*Work supported in part by the National Science Foundation.
[1]R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. G. Dunham (Interscience, New York, 1959), Vol. I, p. 315.
[2]For example, there is a discussion in L. I. Schiff, *Quantum Mechanics*, 3rd edition (McGraw-Hill, New York, 1968), p. 339.
[3]F. W. Byron, C. J. Joachain, and E. H. Mund, Phys. Rev. D 8, 2622 (1973).
[4]R. Blankenbecler and M. L. Goldberger, Phys. Rev. 126, 766 (1962).
[5]H. Abarbanel and C. Itzykson, Phys. Rev. Lett. 23, 53 (1968).
[6]M. Lévy and J. Sucher, Phys. Rev. 186, 1656 (1969).
[7]S. J. Wallace, Phys. Rev. Lett. 27, 622 (1971); Ann. Phys. (N.Y.) 78, 190 (1973); Phys. Rev. D 8, 1846 (1973).
[8]The authors of Ref. 3 anticipate this result on the basis of a careful analysis of the first few terms in the Born and eikonal series. R. J. Moore [Phys. Rev. D 2, 313 (1970)] has a proof that the Glauber eikonal is the high-energy limit of the Born series. His proof is valid only for Yukawa potentials and for large, but finite, values of the momentum transfer. Moreover, since his proof starts from the Glauber eikonal, it in no way demonstrates uniqueness.
[9]R. Eden, P. Landshoff, D. Olive, and J. Polkinghorne, *The Analytic S-Matrix* (Cambridge University Press, New York, 1966), Chap. 3; A. R. Swift, J. Math. Phys. 8, 2420 (1967).
[10]A detailed discussion of the properties of f_n, g_n, and Δ_n can be found in A. R. Swift, Phys. Rev. 176, 1848 (1968).

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Proton-neutron mass difference and the pion mass in a gauge model

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In a variation of the Weinberg SU(2)×SU(2)×U(1) gauge model of the weak and electromagnetic interactions, we study the proton-neutron mass difference, which is calculable, and investigate the appearance of pions as part of the Higgs system. We find that the proton-neutron mass difference is a function of the way in which the symmetry is broken. We exhibit a possible symmetry breaking which produces the correct sign for the mass difference. In the Higgs sector, we have a mass-degenerate pseudoscalar triplet which interacts with nucleons as pions do in the SU(2)×SU(2) σ model. Therefore we identify this triplet with pions. They are massive in zeroth order, but we can calculate the mass difference δm^2. We find that δm^2 is of order $\alpha\mu^2$ which is too large. If we impose a reflection symmetry on the Lagrangian, the symmetry group of the potential is enlarged and we find that the theory contains three pseudo-Goldstone bosons. These are the pion triplet, which are now massless in zeroth order. When we calculate the pion mass in the one-loop approximation, the Π^0 remains massless while the charged pions pick up mass of order $m^2 \sim \alpha\mu^2$. This may perhaps be damped numerically to give a suitable estimate of the pion mass, but the mass difference is still too large.

I. INTRODUCTION

One of the most promising features of gauge theories of the weak and electromagnetic interactions is the possibility of calculating masses and mass differences. Previously, in renormalizable field theories, if a bare mass or mass difference vanished, then either it remained zero to all orders because of an underlying symmetry of the Lagrangian or it was infinite in higher orders. In a spontaneously broken gauge theory, if a mass difference or mass is zero in zeroth order for all possible coupling constants even after the symmetry is broken, then that quantity is necessarily finite and calculable. Since there are no possible counterterms to cancel infinities if the zeroth-order relation holds in the presence of all coupling constants not subject to artificial constraints, all higher corrections must be finite because the theory is renormalizable.[1]

This paper studies particular questions in the domain of this new calculability in the framework of an SU(2)×SU(2)×U(1) model of the weak and electromagnetic interactions. The model is basically due to Weinberg, but our interpretation of it is quite different.[2] One of the aims of this paper is to study the proton-neutron mass difference which is calculable in this model. The second aim is to investigate mechanisms for incorporating pions into gauge theories.

If a gauge theory is to describe the weak and electromagnetic interactions, the gauge symmetry of the Lagrangian must be spontaneously broken to give masses to the gauge vector bosons and some of the fermions. One mechanism for breaking the symmetry is to introduce scalar mesons, called Higgs particles, which acquire nonzero vacuum expectation value.[3] Varying the Higgs content for a given gauge group changes the way in which the symmetry is broken. Another possibility is that symmetry breaking occurs dynamically without the introduction of additional spinless fields. The idea of dynamical symmetry breaking is currently much discussed, particularly in the context of strong-interaction gauge theories, where the introduction of Higgs scalars seems to cause serious problems with respect to asymptotic freedom. In dynamically broken theories, it is anticipated that scalar particles such as pions emerge as bound states of fermions. As yet, there has been no computational implementation of these ideas.[4]

In this paper, we exploit the Higgs mechanism to break the gauge symmetry. In varying the Higgs content in our SU(2)×SU(2)×U(1) model, we explore the effects on calculable quantities and on the physical interpretation of the model of changing the way in which the symmetry is broken. We also investigate the appearance of pions in the weak and electromagnetic sector as part of the Higgs system. In spite of their association with the weak sector, the pions interact strongly with the nucleons via a Yukawa coupling.

The question of the proton-neutron mass difference has haunted particle physicists for decades. Previous calculations based on electromagnetism resulted in either an infinite mass difference or else the wrong sign. However, the discovery of gauge theories provided the possibility that both the weak and electromagnetic interactions could conspire together to produce a neutron heavier than the proton. The Weinberg SU(2)×SU(2)×U(1) model was created as an example of a gauge model in which $\Delta m|_{p-n}$ could be calculated. However, it was commonly held that in this model, as in other models based on the SU(2) group, the one-loop calculation of Δm necessarily gave the wrong sign.[5] However, our study shows that the sign of the proton-neutron mass difference is a function of the Higgs content. We exhibit a possible symmetry breaking which produces a neutron which weighs more than the proton of the theory. The point of the calculation is not so much that one should take one Higgs content more seriously than another, but rather to emphasize that a lot of physics lurks in the Higgs sector of a theory. (This dependence of calculable quantities on the symmetry breaking may well carry over into theories in which the symmetry is broken dynamically.)

Section IV of this paper is concerned with understanding pions in the context of gauge theories. Before the interest in gauge theories, it was often believed that the smallness of the pion mass is due to the spontaneous breakdown of a global chiral SU(2)×SU(2) symmetry. Here the pion is considered as a Goldstone boson; it has nonzero mass because the SU(2)×SU(2) symmetry is only approximately true. However, in spontaneously broken gauge theories, the Goldstone theorem is evaded via the Higgs mechanism—the would-be zero-mass Goldstone scalars get "eaten up" to become the longitudinal modes of the massive Yang-Mills fields. From this viewpoint it is difficult to understand what the pion is and why its mass is so small.

There have been several attempts to integrate pions into weak and electromagnetic gauge theories:

(1) Hagiwara and Lee put pions into the Higgs sector in the Weinberg SU(2)×U(1) model.[6] However, as was stated by these authors, their model is artificial in the technical sense defined in their paper: Parity and isotopic-spin symmetry in the πN coupling are approximate and depend on setting two coupling constants almost equal to one another. This approximate equality is not stable under renormalization since it does not follow from any symmetry argument. In general, even if parity is a natural strong-interaction symmetry in a model, it may be difficult to guarantee that parity is not broken to order α when there are strongly interacting scalar fields in the Lagrangian.[7]

(2) Weinberg's theory of pions as pseudo-Goldstone bosons is a mechanism for producing spinless mesons which are massless in zeroth order but pick up finite calculable masses in higher order.[8] If the potential V of the model is forced because of gauge invariance and renormalizability to be invariant under a larger group than the gauge group, then the model contains pseudo-Goldstone bosons. These masses appear to be of order $m^2 \sim \alpha\mu^2$ (μ a typical vector-meson mass), which may be too large unless they are numerically damped.

Suppose the gauge group is $\overline{G}$, but the potential is invariant under the larger group G. When the Higgs fields acquire vacuum expectation value, the potential remains invariant under a subgroup $\overline{S}$ of $\overline{G}$ for all values of parameters in the Lagrangian[9] and the Lagrangian remains invariant under U(1), the electromagnetic gauge invariance. For each generator of $\overline{G}$ not in $\overline{S}$, there is a scalar meson whose mass is zero in zeroth order. Those mesons corresponding to generators of the true symmetry group G are the true Goldstone bosons of the theory. They become the longitudinal components of the massive vector mesons. Those mesons corresponding to generators of $\overline{G}$ neither in $\overline{S}$ nor in G are the pseudo-Goldstone bosons (see Fig. 1). In Weinberg's theory the pseudo-Goldstone bosons can be either fundamental fields in the Lagrangian or bound states. Attractive though Weinberg's idea is, there have been no models implementing it in the context of unified weak and electromagnetic interactions. (Bars and Lane, however, have a model utilizing the pseudo-Goldstone mechanism in a strong-interaction gauge theory.[10])

In our variation of the Weinberg SU(2)×SU(2) ×U(1) model, we have a mass-degenerate pseudoscalar triplet, with charges +, 0, −, which interacts strongly with nucleons as pions do in the SU(2)×SU(2) σ model. Isotopic spin and parity is a natural strong-interaction symmetry in this model. The triplet, which arises out of linear combinations of the Higgs fields, we identify with the pion triplet. Since it is degenerate in zeroth order, the mass difference $\delta m^2 = m_{\pi^+}{}^2 - m_{\pi^0}{}^2$ is calculable. We find that δm^2 is of order $\alpha\mu^2$, where μ is a typical vector-meson mass. Although it is possible that numerical factors could damp δm^2 by order of 10^{-2}, the estimate is still too large for the pion mass difference.

