PHYSICAL REVIEW D VOLUME 9, NUMBER 6 15 MARCH 1974

f-g version of the SL(6,C) gauge theory

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(Received 11 June 1973)

The recently proposed SL(6,C) gauge-invariant theory describing a nonet of tensor mesons (f) interacting with hadrons is written in a generally covariant formulation. Assuming that leptons interact directly with gravitons (g), a generally covariant as well as SL(6,C) gauge-invariant f-g mixing term is introduced to describe the gravitational interaction of hadrons as well as to provide masses for the SU(3)-octet f's and for a singlet mixture of the f and g fields. The gauge symmetries are broken spontaneously and it is shown that a second orthogonal combination of f-g fields remains massless, representing "true" gravity.

I. INTRODUCTION

The aim of this paper is to indicate how the recently proposed SL(6, C) gauge model for strong interactions[1] can be generalized so as to include the gravitational interaction. The idea of tensor dominance of hadronic form factors can thereby be incorporated within a Lagrangian framework. A scheme of this sort was proposed a few years ago in which a strongly interacting tensor gauge field was caused to mix with the gravitational field.[2] A defect of that scheme, however, was the absence of SU(3) from the hadronic gauge symmetry: The tensor meson was an SU(3) singlet. This point can now be rectified by imposing SL(6, C) gauge symmetry together with general covariance on the strong interactions. A Lagrangian density describing strong SL(6, C) gauge-invariant interaction of hadrons together with the gravitational interaction of hadrons and leptons is proposed which consists of three distinct pieces[3]:

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_m + \mathcal{L}_w \, , \tag{1.1}$$

where $\mathcal{L}_s$ consists entirely of hadronic fields and is invariant with respect to both local SL(6, C) symmetry and general coordinate transformations; $\mathcal{L}_w$ involves the leptonic and gravitational fields and is generally covariant; $\mathcal{L}_m$ is a covariant mixing term which involves both the hadronic gauge fields and the gravitational field. The entire Lagrangian is invariant with respect to both general coordinate transformations and SL(6, C) gauge transformations.

In this simplified model—where weak and electromagnetic interactions are ignored— it is the presence of $\mathcal{L}_m$ which causes the leptonic and hadronic worlds to communicate. This term causes a mixing between the graviton and the singlet part of the SL(6, C) nonet. At the same time, it serves as a mass term for the hadronic gauge fields. In the absence of such a term, both the graviton and the hadron singlet would be without mass since both $\mathcal{L}_s$ and $\mathcal{L}_w$ are required to be generally covariant.

It must be emphasized that both general covariance and local SL(6, C) are to be treated as spontaneously broken gauge symmetries. There are important distinctions however. The well-known Higgs-Kibble mechanism is at least partly operative in the case of SL(6, C) in that it is possible to construct a generally covariant and SL(6, C)-invariant Lagrangian $\mathcal{L}_s$ in such a way that the *octet* part of the gauge field acquires mass. The *singlet* part (if no mixing term $\mathcal{L}_m$ is present) must, however, remain without mass on account of general covariance. Likewise the graviton field, regarded as part of the lepton system, must remain without mass in spite of the spontaneous breakdown of general covariance.[4] However, when the singlet part of the hadronic system is allowed to mix with the graviton, then it can happen that only *one* combination of these fields remains massless while the orthogonal combination becomes massive. In the following we shall exhibit an expression for $\mathcal{L}_m$ which serves both to mix the singlet parts in a suitable way and, at the same time, to give a mass to the octet. We do not claim that this expression is the only possible one.[5]

II. NOTATION

To establish the notation, we briefly review the structure and transformation properties of the gauge fields.[6] The graviton which couples to leptons is characterized by the system

$$L^{\mu}=L^{\mu a}\gamma_a\,,\qquad B_{\mu}=\tfrac{1}{4}B_{\mu[ab]}\sigma_{ab}\,, \tag{2.1}$$

where γ_a, σ_{ab} denote the usual Dirac matrices. Latin indices take the values 0, 1, 2, 3 and are summed relative to the Minkowski metric. Greek indices take the same values, but must be raised and lowered by means of the metric tensor whose contravariant components are defined by

$$g^{\mu\nu}=L^{\mu a}L^{\nu a}=\tfrac{1}{4}\mathrm{Tr}(L^{\mu}L^{\nu})\,. \tag{2.2}$$

