Attempt to generate the angle between the strong and weak interactions*

A. Zee†
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
(Received 2 November 1973)

We construct a toy gauge model in which the Cabibbo angle is generated by the effects of weak interactions. The model is so arranged that in the absence of the weak and electromagnetic interactions the $\mathfrak{N}$ and λ quarks are both massless and degenerate with each other. The weak interaction splits this degeneracy and thus generates an angle between the strong and weak interactions. The value of the angle is determined in terms of certain parameters (experimentally measurable but unknown) that appear in the model. More interestingly, we abstract from this model the following statement: In the limit of vanishing Cabibbo angle, chiral symmetry becomes exact. It is argued that this statement holds for a large class of models. We suggest that in the real world this relation is also valid. A certain phenomenon involving *CP* violation and various options one may have in constructing a gauge theory including *CP* violation are discussed.

I. INTRODUCTION

The origin of the angle[1] between the strong and the weak interactions has long been shrouded in mystery. It would appear that this mystery will remain unresolved until both of these interactions are clearly understood. Meanwhile, we would like to discuss in this paper an attempt to understand a possible mechanism for generating this angle.

Our starting premise is that in the limit when the weak interaction is switched off, the $\mathfrak{N}$ and λ quarks are degenerate (and hence the Cabibbo angle is undefined). We will construct a toy model in which this is so and in which the weak interaction splits this degeneracy and picks out the eigenstates corresponding to the $\mathfrak{N}$ and to the λ quark, thus defining a direction for the strong interaction. The discussion will be in the framework of the gauge theory of weak interactions. It would clearly be too extravagant to hope that at present such a model would reproduce reality, and so, while paying due respect to the prominent selection rules, we will not be overly concerned with a detailed comparison with experiment.

We will not only set the masses of the $\mathfrak{N}$ and the λ quark equal, but we will also set them equal to zero and to the mass of the $\mathcal{P}$ quark. We will have to arrange for this state of affairs in a *natural* way so that the masses acquired by the quarks from the weak interaction are calculable. "Naturalness" is a powerful constraint on the model and has been particularly emphasized by Weinberg in a series of papers.[2] If we are to suppress neutral strangeness-changing currents by adopting the proposal of Glashow *et al.*,[3] we will have to introduce the $\mathcal{P}'$ quark. It turns out that in order to satisfy naturalness and other selection rules, we are obliged to introduce two other quarks, called r and s in what follows. The reader would not be surprised to learn that a multitude of gauge and Higgs[4] fields also demand to be let into the theory. Unfortunately, they bring with them a large number of parameters, in principle measurable but at present unknown, so that we find, annoyingly enough, that we can only calculate the Cabibbo angle θ in terms of these parameters. In particular, θ is but one of a number of Cabibbo-angle-like angles. Our result is thus a relation between these angles. While we do not obtain an absolute prediction for θ, it is possible to learn something from our final equations [Eqs. (4.2) and (4.3)]. In particular, in the limit of exact chiral symmetry, the Cabibbo angle vanishes. Hence, if we insert the observed small amount of chiral-symmetry breaking, we tend to predict a small angle. A rough numerical analysis given in Sec. V shows that the numbers tend to be in the neighborhood of the actual values.

The model we present has the interesting feature that by minimizing the potential in the tree approximation one is unable to determine whether or not CP invariance is broken. This is discussed in Appendix B, which also includes a classification of the different ways in which CP violation may appear in gauge theories. Some readers may wish to skip to Appendix B at this point.

In Sec. II the construction of the model is briefly outlined. We then go on to split the $\mathfrak{N}$-λ degeneracy in Sec. III. The Cabibbo angle is generated in Sec. IV. In Sec. V we analyze our results and in Sec. VI we make a few comments. The Higgs potential is briefly discussed in Appendix A. Appendix B has already been mentioned above.

II. CONSTRUCTION OF THE MODEL

In order for a massless fermion q to acquire mass from the weak and electromagnetic interac-

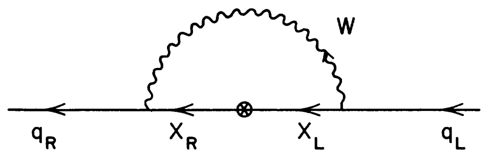

FIG. 1. In order for a massless quark q to acquire mass, q_L has to become X_L by the emission of a gauge particle W and q_R has to become X_R by the absorption of W. X is some massive quark.

tions, it will be necessary to have a gauge boson W which transforms q_L to X_L and q_R to X_R, where X is some massive fermion.[5,6] (See Fig. 1.) Hence $\mathcal{P}_R$, $\mathcal{N}_R$, and λ_R must belong to some nontrivial multiplet under the gauge group. We are thus immediately threatened by the possible presence of $V+A$ currents which will (a) upset low-energy phenomenology, and (b) void[7] the successes current algebra had had with nonleptonic decays. This poses an important constraint on our model construction.

We choose the gauge group to be $G=\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_A \times \mathrm{U}(1)$ and the charge to be $Q=T_{3L}+T_{3R}+T_{3A}+\frac{1}{2}y$ [where y is the generator of U(1)]. As had already been mentioned, we have six quarks,[8] called $\mathcal{P}, \mathcal{P}', r$ (which have charge $+\frac{2}{3}$) and $\mathcal{N}, \lambda, s$ (which have charge $-\frac{1}{3}$). (The charge assignment obviously may be changed according to taste.) These will be assigned to six doublets: three transforming like $(\frac{1}{2},0,0)$, $y=\frac{1}{3}$, under G and called ψ_{iL}, and three transforming like $(0,\frac{1}{2},0)$, $y=\frac{1}{3}$, and called ψ_{iR} $(i=1,2,3)$. A Higgs field φ transforming like the complex $(\frac{1}{2},\frac{1}{2},0)$, $y=0$ representation of G is introduced to join ψ_{iR} and ψ_{iL}. The part of the Lagrangian $\mathcal{L}$ describing this coupling will be taken to be

