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Chiral $SU_4 \otimes SU_4$ and scale invariance*

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Proceeding from the speculation that the real world is approximately $SU_4 \otimes SU_4$- and scale-invariant, we study the mechanism of spontaneous breakdown by considering a σ-type model that embodies both operator partial conservation of axial-vector current and operator partial conservation of dilatation current. Solutions exhibiting spontaneous breakdown are analyzed in relation to the behavior of transition when explicit symmetry breaking is turned on. Allowed domains for the symmetry-breaking parameters are also derived.

I. INTRODUCTION

The group SU_4 made its debut in nuclear physics when Wigner[1] attempted to give a unified treatment of nucleon spin and isospin. The appearance of SU_4 (Refs. 2–7) in particle physics was an outgrowth of the success of SU_3. Of the many features that motivated the study of SU_4 the most attractive is the transparent symmetry between hadrons and leptons[3–6,8,9]. However, the lack of experimental evidence for the existence of many new particles invariably tended to preclude SU_4 from being seriously considered as a symmetry group of the real world. SU_4 was soon swept into limbo after a brief appearance in the waves of SU_3. Nonetheless, the recent emergence of non-Abelian gauge theories seemed to have revived interests in SU_4,[10–14] for instance, as a viable way to avert the strangeness-changing neutral currents. Other experimental indications, such as the rise of the total cross section of pp scattering,[15–17] and the various claims of the discovery of heavy particles,[18] may also point to the charmed particles as embodied in the SU_4 scheme.

The present work is concerned with another facet of the possible relevance of SU_4 to high-energy physics, viz., chiral $SU_4 \otimes SU_4$ (Refs. 8–14, 19, 20) and scale invariance. Instead of the commonly hypothesized relation between scale invariance and $SU_3 \otimes SU_3$,[21–23] we propose that scale invariance is broken together with $SU_4 \otimes SU_4$. In Sec. II a σ model[24] comprising fields that constitute the $(4, \bar{4}) \oplus (\bar{4}, 4)$ representation of $SU_4 \otimes SU_4$ is studied in the "tree approximation." $SU_4 \otimes SU_4$ and scale invariance are broken simultaneously by terms linear in the fields, thence ensue operator partial conservation of axial-vector current (PCAC) and partial conservation of dilatation current (PCDC). In Sec. III the allowed domains for the symmetry-breaking parameters are derived. In Sec. IV solutions in the symmetry limit, and their behavior under conditions of smooth transition, are analyzed.

II. LAGRANGIAN MODEL

The model we shall study is described by the phenomenological Lagrangian

$$\mathcal{L} = \bar{\mathcal{L}} + \mathcal{L}_{\text{S.B.}} ,$$

where

$$\begin{aligned} \bar{\mathcal{L}} &= \mathcal{L}_0 + \mathcal{L}_I , \\ \mathcal{L}_0 &= \tfrac{1}{2}\partial_\mu \mathfrak{M}^\dagger \partial^\mu \mathfrak{M}, \\ \mathcal{L}_I &= f_1(\mathrm{Tr}\,\mathfrak{M}^\dagger \mathfrak{M})^2 + f_2\,\mathrm{Tr}(\mathfrak{M}^\dagger \mathfrak{M})^2 \\ &\quad + g(\det \mathfrak{M} + \det \mathfrak{M}^\dagger), \\ \mathcal{L}_{\text{S.B.}} &= -\epsilon_0\sigma_0 - \epsilon_8\sigma_8 - \epsilon_{15}\sigma_{15} . \end{aligned} \tag{2.1}$$

Here

$$\mathfrak{M} \equiv \sigma + i\phi \equiv \sum_{i=0}^{15} \frac{(\sigma_i + i\phi_i)\lambda_i}{\sqrt{2}} , \tag{2.2}$$

and the λ matrices are given in the Appendix. The fields (σ, ϕ) constitute the $(4, \bar{4}) \oplus (\bar{4}, 4)$ representation of $SU_4 \otimes SU_4$; they transform under the chiral generators F_i and F_i^5 as

$$\begin{aligned} [F_i, \sigma_j] &= i f_{ijk}\sigma_k , \\ [F_i, \phi_j] &= i f_{ijk}\phi_k , \\ [F_i^5, \sigma_j] &= -i d_{ijk}\phi_k , \\ [F_i^5, \phi_j] &= i d_{ijk}\sigma_k , \end{aligned} \tag{2.3}$$

where

$$i = 1, \ldots, 15,$$

$$j, k = 0, \ldots, 15 .$$

The generators F_i and F_i^5 define the Lie algebra

TABLE I. Nonvanishing values of f_{ijk} and d_{ijk}.