We shall show that if we modify our model by imposing a reflection symmetry on the Lagrangian, we enlarge the symmetry group $\overline{G}$ of the potential. We thus have an opportunity to explore the pseudo-Goldstone mechanism in some detail. In this version of the model, $\overline{G}$ is O(4)×O(4)×O(4). When the Higgs fields acquire vacuum expectation value, the potential symmetry is broken down to O(3)×O(3)×O(3). Nine scalar fields have zero mass in zeroth order: Of these, six are true Goldstone bosons and three are pseudo-Goldstone bosons. The pseudo-Goldstone bosons are the same linear combinations of Higgs fields which

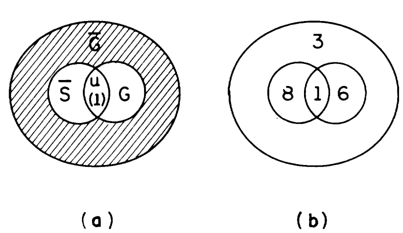

FIG. 1. (a) Pseudo-Goldstone bosons correspond to number of generators in $\overline{G} - (\overline{S} \cup G)$. (b) Pions as pseudo-Goldstone bosons in our model. $G = \mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$; $\overline{G} = \mathrm{O}(4)\times\mathrm{O}(4)\times\mathrm{O}(4)$; $\overline{S} = \mathrm{O}(3)\times\mathrm{O}(3)\times\mathrm{O}(3)$.

we call pions in the non-pseudo-Goldstone realization of the model. However, now they are massless in zeroth order, and therefore we can calculate not only their mass difference but also their masses. As before, they interact with nucleons as in the σ model with strong Yukawa coupling g_π.

When we perform the mass calculation in the one-loop approximation, only the charged members of the triplet acquire mass, while the neutral pion remains massless. Thus in our model the mass differences of pseudo-Goldstone bosons are of the same order as the pseudo-Goldstone-boson masses. We believe that this phenomenon may be more general than for the specific realization considered here and may be a property of pseudo-Goldstone mechanisms embedded in unified weak-electromagnetic gauge theories. This is not a bad approximation for the π-K mass difference, but is not acceptable for the pion triplet.

Compared with the mass-difference problem, the order of magnitude of the pseudo-Goldstone masses does not seem an insurmountable obstacle. The value of m^2 can be damped by numerical factors (in our model damping by factors of 10^{-2} is not implausible; in that case $m^2 \sim 10^{-2}\alpha\mu^2$, which is reasonable). However, if we consider our calculation of $m_{\pi^+}{}^2$ as an estimate for δm^2 (since $m_{\pi^0}{}^2$ is zero) then we again have a result which is too large for the pion triplet. Another possible way around the large estimate for the pseudo-Goldstone masses is uncovered by our calculation. The one-loop calculation of m^2 may be zero for some of the pseudo-Goldstone masses as it is for our neutral pion. In that case, m^2 for those mesons would be nonzero only in the two-loop approximation and hence presumably of order $m^2 \sim \alpha^2\mu^2$. (We could perhaps imagine a model including strange particles in which, in the one-loop approximation, the pion triplet remained massless, while the K and η picked up mass.)

We also raise the question: Is there parity violation of order α? Weinberg has shown that in certain classes of models involving strongly interacting vector mesons parity violation does not occur to order α.[11] However, our model which contains strongly interacting scalar fields in the Lagrangian is not covered by his result. A preliminary investigation of the pion-nucleon form factor reveals that the parity-violating piece of the one-loop radiative corrections is a calculable weak effect of order $\alpha m_{\text{nucleon}}{}^2/\mu^2$. Our calculation only treats the strong interactions perturbatively, but we expect that the result is more generally valid.

The aim of this paper is to study the effects of symmetry breaking on calculable masses and mass differences and on how pions can fit into gauge theories. Of course, we do not pretend that our model is realistic, but we find it an interesting model to test some of these ideas. The limitations of the model are readily apparent. It does not include strange particles and the extension to strange particles is no easy task. Perhaps a more complete model involving strangeness would give us a more realistic mass spectrum for the pseudoscalar octet realized as pseudo-Goldstone bosons.

Furthermore, the complications of the strong interactions have been completely neglected in all of the calculations. Clearly, the strong interactions must be treated nonperturbatively if the calculations are to be "realistic." Future work using the tools of current algebra can perhaps remedy this shortcoming.

In Sec. II we describe the model in detail. The proton-neutron mass difference calculation is performed in Sec. III and its dependence on the Higgs content is displayed. In Sec. IV we explore the two options for incorporating pions in the model: the non-pseudo-Goldstone alternative in which the pion triplet has mass in zeroth order and the pseudo-Goldstone realization of zeroth-order zero-mass pions. In Sec. V we look at the question of parity violation in the pion nucleon form factor.

After the completion of this work, we discovered that Lee, Rawls, and Yu had the same idea of using this model to examine the pseudo-Goldstone mechanism.[12] We come to slightly different conclusions, however. The above-mentioned authors take $g_L = g_R$ (and $v' = v''$), and because of this assumption they arrive at a lower bound on $m_{\pi^+}{}^2$ $[m_{\pi^+}{}^2 \geq (3\alpha/4\pi)\mu^2]$ which causes them to rule out the pseudo-Goldstone mechanism unless the pseudo-Goldstone bosons acquire mass only in the two-loop approximation. However, the assumption $g_L = g_R$, $v' = v''$ is natural (in the technical sense) only if the group is $O(4)\times U(1)$, in which case there is a reflection symmetry between multiplets: For every multiplet $(T_L = m,\ T_R = n, Y)$ there must be a corresponding $(T_L = n,\ T_R = m, Y)$. In the model at hand the reflection symmetry cannot be realized because of the (asymmetric) lepton content of the theory. Therefore, their lower bound on $m_{\pi^+}{}^2$ cannot be consistently derived in the context of the model. On the other hand, no assumptions on g_L vs g_R or v' vs v'' are needed in our work.

II. THE MODEL

We construct the model by writing down the most general renormalizable Lagrangian which is invariant under the gauge group.[13] All particles are assigned to representations of $SU(2)_L\times SU(2)_R \times U(1)$. The electric charge operator is $Q = T_{L3} + T_{R3} + Y$. The left-handed leptons (electron or muon—we treat only the electron sector, but the muon and its neutrino may be added to the model in the same way as the electron) form a doublet:

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} \nu \\ e^- \end{pmatrix},$$

$$T_L = \tfrac{1}{2},\quad T_R = 0,\quad Y = -\tfrac{1}{2}.$$

The right-handed electron is a singlet:

$$R = e_R^- = \tfrac{1}{2}(1-\gamma_5)e^-,\quad T_L = 0,\quad T_R = 0,\quad Y = -1.$$

There are seven gauge vector mesons transforming as the adjoint representation of the group:

$$\vec{A}_L^\mu,\quad T_L = 1,\quad T_R = 0,\quad Y = 0$$

$$\vec{A}_R^\mu,\quad T_L = 0,\quad T_R = 1,\quad Y = 0$$

$$B^\mu,\quad T_L = 0,\quad T_R = 0,\quad Y = 1.$$

To this model we add the left- and right-handed nucleons in a symmetric manner:

$$N_L = \begin{pmatrix} p \\ n \end{pmatrix}_L = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix} p \\ n \end{pmatrix},\quad T_L = \tfrac{1}{2},\quad T_R = 0,\quad Y = \tfrac{1}{2}$$

$$N_R = \begin{pmatrix} p \\ n \end{pmatrix}_R = \tfrac{1}{2}(1-\gamma_5)\begin{pmatrix} p \\ n \end{pmatrix},\quad T_L = 0,\quad T_R = \tfrac{1}{2},\quad Y = \tfrac{1}{2}.$$

Scalar mesons are added to break the symmetry and to give masses to various particles through their nonzero vacuum expectation value. A complex doublet ϕ gives mass to the leptons:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},\quad \langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v' \end{pmatrix},$$

$$T_L = \tfrac{1}{2},\quad T_R = 0,\quad Y = +\tfrac{1}{2}.$$

A real quartet H gives mass to the hadrons:

$$H = \begin{bmatrix} \dfrac{\sigma + i\pi_0}{\sqrt{2}} & i\pi^+ \\ i\pi^- & \dfrac{\sigma - i\pi^0}{\sqrt{2}} \end{bmatrix},\qquad \langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix},$$

$$T_L = \tfrac{1}{2},\quad T_R = \tfrac{1}{2},\quad Y = 0.$$

The notation for the H fields is chosen to be suggestive. The interaction of H with nucleons is precisely that of the σ model. Actually, a "physical" pion triplet will emerge later as that linear combination of all the Higgs fields which is an eigenvector of the mass operator. For later purposes, we also include a doublet ρ:

$$\rho=\begin{pmatrix}\rho^+\\ \rho^0\end{pmatrix},\quad \langle\rho\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v''\end{pmatrix},$$

$$T_L=0,\ T_R=\tfrac{1}{2},\ Y=+\tfrac{1}{2}.$$

By group transformation, we may simultaneously choose $\langle H\rangle$ proportional to the unit matrix with v real and $\langle\phi\rangle=(1/\sqrt{2})\binom{0}{v'}$ with v' real. However, we are not free to choose the phase of v'' concurrently.