$$\sum_{i=1}^{3} f_{i2}\bar{\psi}_{iL}\varphi\psi_{2R}+\sum_{i=1}^{3} f_{i3}\bar{\psi}_{iL}\varphi\psi_{3R}+\sum_{i=1}^{3} h_{i1}\bar{\psi}_{iL}\tilde{\varphi}\psi_{1R}\,, \tag{2.1}$$

where f_{i2}, f_{i3}, and h_{i1} are Yukawa coupling constants. We define

$$\tilde{\varphi}\equiv\tau_2\varphi^*\tau_2\,, \tag{2.2}$$

where the asterisk denotes complex conjugation. $\tilde{\varphi}$ transforms in the same way as φ under G. In order to ensure that Eq. (2.1) is natural we require that $\mathcal{L}$ be also invariant under the discrete symmetry K:

$$\begin{aligned} &\varphi\to+i\varphi\,,\quad \tilde{\varphi}\to-i\tilde{\varphi}\,,\\ &\psi_{iL}\to\psi_{iL}\,,\quad \psi_{1R}\to i\psi_{1R}\,,\\ &\psi_{2R}\to-i\psi_{2R}\,,\quad \psi_{3R}\to-i\psi_{3R}\,. \end{aligned} \tag{2.3}$$

K suppresses nine other terms which otherwise must appear in (2.1).

Let us suppose for the moment that we can construct a Higgs potential such that the vacuum expectation value of φ has the value

$$\langle\varphi\rangle=\begin{pmatrix} v & 0\\ 0 & 0\end{pmatrix} \tag{2.4}$$

[and so also $\langle\tilde{\varphi}\rangle=\begin{pmatrix} 0 & 0\\ 0 & v\end{pmatrix}$]. In Appendix A we will argue that this is indeed possible. In that case it is not hard to see by inspection that (2.1) has been carefully arranged so that one charge $+\frac{2}{3}$ quark and two charge $-\frac{1}{3}$ quarks are massless.

We identify these as the $\mathcal{P}$, $\mathcal{N}_0$, and λ_0 quarks. The subscript 0 on $\mathcal{N}_0$ and λ_0 will remind us that at this stage $\mathcal{N}_0$ and λ_0 are degenerate states and the distinction by strangeness has no meaning as yet.

To study the mass spectrum in detail let us introduce the notation

$$\psi_{iL}=\begin{pmatrix}\xi_{iL}\\ \eta_{iL}\end{pmatrix} \text{ and } \psi_{iR}=\begin{pmatrix}\xi_{iR}\\ \eta_{iR}\end{pmatrix}.$$

Then Eqs. (2.1) and (2.4) generate the mass terms

$$\bar{\xi}_{iL}M^{\xi}_{ij}\xi_{jR}+\bar{\eta}_{iL}M^{\eta}_{ij}\eta_{jR}\,, \tag{2.5}$$

where

$$M^{\xi}=v\begin{pmatrix} 0 & f_{12} & f_{13}\\ 0 & f_{22} & f_{23}\\ 0 & f_{32} & f_{33}\end{pmatrix} \tag{2.6}$$

and

$$M^{\eta}=v\begin{pmatrix} h_{11} & 0 & 0\\ h_{21} & 0 & 0\\ h_{31} & 0 & 0\end{pmatrix}. \tag{2.7}$$

One may now readily verify that there are four rotation matrices $R^{\xi,\eta}_{L,R}$ such that

$$(R^{\xi}_L)^{-1}M^{\xi}R^{\xi}_R=\begin{pmatrix} 0 & 0 & 0\\ 0 & m_{\mathcal{P}'} & 0\\ 0 & 0 & m_r\end{pmatrix}, \tag{2.8}$$

$$(R^{\eta}_L)^{-1}M^{\eta}R^{\eta}_L=\begin{pmatrix} m_s & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \tag{2.9}$$

where $m_{\mathcal{P}'}$, m_r, m_s are determined by M^{ξ} and M^{η}. R^{η}_R and R^{η}_L are determined only up to an arbitrary rotation in the 2-3 plane, which fact again reminds us that $\mathcal{N}_0$ and λ_0 degenerate. An explicit determination of $R^{\xi,\eta}_{L,R}$ reveals that there is no constraint among the masses $m_{\mathcal{P}'}$, m_r, m_s and the angles that appear in $R^{\xi,\eta}_{L,R}$. R^{ξ}_R and R^{η}_R are both rotations about the 1 axis, which tells us that ξ_{1R} and η_{1R} are both eigenstates, clearly to be identified as $\mathcal{P}_R$ and s_R, respectively. By redefining appropriate linear combinations of ψ_{2R} and ψ_{3R} as ψ_{2R} and ψ_{3R}, we can finally write

$$\psi_{1R}=\begin{pmatrix}-\mathcal{P}\\ s\end{pmatrix}_R,\quad \psi_{2R}=\begin{pmatrix}\mathcal{P}'\\ \mathcal{N}_0\end{pmatrix}_R,\quad \psi_{3R}=\begin{pmatrix}r\\ \lambda_0\end{pmatrix}. \tag{2.10}$$

Similarly, we may put ψ_{iL} into the form

$$\psi_{1L}=\begin{pmatrix}-\mathcal{P}\sin\alpha+\cos\alpha(\mathcal{P}'\cos\beta+r\sin\beta)\\ s\end{pmatrix}_L, \tag{2.11}$$

$$\psi_{2L}=\begin{pmatrix}\mathcal{P}\cos\alpha+\sin\alpha(\mathcal{P}'\cos\beta+r\sin\beta)\\ \mathcal{N}_0\end{pmatrix}_L, \tag{2.12}$$

$$\psi_{3L}=\begin{pmatrix}-\mathcal{P}'\sin\beta+r\cos\beta\\ \lambda_0\end{pmatrix}_L. \tag{2.13}$$

Here α and β are two angles simply related to the angles appearing in $R_L^{\xi,\eta}$ and determined by the various Yukawa coupling constants in Eq. (2.1). Notice that we have succeeded in ensuring that each of the $\mathcal{P}, \mathcal{N}, \lambda$ quarks can be transformed into a massive quark by a gauge field, and at the same time in avoiding any unpleasant nonexotic $V+A$ currents.