i	j	k	f_{ijk}	i	j	k	d_{ijk}	i	j	k	d_{ijk}
i	j	0	0	i	j	0	$(1/\sqrt{2})\delta_{ij}$	5	5	8	$-1/2\sqrt{3}$
1	2	3	1	1	1	8	$1/\sqrt{3}$	5	5	15	$1/\sqrt{6}$
1	4	7	$\frac{1}{2}$	1	1	15	$1/\sqrt{6}$	5	9	14	$-\frac{1}{2}$
1	5	6	$-\frac{1}{2}$	1	4	6	$\frac{1}{2}$	5	10	13	$\frac{1}{2}$
1	9	12	$\frac{1}{2}$	1	5	7	$\frac{1}{2}$	6	6	8	$-1/2\sqrt{3}$
1	10	11	$-\frac{1}{2}$	1	9	11	$\frac{1}{2}$	6	6	15	$1/\sqrt{6}$
2	4	6	$\frac{1}{2}$	1	10	12	$\frac{1}{2}$	6	11	13	$\frac{1}{2}$
2	5	7	$\frac{1}{2}$	2	2	8	$1/\sqrt{3}$	6	12	14	$\frac{1}{2}$
2	9	11	$\frac{1}{2}$	2	2	15	$1/\sqrt{6}$	7	7	8	$-1/2\sqrt{3}$
2	10	12	$\frac{1}{2}$	2	4	7	$-\frac{1}{2}$	7	7	15	$1/\sqrt{6}$
3	4	5	$\frac{1}{2}$	2	5	6	$\frac{1}{2}$	7	11	14	$-\frac{1}{2}$
3	6	7	$-\frac{1}{2}$	2	9	12	$-\frac{1}{2}$	7	12	13	$\frac{1}{2}$
3	9	10	$\frac{1}{2}$	2	10	11	$\frac{1}{2}$	8	8	8	$-1/\sqrt{3}$
3	11	12	$-\frac{1}{2}$	3	3	8	$1/\sqrt{3}$	8	8	15	$1/\sqrt{6}$
4	5	8	$\frac{1}{2}\sqrt{3}$	3	3	15	$1/\sqrt{6}$	8	9	9	$1/2\sqrt{3}$
4	9	14	$\frac{1}{2}$	3	4	4	$\frac{1}{2}$	8	10	10	$1/2\sqrt{3}$
4	10	13	$-\frac{1}{2}$	3	5	5	$\frac{1}{2}$	8	11	11	$1/2\sqrt{3}$
5	9	13	$\frac{1}{2}$	3	6	6	$-\frac{1}{2}$	8	12	12	$1/2\sqrt{3}$
5	10	14	$\frac{1}{2}$	3	7	7	$-\frac{1}{2}$	8	13	13	$-1/\sqrt{3}$
6	7	8	$\frac{1}{2}\sqrt{3}$	3	9	9	$\frac{1}{2}$	8	14	14	$-1/\sqrt{3}$
6	11	14	$\frac{1}{2}$	3	10	10	$\frac{1}{2}$	9	9	15	$-1/\sqrt{6}$
6	12	13	$-\frac{1}{2}$	3	11	11	$-\frac{1}{2}$	10	10	15	$-1/\sqrt{6}$
7	11	13	$\frac{1}{2}$	3	12	12	$-\frac{1}{2}$	11	11	15	$-1/\sqrt{6}$
7	12	14	$\frac{1}{2}$	4	4	8	$-1/2\sqrt{3}$	12	12	15	$-1/\sqrt{6}$
8	9	10	$1/2\sqrt{3}$	4	4	15	$1/\sqrt{6}$	13	13	15	$-1/\sqrt{6}$
8	11	12	$1/2\sqrt{3}$	4	9	13	$\frac{1}{2}$	14	14	15	$-1/\sqrt{6}$
8	13	14	$-1/\sqrt{3}$	4	10	14	$\frac{1}{2}$	15	15	15	$-(\frac{2}{3})^{1/2}$
9	10	15	$(\frac{2}{3})^{1/2}$								
11	12	15	$(\frac{2}{3})^{1/2}$								
13	14	15	$(\frac{2}{3})^{1/2}$								

$$[F_i, F_j] = if_{ijk}F_k\,,\qquad [F_i, F^5_j] = if_{ijk}F^5_k\,,\qquad [F^5_i, F^5_j] = if_{ijk}F_k\,, \tag{2.4}$$

where

$$i, j, k = 1, \ldots, 15\,.$$

Nonvanishing values of f_{ijk} and d_{ijk} are tabulated in Table I.

The form of the Lagrangian is the same as the usual SU_3 σ model, yet the underlying group content is now $SU_4 \otimes SU_4$.[25] In contradistinction to the SU_3 case, $\overline{\mathcal{L}}$ is both chiral- and scale-invariant, i.e., the term[26] $g(\det \mathfrak{M} + \det \mathfrak{M}^\dagger)$ that breaks scale invariance in the SU_3 case is now scale-invariant. The breaking of $SU_4 \otimes SU_4$ and scale invariance is due entirely to the same linear terms in $\mathcal{L}_{S.B.}$, and hence we have both operator PCAC and PCDC.

If we introduce the notations

$$a \equiv \frac{\sqrt{2}\,\epsilon_8}{\sqrt{3}\,\epsilon_0}, \quad b \equiv \frac{\epsilon_{15}}{\sqrt{3}\,\epsilon_0}, \tag{2.5}$$

we see that when $a=0$ the Lagrangian is SU_3-symmetric; when $1+a+b=0$ it is $SU_2 \otimes SU_2$-symmetric; when $a=0$ and $b=-1$ it is $SU_3 \otimes SU_3$-symmetric.

The physical fields are defined as

$$\begin{aligned} \sigma_0' &= \sigma_0 - \xi_0, \\ \sigma_8' &= \sigma_8 - \xi_8, \\ \sigma_{15}' &= \sigma_{15} - \xi_{15}, \end{aligned} \tag{2.6}$$

where $\xi_0 \equiv \langle \sigma_0 \rangle$, $\xi_8 \equiv \langle \sigma_8 \rangle$, and $\xi_{15} \equiv \langle \sigma_{15} \rangle$. Using the "tree approximation," we obtain the consistency equations

$$\begin{aligned} \epsilon_0 &= 4\xi_0(\xi_0^2+\xi_8^2+\xi_{15}^2)f_1 \\ &\quad + \frac{1}{\sqrt{3}}(\sqrt{3}\,\xi_0^3 - \sqrt{2}\,\xi_8^3 - 2\xi_{15}^3 + 3\sqrt{3}\,\xi_0\xi_8^2 \\ &\qquad + 3\sqrt{3}\,\xi_0\xi_{15}^2 + 3\xi_{15}\xi_8^2)f_2 \\ &\quad + \frac{1}{6\sqrt{3}}(3\sqrt{3}\,\xi_0^3 - \sqrt{2}\,\xi_8^3 - 2\xi_{15}^3 - 3\sqrt{3}\,\xi_0\xi_8^2 \\ &\qquad - 3\sqrt{3}\,\xi_0\xi_{15}^2 + \xi_{15}\xi_8^2)g, \\ \epsilon_8 &= 4\xi_8(\xi_0^2+\xi_8^2+\xi_{15}^2)f_1 \\ &\quad + \xi_8(3\xi_0^2 + 2\xi_8^2 + \xi_{15}^2 - \sqrt{6}\,\xi_0\xi_8 \\ &\qquad + 2\sqrt{3}\,\xi_0\xi_{15} - \sqrt{2}\,\xi_8\xi_{15})f_2 \\ &\quad - \frac{\xi_8}{2\sqrt{3}}(\sqrt{3}\,\xi_0^2 - \sqrt{3}\,\xi_{15}^2 + \sqrt{2}\,\xi_0\xi_8 \\ &\qquad - 2\xi_0\xi_{15} - \sqrt{6}\,\xi_8\xi_{15})g, \\ \epsilon_{15} &= 4\xi_{15}(\xi_0^2+\xi_8^2+\xi_{15}^2)f_1 \\ &\quad - \tfrac{1}{3}(\sqrt{2}\,\xi_8^3 - 7\xi_{15}^3 - 9\xi_0^2\xi_{15} + 6\sqrt{3}\,\xi_{15}^2\xi_0 \\ &\qquad - 3\sqrt{3}\,\xi_8^2\xi_0 - 3\xi_8^2\xi_{15})f_2 \\ &\quad + \tfrac{1}{6}(\sqrt{2}\,\xi_8^3 - \xi_{15}^3 - 3\xi_0^2\xi_{15} - 2\sqrt{3}\,\xi_{15}^2\xi_0 \\ &\qquad + \sqrt{3}\,\xi_8^2\xi_0 + 3\xi_8^2\xi_{15})g. \end{aligned} \tag{2.7}$$

