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Spin and localization for elementary systems

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One of us (A.J.K.) has found the position operator for the Lorentz-invariant localization of scalar mesons, electrons, neutrinos, and photons. It was found uniquely except for an unknown constant for the electrons. With a set of postulates we find uniquely the spin angular-momentum tensor for the electrons and the value of that constant. We also find some general common properties for all the particles considered.

I. INTRODUCTION

A. General

The Lorentz-invariant localization for elementary systems[1] was considered by one of us (A.J.K.).[2,3] In Ref. 2 (hereafter called I), the general consequences of imposing Lorentz invariance of localization (i.e., the physical consistency of the description of the localization by observers in different inertial frames) were derived and applied to nonzero-mass systems of spin 0 and $\frac{1}{2}$. In Ref. 3 (hereafter called II), the same procedure as in I was applied to zero-mass systems of spin 0, $\frac{1}{2}$, and 1. In all the cases considered, the position operator and its eigenfunctions were uniquely defined, except for a constant in the spin-$\frac{1}{2}$, nonzero-mass case (electrons), and their form was found explicitly.

The basic purpose of this paper is to determine the unknown constant for the electrons and to establish several physical relations between the physical systems considered in I and II. A comparison between the Lorentz-invariant approach and other approaches used to try to solve the localization problem may be found in Ref. 4, where an extensive literature on the subject is also given. For further, more recent works see, e.g., Ref. 5.

In what follows we shall use $\hbar = c = 1$, unless stated explicitly, and the same notation and conventions for the metric in space-time, indices, vectors, and Dirac matrices as in part V of Ref. 6. We consider only one-particle positive-energy states in the p representation, always denoting $p_0 = +(\vec{p}^{\,2} + m^2)^{1/2}$.

B. Outline of the argument

In Sec. II, we will consider the spin of the electrons, determine uniquely its position operator, and find some general properties for them. In Sec. III we establish some common properties of the physical systems considered in papers I and II. In Sec. III we summarize our results.

II. ELECTRONS

A. Spin

In paper I the Bargmann-Wigner formalism[7] for positive-energy states was used. It could be argued that the spin is then already well defined, because Bargmann and Wigner[7] consider that the spin is the antisymmetric second-rank tensor $\frac{1}{2}\sigma^{\mu\nu}$. But, the physical states must belong to the space $\mathcal{H}$ of the allowed wave functions,[1,7] a condition that we can satisfy with a projector Λ,[2] where

$$\Lambda = \Lambda^2, \quad \Lambda = (2m)^{-1}(p_\alpha \gamma^\alpha + m) . \tag{2.1}$$

Then, because of quantum mechanics, a necessary condition that any operator Ω must satisfy to be an observable is

$$[\Omega, \Lambda]\Lambda = 0 , \tag{2.2}$$

which expresses the fact that for any allowed wave function φ, that is, one for which $\Lambda\varphi = \varphi$, this operator does not project it out of $\mathcal{H}$, i.e., that if $\Omega\varphi = \Psi$, then $\Lambda\Psi = \Psi$.

For the antisymmetric second-rank tensor $\sigma^{\mu\nu}$, Eq. (2.2) does not hold, so that $\sigma^{\mu\nu}$ is only a formal spin, but not the observable physical spin; hence we must find the physical spin of the theory.

We shall state what we mean by a physical spin by imposing the following conditions:

(a) It is an antisymmetric second-rank tensor, $\frac{1}{2}\Sigma^{\mu\nu}\Lambda$ (we set here the factor $\frac{1}{2}$ for convenience);

(b) $[\Sigma^{\mu\nu}\Lambda, \vec{p}] = 0;$ (2.3)

(c) $[\Sigma^{\mu\nu}\Lambda, \Lambda]\Lambda = 0\,;$ (2.4)

(d) For eigenstates of momentum in the rest system $\frac{1}{2}\Sigma^{kl}\Lambda = M^{kl}\Lambda$, where[7]

$$M^{\mu\nu}=i(p^\mu\partial^\nu-p^\nu\partial^\mu)+\tfrac{1}{2}\sigma^{\mu\nu} \tag{2.5}$$

is the total angular-momentum tensor.