The Lagrangian density which governs the motion of the graviton fields is given by

$$\mathcal{L}_g=\frac{i}{8\kappa^2}\,g^{1/2}\,\mathrm{Tr}([L^{\mu},L^{\nu}](\partial_{\mu}B_{\nu}-\partial_{\nu}B_{\mu}+i[B_{\mu},B_{\nu}]))\,, \tag{2.3}$$

where $8\pi\kappa^2$ represents the Newtonian constant, and the symbol $g^{1/2}$ denotes the scalar density

$$g^{1/2}=|\det L^{\mu a}|^{-1}\,. \tag{2.4}$$

The Lagrangian density for a typical lepton field (Dirac spinor) in the presence of the gravitational field is given by

$$\mathcal{L}_{\mathrm{lep}}=g^{1/2}[\tfrac{1}{2}i(\bar{\psi}L^{\mu}\nabla_{\mu}\psi-\nabla_{\mu}\bar{\psi}L^{\mu}\psi)-m\bar{\psi}\psi]\,, \tag{2.5}$$

where $\nabla_{\mu}\psi=\partial_{\mu}\psi+iB_{\mu}\psi$ is an SL(2, C)-covariant derivative. The sum of (2.3) and (2.5) is denoted by $\mathcal{L}_w$ in (1.1). This Lagrangian is invariant with respect to the local SL(2, C) transformations:

$$\begin{aligned} &L^{\mu}\to\Omega L^{\mu}\Omega^{-1}\,,\\ &B_{\mu}\to\Omega B_{\mu}\Omega^{-1}+\frac{1}{i}\,\Omega\partial_{\mu}\Omega^{-1}\,,\\ &\psi\to\Omega\psi\,,\\ &\bar{\psi}\to\bar{\psi}\Omega^{-1}\,,\end{aligned} \tag{2.6}$$

where $\Omega=\exp[i\omega_{ab}(x)\sigma_{ab}]$. It is, moreover, a scalar density with respect to the general coordinate transformations:

$$\begin{aligned} &L^{\mu}(x)\to L'^{\mu}(x')=\frac{\partial x'^{\mu}}{\partial x^{\nu}}L^{\nu}(x)\,,\\ &B_{\mu}(x)\to B'_{\mu}(x')=\frac{\partial x^{\nu}}{\partial x'^{\mu}}B_{\nu}(x)\,,\\ &\psi(x)\to\psi'(x')=\psi(x)\,,\\ &\bar{\psi}(x)\to\bar{\psi}'(x')=\bar{\psi}(x)\,.\end{aligned} \tag{2.7}$$

The hadronic gauge system involves the fields $\underline{L}^{\mu}$ and $\underline{B}_{\mu}$ given by

$$\begin{aligned} &\underline{L}^{\mu}=(\underline{L}_j^{\mu a}\gamma_a+\underline{L}_j^{\mu a 5}\,i\gamma_a\gamma_5)\lambda^j\,,\\ &\underline{B}_{\mu}=(\underline{B}^j_{\mu}+\tfrac{1}{2}\underline{B}^j_{\mu[ab]}\sigma_{ab}+\underline{B}^j_{\mu 5}\gamma_5)\tfrac{1}{2}\lambda^j\,,\end{aligned} \tag{2.8}$$

where the components $\underline{B}^j_{\mu}, \underline{B}^j_{\mu 5}$ ($j=1,\ldots,8$), and $\underline{B}^j_{\mu[ab]}, \underline{L}_j^{\mu a}, \underline{L}_j^{\mu a 5}$ ($j=0,1,\ldots,8$), are all real.

The hadronic parts of the Lagrangian density are given by expressions formally identical to (2.3) and (2.5) except for two details. The gravitational parameter κ^2 in (2.3) is replaced by a strong parameter[7] $2\underline{\kappa}^2$ (~ 1 GeV^{-2}), and the density factor $g^{1/2}$ is replaced by the quantity

$$f^{1/2}=|\det f^{\mu\nu}|^{-1/2}\,, \tag{2.9}$$

where the tensor $f^{\mu\nu}$ is defined by

$$f^{\mu\nu}=\tfrac{1}{8}\mathrm{Tr}(\underline{L}^{\mu}\underline{L}^{\nu})\,. \tag{2.10}$$

The trace is carried over both Dirac and SU(3) indices. The Lagrangian so defined is a scalar density with respect to the coordinate transformations analogous to (2.7) and is a scalar with respect to the local SL(6, C) transformations which are analogous to (2.6), with Ω given by

$$\Omega=\exp\{i[\omega^j(x)+\tfrac{1}{2}\omega^j_{ab}(x)\sigma_{ab}+\omega^j_5(x)\gamma_5]\tfrac{1}{2}\lambda^j\}\,,$$

where ω^j, ω_5^j ($j=1,\ldots,8$), and ω^j_{ab} ($j=0,1,\ldots,8$) are all real.