We assume that the strong interaction consists of an Abelian gauge theory or, if one prefers, a non-Abelian gauge theory in which some hidden degrees of freedom ("color") are gauged. This assumption ensures many desirable properties, as was emphasized by Weinberg.[9] In particular if a non-Abelian gauge theory is assumed, we may legitimately calculate the quark mass shifts without taking strong interactions into account. Thus the strong interaction in this model is invariant[10] under G_s = [internal SU(3)] × [gauge SU(3)] × [chiral SU(3)]. The internal and chiral SU(3) are to be broken by the electromagnetic and weak interaction.

There is as yet no contribution to the process in Fig. 1, because with the form of $\langle\varphi\rangle$ given in Eq. (2.4), $W_L^\pm$ and $W_R^\pm$ are mass eigenstates.[11] To mix $W_L^\pm$ and $W_R^\pm$ we introduce two other sets of Higgs field: χ_1 transforming like a complex $(\frac{1}{2}, 0, \frac{1}{2})$, $y=0$ under G, and χ_2 transforming like a complex $(0, \frac{1}{2}, \frac{1}{2})$, $y=0$. We also introduce a complex doublet η which transforms as $\eta\to e^{i\theta_R\tau}e^{i\theta_y}\eta$. These Higgs fields perform an essential service by generating the appropriate mass spectrum for the gauge fields, but do not play any role in low-energy phenomenology since they are all forbidden to couple to the fermions by the gauge group G. The interactions of these Higgs fields with each other and with the gauge fields are considered in Appendix A. The details do not concern us here; suffice it to say that three positively charged gauge mesons emerge as eigenstates, to be denoted by W_β^+ (with masses μ_β^2, β = I, II, III). They are given by

$$W_{B\mu}^+=R_{B\beta}W_{\beta\mu}^+, \tag{2.14}$$

where $B=L,A,R$ and $R_{B\beta}$ is some three-by-three rotation matrix. We are finally prepared to switch on the weak interaction, to compute the quark mass shift, and to observe the splitting of the $\mathcal{N}_0$ and λ_0 degeneracy and the generation of the Cabibbo angle.

III. SPLITTING THE $\mathcal{N}$-λ DEGENERACY

It is clear that the photon and the three neutral massive gauge fields Z_β^μ (β = I, II, III) do not contribute to the mass shift of the $\mathcal{P}$, $\mathcal{N}$, and λ quarks. It suffices to consider in Fig. 1 the contribution of the three $W_{\beta\mu}^\pm$ mesons.

Furthermore, to lowest order in the weak and electromagnetic interaction, we need not be concerned with the mass shift of the (already) massive exotic quarks $\mathcal{P}'$, s, and r. It is not necessary to include the effects of the s quark in evaluating the effect of the perturbation on the degenerate $\mathcal{N}_0$-λ_0 system. Let us introduce the two-by-two mass matrix m for the $\mathcal{N}_0$-λ_0 system by saying that the perturbation produces an effective $\mathcal{L}$ which includes the term

$$\mathcal{L}_{\text{eff}}=-(\bar{\mathcal{N}}_0\ \ \bar{\lambda}_0)_L\, m\begin{pmatrix}\mathcal{N}_0\\ \lambda_0\end{pmatrix}_R+\text{H.c.}\,. \tag{3.1}$$

A straightforward calculation then gives

$$m_{\mathcal{P}}=\sin\alpha I_s, \tag{3.2}$$

$$m=\begin{pmatrix}\sin\alpha\cos\beta I_{\mathcal{P}'} & \sin\alpha\sin\beta I_r\\ -\sin\beta I_{\mathcal{P}'} & \cos\beta I_r\end{pmatrix}. \tag{3.3}$$

Here, for $i=s$, $\mathcal{P}'$, and r,

$$I_i=m_i\frac{g_Lg_R}{2\pi^2}\sum_\beta R_{L\beta}R_{R\beta}\times\int_0^1 d\alpha\ln\frac{\Lambda^2}{\mu_\beta^2+[\alpha^2/(1-\alpha)]m_i^2}, \tag{3.4}$$

where $g_{L,R}$ is the gauge coupling constant associated with $SU(2)_{L,R}$ and Λ is the Pauli-Villars cutoff used to compute the three Feynman diagrams contained in Fig. 1.

A noteworthy feature of the expression for I_i [Eq. (3.4)] is that I_i is independent of Λ^2, since $\sum_\beta R_{L\beta}R_{R\beta}\ln\Lambda^2=0$. This verifies the general conclusion from renormalization theory that corrections to natural symmetries in gauge theories are finite and calculable.

m is easily determined from Eqs. (2.10)–(2.13) by inspection. It is an asymmetric matrix which must now be brought to diagonal form.