Because of assumption (a), the spin, being a covariant entity, has a Lorentz-invariant meaning. Also it is a part of a covariant splitting of the total angular-momentum tensor. Assumption (b) means that the spin is a translationally invariant concept, so that it does not depend on ∂_k. The most general antisymmetrical second-rank tensor in agreement with requirements (a) and (b) must be constructed with p^μ, $g_{\mu\nu}$, $\epsilon_{\mu\nu\alpha\beta}$, I, γ^μ, $\sigma^{\mu\nu}$, $\gamma^5\gamma^\mu$, and γ^5. There are only three independent antisymmetric second-rank tensors; they can be selected as $\sigma^{\mu\nu}$,

$$\epsilon_{\mu\nu\alpha\beta}p^\alpha\gamma^5\gamma^\beta,$$

and

$$p^\mu p_\lambda\sigma^{\lambda\nu}-p^\nu p_\lambda\sigma^{\lambda\mu}.$$

Taking into account requirement (c), we obtain

$$\begin{aligned}\Lambda\Sigma^{\mu\nu}\Lambda&=\Sigma^{\mu\nu}\Lambda\\&=C_1\Lambda\sigma^{\mu\nu}\Lambda+C_2\Lambda\epsilon^{\mu\nu\alpha\beta}p_\alpha\gamma_5\gamma_\beta\Lambda\\&\quad+C_3\Lambda(p^\mu p_\lambda\sigma^{\lambda\nu}-p^\nu p_\lambda\sigma^{\lambda\mu})\Lambda,\end{aligned} \tag{2.6}$$

where C_1, C_2, C_3 are invariants. Using Eq. (A4) and Eq. (A6) of the Appendix, we see that a redefinition of C_1 allows us to impose without loss of generality $C_2=C_3=0$. Then we obtain the result that

$$\Sigma^{\mu\nu}(C)\Lambda\equiv C\Lambda\sigma^{\mu\nu}\Lambda \tag{2.7}$$

is the only antisymmetrical second-rank tensor in the space of the allowed wave functions (that do not depend on ∂_k), and hence because of (a), (b), and (c) the physical spin tensor $\frac{1}{2}\Sigma^{\mu\nu}\Lambda$ for the electrons must be $\frac{1}{2}\Sigma^{\mu\nu}(C)\Lambda$ for some, up to now unknown, value of C.

Now imposing requirement (d) we find that $C=1$, and $\frac{1}{2}\Sigma^{\mu\nu}\Lambda$ is just the Hilgevoord-Wouthuysen spin tensor.[8-16] We must also notice that $\frac{1}{2}\Sigma^{\mu\nu}\Lambda$ is the Bargmann-Wigner[7] spin tensor $\frac{1}{2}\sigma^{\mu\nu}$ projected according to Eq. (2.7) into the space of the allowed wave functions. It is also a conserved quantity[8] and Eq. (A6),

$$\Sigma^{\mu\nu}p_\nu\Lambda=0, \tag{2.8}$$

is the covariant expression of the fact that the temporal components, Σ^{0k}, are zero in the rest system[8,9]; that is, that the electrons behave in accordance with the classical relativistic description.[8,17,18]

The space-space part of $\Sigma^{\mu\nu}$,

$$\vec{\Sigma}=(\Sigma^{23},\Sigma^{31},\Sigma^{12}), \tag{2.9}$$

is a three-vector operator whose components (as well as its norm) have for each eigenfunction of $p_H{}^2$ (as well as for each eigenfunction of $\vec{p}^2$), a bivaluated spectrum, since

$$(\Sigma^{kl})^2=I+m^{-2}p_H{}^2\quad(k\neq l), \tag{2.10}$$

where

$$\begin{aligned}p_H{}^2&=(p^k)^2+(p^l)^2,\\\vec{\Sigma}^2&=3I+2m^{-2}\vec{p}^2\,.\end{aligned} \tag{2.11}$$

From Eq. (2.10) and Eq. (2.11), we see that the commutation relations between the $\frac{1}{2}\vec{\Sigma}\Lambda$ components are not the usual ones. We must note that lack of the usual commutation relations between the $\frac{1}{2}\vec{\Sigma}\Lambda$ components is not in contradiction with the invariance requirement under rotations. The latter only implies that the components of the *total* angular momentum must have the usual commutation relations. We also see from Eq. (2.10) and Eq. (2.11) that the eigenvalues of $\vec{\Sigma}^2$ and of the $\vec{\Sigma}$ components reduce in the rest system to the ordinary values, as it should be.

B. Position operator

In paper I, Eq. (6.28), it was found that the k component of the position operator for electrons is

$$X^k=(p_0)^{-1}[(\tfrac{1}{2}+G)\Sigma^{0k}-M^{0k}]\Lambda, \tag{2.12}$$

where G was an unknown constant, which we want to now find.