In practice it is necessary to introduce a set of auxiliary gauge fields into the hadronic system and to impose a set of covariant constraints. The details of this part of the program, which is rather intricate, can be found in Ref. 1.

III. THE MIXING TERM

Having sketched a procedure for setting up a physics of hadrons with gauged interactions ($\mathcal{L}_s$) and for a separate world of leptons in interaction with gravity ($\mathcal{L}_w$), we come now to the problem of making these systems communicate. Our proposal is that this communication should be through the gravitational field by making it couple to the hadronic gauge fields in a mixing term $\mathcal{L}_m$. To make this interaction as simple as possible, we shall assume that no derivatives are involved in the mixing term, $\mathcal{L}_m=\mathcal{L}_m(L,\underline{L})$.

The mixing term must be a scalar density with respect to general coordinate transformations and it must be invariant with respect to SL(6, C) transformations on $\underline{L}$ and with respect to SL(2, C) transformations on L. That is, we are demanding that all symmetry breaking in this model be of a spontaneous character.

It should be emphasized that the spontaneous breaking of these symmetries is already implicit in the structures $\mathcal{L}_s$ and $\mathcal{L}_w$; it does not depend upon the presence of $\mathcal{L}_m$. To see this, consider, for example, the empty-space classical field equations derived from $\mathcal{L}_g$, (2.3). It is easy to verify that these equations are satisfied by $\langle L^\mu\rangle$ = constant and $\langle B_\mu\rangle = 0$. Such trivial solutions can be shown to be stable against small perturbations provided only that the determinant $\langle g^{1/2}\rangle$ is nonvanishing. These solutions are all equivalent and can be brought, by means of gauge transformations, into the standard form, $\langle L^\mu\rangle = \gamma^\mu$, $\langle B_\mu\rangle = 0$, which clearly exhibits Poincaré invariance but *not* general covariance or local SL(2, C) invariance. The latter symmetries are indeed broken in this classical approximation to the ground-state solution. Similar arguments apply to the hadronic gauge system[8] whose ground-state solution may be given in the standard form, $\langle \underline{L}^\mu\rangle = \gamma^\mu$, $\langle \underline{B}_\mu\rangle = 0$, which violates both general covariance and SL(6,C) but preserves SU(3).

We shall require that SU(3) and Poincaré invariance of the ground state persist when the mixing term is included. The most severe restriction on the form of $\mathcal{L}_m(L, \underline{L})$ comes from the requirement that the ground-state solution discussed above remain stable in the presence of $\mathcal{L}_m$. That is to say, the elementary excitations must carry positive energy. This condition is met by requiring that, for small perturbations about the vacuum values, the first-order variation of the action should vanish while the second-order variation takes the Pauli-Fierz form. One possible mixing term which does this is given by

$$\mathcal{L}_m(L,\underline{L}) = \frac{3M^2}{16\underline{\kappa}^2}\sqrt{-g}\,\mathrm{Tr}(1 - \tfrac{1}{2}g_{\mu\nu}\underline{L}^\mu\underline{L}^\nu - \tfrac{1}{48}g_{\mu\nu}g_{\alpha\beta}[\underline{L}^\mu,\underline{L}^\alpha][\underline{L}^\nu,\underline{L}^\beta]), \tag{3.1}$$

where $g_{\mu\nu} = g_{\mu\nu}(L)$ is obtained by inverting (2.2). This expression is manifestly covariant under general coordinate transformations and under local SL(6, C) and SL(2, C) transformations of $\underline{L}$ and L, respectively.

To test the suitability of (3.1) one must examine the the behavior of the action density under small perturbations about the vacuum solutions. That is, into the Lagrangian for the gauge fields,

$$\mathcal{L}_{\text{gauge}} = \frac{i}{8\kappa^2}(g)^{1/2}\mathrm{Tr}([L^\mu, L^\nu]B_{\mu\nu}) + \frac{i}{16\underline{\kappa}^2}(f)^{1/2}\mathrm{Tr}([\underline{L}^\mu,\underline{L}^\nu]\underline{B}_{\mu\nu}) + \mathcal{L}_m(L,\underline{L}), \tag{3.2}$$

one must substitute[9]

$$(g)^{1/4}L^\mu = \gamma^\mu + \kappa\phi^{\mu a}\gamma_a, \qquad (f)^{1/4}\underline{L}^\mu = \gamma^\mu + \underline{\kappa}\underline{\phi}_j^{\mu a}\gamma_a\lambda^j, \tag{3.3}$$

and treat the fields ϕ, $\underline{\phi}$, B, and $\underline{B}$ as small quantities. One finds (after elimination of the algebraic variables B and $\underline{B}$) the effective free Lagrangian for ϕ and $\underline{\phi}$,

