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Multi-Regge limit of the Neveu-Schwarz model

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The structure of the Neveu-Schwarz model in multi-Regge limits is studied, with particular emphasis on quantum-number effects on Reggeon-particle vertex functions.

I. INTRODUCTION

The Regge analysis of inclusive reactions[1] and its apparent consequences in terms of decoupling theorems for a Pomeron pole at $\alpha(0)=1$ (see Ref. 2) has led to a renewed interest in the structure of Reggeon-particle vertex functions.[3] The prime tool in investigating these has been to exploit specific models to give clues about the general structure of multiparticle amplitudes in various limits. Apart from the ladder-graph model and its more sophisticated version, the Gribov calculus,[3] the most popular model to date has been the conventional dual resonance model (CDRM), which has yielded many insights into the structure of multiparticle amplitudes.[4]

In view of the above, we thought it worthwhile to look at the Regge limits of a more complicated, but theoretically equally successful dual resonance model, the Neveu-Schwarz model[5,6] (NSM). A new feature in the NSM as compared to the CDRM is that there are four leading odd-G-parity trajectories arranged in two exchange-degenerate (EXD) pairs of opposite normality (τP). Thus quantum-number effects on the vertices can be studied.

In Sec. II of the paper we derive the multi-Regge limit of the Neveu-Schwarz amplitude for six pions. In Sec. III the odd-G-parity trajectories are separated by explicitly summing the ω-A_2 contribution in the manner of Van Hove, and the Reggeon vertices are exhibited.

Two of us have repeatedly emphasized that the Pomeron decoupling theorem of Brower and Weis[2] involves an extra assumption about the dependence of the amplitude on the variable ϕ [Eq. (2.3)] that cannot be deduced on general grounds. However, all the simple models studied up to date seem to be in agreement with this assumption, and we want to point out that the NSM does not provide an exception. As explained in Secs. II and III, the NSM seems to have a modified ϕ dependence as compared to the CDRM, but this is in fact due to a quantum-number effect associated with ω-A_2 exchange.

II. THE MULTI-REGGE LIMIT

The six-pion amplitude (Fig. 1) in the NSM is given by[5]

$$A_6=\int_0^1 du_{12}\int_0^1 du_{13}\int_0^1 du_{14}V_6 Y\,, \qquad (2.1)$$

where

$$\begin{aligned}Y=&(2p_4\cdot p_5)(2p_2\cdot p_3)u_{12}u_{34}u_{14}\\&+(2p_2\cdot p_5)(2p_3\cdot p_4)u_{13}u_{12}u_{23}u_{14}u_{45}\\&-(2p_3\cdot p_5)(2p_2\cdot p_4)u_{13}u_{12}u_{14}u_{23}u_{34}u_{45}\,,\end{aligned} \qquad (2.2)$$

and V_6 is the integrand of the six-point function in the CDRM. The well-known duality constraints[7] express the variables u_{ij} in terms of the three variables u_{12}, u_{13}, u_{14}, which we have chosen to be independent.

The multi-Regge limit is defined by $s_{12}\equiv t_1$ = fixed, $s_{13}\equiv t_2$ = fixed, $s_{14}\equiv t_3$ = fixed, $s_{23}\equiv s_1\to-\infty$, $s_{34}\equiv s_2\to-\infty$, $s_{45}\equiv s_3\to-\infty$:

$$\eta_1\equiv\frac{s_1s_2}{s_{24}}=\text{fixed}\,,\quad \eta_2\equiv\frac{s_2s_3}{s_{35}}=\text{fixed}\,,$$
$$\phi\equiv\frac{s_{16}s_2}{s_{24}s_{35}}=\text{fixed}\,. \qquad (2.3)$$

As is well known, in 4-dimensional space-time (or indeed, in any space with only one timelike dimension), ϕ approaches 1 in the multi-Regge limit:

$$1-\phi\sim\frac{\psi}{s_2}\,, \qquad (2.4)$$

ψ being a complicated function of the variables held fixed. As was discussed in Ref. 8 it is, however, important to consider A_6 as a function of ϕ; this corresponds to working in a space of higher

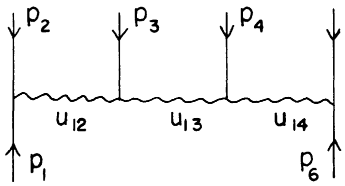

FIG. 1. The six-particle amplitude.

dimensions and appropriate signature; or to continuing analytically into complex values of the momenta $p_1, \ldots, p_6$.