Reexpressed in terms of

$$\begin{aligned} \lambda &\equiv \tfrac{1}{2}\xi_0, \\ c &\equiv \frac{\sqrt{2}\,\xi_8}{\sqrt{3}\,\xi_0}, \\ d &\equiv \frac{\xi_{15}}{\sqrt{3}\,\xi_0}, \end{aligned} \tag{2.8}$$

the equations become

$$\begin{aligned} \tfrac{1}{2}\epsilon_0 &= 8(2+3c^2+6d^2)(f_1\lambda^3) \\ &\quad + 2(2+9c^2+18d^2+9c^2d-3c^3-12d^3)(f_2\lambda^3) \\ &\quad + (2-3c^2-6d^2+3c^2d-c^3-4d^3)(g\lambda^3), \\ \frac{\epsilon_8}{\sqrt{6}} &= 8c(2+3c^2+6d^2)(f_1\lambda^3) \\ &\quad + 12c(1-c+2d-cd+c^2+d^2)(f_2\lambda^3) \\ &\quad - 2c(1+c-2d-3cd-3d^2)(g\lambda^3), \\ \frac{\epsilon_{15}}{2\sqrt{3}} &= 8d(2+3c^2+6d^2)(f_1\lambda^3) \\ &\quad + 2(6d+3c^2-12d^2+3c^2d-c^3+14d^3)(f_2\lambda^3) \\ &\quad - (2d-c^2+4d^2-3c^2d-c^3+2d^3)(g\lambda^3). \end{aligned} \tag{2.9}$$

The masses of the particles are identified as the coefficients of the quadratic terms in the displaced Lagrangian. They are listed in Table II.

III. ALLOWED DOMAINS FOR THE SYMMETRY-BREAKING PARAMETERS

The spectral representations for the current commutators are[27,28]

$$\begin{aligned} \langle [\mathfrak{F}_{\mu i}(x), \mathfrak{F}_{\nu j}(x')] \rangle &= i\int_0^\infty dm^2 \left[\rho_{ij}(m^2)\left(g_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{m^2}\right) - \frac{\sigma_{ij}(m^2)}{m^2}\,\partial_\mu\partial_\nu\right]\Delta(x-x';m^2), \\ \langle [\mathfrak{F}^5_{\mu i}(x), \mathfrak{F}^5_{\nu j}(x')] \rangle &= i\int_0^\infty dm^2 \left[\rho^5_{ij}(m^2)\left(g_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{m^2}\right) - \frac{\sigma^5_{ij}(m^2)}{m^2}\,\partial_\mu\partial_\nu\right]\Delta(x-x';m^2), \end{aligned} \tag{3.1}$$

where ρ_{ij} and σ_{ij} are the transverse and longitudinal spectral weight functions. Taking the divergence of (3.1) and integrating over space gives

$$\begin{aligned} i\langle [\partial^\mu \mathfrak{F}_{\mu i}, F_j] \rangle &= \int_0^\infty dm^2 \sigma_{ij}(m^2) \equiv K_{ij}, \\ i\langle [\partial^\mu \mathfrak{F}^5_{\mu i}, F^5_j] \rangle &= \int_0^\infty dm^2 \sigma^5_{ij}(m^2) \equiv I_{ij}. \end{aligned} \tag{3.2}$$

If we use Gell-Mann's divergence equations[29]

$$\begin{aligned} i\partial^\mu \mathfrak{F}_{\mu i} &= [F_i, \mathcal{H}_{\text{S.B.}}], \\ i\partial^\mu \mathfrak{F}^5_{\mu i} &= [F^5_i, \mathcal{H}_{\text{S.B.}}], \end{aligned} \tag{3.3}$$

we can relate (3.2) to the symmetry-breaking parameters ϵ_0, ϵ_8, ϵ_{15} and the field vacuum expectation values ξ_0, ξ_8, ξ_{15}. The divergence of the vector and the axial-vector current densities are

TABLE II. Mass terms occur in the Lagrangian as $-\frac{1}{2}\mu_i\Phi_i^2 - \frac{1}{2}\mu_j\Phi_j^2 - \mu_{ij}^2\Phi_i\Phi_j$. They are related to the A_i by the expression $\mu^2 = A_1x_1 + A_2x_2 + A_3x_3$, where $x_1 \equiv f_1\lambda^2$, $x_2 \equiv f_2\lambda^2$, and $x_3 \equiv g\lambda^2$.