The most general Lagrangian constructed from these fields is

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}F^{\alpha}_{L\mu\nu}F^{\alpha\mu\nu}_{L}-\tfrac{1}{4}F^{\alpha}_{R\mu\nu}F^{\alpha\mu\nu}_{R}-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\\&-\tfrac{1}{2}D_\mu\phi^\dagger D^\mu\phi-\tfrac{1}{2}D_\mu\rho^\dagger D^\mu\rho-\tfrac{1}{2}D_\mu H^\dagger D^\mu H-V(\phi,\rho,H)\\&-\bar{L}\gamma^\mu D_\mu L-\bar{R}\gamma^\mu D_\mu R-\bar{N}_L\gamma^\mu D_\mu N_L-\bar{N}_R\gamma^\mu D_\mu N_R\\&-g_R(\bar{L}\phi R+\text{H.c.})-g_\pi(\bar{N}_L H N_R+\text{H.c.}),\end{aligned}\tag{1}$$

where

(a) $$\begin{aligned}F^{\alpha}_{L\mu\nu}&=\partial_\mu A^{\alpha}_{L\nu}-\partial_\nu A^{\alpha}_{L\mu}-g_L\epsilon^{\alpha\beta\gamma}A^{\beta}_{L\mu}A^{\gamma}_{L\nu},\\F^{\alpha}_{R\mu\nu}&=\partial_\mu A^{\alpha}_{R\nu}-\partial_\nu A^{\alpha}_{R\mu}-g_R\epsilon^{\alpha\beta\gamma}A^{\beta}_{R\mu}A^{\gamma}_{R\nu},\\F_{\mu\nu}&=\partial_\mu B_\nu-\partial_\nu B_\mu\end{aligned}$$

[g_L is the coupling constant associated with $SU(2)_L$, g_R with $SU(2)_R$, and g_Y with U(1). They are assumed to be small (of order e), whereas g_π, the Yukawa coupling between the "pions" and nucleons, may be large],

(b) the covariant derivatives of the scalar and spinor fields are

$$\begin{aligned}D_\mu\phi&=(\partial_\mu+ig_L\tfrac{1}{2}\vec{\tau}\cdot\vec{A}^L_\mu+\tfrac{1}{2}ig_YB_\mu)\phi,\\D_\mu\rho&=(\partial_\mu+ig_R\tfrac{1}{2}\vec{\tau}\cdot\vec{A}^R_\mu+\tfrac{1}{2}ig_YB_\mu)\rho,\\D_\mu H&=(\partial_\mu H+ig_L\tfrac{1}{2}\vec{\tau}\cdot\vec{A}^L_\mu H-ig_RH\tfrac{1}{2}\vec{\tau}\cdot\vec{A}^R_\mu),\\D_\mu L&=(\partial_\mu+ig_L\tfrac{1}{2}\vec{\tau}\cdot\vec{A}^L_\mu-\tfrac{1}{2}ig_YB_\mu)L,\\D_\mu R&=(\partial_\mu-ig_YB_\mu)R,\\D_\mu N_L&=(\partial_\mu+ig_L\tfrac{1}{2}\vec{\tau}\cdot\vec{A}^L_\mu+\tfrac{1}{2}ig_YB_\mu)N_L,\\D_\mu N_R&=(\partial_\mu+ig_R\tfrac{1}{2}\vec{\tau}\cdot\vec{A}^R_\mu+\tfrac{1}{2}ig_YB_\mu)N_R,\end{aligned}$$

(c) $V(\phi,\rho,H)$ is the most general gauge-invariant fourth-order polynomial in the fields:

$$\begin{aligned}V(\phi,\rho,H)=&a\rho^\dagger\rho+b(\rho^\dagger\rho)^2+c\phi^\dagger\phi+d(\phi^\dagger\phi)^2+e(H^\dagger H)\\&+f(H^\dagger H)^2+h(\phi^\dagger H\rho+\rho^\dagger H^\dagger\phi)+j(\rho^\dagger\rho)(\phi^\dagger\phi)\\&+k(\rho^\dagger\rho)(H^\dagger H)+l(\phi^\dagger\phi)(H^\dagger H).\end{aligned}\tag{2}$$

The condition that the potential be a classical minimum, $\langle\partial V/\partial\psi\rangle_{\text{VEV}}=0$, where ψ is any of the scalar fields, gives relations for the vacuum expectation values:

$$\begin{aligned}&2v'(c+dv'^2+lv^2+\tfrac{1}{2}j|v''|^2)+\sqrt{2}\,hvv''^*=0,\\&2v''^*(a+b|v''|^2+kv^2+\tfrac{1}{2}jv'^2)+\sqrt{2}\,hvv'=0,\\&v(2e+4fv^2+k|v''|^2+lv'^2)+\frac{hv'}{2\sqrt{2}}(v''+v''^*)=0,\\&hv'(v''-v''^*)=0.\end{aligned}\tag{3}$$

These equations must be satisfied for a range of coupling constants $a,b,c,\ldots,l$ if the theory is to be renormalizable. Therefore unless $h\equiv0$, which can be achieved by imposing an added discrete symmetry on the Lagrangian, then v'' is also real.

To calculate physical processes, we shift the scalar fields and define new fields which have no vacuum expectation value:

$$\phi=\phi'+\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v'\end{pmatrix},\quad\langle\phi'\rangle=0,\quad\text{etc.}$$

After shifting fields, all but one of the previously massless vector mesons as well as the electron and the nucleons acquire mass. Six of the scalars, the Goldstone bosons, have zero mass, while the remaining scalars pick up mass.

The zeroth-order vector-meson mass matrix becomes

$$\mu^2=\begin{pmatrix}A&0&0\\0&A&0\\0&0&B\end{pmatrix},$$

where A is the mass matrix for the charged sectors and B is the mass matrix for the three neutral vector mesons:

$$A=\frac{1}{8}\begin{pmatrix}g_L^2(v'^2+2v^2)&-2g_Lg_Rv^2\\-2g_Lg_Rv^2&g_R^2(v''^2+2v^2)\end{pmatrix},$$

$$B=\frac{1}{8}\begin{bmatrix}g_L^2(v'^2+2v^2)&-2g_Lg_Rv^2&-g_Yg_Lv'^2\\-2g_Lg_Rv^2&g_R^2(v''^2+2v^2)&-g_Yg_Rv''^2\\-g_Yg_Lv'^2&-g_Yg_Rv''^2&g_Y^2(v'^2+v''^2)\end{bmatrix}.$$

The mass eigenvalues are

$$\begin{aligned}\mu_{A\pm}^2=\tfrac{1}{16}\big(&g_L^2(v'^2+2v^2)+g_R^2(v''^2+2v^2)\\&\pm\{[g_L^2(v'^2+2v^2)+g_R^2(v''^2+2v^2)]^2\\&-4g_L^2g_R^2(v'^2v''^2+2v^2v'^2+2v^2v''^2)\}^{1/2}\big),\\\mu_{B\pm}^2=\tfrac{1}{16}\big(&g_L^2(v'^2+2v^2)+g_R^2(v''^2+2v^2)+g_Y^2(v'^2+v''^2)\\&\pm\{[g_L^2(v'^2+2v^2)+g_R^2(v''^2+2v^2)\\&+g_Y^2(v'^2+v''^2)]^2\\&-4(g_L^2g_R^2+g_L^2g_Y^2+g_R^2g_Y^2)\\&\times(v'^2v''^2+2v^2v'^2+2v^2v''^2)\}^{1/2}\big).\end{aligned}$$

The massless photon A_μ corresponds to the linear combination

$$A_\mu = \frac{g_R g_Y A_\mu^{L3} + g_L g_Y A_\mu^{R3} + g_L g_R B_\mu}{({g_L}^2 {g_R}^2 + {g_L}^2 {g_Y}^2 + {g_R}^2 {g_Y}^2)^{1/2}} .$$

The electric charge is

$$e = \frac{g_L g_R g_Y}{({g_L}^2 {g_R}^2 + {g_L}^2 {g_Y}^2 + {g_R}^2 {g_Y}^2)^{1/2}} .$$

The electron picks up a mass $m_e = g_e v'/\sqrt{2}$. The proton and neutron have the same zeroth-order mass $m = m_p = m_n = g_\pi v$. Therefore the proton-neutron mass difference is finite and calculable in the model. We will return to this in Sec. III.