The multi-Regge limit of (2.1) is calculated in the standard way by the change of variables

$$u_{12} = -\frac{x_1}{s_1}, \quad u_{13} = -\frac{x_2}{s_2}, \quad \text{and } u_{14} = -\frac{x_3}{s_3}. \tag{2.5}$$

Using

$$\begin{aligned} 2p_2 \cdot p_3 &= s_1 - 2m^2, \\ 2p_3 \cdot p_4 &= s_2 - 2m^2, \\ 2p_4 \cdot p_5 &= s_3 - 2m^2, \\ 2p_2 \cdot p_4 &= s_{24} - s_1 - s_2 + m^2, \\ 2p_3 \cdot p_5 &= s_{35} - s_2 - s_3 + m^2, \\ 2p_2 \cdot p_5 &= s_{16} + s_2 - s_{24} - s_{35}, \end{aligned} \tag{2.6}$$

we see that the leading behavior is given by

$$\begin{aligned} A_6 \sim\, & (-s_1)^{\alpha_1}(-s_2)^{\alpha_2}(-s_3)^{\alpha_3} \\ & \times \int_0^\infty dx_1 \int_0^\infty dx_2 \int_0^\infty dx_3\, x_1^{-\alpha_1} x_2^{-\alpha_2 - 1} x_3^{-\alpha_3} \exp(-x_1 - x_2 - x_3 + x_1 x_2/\eta_1 + x_2 x_3/\eta_2 - x_1 x_2 x_3 \phi/\eta_1\eta_2) \\ & \times \left(1 + \frac{x_2}{\eta_1\eta_2}[(1-\phi)s_2 + 2m^2\phi - \eta_1 - \eta_2] + \frac{x_2^2}{\eta_1\eta_2}\right). \end{aligned} \tag{2.7}$$

In obtaining (2.7) we have written

$$u_{34} = 1 - (1 - u_{34})$$

in the last term of (2.2) and used the duality constraint

$$1 - u_{34} = \frac{u_{13}(1 - u_{12})(1 - u_{14})}{(1 - u_{12}u_{13})(1 - u_{13}u_{14})}.$$

We see that A_6 has a modified ϕ dependence as compared to the CDRM. Due to relation (2.4), A_6 still has the normal Regge behavior (this is, however, not the case in a space with two timelike dimensions, where the behavior would be $s_2^{\alpha_2 + 1}$).

To understand the meaning of the second term in (2.7) we work in the conventional 4-dimensional space-time and impose the Gram-determinant conditions. For this purpose it is convenient to introduce the transverse (spacelike) momentum vectors $p_i^\perp$ $(i = 2, 3, 4, 5)$. Noting that

$$\begin{aligned} \eta_1 &= \frac{s_1 s_2}{s_{24}} = \frac{s_{35} s_1 \phi}{s_{16}} = m^2 - p_3^{\perp 2}, \\ \eta_2 &= \frac{s_2 s_3}{s_{35}} = \frac{s_{24} s_3 \phi}{s_{16}} = m^2 - p_4^{\perp 2}, \\ s_2 - (p_3^\perp + p_4^\perp)^2 &= \frac{s_{24} s_{35}}{s_{16}} = \frac{s_2}{\phi}, \end{aligned} \tag{2.8}$$

we observe that

$$(1-\phi)s_2 + 2m^2\phi - \eta_1 - \eta_2 = -2p_3^\perp \cdot p_4^\perp. \tag{2.9}$$

In 4-dimensional space-time the angle between $p_3^\perp$ and $p_4^\perp$ is uniquely determined by η_1, η_2, t_1, t_2, t_3, where

$$\begin{aligned} t_1 &= p_2^{\perp 2}, \\ t_2 &= (p_2^\perp + p_3^\perp)^2, \\ t_3 &= (p_2^\perp + p_3^\perp + p_4^\perp)^2. \end{aligned} \tag{2.10}$$

(In a space of more than 4 dimensions, the angle is arbitrary.)

