Spontaneous "Cabibbo" suppression*

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We demonstrate the possibility of a unified weak-electromagnetic gauge scheme in which the suppression of strangeness-changing weak amplitudes arises by spontaneous breakdown of an SU(3) symmetry.

The construction of gauge-type models[1] to explain the rich and varied structure of hadron weak reactions is an exciting task and has consequently attracted much attention. Most models have contained the following two features:

(a) acceptance of the Cabibbo[2] rotation as an initially present and hence unexplained effect,

(b) the classification of the fundamental hadrons according to structures different from the usual SU(3) which has been so successful in strong-interaction dynamics [of course these are related in that (a) tends to imply (b)].

Here we point out that a previously given[3] SU(3) $\times$ U(1) gauge model can be modified so as to avoid having to assume (a) and (b). Many variations on the present scheme can also be imagined.

The fundamental hadron fields are considered to include the left-handed usual quark triplet,

$$\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix},$$

while the two lepton triplets are

$$\tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} \nu_e \\ e \\ e' \end{pmatrix}, \quad \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} \nu_\mu \\ \mu \\ \mu' \end{pmatrix}.$$

Note that a (very) heavy electron and a (very) heavy muon are introduced for symmetry with the hadrons. All right-handed objects are taken to be singlets with respect to an SU(3) symmetry group. Other quark-triplet schemes can be similarly accommodated. To emphasize the general features of the model we define the left- and right-handed hadron currents

$$\begin{aligned} (j^l)^b_{a\alpha} &= i\bar{q}_b\gamma_\alpha(1+\gamma_5)q_a\,, \\ (j^r)^b_{a\alpha} &= i\bar{q}_b\gamma_\alpha(1-\gamma_5)q_a \end{aligned} \tag{1}$$

and the analogous lepton currents

$$\begin{aligned} l^b_{a\alpha} &= i\bar{\psi}_b\gamma_\alpha(1+\gamma_5)\psi_a + i\bar{\chi}_b\gamma_\alpha(1+\gamma_5)\chi_a\,, \\ r^b_{a\alpha} &= i\bar{\psi}_b\gamma_\alpha(1-\gamma_5)\psi_a + i\bar{\chi}_b\gamma_\alpha(1-\gamma_5)\chi_a\,, \end{aligned} \tag{2}$$

where ψ_a and χ_a are the electron and muon triplets. In (2), $r^b_{1\alpha} = r^1_{a\alpha} = 0$.

Our gauge symmetry group will involve a U(1) in addition to the SU(3) which acts on the above triplets. The nine vector gauge fields are

$$\begin{aligned} &\text{octet: } (W^b_a)_\alpha, \text{ with } (W^c_c)_\alpha = 0 \\ &\text{singlet: } D_\alpha\,. \end{aligned} \tag{3}$$

It is convenient to call the U-spin singlet member of the octet F_α and the uncharged U-spin triplet member H_α; specifically

$$\begin{aligned} F_\alpha &= -(\tfrac{3}{2})^{1/2}(W^1_1)_\alpha\,, \\ H_\alpha &= (1/\sqrt{2})[(W^2_2)_\alpha - (W^3_3)_\alpha]\,. \end{aligned} \tag{4}$$

The fields F_α and D_α will mix to give the photon A_α and a heavy neutral meson Z_α:

$$\begin{pmatrix} F_\alpha \\ D_\alpha \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} A_\alpha \\ Z_\alpha \end{pmatrix}, \tag{5}$$

where ϕ is a mixing angle *unrelated* to the Cabibbo angle. The (matter current)$\times$ (gauge field) part of the interaction Lagrangian density is

$$\begin{aligned} L = g[&l^b_{a\alpha} + (j^l)^b_{a\alpha}]W^a_{b\alpha} \\ &- g'[l^1_{1\alpha} + l^2_{2\alpha} + l^3_{3\alpha} + \tfrac{3}{2}r^2_{2\alpha} + \tfrac{3}{2}r^3_{3\alpha} \\ &\qquad - (j^r)^1_{1\alpha} + \tfrac{1}{2}(j^r)^2_{2\alpha} + \tfrac{1}{2}(j^r)^3_{3\alpha}]D_\alpha + \cdots\,, \end{aligned} \tag{6}$$

where g and g' are given by

$$g = \frac{-|e|}{\sqrt{6}\cos\phi}\,, \quad g' = (\tfrac{2}{3})^{1/2} g\cot\phi\,. \tag{7}$$

$|e|$ is the magnitude of the electric charge, while (6) and (7) correspond to the unique Yang-Mills scheme which [with the mixing of (5)] leads to the ordinary electromagnetic interaction. The terms multiplying g are independent of the charge assignments of the fundamental triplets, while the terms multiplying g' depend upon them.