The scalar mass matrix is given by

$$M^2{}_{ij} = \left\langle \frac{\partial^2 V(\phi, \rho, H)}{\partial \psi_i \, \partial \psi_j} \right\rangle .$$

The condition that the potential be a true minimum is equivalent to the condition that the scalar mass matrix be semi-positive-definite. Six linear combinations of scalars (two positively charged, two negatively charged, two neutral) have zero mass to all orders. These are the Goldstone bosons which get "eaten up" to become the longitudinal component of the six massive vector mesons. There remain three massive neutrals and a mass-degenerate triplet with charges +, 0, − and with mass

$$m^2 = -\frac{h}{\sqrt{2}} \left(\frac{vv'}{v''} + \frac{vv''}{v'} + \frac{v'v''}{v} \right). \tag{4}$$

The triplet of scalars, which we shall interpret as pions, is

$$\Pi^0 = \frac{v'v''\pi^0 - vv'(\rho^0 - \bar{\rho}^0)/\sqrt{2}\,i + vv''(\phi^0 - \bar{\phi}^0)/\sqrt{2}\,i}{(v^2 v'^2 + v^2 v''^2 + v'^2 v''^2)^{1/2}},$$

$$\Pi^+ = \frac{v'v''\pi^+ - ivv'\rho^+ + ivv''\phi^+}{(v^2 v'^2 + v^2 v''^2 + v'^2 v''^2)^{1/2}}, \quad \Pi^- = (\Pi^+)^\dagger . \tag{5}$$

Since we have a zero-order mass relation, the pion mass difference is also finite and calculable in the model (see Sec. IV). The remaining massive neutral scalars are linear combinations of $(\rho^0 + \bar{\rho}^0)/\sqrt{2}$, $(\phi^0 + \bar{\phi}^0)/\sqrt{2}$, σ with mass-squared matrix

$$M^2 = \begin{bmatrix} 2bv''^2 - \frac{hvv'}{\sqrt{2}\,v''} & jv'v'' + \frac{hv}{\sqrt{2}} & 2kvv'' + \frac{hv'}{\sqrt{2}} \\ jv'v'' + \frac{hv}{\sqrt{2}} & 2dv'^2 - \frac{hvv''}{\sqrt{2}\,v'} & 2lvv' + \frac{hv''}{\sqrt{2}} \\ 2kvv'' + \frac{hv'}{\sqrt{2}} & 2lvv' + \frac{hv''}{\sqrt{2}} & 8fv^2 - \frac{hv'v''}{\sqrt{2}\,v} \end{bmatrix} .$$

Before turning to the nucleon-pion sector which is the main concern of this paper, we will require that the model incorporate universality of the weak interactions. The pieces of the Lagrangian which contribute to β decay and μ decay are

$$\mathcal{L}_\mu = G_\mu [\bar{e}\gamma^\mu \tfrac{1}{2}(1+\gamma_5)\nu_e][\bar{\nu}_\mu \gamma_\mu \tfrac{1}{2}(1+\gamma_5)\mu],$$

$$\mathcal{L}_\beta = G_\beta [\bar{e}\gamma^\mu \tfrac{1}{2}(1+\gamma_5)\nu_e][\bar{p}\gamma_\mu \tfrac{1}{2}(1+A\gamma_5)n].$$

Weak-interaction universality in gauge models means that $G_\mu \approx G_\beta$ and $A \approx 1$ in the approximation that vector-meson momenta in propagators are ignored relative to vector-meson masses. In our model

$$\frac{G_\beta - G_\mu}{G_\mu} = \frac{2v^2}{v''^2 + 2v^2}, \quad A = \frac{v''^2}{v''^2 + 4v^2} .$$

Both conditions are satisfied for $v \ll v''$. Therefore our implementation of universality is unnatural in the technical sense that it depends on one parameter in the theory being much smaller than another.

III. THE PROTON-NEUTRON MASS DIFFERENCE

In this section we calculation the proton-neutron mass difference in several stages. First we calculate $\Delta m|_{p-n}$ for the Higgs content specified in Sec. II. Then we examine the dependence of Δm on the Higgs content for any general irreducible Higgs multiplet. We then exhibit a modification of the Higgs system of Sec. II which gives the correct sign, $\Delta m|_{p-n} < 0$, and also maintains the other features of the model (massive electron, equal nucleon masses, etc.) discussed above.

A. Higgs content of Sec. II

The calculation with fixed Higgs content (ϕ, H) has already been made by others.[5] For this content, it was found that $\Delta m = m_p - m_n > 0$, which is the wrong sign, at least from the point of view of the naive quark model. The sign remains positive

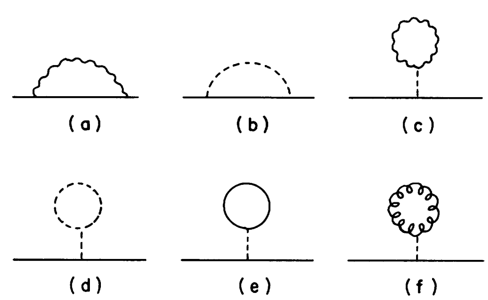

FIG. 2. Diagrams which contribute to the fermion self-mass in the one-loop approximation: (a) vector-meson exchange, (b) Higgs-scalar exchange, (c) vector-meson tadpole, (d) Higgs tadpole, (e) fermion tadpole, (f) ghost tadpole.

when we add the ρ Higgs field.

In the one-loop calculation of the mass difference, there are three types of diagrams that may contribute: vector-meson exchange, scalar-meson exchange, and tadpole diagrams (Fig. 2). In this problem, the tadpoles contribute equally to the proton-neutron self-masses, since contributions proportional to γ_5 only contribute to wave-function renormalization in this order. If we neglect the nucleon mass relative to the vector-meson masses, then we can ignore the scalar-meson exchange term. This leaves vector-meson exchange. The contribution from the $k_\mu k_\nu$ part of the vector-meson propagator vanishes when the fermions are on their mass shells and therefore does not contribute to Δm. Thus we find

$$\Delta m = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\frac{\gamma_\mu(-i(\not p-\not k)+m)\gamma^\mu}{(p-k)^2+m^2} \times[g_Y g_L(k^2+\mu^2)_{L3,B}{}^{-1} + g_Y g_R(k^2+\mu^2)_{R3,B}{}^{-1}]\,. \tag{6}$$

This formula is more general than for the specific Higgs content stated above. The quantity in square brackets has the general form

$$\frac{A+Bk^2}{k^2(k^2+\mu_1{}^2)(k^2+\mu_2{}^2)}\,, \tag{7}$$

where μ_1 and μ_2 are the neutral-vector-meson mass eigenvalues. Simplifying the integral, we find

$$\Delta m = -\frac{m\pi^2}{(2\pi)^4}\frac{1}{\mu_1{}^2-\mu_2{}^2}\left\{\frac{A}{m^2}\left[\frac{J(\mu_1{}^2/m^2)}{\mu_1{}^2/m^2}-\frac{J(\mu_2{}^2/m^2)}{\mu_2{}^2/m^2}\right] - B\left[J\left(\frac{\mu_1{}^2}{m^2}\right)-J\left(\frac{\mu_2{}^2}{m^2}\right)\right]\right\},$$
$$J(\beta)=\int_0^1 dx(1+x)\ln\left(1+\frac{1-x}{x^2}\beta\right). \tag{8}$$

Pi has shown that $J(\beta)$ is a monotonically increasing function of β, whereas $J(\beta)/\beta$ is monotonically decreasing.[5] Therefore the conditions $A\le 0$, $B\le 0$ together are sufficient to guarantee the correct sign for Δm.

For the Higgs content of Sec. II, we find that

$$g_L g_Y(k^2+\mu^2)_{L3,B}{}^{-1} = \frac{g_L{}^2 g_Y{}^2}{8k^2(k^2+\mu_1{}^2)(k^2+\mu_2{}^2)}[v'^2k^2+\tfrac{1}{8}g_R{}^2(v'^2v''^2+2v^2v'^2+2v^2v''^2)]\,,$$

$$g_R g_Y(k^2+\mu^2)_{R3,B}{}^{-1} = \frac{g_R{}^2 g_Y{}^2}{8k^2(k^2+\mu_1{}^2)(k^2+\mu_2{}^2)}[v''^2k^2+\tfrac{1}{8}g_L{}^2(v'^2v''^2+2v^2v'^2+2v^2v''^2)]\,,$$

in which case $A>0$ and $B>0$, so that we find the wrong sign for Δm. This calculation contains the earlier result (without the ρ field) by setting $v''=0$.

