamma(A_1\to\rho\pi)=65$ MeV. This width was computed with a "real" σ in the final state; when the three-body phase space method used previously is applied, the value is 12 MeV.

IV. DECAYS OF THE $E(1422)$ MESON

In an attempt to clarify the role of the $\eta'(958)$, we have investigated the possibility that the $E(1422)$ is the state that should be considered a member of the 0^- nonet. This possibility was compared with the assignment of the E to an $L=1$ state with $J^P=1^+$. Calculations similar to those in Secs. II and III were carried out, with SU(3) assignments of the E to pure singlet, pure octet, and ideal mixing (no strange quarks). The results are summarized in Table II. One striking feature is the factor of 8 in the $\bar{K}^*K+K^*\bar{K}$ rates between singlet and octet assignments, which follows from the rule for evaluating the SU(3) factors. The factor for emission of a pseudoscalar meson P_α is obtained by computing the matrix element of $\lambda_\alpha - \bar{\lambda}_\alpha$ between the initial and final quark-antiquark configurations. Here λ_α is the usual SU(3) generator in the $\underline{3}$ representation, and acts on the quarks in the wave function, while $\bar{\lambda}_\alpha$ is the corresponding matrix in the $\underline{\bar{3}}$ representation, and acts on the antiquarks. For example, for a final state with

TABLE II. Predictions of $E(1422)$ decay widths, in MeV.

Decay mode:	$\eta\sigma$	$\delta(970)\pi$	Total $\eta\pi\pi$	$\bar{K}^*K+K^*\bar{K}$
Experimental width (MeV)	...	...	36 ± 12	12 ± 4
0^- singlet	55	153	208	72
0^- octet	28	77	105	9
0^- ideal mixing	83	230	313	27
1^+ singlet	0.9	67	68	501
1^+ octet	0.5	34	35	63
1^+ ideal mixing	1.4	101	102	188

quark-antiquark composition $\mathfrak{N}\bar{\lambda}$, a singlet initial state provides contributions from both the $\mathfrak{N}\bar{\mathfrak{N}}$ and $\lambda\bar{\lambda}$ parts of the wave function. Each has amplitude $1/\sqrt{3}$ to be present, and the relative signs of the $\underline{3}$ and $\underline{\bar{3}}$ matrices are such that the contributions add to give $2/\sqrt{3}$. For an octet initial state, the $\mathfrak{N}\bar{\mathfrak{N}}$ state is present with amplitude $1/\sqrt{6}$, and the $\lambda\bar{\lambda}$ with $-2/\sqrt{6}$, so the net coupling is $-1/\sqrt{6}$. The ratio of the matrix elements is $-2\sqrt{2}$, and the square of this accounts for the factor of 8 in the rates.

Of the various possibilities in Table II, only the $J^P=1^+$ octet assignment seems compatible with experiment. This does give a rather large fraction for decay into $K^*\bar{K}+\bar{K}^*K$; however, the agreement would be better if all the experimentally observed $\bar{K}K\pi$ mode were due to this channel, which would double the experimental estimate. Furthermore, we solved for the octet-singlet mixture that would reproduce the branching ratio of this mode to the $\delta(970)\pi$ mode, and found that about 2.7% singlet was sufficient. With this mixing, the predicted values for the decay widths are $\Gamma(E\to\delta\pi)=50$ MeV and $\Gamma(E\to K^*\bar{K}+\bar{K}^*K)=17$ MeV, in quite good accord with the present experimental estimates. A prediction of the model is that the $\eta\pi\pi$ mode is almost exclusively $\delta(970)\pi$. If the nearly pure octet $J^P=1^+$ assignment of the $E(1422)$ is correct, then it should be possible to identify the other members of the octet and check the Gell-Mann–Okubo mass formula. A likely candidate for the isovector member of the octet is the $A_1(1100)$, in which case the mass formula calls for a $J^P=1^+$ K^* at about 1340 MeV. This is in the so-called Q region, where some investigators see a broad enhancement, and others see structure, including a reasonably narrow K^* at 1240 MeV. Since the latter fits the mass formula extremely well with the $D(1285)$ as the isoscalar member of the octet, in place of the $E(1422)$, there remains considerable doubt about our identifications. A calculation of the decay rates of the $J^P=1^+$ K^* in this model results in very large values, in particular for the $K^*(892)\pi$ mode, for any mass in the Q region including 1240 MeV. Thus the model favors an interpretation of the Q region as a broad enhancement, and there need be no inconsistency with the Gell-Mann–Okubo mass formula for a pure octet. More detailed knowledge of the meson spectrum is required to settle the question of the model's ability to deal consistently with these problems.