As usual we take the observed weak interactions to result mainly from the exchange of heavy gauge bosons between two currents. Thus the resulting pattern of weak amplitudes as well as the symmetry structure of the theory depends on the gauge meson mass matrix. We arrange the mass terms in the Lagrangian density to include some mixing between $\Delta S=0$ and $|\Delta S|=1$ charged bosons, W_1^2 and W_1^3. The *simplest* form is

$$-m(W^2_{1\alpha}W^1_{2\alpha}+W^3_{1\alpha}W^1_{3\alpha})+p(W^2_{1\alpha}W^1_{3\alpha}+W^3_{1\alpha}W^1_{2\alpha}), \tag{8}$$

where m and p are positive constants. This form is diagonalized by defining the *physical* charged bosons through

$$\begin{aligned} W^2_{1\alpha}&=\frac{1}{\sqrt{2}}(-W'_\alpha+W_\alpha),\\ W^3_{1\alpha}&=\tfrac{1}{2}(W'_\alpha+W_\alpha), \end{aligned} \tag{9}$$

with squared masses

$$\begin{aligned} m^2(W)&=m-p,\\ m^2(W')&=m+p. \end{aligned} \tag{10}$$

Reference to (6) shows that the effective (second-order) Lagrangian density for the usual weak interactions is

$$\begin{aligned} L^w=\frac{G}{\sqrt{2}}\{&l^2_{1\alpha}l^1_{2\alpha}+[(j')^2_{1\alpha}l^1_{2\alpha}+\text{H.c.}]\\ &+r[(j')^3_{1\alpha}l^1_{2\alpha}+(j')^3_{1\alpha}(j')^1_{2\alpha}+\text{H.c.}]\}, \end{aligned} \tag{11}$$

where the Fermi constant G is expressed as

$$\frac{G}{\sqrt{2}}=\frac{g^2}{2}\left[\frac{1}{m^2(W)}+\frac{1}{m^2(W')}\right] \tag{12}$$

and the strangeness suppression factor r ($\simeq\frac{1}{4}$) is given by

$$r=\frac{p}{m}=\frac{m^2(W')-m^2(W)}{m^2(W')+m^2(W)}. \tag{13}$$

From (7), (12), and (13) we obtain

$$m^2(W)=\frac{\sqrt{2}\,e^2}{6G(1+r)\cos^2\phi}, \tag{14}$$

which establishes a lower bound for the lighter of the two charged bosons $m(W)\gtrsim 38.6$ GeV. All in all, (11) is the same as the Cabibbo scheme except that the $\Delta S=0$ semileptonic amplitudes[4] have a relative strength 1 instead of $\cos\theta_C\simeq 0.98$, θ_C being the Cabibbo angle. While the additional 2% suppression[5] has some experimental plausibility, it is a rather small effect and does involve the difficult estimate of electromagnetic corrections in addition to high-accuracy experiments. Considering the present state of flux of weak-interaction theory, it seems worthwhile to keep an open mind on this question.[6]

In addition to (11), the basic interaction (6) gives rise in second order to neutral-current effects. All of these satisfy $\Delta S=0$ because the gauge meson W^3_2 does not mix with any others. For $e\nu_e$ and $e\nu_\mu$ scattering the formulas are given in (2.20) of Ref. 3, except that we must replace $m^{-2}(W^2_1)$ by the average of $m^{-2}(W)$ and $m^{-2}(W')$. For the $\nu+$ hadron $\to\nu+$ hadron reactions of recent interest, we have the simple result

$$\begin{aligned} L_{\text{eff}}=&\frac{g^2}{\sin^2\phi\, m^2(Z)}\,i\bar{\nu}\gamma_\alpha(1+\gamma_5)\nu\\ &\times(-\cos2\phi\, j^{\text{em}}_\alpha+j^{\text{em}}_{5\alpha}), \end{aligned}$$

where j^{em}_α is the hadron electromagnetic current and $j^{\text{em}}_{5\alpha}$ is obtained by replacing γ_α by $\gamma_\alpha\gamma_5$ in j^{em}_α.