B. One irreducible Higgs representation

We next ask the question: Is the sign of Δm typical for this gauge group or does it depend on the representation content chosen and vary with the way the symmetry is broken? Since the only kind of symmetry breaking we know how to calculate is via the Higgs mechanism, we look at the sign of Δm for general Higgs content. Equations (6)–(8) for the mass difference are valid for any Higgs system, except that A and B depend on the Higgs sector through the neutral-vector-meson mass matrix:

$$A = g_L g_Y(\mu_{L3,R3}{}^2\mu_{R3,B}{}^2-\mu_{L3,B}{}^2\mu_{R3,R3}{}^2) + g_R g_Y(\mu_{L3,R3}{}^2\mu_{L3,B}{}^2-\mu_{R3,B}{}^2\mu_{L3,L3}{}^2)\,,$$
$$B = -g_L g_Y\mu_{L3,B}{}^2 - g_R g_Y\mu_{R3,B}{}^2\,. \tag{9}$$

The condition $B\le 0$ is linear in μ^2 and therefore additive with respect to different Higgs multiplets, whereas the condition $A\le 0$ is not. Therefore, for simplicity we suppose that the Higgs fields all belong to a single irreducible representation K_{ij} of the gauge group with quantum numbers $T_L=m$, $T_R=n$, Y. Then

$$T_{L3}^{ij}=\delta_{ij}T_i^{L3}=(m-i+1)\delta_{ij}\,,\quad i=1,2,\ldots,2m+1$$
$$T_{R3}^{ij}=\delta_{ij}T_i^{R3}=(n-i+1)\delta_{ii}\,,\quad i=1,2,\ldots,2n+1\,.$$

The contribution of K_{ij} to the neutral vector-meson masses (NVMM) comes from the term in the Lagrangian

$$\mathcal{L}_{\text{NVMM}} = -\tfrac{1}{2}|g_L T_{ik}^{L3}\langle K_{kj}\rangle A_\mu^{L3} - g_R\langle K_{ik}\rangle T_{kj}^{R3}A_\mu^{R3} + g_Y Y\langle K_{ij}\rangle B_\mu|^2$$
$$= -\tfrac{1}{2}|g_L(m-i+1)\langle K_{ij}\rangle A_\mu^{L3} - g_R(n-j+1)\langle K_{ij}\rangle A_\mu^{R3} + g_Y Y\langle K_{ij}\rangle B_\mu|^2\,.$$

Since $\langle K_{ij}\rangle = 0$ unless K_{ij} is neutral, we use the relation $Q = T_{L3} + T_{R3} + Y$ to simplify the mass term: $QK_{ij} = (n + m + Y + 2 - i - j)K_{ij}$. Eliminating $j = n + m + Y + 2 - i$, we define

$$K_i \equiv |\langle K_i, n + m + Y + 2 - i\rangle|. \tag{10}$$

Then

$$\mathcal{L}_{\text{NVMM}} = -\tfrac{1}{2}|[g_L(m - i + 1)A_\mu^{L3} - g_R(i - m - Y - 1)A_\mu^{R3} + g_Y Y B_\mu]K_i|^2 .$$

Using this equation to substitute the values of $\mu^2{}_{ij}$ into Eq. (9) for A, we find that $A \equiv 0$ for an irreducible representation. That leaves condition B which becomes

$$[Y^2 g_R{}^2 + Y(m - i + 1)(g_R{}^2 + g_L{}^2)]K_i{}^2 > 0 . \tag{11}$$

It is not difficult to satisfy this equation. Suppose, for instance, that we add a Higgs multiplet with quantum numbers $T_L = 1$, $T_R = \frac{1}{2}$, $Y = \frac{1}{2}$. Then B becomes

$$g_R{}^2 K_2{}^2 - (g_R{}^2 + 2g_L{}^2)K_3{}^2 > 0 .$$

This is positive if K_2 is chosen sufficiently larger than K_3.

C. Modification of Sec. II Higgs content to give $\Delta m < 0$

The model of Sec. II can be modified by adding the scalar K ($T_L = 1$, $T_R = \frac{1}{2}$, $Y = \frac{1}{2}$) to the Higgs sector. The other Higgs fields remain to give mass to the fermions. However, if K_2 is sufficiently large compared with the other vacuum expectation values, Δm has the correct sign. In that case, we also preserve weak universality. Moreover, since K does not couple to the fermions in the theory, it does not alter the zeroth-order fermion mass relations. However, we see that it has a profound influence on dynamics via the change it introduces into the vector-meson mass matrix. It will also change the physical interpretation of scalars in the Higgs sector since we must include in the potential a term $(K^\dagger K)(\psi^\dagger\psi)$ for all other Higgs representations ψ.

The point of the calculation is not so much that we should add this particular Higgs scalar K to our model, for criteria of economy in model building make us reluctant to do so. More importantly, the lesson we learn is that by changing the Higgs content in a theory, even when the new scalars do not interact directly with fermions, we can radically alter the calculable masses and mass differences.

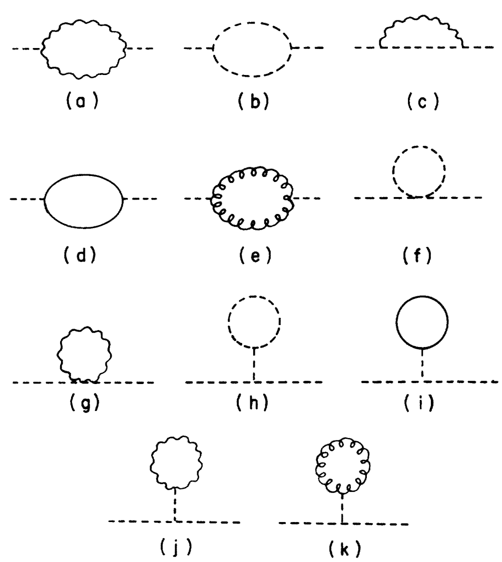

FIG. 3. Diagrams which contribute to scalar self-mass in the one-loop approximation: (a) vector-meson exchange, (b) scalar exchange, (c) vector-meson–scalar exchange, (d) fermion exchange, (e) ghost exchange, (f) scalar seagull, (g) vector-meson seagull, (h) scalar tadpole, (i) fermion tadpole, (j) vector-meson tadpole, (k) ghost tadpole.

IV. PIONS

A. Massive pions

Returning to the model of Sec. II, we recall that we have a degenerate triplet of pions with zeroth-order mass given by Eq. (4). Therefore, to ensure $m_\pi{}^2 \approx 0$ the trilinear Higgs coupling constant h would have to be small (of order $h \sim em_\pi{}^2/\mu$). Although this implementation of the model can give a realistic pion mass, we think that an acceptable model of pions should explain why they are so light. Since we have a zeroth-order mass relation, we can calculate the mass difference $\delta m^2 = m_{\pi^+}{}^2 - m_{\pi^0}{}^2$. The diagrams which contribute to the pion self-energy are shown in Fig. 3. We expect them to be of order $\alpha m_\pi{}^2$ or $\alpha\mu^2$. Since there are so many diagrams to calculate, we would like to be able to pick out the $\alpha\mu^2$ diagrams and ignore the $\alpha m_\pi{}^2$ diagrams. In Sec. IV B we will discover a trick to enable us to do this, and therefore we will complete the calculation of δm^2 in Sec. IV C.

B. Pseudo-Goldstone boson realization

We now look more closely at the potential in the Lagrangian [Eq. (2)]. The invariance group of V coincides with the gauge group, $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$. However, if we demand that the Lagrangian be invariant under a reflection symmetry R, which sends ρ into $-\rho$ but sends all other fields into themselves, then the trilinear term $h(\phi^\dagger H\rho + \rho^\dagger H^\dagger\phi)$ drops from the potential. In that case V is invariant under a larger symmetry group. If we rewrite the fields in terms of real fields,

$$P=\frac{\rho^0+\bar\rho^0}{\sqrt2}\,,\quad P'=\frac{\rho^0-\bar\rho^0}{\sqrt2\,i}\,,$$
$$S=\frac{\rho^++\rho^-}{\sqrt2}\,,\quad S'=\frac{\rho^+-\rho^-}{\sqrt2\,i}\,,$$
$$F=\frac{\phi^0+\bar\phi^0}{\sqrt2}\,,\quad F'=\frac{\phi^0-\bar\phi^0}{\sqrt2\,i}\,, \tag{12}$$
$$T=\frac{\phi^++\phi^-}{\sqrt2}\,,\quad T'=\frac{\phi^+-\phi^-}{\sqrt2\,i}\,,$$
$$\sigma,\qquad \pi^0,$$
$$U=\frac{\pi^++\pi^-}{\sqrt2}\,,\quad U'=\frac{\pi^+-\pi^-}{\sqrt2\,i}\,,$$

then each of the Higgs fields ϕ, H, ρ is a real 4-vector and the potential is a function only of their lengths:

$$V(\phi,\rho,H)=V(P^2+P'^2+S^2+S'^2,\ F^2+F'^2+T^2+T'^2,\ \sigma^2+\pi^{02}+U^2+U'^2)\,.$$