Simple, but messy, calculations give

$$\begin{aligned} -2p_3^\perp \cdot p_4^\perp = & \frac{(t_1 - t_2 + \eta_1 - m^2)(t_3 - t_2 + \eta_2 - m^2)}{2t_2} \\ & - \frac{\{[(\eta_1 - m^2)^2 + 2(\eta_1 - m^2)(t_1 + t_2) + (t_1 - t_2)^2][(\eta_2 - m^2)^2 + 2(\eta_2 - m^2)(t_2 + t_3) + (t_3 - t_2)^2]\}^{1/2}}{2t_2}. \end{aligned} \tag{2.11}$$

Thus the second term in the large parentheses in (2.7) breaks up into a sum of two factorized terms. The second involves the factor

$$Z_1 = \{-[(\eta_1 - m^2)^2 + 2(\eta_1 - m^2)(t_1 + t_2) + (t_1 - t_2)^2]\}^{1/2}$$
$$= [-\eta_1 + m^2 + (\sqrt{-t_1} + \sqrt{-t_2})^2]^{1/2}[\eta_1 - m^2 - (\sqrt{-t_1} - \sqrt{-t_2})^2]^{1/2},$$

showing the thresholds and the pseudothresholds in the transverse momenta. If we reexpress Z_1 in terms of the Toller angle ω_1, related to η_1 by

$$\eta_1 = \frac{\lambda(t_1, t_2, m^2)}{2\sqrt{-t_1}\sqrt{-t_2}\cos\omega_1 - t_1 - t_2 + m^2},$$

we obtain

$$\frac{Z_1}{\eta_1} = 2\sqrt{-t_2}\,\sin\omega_1\left(\frac{-t_1}{\lambda(t_1, t_2, m^2)}\right)^{1/2}, \tag{2.12}$$

and so (2.7) can be rewritten as

$$A_6 \sim (-s_1)^{\alpha_1}(-s_2)^{\alpha_2}(-s_3)^{\alpha_3}\int_0^\infty dx_1\int_0^\infty dx_2\int_0^\infty dx_3\, x_1^{-\alpha_1}x_2^{-\alpha_2-1}x_3^{-\alpha_3}$$
$$\times\exp(-x_1 - x_2 - x_3 + x_1x_2/\eta_1 + x_2x_3/\eta_2 - x_1x_2x_3\phi/\eta_1\eta_2)$$
$$\times\left[1 + \frac{x_2}{2t_2}\frac{(t_1 - t_2 + \eta_1 - m^2)}{\eta_1}\frac{(t_3 - t_2 + \eta_2 - m^2)}{\eta_2}\right.$$
$$\left. + \frac{x_2^2}{\eta_1\eta_2} + 2x_2\sin\omega_1\sin\omega_2\left(\frac{-t_1}{\lambda(t_1, t_2, m^2)}\right)^{1/2}\left(\frac{-t_3}{\lambda(t_3, t_2, m^2)}\right)^{1/2}\right]. \tag{2.13}$$

The interpretation of various terms in (2.13) is as follows: In the even-G-parity channels α_1, α_3 we see the exchange of the EXD ρ-f_0 trajectories, including the eliminating of the ρ-tachyon poles at $\alpha_1 = 0$, $\alpha_3 = 0$. The analysis of the spectrum of states in the NSM[5,6] shows that in the odd-G-parity channel α_2 there are four leading trajectories, all with the trajectory function $\alpha_2(t_2) = \frac{1}{2} + t_2$, grouped into two EXD pairs with opposite normality (τP): the ω-A_2 trajectories with $\tau P = +1$ and the π trajectory and its partner, the first particle of which occurs at $\alpha_2 = 3$, with $\tau P = -1$. An ω-A_2 exchange in the α_2 channel clearly involves normality change at the internal Reggeon-Reggeon-particle vertices, and is thus expected to lead to a factor $\sin\omega_1\sin\omega_2$.[9] Thus we may identify the last term in (2.13) as representing the ω-A_2 exchange in the α_2 channel, whereas the exchange of the π and its partner (?) trajectory gives rise to the first three terms. That this is indeed the case is explicitly shown in Sec. III.