IV. GENERATING A CABIBBO ANGLE

The matrix m determines for us which linear combination of $\mathcal{N}_{0R}$ and λ_{0R} and of $\mathcal{N}_{0L}$ and λ_{0L} in fact corresponds to the physical $\mathfrak{N}$ and λ quarks. To discover this we have to find two two-by-two rotation matrices R_L and R_R such that

$$R_L^T m R_R = \text{the diagonal matrix } \begin{pmatrix} m_{\mathfrak{N}} & 0 \\ 0 & m_\lambda \end{pmatrix}. \tag{4.1}$$

(Incidentally, all these rotations we are performing also ensure that there is no parity violation or strangeness nonconservation in order α.) The matrices R_L and R_R and the masses $m_{\mathfrak{N}}$ and m_λ are uniquely determined by m. The angle of rotation in R_L is of course the Cabibbo angle θ since

$$\mathfrak{N}_0 = \mathfrak{N}\cos\theta + \lambda\sin\theta .$$

(The angle of rotation in R_R, also determined here, is a sort of right-handed Cabibbo angle which governs the relative ratio of various as-yet-unobserved exotic pieces in the right-handed current such as $\overline{\mathcal{P}}'_R\gamma_\mu\mathfrak{N}_R$ and $\overline{\mathcal{P}}'_R\gamma_\mu\lambda_R$ but which may become of interest in the unpredictable future.)

Let us consider only the ratio $m_{\mathfrak{N}}/m_\lambda$. Solving Eq. (4.1), we would then find θ and $m_{\mathfrak{N}}/m_\lambda$ as functions of the three variables α, β, and $x \equiv I_{\mathcal{P}'}/I_r$. Tedious but straightforward arithmetic gives

$$\tan 2\theta = \sin\alpha \sin 2\beta\left(\frac{1-x^2}{Kx^2+L}\right), \tag{4.2}$$

$$\frac{m_{\mathfrak{N}}}{m_\lambda} = \frac{V\cos 2\theta - 1}{V\cos 2\theta + 1}. \tag{4.3}$$

To simplify our formulas we have introduced the notation

$$V \equiv \frac{Px^2 + 2\sin\alpha\, x + Q}{Kx^2 + L}, \tag{4.4}$$

$$K \equiv \sin^2\beta - \sin^2\alpha\cos^2\beta, \tag{4.5}$$

$$L \equiv \cos^2\beta - \sin^2\alpha\sin^2\beta, \tag{4.6}$$

$$P \equiv \sin^2\beta + \sin^2\alpha\cos^2\beta, \tag{4.7}$$

$$Q \equiv \cos^2\beta + \sin^2\alpha\sin^2\beta. \tag{4.8}$$

Equations (4.2) and (4.3) represent our main results. Unfortunately, the angle θ is given in terms of a number of experimentally unknown quantities. Note that α and β are also Cabibbo-like angles which govern the ratio of various exotic pieces in the left-handed current [Eqs. (2.11)–(2.13)]. To sum up, we have determined $m_{\mathfrak{N}}/m_\lambda$, $m_{\mathcal{P}}/m_\lambda$, and two of the four Cabibbo-like angles in the theory in terms of the other two angles and m_s/m_r, $m_{\mathcal{P}'}/m_r$, unfortunately all unknown at present. (Typically, $\mu_B{}^2 \gg m_i{}^2$, so that $I_i \simeq \text{const} \times m_i$.)

The ratio $m_{\mathfrak{N}}/m_\lambda$ is a fundamental quantity governing the amount of chiral-symmetry breaking. In the standard $(3,\bar{3})+(\bar{3},3)$ model[12] of symmetry breaking

$$\mathcal{L}_{SB} = u_0 + \epsilon_8 u_8 + \epsilon_3 u_3 \tag{4.9}$$

the deviation of ϵ_8 from $-\sqrt{2}$ (the value corresponding to $m_\pi{}^2 = 0$ and exact chiral symmetry) is related to $m_{\mathfrak{N}}/m_\lambda$. The result usually quoted[13] is that $\epsilon_8 \simeq -1.25$ and $\epsilon_3 \simeq -0.02$, which corresponds to $m_{\mathfrak{N}}/m_\lambda \sim \frac{1}{30}$ and $m_{\mathcal{P}}/m_{\mathfrak{N}} \sim 0.4$. (The reader will recall that the determination of these numbers suffers from a great deal of uncertainties.)

V. DISCUSSION AND A BRIEF ANALYSIS

Although θ is not determined, one may consider undertaking the following analysis: Take $m_{\mathfrak{N}}/m_\lambda$ as given (to be, say, $\sim\frac{1}{30}$) and eliminate x between Eq. (4.2) and Eq. (4.3) so that $\theta = \theta(\alpha, \beta)$. Letting α and β run through their natural domain we find that θ is in fact bounded. Such an analysis[14] turns out to be quite complicated and is probably not warranted by our model. We shall opt for a less systematic analysis.

First, let us consider some special limits. For instance, let $\alpha \to 0$. In this limit[15] we find that $\theta \to 0$ and $m_{\mathfrak{N}}/m_\lambda \to 0$, $m_{\mathcal{P}}/m_\lambda \to 0$. This is encouraging since experimentally θ and $m_{\mathfrak{N}}/m_\lambda$ are both small. Referring back to Eqs. (2.11)–(2.13), we see that this situation is not at all surprising. As $\alpha \to 0$, $\mathcal{P}_L$ and $\mathfrak{N}_{0L}$ find themselves in the same multiplet. As a result, neither of them can become massive, and $\mathfrak{N}_{0L} = \mathfrak{N}_L$, giving $\theta = 0$. This conclusion, that in *the limit of vanishing Cabibbo angle chiral symmetry becomes exact*, in fact appears to be generally true, as long as the nonexotic quarks are assigned to *doublet* representations of SU(2) and acquire masses from radiative corrections. We have explicitly verified this in a number of models.[16] A moment's thought will convince the reader of the plausibility of this theorem.