μ^2	A_1	A_2	A_3
m_π^2	$-8(3c^2+6d^2+2)$	$-4(c+d+1)^2$	$2(3d^2-6cd+2c+2d-1)$
m_K^2	$-8(3c^2+6d^2+2)$	$-4(7c^2+d^2-cd-c+2d+1)$	$2(3d^2+3cd-c+2d-1)$
m_X^2	$-8(3c^2+6d^2+2)$	$-4(c^2+13d^2+5cd+c-2d+1)$	$2(2c^2-d^2+cd+c-2d-1)$
m_Y^2	$-8(3c^2+6d^2+2)$	$-4(4c^2+13d^2-10cd-2c-2d+1)$	$-2(c+d+1)^2$
m_0^2	$-8(3c^2+6d^2+2)$	$-2(3c^2+6d^2+2)$	$-3(c^2+2d^2-2)$
m_8^2	$-8(3c^2+6d^2+2)$	$-4(3c^2+d^2-2cd-2c+2d+1)$	$2(3d^2+6cd-2c+2d-1)$
m_{15}^2	$-8(3c^2+6d^2+2)$	$-2(c^2+14d^2-8d+2)$	$(3c^2-6d^2-8d-2)$
$m_{0,8}^2$		$2\sqrt{6}c(c-2d-2)$	$-\sqrt{6}c(2+c-2d)$
$m_{0,15}^2$		$-2\sqrt{3}(c^2-4d^2+4d)$	$\sqrt{3}(c^2-4d^2-4d)$
$m_{8,15}^2$		$2\sqrt{2}c(c-2d-2)$	$\sqrt{2}c(3c+6d+2)$
σ_π^2	$-8(3c^2+6d^2+2)$	$-12(c+d+1)^2$	$-2(3d^2-6cd+2c+2d-1)$
σ_K^2	$-8(3c^2+6d^2+2)$	$-12(c^2+d^2-cd-c+2d+1)$	$-2(3d^2+3cd-c+2d-1)$
σ_X^2	$-8(3c^2+6d^2+2)$	$-4(c^2+7d^2-cd+3c-6d+3)$	$-2(2c^2-d^2+cd+c-2d-1)$
σ_Y^2	$-8(3c^2+6d^2+2)$	$-4(4c^2+7d^2+2cd-6c-6d+3)$	$2(c+d+1)^2$
σ_0^2	$-24(c^2+2d^2+2)$	$-6(3c^2+6d^2+2)$	$3(c^2+2d^2-2)$
σ_8^2	$-8(9c^2+6d^2+2)$	$-12(3c^2+d^2-2cd-2c+2d+1)$	$-2(3d^2+6cd-2c+2d-1)$
σ_{15}^2	$-8(3c^2+18d^2+2)$	$-6(c^2+14d^2-8d+2)$	$-(3c^2-6d^2-8d-2)$
$\sigma_{0,8}^2$	$-16\sqrt{6}c$	$6\sqrt{6}c(c-2d-2)$	$\sqrt{6}c(2+c-2d)$
$\sigma_{0,15}^2$	$-32\sqrt{3}d$	$-6\sqrt{3}(c^2-4d^2+4d)$	$-\sqrt{3}(c^2-4d^2-4d)$
$\sigma_{8,15}^2$	$-48\sqrt{2}cd$	$6\sqrt{2}c(c-2d-2)$	$-\sqrt{2}c(3c+6d+2)$

listed in Table III; the nonvanishing K_{ij} and I_{ij} are given in Table IV.

We notice that K_{ij} and I_{ij} are symmetric under the simultaneous interchange of $a \leftrightarrow c$ and $b \leftrightarrow d$.

The positivity conditions require that[27,28]

$$\sum_{i,j=1}^{15} c_i^* K_{ij} c_j \geq 0\,, \qquad \sum_{i,j=1}^{15} c_i^* I_{ij} c_j \geq 0 \tag{3.4}$$

for any choice of the constants c_i. These conditions are satisfied provided[28]

$$K_{ii} \geq 0\,, \quad I_{ii} \geq 0\,, \quad I_{88} I_{15,15} \geq I_{8,15}^2\,. \tag{3.5}$$

Allow domains for the cases $a=0$, $b=-1$ ($SU_3 \otimes SU_3$-symmetric Lagrangian), $a=c=0$ (SU_3-symmetric Lagrangian and vacuum), and $1+a+b=0$, $c=0$ ($SU_2 \otimes SU_2$-symmetric Lagrangian and SU_3-symmetric vacuum) are given in Table V; their graphical representations are shown in Figs. 1–3.

IV. ANALYSIS OF THE MECHANISM OF SPONTANEOUS BREAKDOWN

A. Spontaneous-breakdown solutions

In this section we first study the possible solutions of (ξ_0, ξ_8, ξ_{15}) in the symmetry limit $\epsilon_0 = \epsilon_8 = \epsilon_{15} = 0$, and then we turn on symmetry breaking and examine their behavior in the course of transition.[23,30]

We consider the following cases.

Case 1. $\xi_0 = 0$, $\xi_8 = 0$, $\xi_{15} \neq 0$. In this case Eqs. (2.7) simplify to

$$6f_2 + g = 0\,, \qquad 24f_1 + 14f_2 - g = 0\,. \tag{4.1}$$

The ratios between the coupling constants are thus fixed:

TABLE III. Divergences of the $SU_4 \otimes SU_4$ current densities.

i	$\partial^\mu \mathfrak{F}_{\mu i} = (\epsilon_8 f_{i8k} + \epsilon_{15} f_{i15k})\sigma_k$	$\partial^\mu \mathfrak{F}^5_{\mu i} = -[(\epsilon_0/\sqrt{2})\delta_{ik} + \epsilon_8 d_{i8k} + \epsilon_{15} d_{i15k}]\phi_k$
1	0	$-\frac{1}{\sqrt{6}}(\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 + \epsilon_{15})\phi_1$
2	0	$-\frac{1}{\sqrt{6}}(\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 + \epsilon_{15})\phi_2$
3	0	$-\frac{1}{\sqrt{6}}(\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 + \epsilon_{15})\phi_3$
4	$-\frac{\sqrt{3}}{2}\epsilon_8\sigma_5$	$-\frac{1}{2\sqrt{6}}(2\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 + 2\epsilon_{15})\phi_4$
5	$\frac{\sqrt{3}}{2}\epsilon_8\sigma_4$	$-\frac{1}{2\sqrt{6}}(2\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 + 2\epsilon_{15})\phi_5$
6	$-\frac{\sqrt{3}}{2}\epsilon_8\sigma_7$	$-\frac{1}{2\sqrt{6}}(2\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 + 2\epsilon_{15})\phi_6$
7	$\frac{\sqrt{3}}{2}\epsilon_8\sigma_6$	$-\frac{1}{2\sqrt{6}}(2\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 + 2\epsilon_{15})\phi_7$
8	0	$-\frac{1}{\sqrt{6}}[\sqrt{3}\,\epsilon_8\phi_0 + (\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 + \epsilon_{15})\phi_8 + \epsilon_8\phi_{15}]$
9	$-\frac{1}{2\sqrt{3}}(\epsilon_8 + 2\sqrt{2}\,\epsilon_{15})\sigma_{10}$	$-\frac{1}{2\sqrt{6}}(2\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 - 2\epsilon_{15})\phi_9$
10	$\frac{1}{2\sqrt{3}}(\epsilon_8 + 2\sqrt{2}\,\epsilon_{15})\sigma_9$	$-\frac{1}{2\sqrt{6}}(2\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 - 2\epsilon_{15})\phi_{10}$
11	$-\frac{1}{2\sqrt{3}}(\epsilon_8 + 2\sqrt{2}\,\epsilon_{15})\sigma_{12}$	$-\frac{1}{2\sqrt{6}}(2\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 - 2\epsilon_{15})\phi_{11}$
12	$\frac{1}{2\sqrt{3}}(\epsilon_8 + 2\sqrt{2}\,\epsilon_{15})\sigma_{11}$	$-\frac{1}{2\sqrt{6}}(2\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 - 2\epsilon_{15})\phi_{12}$
13	$\frac{1}{\sqrt{3}}(\epsilon_8 - \sqrt{2}\,\epsilon_{15})\sigma_{14}$	$-\frac{1}{\sqrt{6}}(\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 - \epsilon_{15})\phi_{13}$
14	$-\frac{1}{\sqrt{3}}(\epsilon_8 - \sqrt{2}\,\epsilon_{15})\sigma_{13}$	$-\frac{1}{\sqrt{6}}(\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 - \epsilon_{15})\phi_{14}$
15	0	$-\frac{1}{\sqrt{6}}[\sqrt{3}\,\epsilon_{15}\phi_0 + \epsilon_8\phi_8 + (\sqrt{3}\,\epsilon_0 - 2\epsilon_{15})\phi_{15}]$