V. CONCLUSIONS

We have attempted to apply a particular version of the relativistic quark model to some rare meson decays. This model has already been successfully applied to a number of baryon and meson-decay processes; in the course of that application, the basic parameters (coupling constants) of the model have been fixed, and a set of phenomenological form factors and prescriptions has been devised. In the present paper we have applied this formalism to a number of rare meson decays and find that the agreement with experiment remains generally good. It would appear that the model offers a hope of providing a reasonably consistent description of hadronic decay processes based on some sound theoretical ideas.

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†On leave of absence from Atma Ram Sanatan Dharma College, University of Delhi, Delhi, India.

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Analysis of $\bar{p}p$ elastic scattering using two coherent exponentials

H. B. Crawley, N. W. Dean, E. S. Hafen, W. J. Kernan, and F. Ogino
Ames Laboratory-USAEC *and Department of Physics, Iowa State University, Ames, Iowa 50010*
(Received 19 September 1973)

Published data from many groups on $\bar{p}p$ elastic scattering at twenty-eight laboratory momenta have been analyzed with a parametrization corresponding to two coherent interfering exponentials. The data from almost all of these experiments can be adequately represented by this parametrization. In most of the experiments the data are such that the fit does not uniquely determine all five parameters with reasonable errors. In fits to at least eight experiments the parameters are well determined. Using these parameters as guides, attempts are made to reduce the total number of parameters and still fit all the data.

I. INTRODUCTION

In studies of $\bar{p}p$ elastic scattering at incident laboratory momenta of 1.11, 1.33, and 1.52 GeV/c, Kalbfleisch *et al.*[1] used two coherent interfering exponentials to parametrize the dependence of the differential scattering cross section on t, the square of the four-momentum transfer. Later experiments at 2.33 (Ref. 2) and 2.85 GeV/c (Ref. 3) have also been successfully represented over a large t range with this parametrization as well. An additional experiment[4] at 2.32 GeV/c has also been successfully analyzed[2] using this formula. This led to speculation that this form might represent, at least qualitatively, the data over a large range of center-of-mass energy as well as a large range in t. The change in the qualitative appearance of the data as the laboratory momentum varies from about 1 GeV/c to about 16 GeV/c, the range over which data on this reaction are available, might then be understood as a decrease in the amplitude of the second exponential term relative to the first, or as a change in the amplitude combined with a change in the phase between the two terms.

In order to test this idea, the data summary on this reaction from the Particle Data Group[5] was used to select a beginning series of experiments to fit. The initial selection criteria were quite simple. Each experiment had to cover the t range of the diffraction peak, the first minimum corresponding to $-t\approx 0.35$ (GeV/c)^2, and the region out to $-t\approx 0.6$ (GeV/c)^2, where the second maximum occurs. The most useful data sets also go beyond the second maximum to show a secondary decrease in the differential cross section. To this group of data we added the 2.32-GeV/c data of the Oxford-Argonne collaboration,[4] the data of our own experiments at 2.33 and 2.85 GeV/c, and the data of the Argonne EMS (effective mass spectrometer) group at 3.0, 3.65, 5.0, and 6.0 GeV/c.[6,7] This yielded a data sample of 28 energies for which the differential cross section has been measured and the above criteria satisfied. These 28 energies and the references[1-14] are summarized in Table I.

It has also been necessary to limit the t range of the data used at any energy. If the measured values correspond to t values that go beyond the second-peak structure, a maximum-t cutoff on the data has been used. If the experiment has reported a cross section at very low t values where detection efficiencies may be causing an apparent decrease in the differential cross section, then a minimum-t-value cutoff on the data has been applied. In order to completely specify what has been done, the t range of each data sample used is also shown in Table I.

II. THE FIVE-PARAMETER FITS

Two coherent interfering exponentials correspond to the equation