On the other hand, it is rather easy to construct models in which $\mathcal{P}$ and $\mathfrak{N}$ are assigned to larger multiplets together with other quarks and in which the "theorem" is not true. We would like to conjecture that in the real world the doublet representation of SU(2) plays an important role and that this "theorem" is in fact true.[17]

There is a suggestive argument that α may be small. So far we have not said anything about the lepton sector. Clearly, the lepton sector will influence the hadron sector if lepton-hadron universality is to be maintained. The simplest theory of leptons known is that of Weinberg and Salam.[18] In this model, we have

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \text{ and } \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$$

transforming like $(\frac{1}{2}, 0, 0)$, e_R^-, μ_R^- as $(0,0,0)$. Define $\delta \equiv 1 - G_\beta/G_\mu$ = the deviation of the rates of β decay and μ decay from equality. In our theory

$\delta = 1 - \cos\alpha \cos\theta$ and so α must be small. The determination of δ suffers from severe uncertainties stemming from theoretical difficulty in calculating the radiative correction to β decay. A typical value quoted[19] is $\delta \sim 2\%$. The uncertainty in θ as determined from K decay $(0.21 \lesssim \theta \lesssim 0.27)$ certainly allows a value of α as big as 0.1, say.

Let us proceed with the assumption that α is indeed small and ~0.1. With $\alpha \sim 0.1$, the quantities K and L defined in Eqs. (4.5) and (4.6) are both positive unless the angle β is very small $(\beta \lesssim 0.1)$ or very close to $\pi/2$ $[(\pi/2 - \beta) \lesssim 0.1]$. We now refer back to Eq. (4.2),

$$\tan 2\theta = \sin\alpha \sin 2\beta \left(\frac{1 - x^2}{Kx^2 + L} \right),$$

and note that the factor $\eta(x) \equiv (1 - x^2)/(Kx^2 + L)$ is bounded above and below:

$$-\frac{1}{K} = \eta(\infty) \leq \eta(x) \leq \eta(0) = \frac{1}{L}. \tag{5.1}$$

Since $K \simeq \sin^2\beta$ and $L \simeq \cos^2\beta$, to a good approximation $\tan 2\theta$ is bounded (given the various assumptions we have made):

$$-2\alpha \cot\beta \leq \tan 2\theta \leq 2\alpha \tan\beta. \tag{5.2}$$

With $\alpha \sim 0.1$ the bounds are very stringent unless either $\cot\beta$ or $\tan\beta$ is large, of the order ~2.5. For the purpose of illustration and in order to get a feeling for the equations governing θ and $m_{\mathfrak{N}}/m_\lambda$, we will choose $\tan\beta \sim 2$ or 3, which gives $\beta \simeq \frac{1}{2}\pi - 0.33$. With this choice $\tan 2\theta = 0.066\eta(x)$, which forces $\eta(x)$ to be large (~10) if θ is to have roughly the right value. This forces $x \simeq m_{\mathcal{P}'}/m_r$ to be small. For illustrations some typical numbers are as follows: $x = 0.14$ implies $\sin\theta \sim 0.24$ and $m_{\mathfrak{N}}/m_\lambda \sim \frac{1}{9}$, and $x = 0.1$ implies $\sin\theta \sim 0.26$ and $m_{\mathfrak{N}}/m_\lambda \sim \frac{1}{12}$. The values for $m_{\mathfrak{N}}/m_\lambda$ that emerge are perhaps a factor of $\frac{1}{3}$ too small, but the purpose of this (very rough) analysis is not to fit "data" (which is certainly possible). Rather, we have illustrated how it is possible that the small value of θ is correlated with small chiral symmetry breaking.

VI. COMMENTS

(1) The mass acquired by the $\mathcal{P}$ quark is given in Eq. (3.2). Taking the determinant of Eq. (4.1) we find that

$$m_{\mathfrak{N}} m_\lambda = \sin\alpha\, I_{\mathcal{P}'} I_r. \tag{6.1}$$

We thus obtain a simple relation between the masses of the exotic quarks and the masses of the nonexotic quarks:

$$(m_{\mathcal{P}}^2/m_{\mathfrak{N}} m_\lambda) = \sin\alpha (m_s^2/m_{\mathcal{P}'} m_r). \tag{6.2}$$

With α small and ~0.1 we see that we can fix $(m_{\mathcal{P}}/m_{\mathfrak{N}})$ to be $\sim \frac{1}{2}$ if $(m_s/m_{\mathcal{P}'})$ is of order 1. Hence (with the various choices we have made) a hierarchy of quark masses emerges in this model, namely, that

$$m_{\mathcal{P}} \lesssim m_{\mathfrak{N}} \ll m_\lambda \ll m_{\mathcal{P}'} \lesssim m_s \ll m_r. \tag{6.3}$$

(2) Perhaps one unsatisfactory feature of this model is that if we take $g_L \sim g_R \sim e$ we have $m_{\mathcal{P}} \sim \alpha m_s$, $m_{\mathfrak{N}}, m_\lambda \sim \alpha m_{\mathcal{P}'}$, αm_r (α is the fine-structure constant). μ_β will typically range from ~30 GeV to ~100 GeV. The exotic quark masses $m_s, m_{\mathcal{P}'}, m_r$ may range from anywhere up to ~10 GeV or ~30 GeV, so that $(m^2)_{\text{exotic quark}}/\mu_\beta^2 \ll 1$. The nonexotic quark masses will then be anywhere up to ~100 MeV or ~300 MeV. Since we do not know a reliable way to relate quark masses to hadron masses, we feel that the only reliable measures are the ratios $m_{\mathfrak{N}}/m_\lambda$ and $m_{\mathcal{P}}/m_{\mathfrak{N}}$ which are determined by current algebra. Some readers may be perturbed by the fact that the proton-neutron mass difference is due to electromagnetism and to weak interaction (via u_3), while the nuclear mass itself comes from the weak interaction. This need not be an inconsistency, of course, since no one has exhibited a connection between nucleon mass and quark masses. Indeed, if the quarks are not physical particles then only the group-theoretic pattern $u_0 + \epsilon_3 u_3 + \epsilon_8 u_8$ governed by the ratio of their (unphysical) Lagrangian masses is relevant. Gell-Mann[20] has postulated the existence of a scale-breaking but chiral-symmetric term in the Lagrangian. Perhaps some such term is necessary here.