TABLE IV. Expressions defining the allowed domains; $\gamma \equiv -\frac{1}{8}\epsilon_0\xi_0$.

K_{44}	$-\frac{3}{4}\epsilon_8\epsilon_8$	$9\gamma ac$
K_{99}	$-\frac{1}{12}[\epsilon_8\xi_8 + 2\sqrt{2}(\epsilon_8\xi_{15} + \epsilon_{15}\xi_8) + 8\epsilon_{15}\xi_{15}]$	$\gamma(a+4b)(c+4d)$
$K_{13,13}$	$-\frac{1}{3}[\epsilon_8\xi_8 - \sqrt{2}(\epsilon_8\xi_{15} + \epsilon_{15}\xi_8) + 2\epsilon_{15}\xi_{15}]$	$4\gamma(a-2b)(c-2d)$
I_{11}	$-\frac{1}{6}(\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 + \epsilon_{15})(\sqrt{3}\,\xi_0 + \sqrt{2}\,\xi_8 + \xi_{15})$	$4\gamma(1+a+b)(1+c+d)$
I_{44}	$-\frac{1}{24}(2\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 + 2\epsilon_{15})(2\sqrt{3}\,\xi_0 - \sqrt{2}\,\xi_8 + 2\xi_{15})$	$\gamma(2-a+2b)(2-c+2d)$
I_{99}	$-\frac{1}{24}(2\sqrt{3}\,\epsilon_0 + \sqrt{2}\,\epsilon_8 - 2\epsilon_{15})(2\sqrt{3}\,\xi_0 + \sqrt{2}\,\xi_8 - 2\xi_{15})$	$\gamma(2+a-2b)(2+c-2d)$
$I_{13,13}$	$-\frac{1}{6}(\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 - \epsilon_{15})(\sqrt{3}\,\xi_0 - \sqrt{2}\,\xi_8 - \xi_{15})$	$4\gamma(1-a-b)(1-c-d)$
I_{88}	$-\frac{1}{6}[4\epsilon_8\xi_8 + (\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 + \epsilon_{15})(\sqrt{3}\,\xi_0 - \sqrt{2}\,\xi_8 + \xi_{15})]$	$4\gamma[(1-a+b)(1-c+d)+2ac]$
$I_{15,15}$	$-\frac{1}{6}[\epsilon_8\xi_8 + 3\epsilon_{15}\xi_{15} + (\sqrt{3}\,\epsilon_0 - 2\epsilon_{15})(\sqrt{3}\,\xi_0 - 2\xi_{15})]$	$2\gamma[2(1-2b)(1-2d)+ac+6bd]$
$I_{8,15}$	$-\frac{1}{6}[\epsilon_8(\sqrt{3}\,\xi_0 + \xi_{15}) + (\sqrt{3}\,\epsilon_0 - \sqrt{2}\,\epsilon_8 + \epsilon_{15})\xi_8]$	$2\sqrt{2}\gamma[c(1-a+b)+a(1+d)]$

TABLE V. K_{ij} and I_{ij} for the three cases considered in Sec. III.

K_{ij}, I_{ij}	$a=0,\ b=-1$	$a=c=0$	$1+a+b=0,\ c=0$
K_{99}	$-4\gamma(c+4d)$	$16\gamma bd$	$-4\gamma d(1-3b)$
$K_{13,13}$	$8\gamma(c-2d)$		$8\gamma d(1+3b)$
I_{11}		$4\gamma(1+b)(1+d)$	
I_{44}			$6\gamma(1+b)(1+d)$
I_{99}	$4\gamma(2+c-2d)$	$4\gamma(1-b)(1-d)$	$2\gamma(1-3b)(1-d)$
$I_{13,13}$	$8\gamma(1-c-d)$		$8\gamma(1-d)$
I_{88}			$8\gamma(1+b)(1+d)$
$I_{15,15}$	$12\gamma(1-3d)$	$4\gamma(1-2b-2d+7bd)$	$4\gamma(1-2b-2d+7bd)$
$I_{8,15}$			$-2\sqrt{2}\gamma(1+b)(1+d)$

$$F=-\tfrac{6}{5}, \qquad G=\tfrac{36}{5}, \tag{4.2}$$

where

$$F\equiv\frac{f_2}{f_1}, \qquad G\equiv\frac{g}{f_1}. \tag{4.3}$$

Case 2. $\xi_0\neq0$, $\xi_8=0$, $\xi_{15}=0$.

$$8f_1+2f_2+g=0. \tag{4.4}$$

Hence, $2F+G=-8$.