Furthermore (6) contains terms which describe the interactions of the heavy leptons.

To implement the spontaneous-breakdown mechanism we may introduce a set of auxiliary scalar fields whose nonvanishing vacuum expectation values result in nonzero masses for the gauge mesons. Achieving an arbitrary vector-meson mass matrix in this way is a nontrivial problem.[7] For the present case the simplest solution is to add an octet (ϕ^b_a with $\phi^c_c=0$) and a complex sextet ($f_{ab}=f_{ba}$). We thus include in the Lagrangian density the following SU(3)$\times$U(1)-invariant forms:

$$-\tfrac{1}{2}\mathfrak{D}_\mu\phi^b_a\,\mathfrak{D}_\mu\phi^a_b-(\mathfrak{D}_\mu f_{ab})^*\mathfrak{D}_\mu f_{ab}, \tag{15}$$

where the gauge-covariant derivatives are

$$\begin{aligned} \mathfrak{D}_\mu\phi^b_a&=\partial_\mu\phi^b_a+2ig(\phi^c_a W^b_{c\mu}-W^c_{a\mu}\phi^b_c),\\ \mathfrak{D}_\mu f_{ab}&=\partial_\mu f_{ab}-2ig'D_\mu f_{ab}\\ &\quad-2ig(W^c_{a\mu}f_{cb}+W^c_{b\mu}f_{ca}). \end{aligned} \tag{16}$$

The vector-meson mass terms to lowest order are obtained by setting $\phi^b_a=\langle\phi^b_a\rangle_0$ and $f_{ab}=\langle f_{ab}\rangle_0$ in (15) and (16) and keeping terms bilinear in $W^b_{a\alpha}$ and D_α. We choose[8]

$$\begin{aligned} &\langle f_{22}\rangle_0=\langle f_{33}\rangle_0=K/\sqrt{2},\\ &\langle\phi^1_1\rangle_0=-2\langle\phi^2_2\rangle_0=-2\langle\phi^3_3\rangle_0=C,\\ &\langle\phi^3_2\rangle_0=K, \end{aligned} \tag{17}$$

where K and C are two positive constants. The others (except for complex conjugates) are taken to be zero. The gauge-field mass terms in L then become

$$\begin{aligned} &-g^2(8K^2+9C^2)(W^2_{1\alpha}W^1_{2\alpha}+W^3_{1\alpha}W^1_{3\alpha})\\ &\quad+12g^2CK(W^2_{1\alpha}W^1_{3\alpha}+\text{H.c.})\\ &\quad-16g^2K^2W^3_{2\alpha}W^2_{3\alpha}\\ &\qquad-16g^2K^2H_\alpha H_\alpha-4K^2[g'D_\alpha-g(\tfrac{2}{3})^{1/2}F_\alpha]^2. \end{aligned} \tag{18}$$

The choice (17) guarantees that no $|\Delta S|=2$ terms appear in (18). From (18) we may solve for all the masses in terms of $m^2(W)$ and r [see (13)]. The result is

$$\begin{aligned} m^2(W') &= \frac{1+r}{1-r} m^2(W), \\ m^2(A) &= 0, \\ m^2(W_2^3) &= \frac{1 \pm (1-2r^2)^{1/2}}{1-r} m^2(W), \\ m^2(H) &= 6\sin^2\phi m^2(Z) = 2m^2(W_2^3), \end{aligned} \tag{19}$$

where ϕ is given in (14). The $\pm$ sign corresponds to the two possibilities in $K/C = (3/4r)[1 \pm (1-2r^2)^{1/2}]$. Lower bounds are obtained on these masses from (14) and (19):

$$\begin{aligned} m(W') &\gtrsim 50.0 \text{ GeV}, \\ m(W_2^3) &\gtrsim 60.9 \ (10.8) \text{ GeV}, \\ m(H) &\gtrsim 86.2 \ (21.6) \text{ GeV}, \\ m(Z) &\gtrsim 70.5 \ (20.0) \text{ GeV}, \end{aligned}$$

where the values in parentheses correspond to taking the minus sign in (19).