(3) The sign of the Cabibbo angle in this model may be changed by making a γ_5 transformation: $\lambda_L \to -\lambda_L$ and $\lambda_R \to -\lambda_R$. The γ_5 transformation also changes the right-handed current and the exotic pieces of the left-handed current.

ACKNOWLEDGMENTS

It is a pleasure to thank H. Georgi for many stimulating discussions and S. Treiman for his interest. Part of this work was performed at the Brookhaven National Laboratory. We thank R. Peierls for his hospitality.

APPENDIX A: THE HIGGS POTENTIAL

We have four sets of gauge fields: φ, χ_1, χ_2, and η. The Higgs potential V consists of all the gauge-invariant terms that can be formed out of the Higgs fields consistent with the discrete symmetries we impose. We shall impose three independent discrete symmetries: K, K_1, and K_2. Under K, $\varphi \to i\varphi$. Under K_1, $\chi_1 \to i\chi_1$. Under K_2, $\chi_2 \to i\chi_2$. We decompose V as follows: $V = V_0 + \tilde{V} + V' + V_\eta$. V_0 contains 15 terms, viz.,

$$V_0 = c_1 \mathrm{tr}\varphi^\dagger\varphi + c_2 \mathrm{tr}\chi_1^\dagger\chi_1 + c_3 \mathrm{tr}\chi_2^\dagger\chi_2 + c_4[\mathrm{tr}(\varphi^\dagger\varphi)]^2 + c_5(\mathrm{tr}\chi_1^\dagger\chi_1)^2 + c_6(\mathrm{tr}\chi_2^\dagger\chi_2)^2 + c_7 \mathrm{tr}\varphi^\dagger\varphi\,\mathrm{tr}\chi_1^\dagger\chi_1$$
$$+ c_8 \mathrm{tr}\varphi^\dagger\varphi\,\mathrm{tr}\chi_2^\dagger\chi_2 + c_9 \mathrm{tr}\chi_1^\dagger\chi_1\,\mathrm{tr}\chi_2^\dagger\chi_2 + c_{10}\mathrm{tr}\varphi^\dagger\varphi\varphi^\dagger\varphi + c_{11}\mathrm{tr}\chi_1^\dagger\chi_1\chi_1^\dagger\chi_1 + c_{12}\mathrm{tr}\chi_2^\dagger\chi_2\chi_2^\dagger\chi_2$$
$$+ c_{13}\mathrm{tr}\varphi^\dagger\varphi\chi_2^\dagger\chi_2 + c_{14}\mathrm{tr}\varphi\varphi^\dagger\chi_1\chi_1^\dagger + c_{15}\mathrm{tr}\chi_1^\dagger\chi_1\chi_2\chi_2^\dagger\,. \quad \text{(A1)}$$

$\tilde{V}$ consists of those terms gotten by putting tildes on the terms in V_0. Recall that $\tilde{\chi}_1 = \tau_2\chi_1^*\tau_2$, $\tilde{\chi}_2 = \tau_2\chi_2^*\tau_2$, $\tilde{\varphi} = \tau_2\varphi^*\tau_2$. Because of K, K_1, K_2, each term in $\tilde{V}$ can display only even number of tildes. V' contains terms like $\mathrm{tr}\,\varphi^\dagger\tau^a\varphi\,\mathrm{tr}\chi_1^\dagger\tau_a\chi_1$ and $\mathrm{tr}\tau_b\,\varphi^\dagger\tau_a\varphi\,\mathrm{tr}\tau_b\,\varphi^\dagger\tau_a\varphi$ (plus terms gotten from them by adding tildes) which are allowed because we use complex $(\frac{1}{2}, \frac{1}{2})$ representations. Finally, V_η contains terms involving η. A number of possible terms are ruled out by imposing CP invariance.

In view of the large number of terms in V, a simplifying assumption is needed. The standard assumption is that given the charge Q (in this case $Q = T_{3L} + T_{3R} + T_{3A} + \frac{1}{2}y$) only those fields neutral under Q develop vacuum expectation values. It is clearly necessary in order to have light in the theory. One can easily see that the point at which all $Q \neq 0$ fields have vanishing vacuum expectation value is an extremum of the Higgs potential. It has been implicitly conjectured by workers[21] in gauge theory that this extremum may be made into a minimum by restricting the parameters in V to suitable domains. This is plausible because of the large number of parameters.

We can thus write, without loss of generality,

$$\langle\chi_1\rangle = e^{i\varphi_1}\begin{pmatrix} a & 0 \\ 0 & b\end{pmatrix}, \quad \text{(A2)}$$

$$\langle\chi_2\rangle = e^{i\varphi_2}\begin{pmatrix} \alpha & 0 \\ 0 & \beta\end{pmatrix}, \quad \text{(A3)}$$

$$\langle\varphi\rangle = \begin{pmatrix} ve^{i\varphi_v} & 0 \\ 0 & ze^{i\varphi_z}\end{pmatrix}, \quad \text{(A4)}$$

$$\langle\eta\rangle = \begin{pmatrix} we^{i\varphi_w} \\ h\end{pmatrix}, \quad \text{(A5)}$$

where $a, b, \alpha, \beta, v, z, w, h$ are all real. The potential is a quartic polynomial in $a, b, \alpha, \beta, v, z, w, h$ and cosines of various combinations of the phase angles. We assume that we cannot, with no loss of generality, look at those terms not depending on v, z when determining the value of v and z at the minimum of V. We thus consider the prototypical potential

$$V_p = f_0 z^2 + f_1(v^2+z^2) + f_2(v^2+z^2)^2 + f_3 v^2 z^2\,, \quad \text{(A6)}$$

where f_i are themselves functions of $a, b, \alpha, \beta, w, h$, and the phase angles. It is easy to see that by suitably adjusting the f_i's the minimum of V_p occurs for $z=0$ and $v^2 = -f_1/2f_2$.