Case 3. $\xi_0\neq0$, $\xi_8=0$, $\xi_{15}\neq0$. Equations (2.9) become

$$8(1+3d^2)x_1+2(3-6d+7d^2)x_2-(1+d)^2x_3=0,$$
$$8(1+3d^2)x_1+2(1+9d^2-6d^3)x_2+(1+d)^2(1-2d)x_3=0. \tag{4.5}$$

For $d\neq\pm1$,

$$F=\frac{-2(1+3d^2)}{1-2d+5d^2}, \qquad G=\frac{-4(1-3d)(1+3d^2)}{(1+d)(1-2d+5d^2)}; \tag{4.6}$$

when $d=1$, $2F-G=-8$; when $d=-1$, $F=-1$, and G is undetermined.

Case 4. $\xi_0=0$, $\xi_8\neq0$, $\xi_{15}=0$. Equations (2.7) are reduced to

$$6f_2+g=0, \qquad 2f_1+f_2=0, \qquad 2f_2-g=0. \tag{4.7}$$

Hence the only possible solution is $f_1=f_2=g=0$, i.e., we are left with a free-field theory—or else

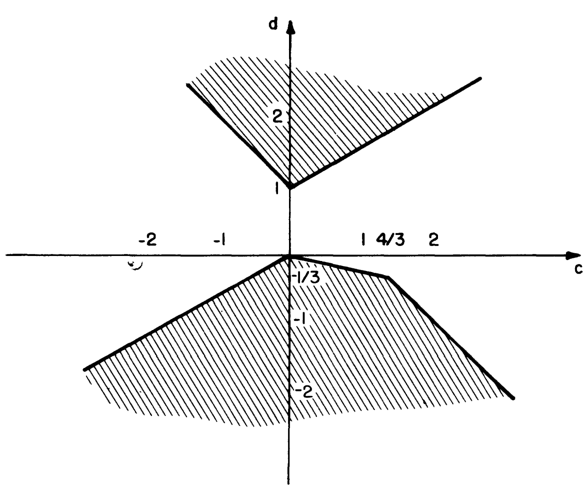

FIG. 1. An $SU_3\otimes SU_3$-symmetric Lagrangian ($a=0$, $b=-1$). Allowed domains are indicated by the shaded regions.

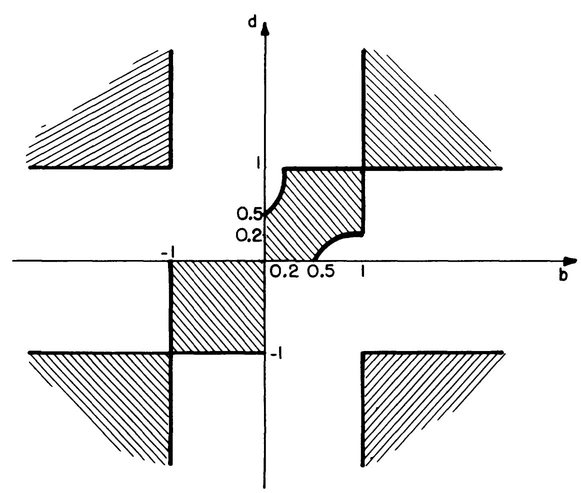

FIG. 2. Allowed domains for an SU_3-symmetric Lagrangian and vacuum ($a=c=0$).

$\xi_0=\xi_8=\xi_{15}=0$, which is the normal solution.

Case 5. $\xi_0 \neq 0$, $\xi_8 \neq 0$, $\xi_{15}=0$.

$$8(2+3c^2)x_1+2(2+9c^2-3c^3)x_2+(2-3c^2-c^3)x_3=0,$$
$$4(2+3c^2)x_1+6(1-c+c^2)x_2-(1+c)x_3=0, \tag{4.8}$$
$$2(3-c)x_2+(1+c)x_3=0.$$

For $f_2\neq 0$, c can only take on the values

$$c=1,2,-2; \tag{4.9}$$

the corresponding values of F and G are

$$F=-2,-\tfrac{14}{5},-\tfrac{14}{13},$$
$$G=4,\tfrac{28}{15},-\tfrac{140}{13}; \tag{4.10}$$

for $f_2=0$, $c=-1$, and $f_1=f_2=g=0$.

Case 6. $\xi_0=0$, $\xi_8\neq 0$, $\xi_{15}\neq 0$.

$$6(1+\delta)(1-2\delta)^2 f_2+(1-\delta+4\delta^3)g=0,$$
$$4(1+2\delta^2)f_1+2(1-\delta+\delta^2)f_2+\delta(1+\delta)g=0,$$
$$24\delta(1+2\delta^2)f_1-2(1-3\delta-14\delta^3)f_2 \tag{4.11}$$
$$+(1+\delta)(1+2\delta-2\delta^2)g=0,$$

where

$$\delta\equiv d/c. \tag{4.12}$$

For $g\neq 2f_2$,

$$\delta=-1,\tfrac{1}{2},-\tfrac{1}{4},$$
$$F=-2,-4,-\tfrac{114}{107}, \tag{4.13}$$
$$G=0,0,\tfrac{972}{107};$$

when $g=2f_2$, $f_1=f_2=g=0$.

From the foregoing analysis we see that, in order for the spontaneous-breakdown mode to be possible, the field vacuum expectation values, and subsequently the ratios of the coupling constants, can only assume certain definite values. This situation is similar to that of the $U_3\otimes U_3$ case studied in Ref. 30.

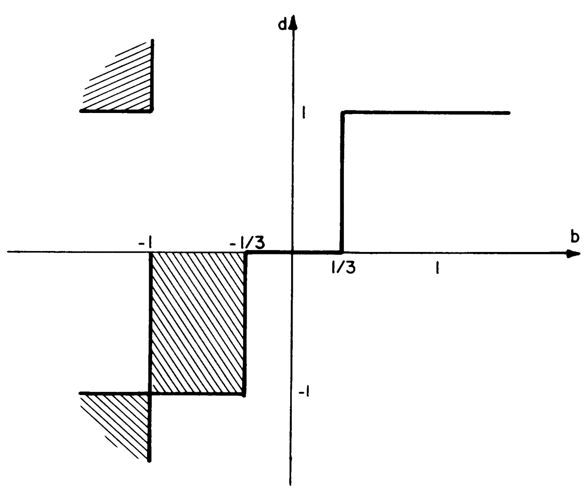

FIG. 3. Allowed domains for an $SU_2\otimes SU_2$-symmetric Lagrangian and an SU_3-symmetric vacuum ($1+a+b=0$, $c=0$).