III. SEPARATING THE ODD-G-PARITY TRAJECTORY CONTRIBUTIONS

In the Fock space $\mathfrak{F}_2$,[6] a state on the (ω-A_2) trajectories of momentum k and spin J is given by

$$\frac{C}{[k^2(J-1)!]^{1/2}}$$
$$\times e(k)^{\mu_1\cdots\mu_J}\epsilon_{\mu_1}^{\alpha\beta\gamma}k_\alpha b_{0\beta}^\dagger b_{0\gamma}^\dagger a_{1\mu_2}^\dagger\cdots a_{1\mu_J}^\dagger|0, k\rangle, \tag{3.1}$$

where $e(k)^{\mu_1\cdots\mu_J}$ is the spin-J polarization tensor and C is an over-all normalization constant. Using (3.1) and the operator forms of the vertex and the propagator, we can calculate the amplitude A_J for the coupling of the state (3.1) to three pions (Fig. 2) and we obtain

$$A_J^m(p_1, p_2, p_3, k) = \frac{2C(-\sqrt{2})^{J-1}}{[k^2(J-1)!]^{1/2}}e(k, m)^{\mu_1\cdots\mu_J}\epsilon_{\mu_1}^{\alpha\beta\gamma}p_{1\alpha}p_{2\beta}p_{3\gamma}\int_0^1 dx\, x^{-\alpha_\rho(s_{12})}(1-x)^{-\alpha_\rho(s_{23})}$$
$$\times(p_3 + xp_2)_{\mu_2}\cdots(p_3 + xp_2)_{\mu_J}, \tag{3.2}$$

where m labels the different polarizations. $\alpha_\rho(s)$ is the ρ trajectory $1 + s$. We can now calculate the contribution of the ω-A_2 trajectory to A_6 by summing over all states (Fig. 3):

$$A_6^{\omega\text{-}A_2} = \sum_{J=1}^\infty \frac{1}{J - \alpha_2(t_2)}\sum_m A_J^m(p_1, p_2, p_3, k)$$
$$\times A_J^m(p_6, p_5, p_4, -k)^*. \tag{3.3}$$

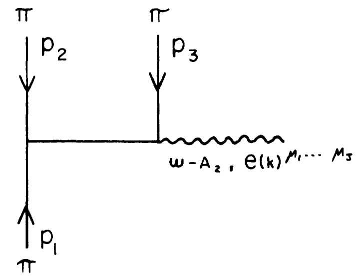

FIG. 2. The coupling of (ω-A_2) to three pions.

In (3.3) we replace the sum over the polarizations

$$\sum_m e(k,m)^{\mu_1\cdots\mu_J}e^*(k,m)^{\nu_1\cdots\nu_J}$$

by $g^{\mu_1\nu_1}g^{\mu_2\nu_2}\cdots g^{\mu_J\nu_J}$. This will of course admit states of lower spin than J into the Jth term in (3.3), but as we will later let $s_2\to-\infty$, only the highest spin states will survive, and the procedure is thus justified. With this understanding, we see that

$$A_6^{\omega\text{-}A_2}=\frac{4C^2}{t_2}\,\epsilon_\mu^{\alpha\beta\gamma}p_{1\alpha}p_{2\beta}p_{3\gamma}\epsilon^{\mu\lambda\rho\sigma}p_{6\lambda}p_{5\rho}p_{4\sigma}\times\int_0^1 dx_1x_1^{-\alpha_\rho(t_1)}(1-x_1)^{-\alpha_\rho(s_1)}\times\int_0^1 dx_3x_3^{-\alpha_\rho(t_3)}(1-x_3)^{-\alpha_\rho(s_3)}\times\sum_{J=1}^{\infty}\frac{1}{(J-1)!}\,\frac{1}{J-\alpha_2(t_2)}\times[2(p_3+x_1p_2)\cdot(p_4+x_3p_5)]^{J-1}. \quad (3.4)$$