Thus the main features have been outlined for a rather straightforward type of weak-electromagnetic gauge scheme which is based on the usual (left-handed) SU(3) symmetry group of the hadrons. As a bonus the suppression of $|\Delta S| = 1$ amplitudes occurs by a spontaneous-breakdown mechanism. Note that this suppression mechanism does not affect the validity of Gell-Mann's SU(3) $\times$SU(3) current algebra. Hence it differs essentially from early attempts[9] to explain the suppression as a strong SU(3) violation [of order $m(\pi)/m(K)$] of the strangeness-changing current's matrix elements.

We conclude with a brief discussion of some technical points. The present theory should be renormalizable[10] modulo anomalies.[11] Note that renormalizable Yukawa couplings between the auxiliary scalar fields (sextet and octet) and the basic fermions of the theory cannot be constructed. Hence if we wish the fermion masses to already arise in lowest order, we may add, for example, three scalar triplets to the theory. This would modify the gauge meson masses and mixings in a nonessential way. Further results on this model will be reported elsewhere.

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[1]See, for example, H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972); J. Prentki and B. Zumino, Nucl. Phys. B47, 99 (1972); B. W. Lee, J. R. Primack, and S. B. Treiman, Phys. Rev. D 7, 510 (1973); C. Itoh, T. Minamikawa, K. Miura, and T. Watanabe, Asia Univ. (Tokyo) report (unpublished); J. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973); A. Pais, *ibid.* 8, 625 (1973); I. Bars, M. Halpern, and M. Yoshimura, *ibid.* 7, 1233 (1973); T. P. Cheng, *ibid.* 8, 496 (1973); N. Chang, and E. Ma, *ibid.* 7, 3808 (1973); T. C. Yang, Nucl. Phys. B58, 283 (1973); M. A. B. Bég and A. Zee, Phys. Rev. Lett. 30, 675 (1973).

[2]N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

[3]J. Schechter and Y. Ueda, Phys. Rev. D 8, 484 (1973).

[4]A different model leading to this result has been proposed by J. Schwinger, Phys. Rev. D 8, 960 (1973).

[5]Discussion is given by R. Marshak, Riazuddin, and C. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969).

[6]Also, in unified weak-electromagnetic gauge schemes, we expect additional corrections to the "current-current" weak interaction.

[7]For example, the model of Ref. 3 appears to require, on the basis of unpublished work done in collaboration with A. P. Balachandran and Y. Ueda, an extremely large number of auxiliary scalar particles. Some general discussion of the problem has been given by J. Lieberman, Phys. Rev. D 8, 2545 (1973).

[8]The *simplest* quartic (i.e., renormalizable) polynomial function whose stable minimum occurs when (17) is satisfied is

$$V = A_1\left[I_1 - \frac{1}{2K^2}(I_1)^2\right] + A_2\left[I_2 - \frac{1}{3C^2 + 4K^2}(I_2)^2\right],$$

where A_1 and A_2 are real constants and $I_1 = (f_{ab})^* f_{ab}$, $I_2 = \phi_a^b \phi_b^a$. However, this choice leads to some pseudo-Goldstone bosons or Higgs particles which will only receive mass beyond zeroth order. See S. Weinberg, Phys. Rev. D 7, 2887 (1973); 8, 605 (1973).

[9]These attempts appear to contradict the Adler-Weisberger-type sum rules for $|\Delta S| = 1$ axial-vector currents which follow from SU(3) $\times$SU(3) algebra. A review of the situation is given in Ref. 5.

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[11]C. Bouchiat, J. Iliopoulos, and P. Meyer, Phys. Lett. 38B, 519 (1972); H. Georgi and S. L. Glashow, Phys. Rev. D 6, 429 (1972); D. Gross and R. Jackiw, *ibid.* 6, 477 (1972).