B. Turning on symmetry breaking; fixed couplings

We proceed to examine the solutions when the explicit symmetry-breaking interaction is turned on. First we assume fixed couplings.

Case 1. Equations (2.9) now become

$$\epsilon_0=-\frac{\xi_{15}{}^3}{3\sqrt{3}}(6f_2+g),$$
$$\epsilon_8=0, \tag{4.14}$$
$$\epsilon_{15}=\tfrac{1}{6}\xi_{15}{}^3(24f_1+14f_2-g).$$

Since $\xi_8=0$ necessarily entails $\epsilon_8=0$, it is not possible for the spontaneous-breakdown solution to undergo a smooth transition for $\epsilon_8\neq 0$. Moreover, even if we let $\epsilon_8=0$, there is still no solution[23] in the presence of nonvanishing ϵ_0 and ϵ_{15}, inasmuch as $6f_2+g=0$, and $24f_1+14f_2-g=0$, for which spontaneous breakdown was possible.

Case 2.

$$\epsilon_0=\tfrac{1}{2}\xi_0{}^3(8f_1+2f_2+g),$$
$$\epsilon_8=0, \tag{4.15}$$
$$\epsilon_{15}=0.$$

Again, there is no solution for $\epsilon_0\neq 0$, since $8f_1+2f_2+g=0$, even if we allow for $\epsilon_8=\epsilon_{15}=0$.

Case 3.

$$\tfrac{1}{2}\epsilon_0=8(1+3d^2)(f_1\lambda^3)$$
$$+2(1+9d^2-6d^3)(f_2\lambda^3)+(1+d)^2(1-2d)(g\lambda^3),$$
$$\epsilon_8=0, \tag{4.16}$$
$$\frac{\epsilon_{15}}{2\sqrt{3}}=d[8(1+3d^2)(f_1\lambda^3)$$
$$+2(3-6d+7d^2)(f_2\lambda^3)-(1+d)^2(g\lambda^3)].$$

$\xi_8=0$ precludes $\epsilon_8\neq 0$, whereas $b=\epsilon_{15}/\sqrt{3}\,\epsilon_0$ is indeterminable, as expected.

Case 5.

$$a=\frac{2c[4(2+3c^2)+6(1-c+c^2)F-(1+c)G]}{8(2+3c^2)+2(2+9c^2-3c^3)+(2-3c^2-c^3)G},$$

$$b=\frac{c^2[2(3-c)F+(1+c)G]}{8(2+3c^2)+2(2+9c^2-3c^3)F+(2-3c^2-c^3)G}. \tag{4.17}$$

We know that in the symmetry limit they will take on the form of 0/0. To obtain the limiting values for a and b we apply l'Hospital's rule and find[30]

c	1	2	−2
a	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$
b	$-\frac{1}{3}$	$\frac{7}{12}$	$-\frac{7}{12}$

(4.18)

Case 6.

$$a=\frac{-6[4(1+2\delta^2)+2(1-\delta+\delta^2)F+\delta(1+\delta)G]}{6(1+\delta)(1-2\delta^2)F+(1-\delta+4\delta^3)G},$$

$$b=\frac{-24\delta(1+2\delta^2)+2(1-3\delta-14\delta^3)F-(1+\delta)(1+2\delta-2\delta^2)G}{6(1+\delta)(1-2\delta)^2F+(1-\delta+4\delta^3)G}. \tag{4.19}$$

Similarly, we obtain the following limiting values:

δ	−1	$\frac{1}{2}$	$-\frac{1}{4}$
a	$-\frac{2}{9}$	∞	$-\frac{50}{27}$
b	$-\frac{1}{9}$	∞	$-\frac{100}{27}$
β	$\frac{1}{2}$	−1	2

(4.20)

where

$$\beta \equiv b/a. \tag{4.21}$$

C. Variable couplings

However, if we substitute the symmetry-limit values obtained in Sec. IV B into Table III, we find that they lie outside the allowed domains. To remedy this situation, we have to allow for variable couplings, and consequently extra parameters have to be introduced. For instance, we can follow Carruthers and Haymaker[30] by assuming a linear variation of F and G with respect to the vacuum expectation values as symmetry breaking is turned on[31]:

$$\frac{dF(\bar{\xi}_i)}{d\xi_i}=\frac{dG(\bar{\xi}_i)}{d\xi_i}=\rho, \tag{4.22}$$

where $\bar{\xi}_i$ denote the symmetry-limit values.

This device enables us to get

c	1	2	−2
a	$-4\left(\frac{2+\rho}{12-7\rho}\right)$	$2\left(\frac{-64+225\rho}{384+75\rho}\right)$	$2\left(\frac{64-559\rho}{384+793\rho}\right)$
b	$-\left(\frac{4+3\rho}{12-7\rho}\right)$	$2\left(\frac{112+75\rho}{384+75\rho}\right)$	$-2\left(\frac{112-117\rho}{384+793\rho}\right)$

(4.23)

for case 5; and

δ	−1	$\frac{1}{2}$	$-\frac{1}{4}$
a	$-6\left(\frac{2-3\rho}{54+\rho}\right)$	$-\frac{3}{2}\left(\frac{32+9\rho}{\rho}\right)$	$-6\left(\frac{6400+4173\rho}{20736+19367\rho}\right)$
b	$-6\left(\frac{1+3\rho}{54+\rho}\right)$	$\frac{3}{4}\left(\frac{64-9\rho}{\rho}\right)$	$-\frac{1}{2}\left(\frac{153600-12519\rho}{20736+19367\rho}\right)$
β	$\frac{1+3\rho}{2-3\rho}$	$-\frac{1}{2}\left(\frac{64-9\rho}{32+9\rho}\right)$	$\frac{1}{12}\left(\frac{153600-12519\rho}{6400+4173\rho}\right)$

(4.24)

for case 6.

As $\rho \to 0$ the previous limiting values are regained. By adjusting ρ, we can now restore them to the allowed regions. The goal of a smooth transition is thus attained.