Similarly, we consider

$$V_p' = f_0' b^2 + f_1'(a^2+b^2) + f_2'(a^2+b^2)^2 + f_3' a^2 b^2\,. \quad \text{(A7)}$$

By adjusting f_i' one may arrange the minimum of V_p' to occur for $a \neq 0$ and $b \neq 0$. The same remarks apply to α and β. It is also presumably possible to adjust things so that $w=0$. We defer a discussion of the phase angles to Appendix B.

With this set of vacuum expectation values, the mass spectrum discussed in the text is readily generated.

APPENDIX B

The various phase angles are intimately connected to possible CP violation. We require that our Lagrangian be invariant under the CP transformation which sends $\varphi \to \varphi^*$, etc. This is equivalent to changing the sign of the phase angles $\varphi_v, \varphi_z, \varphi_1, \varphi_2, \varphi_w$. Thus the Higgs potential is an even function of $\varphi_v, \varphi_z, \varphi_1, \varphi_2, \varphi_w$ for small phase angles. The point at which all these phase angles vanish is evidently an extremum and presumably may be made into a minimum of the potential. In that case CP invariance of the theory will be ensured.

The following phenomenon arises here because of the large number of Higgs fields. Clearly $\varphi_z{}^2$ always occurs in combination with z^2. In the tree approximation we have arranged things so that z vanishes. Thus φ_z is undetermined by minimizing the potential in the tree approximation. One would then be obliged to calculate the potential beyond[22] the tree approximation. The potential in some higher-loop approximation is minimized at a point at which $z \neq 0$. The question of whether or not $\varphi_z = 0$ is a minimum and consequently of whether or not CP is a good symmetry will then, and only then, be decided. (In this model, it appears that by a simple choice of the sign of a certain coupling constant in the Higgs Lagrangian we can ensure that the point $\varphi_z = 0$ is a minimum.)

So far, the gauge theory of weak interactions has not clarified the origin of CP violation. We would like to take this opportunity to list the various options[23] one has regarding the inclusion of CP violation in a gauge model. Firstly, one may simply write down a CP-noninvariant Lagrangian. This is illustrated, for example, by the model of Mohapatra[24] and the model of Pais.[25] A more interesting possibility is that the Lagrangian is in fact CP-invariant. Theories in which this is so may be subdivided into four types:

Type I. When one minimizes the potential in the tree approximation, one finds that the minimum is not invariant under CP. An example is the model of Lee.[26]

Type II. The minimum of the potential in the tree approximation does conserve CP. Furthermore, the minimum of the exact potential is also guaranteed to conserve CP. An example is the original model of Weinberg.[18] One particular way in which this phenomenon can come about is illustrated by Weinberg's model: The theory is so simple that it admits of a rotation which ensures that no phase angle can appear.

Type III. As in Type II, the minimum of the potential in the tree approximation respects CP. However, when the potential is computed including loops the minimum no longer respects CP.

Type IV. Finally, there is the interesting possibility that one does not know whether the minimum of the potential in the tree approximation respects CP or not. The question has to be decided by computing the potential to include loops. The model presented in this paper is an example of this type. It is unfortunately, in same sense, a trivial one since one apparently may ensure the absence of CP violation by the simple expedient of choosing the sign of a certain coupling constant.

In conclusion, we would like to emphasize the following point. Theories of Type II are characterized by the small number of Higgs fields and consequently by a freedom to utilize the gauge group to rotate any phase angle away. On the other hand, if one constructs a model involving many Higgs fields and if one labors to ensure that the minimum of the potential in the tree approximation respects CP, then more likely than not the model would be of Types III and IV and may generate CP violation in higher order. This seems to us an attractive possibility in trying to explain CP violation, particularly if one subscribes to the philosophy that Higgs fields are not fundamental fields but are dynamical manifestations.

---

*Research supported by the National Science Foundation under Grant No. GP 30738X.

†Alfred P. Sloan Foundation Fellow.

[1]N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

[2]S. Weinberg, Phys. Rev. Lett. 29, 388 (1972); 29, 1698 (1972); Phys. Rev. D 7, 2887 (1973). Some of the points discussed by Weinberg had also been noted by other authors, e.g., H. Georgi and S. Glashow, *ibid*, 6, 2977 (1972); R. N. Mohapatra and P. Vinciarelli, *ibid*. 8, 481 (1973); G. 't Hooft, Nucl. Phys. B35, 167 (1971).

[3]S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[4]P. W. Higgs, Phys. Lett. 12, 132 (1964).

[5]We only consider the contribution pictured in Fig. 1, assuming that the contributions from Higgs fields are suppressed.

[6]The calculation we will describe clearly bears some resemblance to the μ/e calculation of H. Georgi and S. Glashow [Phys. Rev. D 7, 2457 (1973)]. However, the dissimilarities are important. For the μ/e problem one must guarantee that the neutrinos remain massless. Here, no massless particles may remain. The selection rules to be built in are also vastly different.

[7]M. A. B. Bég and A. Zee, Phys. Rev. D 8, 1460 (1973); D. Bailin, A. Love, D. V. Nanopoulos, and G. G. Ross, Rutherford Laboratory report (unpublished).

[8]We assume in this paper that all observed hadrons are made up of the nonexotic quarks $\mathcal{P}$, $\mathfrak{N}$, λ.

[9]S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); Phys. Rev. D 8, 4482 (1973).