Thus V is invariant under the larger group $\bar G=O(4)\times O(4)\times O(4)$. Each 4-vector picks a direction when it acquires nonzero vacuum expectation value and therefore $\bar S=O(3)\times O(3)\times O(3)$. Counting dimensions, since $d(\bar G)=18$, $d(\bar S)=9$, and $G\cap\bar S=U(1)$, we find that

number of pseudo-Goldstone bosons

$$\begin{aligned}&=d(\bar G-\bar S\cup G)\\&=d(\bar G)-[d(\bar S)+d(G)-d(\bar S\cap G)]\\&=18-(9+7-1)=3.\end{aligned}$$

Thus we have precisely three pseudo-Goldstone bosons which we are going to connote as pions. They are the Π triplet of Sec. II. Their zero-order mass,

$$m^2=\frac{-h}{\sqrt2}\left(\frac{vv'}{v''}+\frac{vv''}{v'}+\frac{v'v''}{v}\right),$$

becomes zero when the reflection symmetry R is imposed ($h\equiv0$).

Thus we have two options to choose from: our model of Sec. II with or without symmetry R. In the first case we can calculate the pion masses and expect them to be small; in the second case the triplet mass is arbitrary and there is no reason for it to be small, but we can calculate δm^2.

In the pseudo-Goldstone-boson realization, we calculate the pion mass by examining the eleven diagrams of Fig. 3. Weinberg has shown that a number of cancellations occur when we specialize to the pseudo-Goldstone self-masses and that the result is ξ-independent. Furthermore, the fermion diagrams [(d) and (i)] do not contribute when the Yukawa interactions are invariant under $\bar G$, as is the case in our model. The only diagrams which contribute are (a), (g), and (j) with the ξ-independent piece of the vector-meson propagator instead of the total propagator used in the calculation.[8] The necessary Higgs–vector-meson vertices are given in the Appendix.

For the neutral-pion mass, diagram (a) is absent because the Π^0 does not couple to two vector mesons. Moreover, diagrams (g) and (j) exactly cancel, so that $m_{\pi^0}{}^2=0$ to first order as well. Using the relation (5), we write

$$G(\Pi^0,\Pi^0)=\frac{1}{v^2v'^2+v^2v''^2+v'^2v''^2}\,[v'^2v''^2G(\pi^0,\pi^0)+v^2v'^2G(P',P')+v^2v''^2G(F',F')]\,,$$
$$J(\Pi^0,\Pi^0)=\frac{1}{v^2v'^2+v^2v''^2+v'^2v''^2}\,[v'^2v''^2J(\pi^0,\pi^0)+v^2v'^2J(P',P')+v^2v''^2J(F',F')]\,.$$

For simplicity we shall assume that the coupling constants j, k, l in V [see Eq. (2)] are absent—this makes the F, P, σ mass matrix diagonal. Looking at the F' contribution to G and J we find

$$\begin{aligned}G(F',F')=&-\tfrac{1}{16}(3\times2)\int\frac{d^4k}{(2\pi)^4}\{g_L{}^2[(k^2+\mu^2)_{L1,L1}{}^{-1}+(k^2+\mu^2)_{L2,L2}{}^{-1}+(k^2+\mu^2)_{L3,L3}{}^{-1}]\\&+g_Y{}^2(k^2+\mu^2)_{B,B}{}^{-1}-2g_Lg_Y(k^2+\mu^2)_{L3,B}{}^{-1}\}\,,\\J(F',F')=&-\frac{g_{F'F'F}v'}{m_F{}^2 8}(3\times2)\int\frac{d^4k}{(2\pi)^4}\{g_L{}^2[(k^2+\mu^2)_{L1,L1}{}^{-1}+(k^2+\mu^2)_{L2,L2}{}^{-1}+(k^2+\mu^2)_{L3,L3}{}^{-1}]\\&+g_Y{}^2(k^2+\mu^2)_{B,B}{}^{-1}-2g_Lg_Y(k^2+\mu^2)_{L3,B}{}^{-1}\}\,.\end{aligned} \tag{13}$$

The factor 3 comes from the ξ-independent piece of

$$g^{\mu\nu}\Delta^A_{\alpha\mu,\beta\nu}=\left(\frac{3}{k^2+\mu^2}+\frac{1}{\xi k^2+\mu^2}\right)_{\alpha\beta}$$

and the factor 2 comes from counting in the vector-meson loop. Since $g_{F'F'F}=-dv'$ and $m_F{}^2=2dv'^2$, the two diagrams cancel. Similarly, the Π^0 and P', G, and J diagrams cancel. For general j, k, l, Eqs. (13) are modified, but the calculation gives the same result: $-m_{\pi^0}{}^2\equiv 0$ in the one-loop approximation.

When we calculate $m_{\pi^+}{}^2$, diagram (a) is no longer zero. Moreover, there are cross terms which contribute to the physical Π mass from diagram (a). For example, consider

$$\Pi^1=\frac{\Pi^+ + \Pi^-}{\sqrt{2}}=\frac{1}{(v^2v'^2+v^2v''^2+v'^2v''^2)^{1/2}}\,(v'v''U+vv'S'-vv''T')\,.$$

Then we have

$$A(\Pi^1,\Pi^1)=\frac{1}{v^2v'^2+v^2v''^2+v'^2v''^2}\big[v'^2v''^2A(U,U)+v^2v'^2A(S',S')+v^2v''^2A(T',T')$$
$$+2vv'^2v''A(U,S')-2vv'v''^2A(U,T')-2v^2v'v''A(S',T')\big]\,.$$

For example,

$$A(T',T')=\tfrac{3}{16}v'^2g_L{}^2g_Y{}^2\int\frac{d^4k}{(2\pi)^4}\,(k^2+\mu^2)_{L1,L1}{}^{-1}(k^2+\mu^2)_{B,B}{}^{-1}\,.$$

Similarly, in the approximation in which we set $j, k, l=0$,

$$G(T',T')=-\frac{3\times 2}{16}\int\frac{d^4k}{(2\pi)^4}\,\big[g_L{}^2\big((k^2+\mu^2)_{L1,L1}{}^{-1}+(k^2+\mu^2)_{L2,L2}{}^{-1}+(k^2+\mu^2)_{L3,L3}{}^{-1}\big)$$
$$+g_Y{}^2(k^2+\mu^2)_{B,B}{}^{-1}+2g_Lg_Y(k^2+\mu^2)_{L3,B}{}^{-1}\big]\,,$$

$$J(T',T')=\frac{3\times 2}{16}\int\frac{d^4k}{(2\pi)^4}\,\big[g_L{}^2\big((k^2+\mu^2)_{L1,L1}{}^{-1}+(k^2+\mu^2)_{L2,L2}{}^{-1}+(k^2+\mu^2)_{L3,L3}{}^{-1}\big)$$
$$+g_Y{}^2(k^2+\mu^2)_{B,B}{}^{-1}-2g_Lg_Y(k^2+\mu^2)_{L3,B}{}^{-1}\big]\,.$$

Substituting into these equations the values of the inverse propagator matrix, we find

$$\Sigma(T',T')=A(T',T')+G(T',T')+J(T',T')$$
$$=-\tfrac{3}{2}g_Lg_Y\int\frac{d^4k}{(2\pi)^4}\left[(k^2+\mu^2)_{L3,B}{}^{-1}-g_Lg_Y\frac{v'^2}{8}\,(k^2+\mu^2)_{L1,L1}{}^{-1}(k^2+\mu^2)_{B,B}{}^{-1}\right]$$
$$=-\frac{3}{2}\,\frac{g_L{}^2g_R{}^2g_Y{}^2v^2v''^2}{32}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2(k^2+\mu_1{}^2(k^2+\mu_2{}^2)}$$
$$=-\tfrac{3}{64}g_L{}^2g_R{}^2g_Y{}^2v^2v''^2\left(\frac{i\pi^2}{(2\pi)^4}\right)\frac{\ln(\mu_1{}^2/\mu_2{}^2)}{\mu_1{}^2-\mu_2{}^2}\,,$$

$$m^2{}_{T',T'}=i\Sigma(T',T')=\frac{3\pi^2}{64(2\pi)^4}\,g_L{}^2g_R{}^2g_Y{}^2v^2v''^2\,\frac{\ln(\mu_1{}^2/\mu_2{}^2)}{\mu_1{}^2-\mu_2{}^2}\,.$$

In a similar way, we can compute the remaining matrix elements of $m^2{}_{ij}$ $(i,j=S',T',U)$:

$$m^2=\begin{pmatrix} v^2v'^2 & -v'v''v^2 & -vv'^2v'' \\ -v'v''v^2 & v^2v''^2 & vv'v''^2 \\ -vv'^2v'' & vv'v''^2 & v'^2v''^2 \end{pmatrix}\frac{3\pi^2}{64(2\pi)^4}\,g_L{}^2g_R{}^2g_Y{}^2\,\frac{\ln(\mu_1{}^2/\mu_2{}^2)}{\mu_1{}^2-\mu_2{}^2}\,.$$

We note that m^2 is proportional to the zero-order mass matrix when $h\neq 0$—namely, the linear combinations of Higgs scalars which were true Goldstone bosons pick up no mass in one loop (and to all higher orders as well). The charged pions acquire mass:

$$m_{\pi^+}{}^2=\frac{3\pi^2}{64(2\pi)^4}\,g_L{}^2g_R{}^2g_Y{}^2\,\frac{\ln(\mu_1{}^2/\mu_2{}^2)}{\mu_1{}^2-\mu_2{}^2}$$
$$\times(v^2v'^2+v^2v''^2+v'^2v''^2)\,.$$

This is of order $\alpha\mu^2$, but perhaps we should not rule it out too soon since by experimenting with different values of the parameters $(g_L, g_R, g_Y, v, v', v'')$ we find that we can damp this by factors of 10^{-2}.

