which is found to be of the order m^2/E^2 and is negligible.

The total spin-dependent decay rate is then obtained by summing Eqs. (15) and (21):

$$\gamma_{\zeta'\bar{\zeta}'} = \tfrac{1}{2}(1+\zeta'\bar{\zeta}')\gamma^{(\mathrm{nf})} + \tfrac{1}{2}(1-\zeta'\bar{\zeta}')\gamma^{(\mathrm{f})}, \qquad (26)$$

where the spin-flip rate is

$$\gamma^{(\mathrm{f})} = \gamma', \qquad (27)$$

while the spin-nonflip rate is the difference between the total and the spin-flip rate:

$$\gamma^{(\mathrm{nf})} = \gamma - \gamma', \qquad (28)$$

as expected.

In the limit when

$$\lambda = \frac{3}{2}\frac{E}{m}\frac{eH}{m^2} \ll 1, \qquad (29)$$

we may expand Eq. (22) in power series in λ and obtain

$$\gamma^{(\mathrm{f})} = \frac{5\alpha}{18\sqrt{3}}\frac{m^2}{E}\lambda^3\left(1-\frac{8}{5\sqrt{3}}\zeta'\right); \qquad (30)$$

the spin-nonflip rate is

$$\gamma^{(\mathrm{nf})} = \frac{5\alpha}{3\sqrt{3}}\frac{m^2}{E}\lambda\left(1-\frac{16}{15\sqrt{3}}\lambda + \tfrac{1}{5}\zeta'\lambda + \tfrac{25}{18}\lambda^2 - \frac{4}{3\sqrt{3}}\zeta'\lambda^2\right), \qquad (31)$$

in agreement with that obtained in Refs. 1–3.

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[5]Without loss of generality, we choose $\Pi_3 = 0$.

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$$(m+\gamma\Pi)\psi_{\zeta'} = 0$$

and the eigenvalue equation

$$(\gamma^0 q\sigma_3)\psi_{\zeta'} = \zeta'\psi_{\zeta'}$$

when $(m+\gamma\Pi)$ and $(\gamma^0 q\sigma_3)$ appear on the left- or the right-hand-side in $M_{\zeta'\bar{\zeta}'}$.

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How to calculate low-energy partial-wave parameters of dual resonance amplitudes*

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It is shown that the low-energy parameters for *all* classes of dual, elastic scattering amplitudes can be readily derived by simple use of Rodrigues's formula for the Legendre polynomials.

In an earlier publication,[1] we showed that the low-energy parameters of the dual $\pi\pi$ scattering amplitudes can be easily extracted from an integral representation of the partial waves. This particular representation is applicable to most one-term (without satellites) Veneziano-type meson-meson dual resonance amplitudes[2] which have thus far been proposed in the literature. However, if we add satellite terms, or if we consider meson-baryon[3] or baryon-baryon[4] dual scattering amplitudes, this method is not applicable for certain ranges of the trajectory parameters (particularly, the intercepts).

In this note, we would like to show that a more compact and direct way of studying the low-energy parameters for *all* classes of dual resonance amplitudes[2–4] is to make use of Rodrigues's formula for the Legendre polynomials. Let us consider, without loss of generality, a dual, elastic scattering amplitude (including satellite terms) for the process $1+2\to 1+2$,

$$V^{xy}(s,t)=\frac{\Gamma(i-\alpha_x(s))\Gamma(j-\alpha_y(t))}{\Gamma(k-\alpha_x(s)-\alpha_y(t))}, \quad (1)$$

where $\alpha_x(s)=as+b_x$; a is the universal slope of the Regge trajectory and b_x is its intercept; and i, j, and k are positive integers for meson-meson scattering[2] and may have half-integral values for meson-baryon[3] or baryon-baryon[4] scattering. Also, s and t are the usual Mandelstam variables, with

$$s=2\nu+m_1^2+m_2^2+2[(\nu+m_1^2)(\nu+m_2^2)]^{1/2}$$

and

$$t=-2\nu(1-z),$$

where $\nu\equiv\vec{q}^{\,2}$, $z\equiv\cos\theta$, $\vec{q}$ is the three-momentum, and θ is the scattering angle in the c.m. system. The general dual amplitudes involve particular combinations of $V^{xy}(s,t)$, $V^{xy}(t,u)$, and $V^{xy}(u,s)$.