[10]The problem of the unwanted conserved axial-vector baryonic current, which plagues Lagrangian models of quarks and vector gluons, is of course also present here.

[11]The same phenomenon occurs in a calculation of the electron-muon mass ratio (Ref. 6).

[12]M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968).

[13]For example, M. Gell-Mann, lectures delivered at the Summer School of Theoretical Physics, University of Hawaii [Caltech Report No. CALT-68-224 (unpublished)].

[14]This analysis was suggested to us by S. Treiman.

[15]Another interesting limit is the limit $x\rightarrow 1$ ($m_{\mathcal{P}'}\simeq m_r$) in which case $\tan 2\theta \rightarrow 0$ and $m_{\mathfrak{N}}/m_\lambda \rightarrow \sin\alpha$. This result follows since in the limit $x=0$ Φ' and r become degenerate.

[16]H. Georgi and the author have considered a number of models in which $m_{\mathcal{P}}=m_{\mathfrak{N}}=0$ and $m_\lambda \neq 0$ (unpublished work). The theorem also holds in these models as long as one uses SU(2) doublets.

[17]There is a calculation some years ago of θ based on an (arbitrary) ansatz about the leading quadratic divergences appearing in quark mass shifts [R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. 28B, 128 (1968); N. Cabibbo and L. Maiani, *ibid*. 28B, 131 (1968)]. The "theorem" also holds for this calculation.

[18]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No. 8), edited by N. Svartholm (Almqvist, Stockholm, 1968), p. 367.

[19]For example, T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. 15, 381 (1965). For a recent discussion of the determination of G_β from data and for a list of earlier references, see M. A. B. Bég, J. Bernstein, and A. Sirlin, Phys. Rev. D 6, 2597 (1973), and A. Sirlin, NYU Report No. NYU/TR2/73 (unpublished).

[20]M. Gell-Mann, Ref. 13.
[21]We thank H. Georgi for discussions on this point. The conjecture was enunciated by him in conversation with the author.
[22]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[23]We thank Professor S. Treiman for a clarifying discussion of this point.
[24]R. Mohapatra, Phys. Rev. D 6, 2023 (1972).
[25]A. Pais, Phys. Rev. Lett. 29, 1719 (1972); 30, 114(E) (1973).
[26]T. D. Lee, Phys. Rev. D 8, 1226 (1973).

PHYSICAL REVIEW D VOLUME 9, NUMBER 6 15 MARCH 1974

Electromagnetic mass difference of vector mesons*

Laurie M. Brown[†]
Instituto de Física, Universidade de São Paulo, São Paulo 8.219, Brazil

J. D. Mich
Northwestern University, Evanston, Illinois 60201
(Received 31 July 1972; revised manuscript received 12 June 1973)

We discuss an effective Lagrangian model of vector mesons interacting with themselves and photons, which yields finite lowest-order dynamical electromagnetic corrections to the vector-meson masses. The strong-interaction part of the Lagrangian is given by a renormalizable model of 't Hooft, based on massless Yang-Mills fields which acquire mass by the Higgs-Kibble mechanism. The relationship of this vector field to the usual massive Yang-Mills field and of our electromagnetic interaction to that of Kroll, Lee, and Zumino is discussed in the light of a suggestion made by Lee and Zinn-Justin that the renormalizable vector field may be a regularized version of the usual massive vector field. The mass shifts of the charged and neutral ρ mesons are found to be small and nearly equal, well within present experimental uncertainties in the ρ-meson masses.

I. INTRODUCTION

Theories based on vector-meson dominance of the hadronic electromagnetic form factors have yielded finite electromagnetic mass differences for spinless mesons,[1] but generally suffer from the serious defect that the radiative corrections to the vector-meson masses themselves diverge.[2] Recently, renormalizable Lagrangian models have been proposed, based on massless Yang-Mills fields which acquire a mass as a consequence of spontaneous symmetry breaking.[3–5] In such models the free-field propagators of the gauge fields are more convergent than in the Kroll-Lee-Zumino (KLZ) model[6] of vector-meson dominance, and may lead to less divergent or finite electromagnetic mass differences.

For the Lagrangian of strong interactions we shall make use of a model proposed by 't Hooft and shown by him and Veltman to be renormalizable.[4] This model is based on a Lagrangian locally invariant under a group U(1)×SU(2), which we shall call $G^{(1)}$, and contains a triplet of massless vector fields $\vec{V}_\mu$ and a scalar field which is a complex spinor representation of $G^{(1)}$. The four Hermitian components of the scalar can be regarded as a singlet σ, which acquires a nonzero vacuum expectation value σ_0 as a consequence of spontaneous symmetry breaking, and a triplet $\vec{\psi}$, which contributes the longitudinal component of the vector field when the latter acquires mass, as explained by Higgs and Kibble.[7]

We wish to use the Lagrangian referred to as an effective Lagrangian, introducing electromagnetic interaction in a way analogous to KLZ. However, this raises some problems of physical interpretation which are connected with the presence of the auxiliary scalar fields introduced to implement the Higgs-Kibble mechanism. For example, the most convenient gauge is one in which the propagators of the $\vec{\psi}$, σ, and $\vec{V}_\mu$ fields are diagonalized and in which the vector propagator has a manifestly gauge-invariant form, but the last appears to have a pole at zero mass. Although it has been shown in similar cases that this pole is not really present in the renormalizable theory,[5] it is not obvious how to make this argument succeed if we limit ourselves to tree diagrams (in the strong-interaction sense), as in the present case. However, following the suggestion of Lee and Zinn-Justin that the scalar fields may be only a device to regularize the usual massive Yang-Mills theory, which may be renormalizable even though power-counting arguments suggest that it is not, we give a possible resolution to the problems of physical interpretation in Sec. III. This makes use of the