V. SUMMARY

We have studied an SU_4 σ model that incorporates both operator PCAC and PCDC. The mechanism of spontaneous breakdown is investigated in the "tree approximation." In the symmetry limit, the ratios of the coupling constants are found to assume only certain definite values in order for the spontaneous breakdown solutions to be possible. However, these solutions, when explicit symmetry-breaking interaction is turned on, lie outside the allowed domains dictated by positivity, and hence are unstable. To remedy this situation, we allow for variable couplings, and an extra parameter is introduced. By adjusting this parameter, the solutions can be restored to the allowed regions to ensure a smooth transition in the wake of symmetry breaking.

ACKNOWLEDGMENTS

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APPENDIX

The λ matrices are defined as follows:

$$\lambda_0=\frac{1}{\sqrt{2}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\quad \lambda_1=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},$$

$$\lambda_2=\begin{bmatrix}0&-i&0&0\\i&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\quad \lambda_3=\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},$$

$$\lambda_4=\begin{bmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix},\quad \lambda_5=\begin{bmatrix}0&0&-i&0\\0&0&0&0\\i&0&0&0\\0&0&0&0\end{bmatrix},$$

$$\lambda_6=\begin{bmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{bmatrix},\quad \lambda_7=\begin{bmatrix}0&0&0&0\\0&0&-i&0\\0&i&0&0\\0&0&0&0\end{bmatrix},\tag{A1}$$

$$\lambda_8=\frac{1}{\sqrt{3}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-2&0\\0&0&0&0\end{bmatrix},\quad \lambda_9=\begin{bmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix},$$

$$\lambda_{10}=\begin{bmatrix}0&0&0&-i\\0&0&0&0\\0&0&0&0\\i&0&0&0\end{bmatrix},\quad \lambda_{11}=\begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{bmatrix},$$

$$\lambda_{12}=\begin{bmatrix}0&0&0&0\\0&0&0&-i\\0&0&0&0\\0&i&0&0\end{bmatrix},\quad \lambda_{13}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix},$$

$$\lambda_{14}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&-i\\0&0&i&0\end{bmatrix},\quad \lambda_{15}=\frac{1}{\sqrt{6}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{bmatrix}.$$

They satisfy the relations

$$\begin{aligned}&\mathrm{Tr}\,\lambda_i\lambda_j=2\delta_{ij}\,,\\&[\lambda_i,\lambda_j]=2if_{ijk}\lambda_k\,,\\&\{\lambda_i,\lambda_j\}=2d_{ijk}\lambda_k\,.\end{aligned}\tag{A2}$$

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[1]E. P. Wigner, Phys. Rev. 51, 106 (1937).
[2]P. Tarjanne and V. L. Teplitz, Phys. Rev. Lett. 11, 447 (1963).
[3]W. Królikowski, Nucl. Phys. 52, 342 (1964).
[4]Y. Hara, Phys. Rev. 134, B701 (1964).
[5]J. D. Bjorken and S. L. Glashow, Phys. Lett. 11, 255 (1964).
[6]D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Nuovo Cimento 34, 1732 (1964).
[7]Yeh Peng-sheng, Acta Phys. Sinica 22, 163 (1966) [Chinese J. Phys. 22, 131 (1966)].
[8]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).
[9]P. Dittner, S. Eliezer, and T. K. Kuo, Phys. Rev. Lett. 30, 1274 (1973).
[10]H. Georgi and S. L. Glashow, Phys. Rev. D 6, 2977 (1972).
[11]D. A. Dicus and V. S. Mathur, Phys. Rev. D 7, 525 (1973).
[12]H. Georgi and S. L. Glashow, Harvard Univ. report (unpublished).
[13]A. Duncan, MIT report (unpublished).

[14]S. Eliezer and D. A. Ross, Imperial College report (unpublished).

[15]U. Amaldi *et al.* (unpublished).

[16]S. R. Amendolia *et al.* (unpublished).

[17]J. D. Hansen *et al.*, CERN report (unpublished).

[18]K. Niu, E. Mikumo, and Y. Maeda, Prog. Theor. Phys. 46, 1644 (1971).

[19]Z. Maki, T. Maskawa, and I. Umemura, Prog. Theor. Phys. 47, 1682 (1972).

[20]P. Dittner and S. Eliezer, Phys. Rev. D 8, 1929 (1973).

[21]K. Wilson, Phys. Rev. 179, 1499 (1969).

[22]K. Wilson, Phys. Rev. D 3, 1818 (1971).

[23]P. Carruthers, Phys. Rev. D 2, 2265 (1970).

[24]M. Lévy, Nuovo Cimento 52A, 23 (1967).

[25]This is *not* the most general phenomenological Lagrangian that can be constructed for an $SU_4 \otimes SU_4$ theory: whence a term $f_3\,\mathrm{Tr}(\mathfrak{M}^\dagger\mathfrak{M})^3$ should have been included. [It can be shown in general, from the Cayley-Hamilton theorem, that there are $n-2$ interaction terms of the form $f_i\,\mathrm{Tr}(\mathfrak{M}^\dagger\mathfrak{M})^i$ $(i=2,\ldots,n-1)$ in an $SU_n \otimes SU_n$ phenomenological Lagrangian.] However, so far as the intent of this paper is concerned, the present Lagrangian suffices. Renormalizability also excludes terms of higher order of products of fields.

[26]The following relation is useful in later derivations:

$$\det\mathfrak{M} = \tfrac{1}{24}(\mathrm{Tr}\mathfrak{M})^4 + \tfrac{1}{3}(\mathrm{Tr}\mathfrak{M})(\mathrm{Tr}\mathfrak{M}^3) - \tfrac{1}{4}(\mathrm{Tr}\mathfrak{M})^2(\mathrm{Tr}\mathfrak{M}^2) + \tfrac{1}{8}(\mathrm{Tr}\mathfrak{M}^2)^2 - \tfrac{1}{4}\mathrm{Tr}\mathfrak{M}^4.$$

[27]S. Okubo and V. Mathur, Phys. Rev. D 1, 2046 (1970).

[28]P. Carruthers and R. W. Haymaker, Phys. Rev. D 4, 406 (1971).

[29]M. Gell-Mann, Physics 1, 63 (1964).

[30]P. Carruthers and R. W. Haymaker, Phys. Rev. D 4, 1815 (1971).

[31]The *ad hoc* and nonunique nature of this prescription should be noted.