$$\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\text{PF}}(\hat{\phi}, 0) + \mathcal{L}_{\text{PF}}(\hat{\underline{\phi}}, M_1) + \sum_{j=1}^{8}\mathcal{L}_{\text{PF}}(\underline{\phi}_j, M_8), \tag{3.4}$$

where $\mathcal{L}_{\text{PF}}(\phi, M)$ denotes the Pauli-Fierz form,

$$\mathcal{L}_{\text{PF}}(\phi,M) = \tfrac{1}{2}(\partial_\lambda\phi_{(\mu\nu)}\partial_\lambda\phi_{(\mu\nu)} - 2\partial_\mu\phi_{(\mu\lambda)}\partial_\nu\phi_{(\nu\lambda)} - \tfrac{1}{2}\partial_\lambda\phi_{(\mu\mu)}\partial_\lambda\phi_{(\nu\nu)}) - \tfrac{1}{2}M^2(\phi_{(\mu\nu)}\phi_{(\mu\nu)} - \phi_{(\mu\mu)}\phi_{(\nu\nu)}), \tag{3.5}$$

and involves only the symmetric part of the field $\phi_{\mu\nu}$. The orthogonal singlet combinations appearing in (3.4) are given by

$$\hat{\underline{\phi}}^{\mu\nu} = \frac{\sqrt{2}\,\underline{\kappa}\underline{\phi}_0^{\mu\nu} - \sqrt{3}\,\kappa\phi^{\mu\nu}}{(2\underline{\kappa}^2 + 3\kappa^2)^{1/2}}, \qquad \hat{\phi}^{\mu\nu} = \frac{\sqrt{3}\kappa\underline{\phi}_0^{\mu\nu} + \sqrt{2}\,\underline{\kappa}\phi^{\mu\nu}}{(2\underline{\kappa}^2 + 3\kappa^2)^{1/2}}. \tag{3.6}$$

The singlet and octet masses are given, respectively, by

$$M^2_{\text{singlet}} = \left(1 + \frac{3}{2}\frac{\kappa^2}{\underline{\kappa}^2}\right)M^2, \qquad M^2_{\text{octet}} = M^2. \tag{3.7}$$

We therefore conclude that the Lagrangian (3.2) indeed leads to a stable vacuum which is consistent with Poincaré and SU(3) invariance. The elementary excitations consist of a spin-2 octet with mass M_{octet}, a spin-2 singlet with mass M_{singlet} given by (3.7) and a helicity-2 singlet (or graviton). It is not clear at present whether the stability will persist when higher-order effects are taken into account.[5]

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[1]C. J. Isham, Abdus Salam, and J. Strathdee, Phys. Rev. D 8, 2600 (1973).

[2]C. J. Isham, Abdus Salam, and J. Strathdee, Phys. Rev. D 3, 867 (1971); B. Zumino, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser *et al.* [MIT Press, Cambridge, Mass. (1971)], Vol. II, p. 441.

[3]This scheme is the tensor analog of the vector-dominance model of T. D. Lee, N. M. Kroll, and B. Zumino [Phys. Rev. 157, 1376 (1967)]. Our model, however, is not renormalizable.

[4]See discussion in Sec. III.

[5]An alternative possibility is to replace $(g)^{1/2}$ in (3.1) by $(f)^{1/2}$ or indeed by a linear combination $[\det(\alpha g^{\mu\nu} + (1-\alpha) f^{\mu\nu})]^{-1/2}$. The mass relations (3.7) for this particular modification of the mass term are unaltered. However, there is the possibility of inventing other SL(6,C)-invariant modifications which would alter the ratio of singlet versus octet masses.

[6]Details can be found in Ref. 1 and, more briefly, in C. J. Isham, Abdus Salam, and J. Strathdee, Nuovo Cimento Lett. 5, 969 (1972).

[7]The replacement of κ^2 by $2\underline{\kappa}^2$ is indicated by the use of the Gell-Mann matrices λ^j in (2.8). These 3×3 matrices are conventionally normalized so that $\mathrm{Tr}(\lambda^i\lambda^j) = 2\delta^{ij}$, $i, j = 0, 1, \ldots, 8$.

[8]In the hadronic system it is necessary to take account of the covariant constraints discussed in Ref. 1. The main consequence of these constraints is the equation $\underline{L}_j^{\mu a 5} = 0$ which is valid in a suitably chosen gauge.

[9]The axial-vector part of L^μ can be omitted here in view of the constraint $\underline{L}_j^{\mu a 5} = 0$.