If, on the other hand, we wish to interpret this calculation of $m_{\pi^+}{}^2$ as a calculation of δm^2 (since $m_{\pi^0}{}^2=0$), we seem to be in trouble. We would then need damping by at least 10^{-3}, which seems highly unlikely in this model. Furthermore, to agree

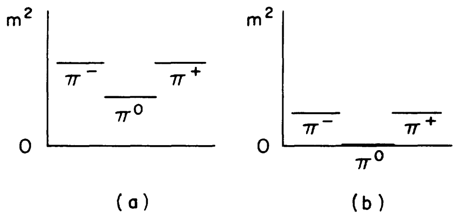

FIG. 4. Pion mass spectrum in one-loop approximation. (a) Presence of zeroth-order pion mass (no reflection symmetry R in Lagrangian). (b) Pseudo-Goldstone-boson realization (reflection symmetry R).

with experiment, the mass of the Π^0, a two-loop calculation, would have to be an order of magnitude greater than δm^2, a one-loop quantity. This does not make much sense.

Thus in our model we find that the mass differences of pseudo-Goldstone bosons are of the same order as the pseudo-Goldstone boson masses. We expect that this may be a general feature of models which implement the pseudo-Goldstone mechanism in weak and electromagnetic gauge theories.

C. δm^2 for massive pions

We now return to the calculation of δm^2 in the version of the model with massive zeroth-order pions. We can apply Weinberg's analysis of the scalar self-mass in the pseudo-Goldstone boson case to eliminate those diagrams of order αm_π^2. They are precisely those diagrams which cancel out when $h=0$. Since $h=0$ corresponds to the pseudo-Goldstone realization, we can use our calculation of the pseudo-Goldstone masses to find the leading contribution to $\delta m^2_{(\text{massive})}$:

$$\delta m^2_{(\text{massive})} \approx (m_{\pi^+}{}^2 - m_{\pi^0}{}^2)_{\text{pseudo-Goldstone}}$$
$$= \frac{3\pi^2}{64(2\pi)^4} g_L{}^2 g_R{}^2 g_Y{}^2 \left(\frac{\ln(\mu_1{}^2/\mu_2{}^2)}{\mu_1{}^2 - \mu_2{}^2}\right)$$
$$\times (v^2 v'^2 + v^2 v''^2 + v'^2 v''^2)\,.$$

Thus δm^2 is of order $\alpha\mu^2$ and it is unlikely that this can be damped sufficiently to account for the Π^+-Π^0 mass difference. (See Fig. 4.)

V. PARITY VIOLATION

Gauge models with strongly interacting scalar fields appear to violate parity to order α, which is unacceptably large. In our model we look at this question for the pion-nucleon form factor as a preliminary study. The diagrams which may contribute in the one-loop approximation to the proper pion-nucleon vertex are shown in Fig. 5; we are interested only in the parity-nonconserving piece of each diagram.

We look at the special case $\Pi^0 pp$ for simplicity. Diagram (a) does not contribute to parity violation because of the symmetry of the vector-meson coupling. Diagrams (c) and (d) do not contribute because the Higgs-fermion system is parity-conserving. Diagram (e) is absent because the Π^0 does not couple to two vector mesons. The parity-violating piece of the (b) diagram is

$$B_{\text{I}} = -\tfrac{1}{4} i g_\pi \int \frac{d^4 l}{(2\pi)^4} \frac{(2k-l)_\nu}{(k-l)^2 + m_\sigma{}^2} \frac{\gamma_\mu[-i(\not p - \not l) + m]}{(p-l)^2 + m^2} \big[-\tfrac{1}{2} g_L{}^2 \Delta^{\mu\nu}_{L3,L3}(l) - g_L g_Y \Delta^{\mu\nu}_{B,L3}(l)$$
$$+\tfrac{1}{2} g_R{}^2 \Delta^{\mu\nu}_{R3,R3}(l) + g_R g_Y \Delta^{\mu\nu}_{B,R3}(l)\big]\,,$$

$$B_{\text{II}} = -\tfrac{1}{4} i g_\pi \int \frac{d^4 l}{(2\pi)^4} \frac{(2k-l)_\nu}{(k-l)^2 + m_\sigma{}^2} \frac{[-i(\not p' + \not l) + m]}{(p'+l)^2 + m^2} \gamma_\mu \big[-\tfrac{1}{2} g_L{}^2 \Delta^{\mu\nu}_{L3,L3}(l) - g_L g_Y \Delta^{\mu\nu}_{B,L3}(l)$$
$$+\tfrac{1}{2} g_R{}^2 \Delta^{\mu\nu}_{R3,R3}(l) + g_R g_Y \Delta^{\mu\nu}_{B,R3}(l)\big]\,.$$

Taking the sum, in the $\xi = 1$ gauge for simplicity and substituting the vector-meson masses into the formulas, we find

$$B = \tfrac{1}{4} i g_\pi \int \frac{d^4 l}{(2\pi)^4} \big[l^2 \tfrac{1}{2}(g_L{}^2 - g_R{}^2) + \tfrac{1}{16} g_L{}^2 g_R{}^2 (v'^2 - v''^2) - \tfrac{1}{16} g_L{}^2 g_Y{}^2 (3v'^2 + v''^2) + \tfrac{1}{16} g_R{}^2 g_Y{}^2 (v'^2 + 3v''^2)\big]$$
$$\times \left(\frac{[(p'+l)^2 + m^2](2\not k - \not l)(i\not p - i\not l - m) + [(p-l)^2 + m^2](i\not p' + i\not l - m)(2\not k - \not l)}{[(k-l)^2 + m_\sigma{}^2](l^2 + \mu_1{}^2)(l^2 + \mu_2{}^2)[(p-l)^2 + m^2][(p'+l)^2 + m^2]}\right)\,.$$

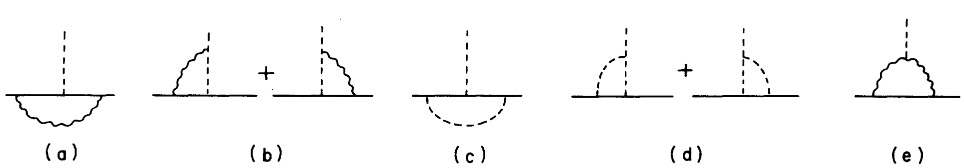

FIG. 5. One-loop corrections to the pion-nucleon form factor.

The apparent logarithmically divergent piece disappears and the leading contribution behaves like $\alpha(m^2/\mu^2)$ in the region where the external momenta are small compared with the vector-meson masses. [We note that $B \equiv 0$ when the left-right symmetry of the model is artificially realized $(g_L = g_R,\ v' = v'')$.] To arrive at this estimate we consider only the leading behavior of the numerator and denominator, which is legitimate since the denominator contains only massive particle propagators and is not plagued by infrared singularities. (A good check on our result is that $B \equiv 0$ when $m = 0$.) Thus in the form factor, parity violation is a calculable, weak phenomenon. This is certainly fine if we look at the term proportional to g_π, as shown here. However, this is not sufficient since we do not wish to treat g_π as a perturbative constant. However, we expect that our result is more generally valid.

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APPENDIX

We now give the vertices necessary for the pion mass calculation.