The important thing to note is that (3.4) is proportional to the determinant

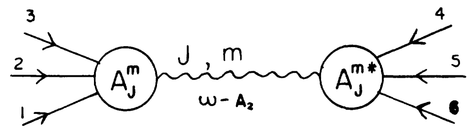

FIG. 3. The (ω-A_2) contribution to the six-particle amplitude.

$$\omega=\epsilon^{\mu\alpha\beta\gamma}p_{1\alpha}p_{2\beta}p_{3\gamma}\epsilon_\mu^{\lambda\rho\sigma}p_{6\lambda}p_{5\rho}p_{4\sigma}=\text{Det}(p_i\cdot p_j)\quad(i=1,2,3,\ j=4,5,6)\,. \quad (3.5)$$

The sum in (3.4) can be performed explicitly, using the formula

$$\sum_{n=0}^{\infty}\frac{1}{n+a}\,\frac{z^n}{n!}=(-z)^{-a}\gamma(a,-z)\,, \quad (3.6)$$

where $\gamma(a,z)$ is the incomplete γ function:

$$\gamma(a,z)=\int_0^z dt\,e^{-t}t^{a-1}\,. \quad (3.7)$$

Thus

$$A_6^{\omega\text{-}A_2}=-\frac{4C^2}{t_2}\,\omega\int_0^1 dx_1x_1^{-\alpha_\rho(t_1)}(1-x_1)^{-\alpha_\rho(s_1)}\times\int_0^1 dx_3x_3^{-\alpha_\rho(t_3)}(1-x_3)^{-\alpha_\rho(s_3)}\times[-2(p_3+x_1p_2)\cdot(p_4+x_3p_5)]^{\alpha_\omega(t_2)-1}\times\gamma(1-\alpha_\omega(t_2),\ -2(p_3+x_1p_2)\cdot(p_4+x_3p_5))\,. \quad (3.8)$$

The multi-Regge limit of the integral in (3.8) is readily computed by observing that the incomplete γ function approaches a complete γ function as $|s_2|\to\infty$. A tedious calculation shows that the limiting behavior of ω is given by

$$8\omega\sim s_1s_2s_3\left(\frac{2t_2}{\eta_1\eta_2}(1-\phi)s_2-\frac{1}{\eta_1\eta_2}[m^4-m^2(t_1+2t_2-t_3)+(t_1-t_2)(t_3-t_2)]+\frac{1}{\eta_1}(m^2-t_1-t_2)+\frac{1}{\eta_2}(m^2-t_2-t_3)-1\right)$$
$$\sim s_1s_2s_34t_2\sin\omega_1\sin\omega_2\left(\frac{-t_1}{\lambda(t_1,t_2,m^2)}\right)^{1/2}\left(\frac{-t_3}{\lambda(t_3,t_2,m^2)}\right)^{1/2}, \quad (3.9)$$

upon using the expressions of Sec. II. Finally, then, the contributions of the ω-A_2 trajectories are given by

$$A_6^{\omega\text{-}A_2}\sim C^2(-s_1)^{\alpha_1}(-s_2)^{\alpha_2}(-s_3)^{\alpha_3}2\sin\omega_1\sin\omega_2\left(\frac{-t_1}{\lambda(t_1,t_2,m^2)}\right)^{1/2}\left(\frac{-t_3}{\lambda(t_2,t_3,m^2)}\right)^{1/2}$$
$$\times\int_0^\infty dx_1\int_0^\infty dx_2\int_0^\infty dx_3\;x_1^{-\alpha_1}x_2^{-\alpha_2}x_3^{-\alpha_3}\exp(-x_1-x_2-x_3+x_1x_2/\eta_1+x_2x_3/\eta_2-x_1x_2x_3\phi/\eta_1\eta_2)\,. \quad (3.10)$$

This corresponds exactly to the last term of Eq. (2.13), and our claim that it was due to ω-A_2 exchange has thus been justified. It is interesting to note that the term containing the factor $(1-\phi)s_2$ arises entirely from the (ω-A_2)-exchange contribution. The modification of the ϕ dependence is thus attributed to the change of normality at the Reggeon-Reggeon-particle vertices.