Now we carry out a partial-wave projection of $V^{xy}(s,t)$:

$$V_l^{xy}(\nu)=\int_{-1}^{1}dz\,P_l(z)\frac{\Gamma(i-as-b_x)\Gamma(j+2a\nu(1-z)-b_y)}{\Gamma(k-as+2a\nu(1-z)-b_x-b_y)}. \quad (2)$$

Substituting

$$P_l(z)=\frac{1}{2^l l!}\left(\frac{d}{dz}\right)^l(z^2-1)^l,$$

we obtain

$$V_l^{xy}(\nu)=\frac{1}{2^l l!}\int_{-1}^{1}dz(1-z^2)^l\left(\frac{d}{dz}\right)^l\frac{\Gamma(i-as-b_x)\Gamma(j+2a\nu(1-z)-b_y)}{\Gamma(k-as+2a\nu(1-z)-b_x-b_y)}$$
$$=\frac{(2a\nu)^l}{2^l l!}\int_{-1}^{1}dz(z^2-1)^l\left(\frac{d}{dw}\right)^l\frac{\Gamma(i-as-b_x)\Gamma(j+2a\nu(1-z)-b_y+w)}{\Gamma(k-as+2a\nu(1-z)-b_x-b_y+w)}\bigg|_{w=0}. \quad (3)$$

To determine the low-energy parameters, we expand $V_l^{xy}(\nu)$ around $\nu=0$:

$$V_l^{xy}(\nu)=\nu^l[C_l^{xy}+\nu D_l^{xy}+O(\nu^2)]. \quad (4)$$

Then it follows immediately from (3) that

$$C_l^{xy}=\frac{2(-2a)^l\Gamma(i-aM^2-b_x)}{(2l+1)!!}\left(\frac{d}{dw}\right)^l\frac{\Gamma(j-b_y+w)}{\Gamma(k-aM^2-b_x-b_y+w)},$$
$$D_l^{xy}=\frac{2(-2a)^{l+1}\Gamma(i-aM^2-b_x)}{(2l+1)!!}\left(\frac{d}{dw}\right)^l\frac{\Gamma(j-b_y+w)}{\Gamma(k-aM^2-b_x-b_y+w)}$$
$$\times\left[\frac{M^2}{2m_1m_2}\psi(i-aM^2-b_x)-\psi(j-b_y+w)-\left(\frac{M^2}{2m_1m_2}-1\right)\psi(k-aM^2-b_x-b_y+w)\right], \quad (5)$$

where $w=0$, $M\equiv m_1+m_2$, and $\psi(z)=(d/dz)\ln\Gamma(z)$.

Equation (4) exhibits the normal threshold behavior of the amplitude, and (5) expresses the threshold coefficients for arbitrary angular momentum l in terms of the trajectory parameters. It is now a straightforward matter to use (5) to calculate the low-energy parameters, such as the scattering lengths and effective ranges for *any* l, in terms of the trajectory parameters.[1]

We can apply the same method to $V^{xy}(t,u)$ and $V^{xy}(u,s)$, to a class of "smeared" dual resonance amplitudes,[5] and also to polynomial-times-beta-function types of dual amplitudes; furthermore, the extension to higher orders of ν is trivial. The simplification embodied in (5) comes, of course, from the use of the auxiliary differential variable w and the fact that $\cos\theta$, in dual amplitudes, always appears in the form $\nu\cos\theta$. In point of fact, the procedure can be carried through quite generally so long as one is dealing with an amplitude which is a function of the Mandelstam variables s,t,u and which is singularity-free in the closed interval $-1\le\cos\theta\le 1$. In essence, therefore, the procedure described in this note is rather elementary, but it appears to have been overlooked in the literature.

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