1. Coupling of ϕ with vector mesons

$$\phi A^2 \text{ terms:}\quad -\tfrac{1}{8}Fv'[g_L{}^2(A_{L1}{}^2+A_{L2}{}^2+A_{L3}{}^2)+g_Y{}^2B^2-2g_Lg_YA_{L3}B]-\tfrac{1}{4}Tv'g_Lg_YA_{L1}B+\tfrac{1}{4}T'v'g_Lg_YA_{L2}B,$$

$$\begin{aligned}\phi^2A^2 \text{ terms:}\quad &-\tfrac{1}{16}(F^2+F'^2)[g_L{}^2(A_{L1}{}^2+A_{L2}{}^2+A_{L3}{}^2)+g_Y{}^2B^2-2g_Lg_YA_{L3}B]\\ &-\tfrac{1}{16}(T^2+T'^2)[g_L{}^2(A_{L1}{}^2+A_{L2}{}^2+A_{L3}{}^2)+g_Y{}^2B^2+2g_Lg_YA_{L3}B].\end{aligned}$$

2. Coupling of ρ with vector mesons

$$\rho A^2 \text{ terms:}\quad -\tfrac{1}{8}Pv''[g_R{}^2(A_{R1}{}^2+A_{R2}{}^2+A_{R3}{}^2)+g_Y{}^2B^2-2g_Rg_YA_{R3}B]-\tfrac{1}{4}Sv''g_Rg_YA_{R1}B+\tfrac{1}{4}S'v''g_Rg_YA_{R2}B,$$

$$\begin{aligned}\rho^2A^2 \text{ terms:}\quad &-\tfrac{1}{16}(P^2+P'^2)[g_R{}^2(A_{R1}{}^2+A_{R2}{}^2+A_{R3}{}^2)+g_Y{}^2B^2-2g_Rg_YA_{R3}B]\\ &-\tfrac{1}{16}(S^2+S'^2)[g_R{}^2(A_{R1}{}^2+A_{R2}{}^2+A_{R3}{}^2)+g_Y{}^2B^2+2g_Rg_YA_{R3}B].\end{aligned}$$

3. Coupling of H with vector mesons

$$\begin{aligned}HA^2 \text{ terms:}\quad &-\tfrac{1}{4}v\sigma[g_L{}^2(A_{L1}{}^2+A_{L2}{}^2+A_{L3}{}^2)+g_R{}^2(A_{R1}{}^2+A_{R2}{}^2+A_{R3}{}^2)+2g_Lg_R(A_{R1}A_{L1}+A_{R2}A_{L2}-A_{R3}A_{L3})]\\ &+\tfrac{1}{2}vUg_Lg_R(A_{L2}A_{R3}-A_{R2}A_{L3})+\tfrac{1}{2}vU'g_Lg_R(A_{L1}A_{R3}-A_{R1}A_{L3}),\end{aligned}$$

$$\begin{aligned}H^2A^2 \text{ terms:}\quad &-\tfrac{1}{8}(\pi_0{}^2+\sigma^2)[g_L{}^2(A_{L1}{}^2+A_{L2}{}^2+A_{L3}{}^2)+g_R{}^2(A_{R1}{}^2+A_{R2}{}^2+A_{R3}{}^2)+2g_Lg_R(A_{R1}A_{L1}+A_{R2}A_{L2}-A_{R3}A_{L3})]\\ &-\tfrac{1}{8}U^2[g_L{}^2(A_{L1}{}^2+A_{L2}{}^2+A_{L3}{}^2)+g_R{}^2(A_{R1}{}^2+A_{R2}{}^2+A_{R3}{}^2)+2g_Lg_R(-A_{R1}A_{L1}+A_{R2}A_{L2}+A_{R3}A_{L3})]\\ &-\tfrac{1}{8}U'^2[g_L{}^2(A_{L1}{}^2+A_{L2}{}^2+A_{L3}{}^2)+g_R{}^2(A_{R1}{}^2+A_{R2}{}^2+A_{R3}{}^2)+2g_Lg_R(A_{R1}A_{L1}-A_{R2}A_{L2}+A_{R3}A_{L3})].\end{aligned}$$

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[1]S. Weinberg, Phys. Rev. D 5, 1962 (1972); H. Georgi and S. L. Glashow, *ibid.* 6, 2977 (1972).

[2]S. Weinberg, Phys. Rev. Lett. 29, 388 (1972).

[3]P. Higgs, Phys. Lett. 12, 132 (1964); Phys. Rev. Lett. 13, 508 (1964); Phys. Rev. 145, 1156 (1966); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, *ibid.* 13, 585 (1964); T. W. B. Kibble, Phys. Rev. 155, 1554 (1967).

[4]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973); H. Pagels, *ibid.* 7, 3689 (1973); R. Jackiw and K. Johnson, *ibid.* 8, 2386 (1973); J. M. Cornwall and R. E. Norton, *ibid.* 8, 3338 (1973).

[5]D. Z. Freedman and W. Kummer, Phys. Rev. D 7, 1829 (1973); A. Duncan and P. Schattner, *ibid.* 7, 1861 (1973); S.-Y. Pi, Phys. Rev. D 7, 3750 (1973).

[6]T. Hagiwara and B. W. Lee, Phys. Rev. D 7, 459 (1973).

[7]For other attempts to put pions into weak-electromagnetic gauge theories see J. Schechter and Y. Ueda, Phys. Rev. D 5, 2846 (1972); W. F. Palmer, *ibid.* 6, 1190 (1972); M. Weinstein, *ibid.* 7, 1854 (1973); D. A. Dicus and V. S. Mathur, *ibid.* 7, 525 (1973).

[8]S. Weinberg, Phys. Rev. Lett. 29, 1698 (1972); S. Weinberg, Phys. Rev. D 7, 2887 (1973).

[9]It need remain invariant only for some finite neighborhood of the parameters.

[10]I. Bars and K. Lane, Phys. Rev. D 8, 1169 (1973); 8, 1257 (1973).
[11]S. Weinberg, Phys. Rev. D 8, 605 (1973); 8, 4482 (1973).
[12]S. Y. Lee, J. M. Rawls, and L.-P. Yu, University of California (San Diego) report, 1973 (unpublished).
[13]We use the metric $g^{\mu\nu}$ with nonzero elements $(-1, +1, +1, +1)$ for $\mu = \nu = 0, 1, 2, 3$.

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Minimal hierarchy of Faddeev-type equations for N-particle scattering*

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Operators are introduced, which make up the kernels of all N-body Faddeev-type scattering equations. The hierarchy of these operators, and the equations they satisfy, are shown to provide the minimal description of all Faddeev-type formalisms. By means of this hierarchy, the N-body formalisms proposed by Yakubovskii and by Alt, Grassberger, and Sandhas are shown to be equivalent. The four-body case is treated in some detail.

I. INTRODUCTION

After the pioneering work of Faddeev on the formulation and solution of the three-body problem,[1] several attempts have been made to obtain viable systems of equations for N particles. Among these, two approaches are particularly interesting, namely the ones presented by Yakubovskii[2] and by Alt, Grassberger, and Sandhas[3] (AGS).

The method of Yakubovskii relies on a powerful index notation to handle the channel structure arising from the separation of N particles into subgroups. It can be understood as a repeated application of the Faddeev procedure of removing from the kernel of the N-body Lippmann-Schwinger (LS) equations the pieces representing disconnected subprocesses.

The AGS approach is based on a scheme for writing down three-body relations as matrix versions of two-body relations; in particular the three-body Faddeev equations correspond to matrix Lippmann-Schwinger-type equations. For the four-body case, a matrix version of the Faddeev procedure is applied to such LS equations, and the resulting Faddeev-type matrix equations are again written in two-body-like form. In this way, an inductive prescription is established for the generation of matrices of operators for the N-body case; their equations are obtained by simply writing down the N-body matrix version of the corresponding two-body relations.

In this paper we show that these two approaches are equivalent, but that neither of them provides the most concise description of the hierarchy of N-body equations with Faddeev-type kernels. By generalizing to the N-body case an alternative formalism based on the three-body K operators,[4] we obtain a hierarchy of equations for precisely the operators of the kernels, and we identify these as the minimal hierarchy for the N-body problem.

A detailed description of the Faddeev-Yakubovskii (FY) procedure for $N=4$ is given in Sec. II, where, in addition to reproducing the Yakubovskii[2] results, we obtain symmetric four-body M operators that form a more natural generalization of the three-body Faddeev $M_{\beta\alpha}$'s than the operators obtained by Yakubovskii.

The AGS formalism for $N=4$ is outlined in Sec. III; M operators are also obtained within this scheme.

In Sec. IV, the three-body K operators and their equations are generalized to the four-body case. In order to see the relevance of the K operators, we show that all the four-body equations obtained in Secs. II to IV have the same kernel, namely maximal[5] subsystem K operators. In other words, just as the two-body t operators make up the three-body Faddeev kernel, the (3+1)- and (2+2)-subsystem K operators make up the four-body Faddeev-type kernel. Since they produce four-body equations with identical kernels, we conclude that the AGS and FY four-body formalisms are equivalent. We end Sec. IV with a detailed explanation of the minimal characteristics of the N-body K-operator hierarchy and its relation to other hierarchies.

The N-body scattering problem is treated in Sec. V. The K-operator hierarchy is constructed within both the AGS and FY formalisms, thereby proving that the N-body equations of both formalisms have identical kernels. The equivalence of