The remaining three terms in (2.13) are due to the exchange of the π and its partner trajectory. To show that they collapse into one factorized term, we do the x_1, x_2, x_3 integrations, getting an expression in terms of Tricomi's Ψ functions[10]:

$$A^{\pi-?} \sim (-s_1)^{\alpha_1}(-s_2)^{\alpha_2}(-s_3)^{\alpha_3}(-\eta_1)^{1-\alpha_1}(-\eta_2)^{1-\alpha_3}\Gamma(1-\alpha_1)\Gamma(1-\alpha_3)\Gamma(-\alpha_2)$$
$$\times\Big[\Psi(1-\alpha_1, 2-\alpha_1+\alpha_2; -\eta_1)\Psi(1-\alpha_3, 2-\alpha_3+\alpha_2; -\eta_2)$$
$$-\frac{\alpha_2(\alpha_1-\alpha_2+\eta_1)(\alpha_3-\alpha_2+\eta_2)}{(2\alpha_2-1)\eta_1\eta_2}\Psi(1-\alpha_1, 1-\alpha_1+\alpha_2; -\eta_1)\Psi(1-\alpha_3, 1-\alpha_3+\alpha_2; -\eta_2)$$
$$+\frac{\alpha_2(\alpha_2-1)}{\eta_1\eta_2}\Psi(1-\alpha_1, -\alpha_1+\alpha_2; -\eta_1)\Psi(1-\alpha_3, -\alpha_3+\alpha_2; -\eta_2)\Big]. \tag{3.11}$$

Using a recursion relation between the three Ψ functions of the same argument[10] we can factorize (3.11) into the form

$$A_6^{\pi-?} \sim (-s_1)^{\alpha_1}(-s_2)^{\alpha_2}(-s_3)^{\alpha_3}(-\eta_1)^{-\alpha_1}(-\eta_2)^{-\alpha_3}\Gamma(1-\alpha_1)\Gamma(1-\alpha_3)\Gamma(-\alpha_2)\left(\frac{1-\alpha_2}{1-2\alpha_2}\right)$$
$$\times[(\alpha_1-\alpha_2+\eta_1)\Psi(1-\alpha_1, 1-\alpha_1+\alpha_2; -\eta_1)+(2\alpha_2-1)\Psi(1-\alpha_1, -\alpha_1+\alpha_2; -\eta_1)]$$
$$\times[(\alpha_3-\alpha_2+\eta_2)\Psi(1-\alpha_3, 1-\alpha_3+\alpha_2; -\eta_2)+(2\alpha_2-1)\Psi(1-\alpha_3, -\alpha_3+\alpha_2; -\eta_2)], \tag{3.12}$$

which explicitly shows the absence of a spin-1 state on the pion partner trajectory. The apparent pole at $\alpha_2=\frac{1}{2}$ is spurious. To see this, notice that as $\alpha_2=\frac{1}{2}+t_2$, $\alpha_2=\frac{1}{2}$ means $t_2=0$. At this point $\alpha_1-\alpha_2+\eta_1=\eta_1+t_1-m_\pi{}^2=0$, as $\eta_1=m_\pi{}^2-t_1+O(t_2)$. The same holds, of course, also for $\alpha_3-\alpha_2+\eta_2$.

We have checked that the residues of the poles of (3.12) in α_2 factorize into the appropriate limiting forms of the amplitudes coupling the states on the π-? trajectory, the spin-J on-shell state of which is given by

$$e^{\mu_1\cdots\mu_J}\left[\left(\frac{J-1}{(2J-1)J!}\right)^{1/2}a^\dagger_{1\mu_1}a^\dagger_{1\mu_2}+\left(\frac{J}{(2J-1)(J-2)!}\right)^{1/2}b^\dagger_{0\mu_1}b^\dagger_{1\mu_2}\right]a^\dagger_{1\mu_3}\cdots a^\dagger_{1\mu_J}|0\rangle, \tag{3.13}$$

to three pions. The identification of the terms in (2.13) is thus completed.

From Eqs. (3.10) and (3.12) we can now read off the forms of the Reggeon propagators and Reggeon-Reggeon-particle vertices. We write

$$A_6 \sim \beta^{\rho-f_0}_{\pi\pi}(t_1)P_{\rho-f_0}(s_1, \alpha_\rho(t_1))$$
$$\times[V^\pi_{(\rho-f_0)(\omega-A_2)}(\alpha_\rho(t_1), \alpha_\omega(t_2), \eta_1)P_{\omega-A_2}(s_2, \alpha_\omega(t_2))V^\pi_{(\omega-A_2)(\rho-f_0)}(\alpha_\omega(t_2), \alpha_\rho(t_3), \eta_2)$$
$$+V_{(\rho-f_0)(\pi-?)}(\alpha_\rho(t_1), \alpha_\pi(t_2), \eta_2)\,P_{\pi-?}\,(s_2, \alpha_\pi(t_2))V^\pi_{(\pi-?)(\rho-f_0)}(\alpha_\pi(t_2), \alpha_\rho(t_3), \eta_2)]P_{\rho-f_0}(s_3, \alpha_\rho(t_3))\beta^{\rho-f_0}_{\pi\pi}(t_3), \tag{3.14}$$

and find for the Reggeon propagators

$$P_{\rho-f_0}(s, \alpha)=\Gamma(1-\alpha_\rho)(-s)^{\alpha_\rho},$$
$$P_{\omega-A_2}(s, \alpha_\omega)=\Gamma(1-\alpha_\omega)(-s)^{\alpha_\omega}, \tag{3.15}$$
$$P_{\pi-?}(s, \alpha_\pi)=(1-\alpha_\pi)(1-2\alpha_\pi)\Gamma(-\alpha_\pi)(-s)^{\alpha_\pi}.$$

The vertex functions take the simplest form when they are written down in terms of complex (helicity) integrals:

$$V^\pi_{(\rho-f_0)(\omega-A_2)}(\alpha_\rho, \alpha_\omega, \eta)=\sin\omega\left(\frac{-2t_\rho}{\lambda(t_\rho, t_\omega, m^2)}\right)^{1/2}\frac{1}{\Gamma(1-\alpha_\rho)}\frac{1}{\Gamma(1-\alpha_\omega)}$$
$$\times\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}dm\,\Gamma(-m)\Gamma(m-\alpha_\rho+1)\Gamma(m-\alpha_\omega+1)(-\eta)^{-m} \tag{3.16}$$

$$V_{(\rho-f_0)(\pi-?)}(\alpha_\rho,\alpha_\pi,\eta)=-\frac{1}{\Gamma(1-\alpha_\rho)\Gamma(1-\alpha_\pi)}\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}dm\,\Gamma(-m)(-\eta)^{-m}\,\Gamma(m-\alpha_\rho)\Gamma(m-\alpha_\pi)\times\left[\frac{(m-\alpha_\rho)(m-\alpha_\pi)}{2\alpha_\pi-1}+\frac{m(\alpha_\rho-\alpha_\pi)}{2\alpha_\pi-1}+\frac{m(m-\alpha_\pi)}{1-\alpha_\pi}\right]. \quad (3.17)$$

Finally, as in the CDRM

$$\beta^{\rho}_{\pi\pi}(t)=1\;. \quad (3.18)$$

As has been mentioned previously, only the (ω-A_2)-exchange contribution contains factors modifying the ϕ dependence. However, this modification is associated with the change of normality and comes from the determinant ω (3.5). In the limit relevant for the inclusive reactions, $p_5=p_6$, and this determinant vanishes and thus decouples the (ω-A_2) contribution. A straightforward analysis along the lines of Ref. 8 shows that in this limit only the π-? trajectory contribution survives. As its ϕ dependence is essentially of the standard form, this contribution does not provide a counterexample to the decoupling theorems.

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