This is the $t=0$ position operator in the Heisenberg picture; that is, the value at $t=0$ of the operator $X^k(t)$. Then, $X^k(t)=\exp(ip_0t)X^k\exp(-ip_0t)$ because $X^k=\Lambda X^k\Lambda$ and $H\Lambda=p_0\Lambda$ [where $p_0=+(\vec{p}^2+m^2)^{1/2}$]; we obtain

$$X^k(t)=X^k+(p_0)^{-1}p^kt\Lambda\,. \tag{2.13}$$

To determine G we shall only impose

$$\tfrac{1}{2}\vec{\Sigma}(C)\,\Lambda=\vec{M}\Lambda-\vec{X}(t)\times\vec{p}, \tag{2.14}$$

where $\vec{M}$ is the space-space part of the total angular-momentum tensor $M^{\mu\nu}$.

Notice that we are not using the previously found value $C=1$. Notice also that $\vec{\Sigma}(C)\Lambda$ is the space-space part of Eq. (2.7) and that it is not the physical spin because it remains to apply requirement (d) to determine the constant C. Equation (2.14) contains two undetermined parameters, C and G, but this equation is enough to determine them. Despite the appearance of Eq. (2.14), remember that $\vec{\Sigma}(C)$, $\vec{p}$, and $\vec{M}$ are constants of the motion and hence do not depend on time t.

The substitution of the expressions of $\vec{M}$ [Eq. (2.5)], $\vec{\Sigma}(C)\Lambda$ [Eq. (2.7)], and $\vec{X}(t)$ [Eq. (2.12)] into Eq. (2.14) gives

$$\tfrac{1}{2}(C-1)\Lambda\sigma^{kl}\Lambda=-Gp_0{}^{-1}i\Lambda(\alpha^kp^l-\alpha^lp^k)\Lambda\,. \tag{2.15}$$

Setting $\vec{p}=0$, Eq. (2.15) reads $\frac{1}{2}(C-1)\Lambda\sigma^{kl}\Lambda=0$; hence we obtain once more $C=1$ because $\Lambda\sigma^{kl}\Lambda|_{\vec{p}=0} \neq 0$. We keep $\vec{p}$ free again and replace $C=1$ in Eq. (2.15). Then, because $\Lambda(\alpha^k p^l - \alpha^l p^k)\Lambda \neq 0$, we can conclude that $G=0$. Then, the single imposition of Eq. (2.14) has led us to the conclusion that

$$C=1, \quad G=0. \tag{2.16}$$

We find that the orbital angular momentum

$$\vec{\mathcal{L}} = \vec{X}(t)\times\vec{p} = \vec{X}\times\vec{p} \tag{2.17}$$

is the space-space part of the orbital angular-momentum tensor defined by

$$\mathcal{L}^{\mu\nu} = M^{\mu\nu}\Lambda - \tfrac{1}{2}\Sigma^{\mu\nu}\Lambda . \tag{2.18}$$

We know that $\Lambda M^{\mu\nu}\Lambda = M^{\mu\nu}\Lambda$. Then by Eq. (2.5) and Eq. (2.7) we notice that $\mathcal{L}^{\mu\nu}$ defined by Eq. (2.18) is the Bargmann-Wigner orbital angular-momentum tensor

$$L^{\mu\nu} = i(p^\mu\partial^\nu - p^\nu\partial^\mu) \tag{2.19}$$

projected according to Λ into the space of the allowed wave functions.

For the $\mathcal{L}^{0k}$ components, if we define $X^0(t)$ such that

$$\mathcal{L}^{0k} = p^k X^0(t) - p^0 X^k(t), \tag{2.20}$$

then we find that

$$X^0(t) = t\Lambda . \tag{2.21}$$

This last result strongly depends on the ordering of factors in Eq. (2.20). For example, a symmetrization of the type (see, e.g., Refs. 19 and 16)

$$A^\mu : B^\nu \equiv \tfrac{1}{2}(A^\mu B^\nu + B^\nu A^\mu) \tag{2.22}$$

could be used. It must be noted that the ordering of factors in Eq. (2.17) [and hence in Eq. (2.14)] is irrelevant.

With $X^0(t)=t\Lambda$ we recover the c-number time that has been used in paper I. If we write $X^\mu(t)$, where $X^k(t)$ is given by Eq. (2.13) and $X^0(t)=t\Lambda$, we know that *it is not a 4-vector*,[2,20] but that it can be extended to a formally covariant operator[20] in the sense of Fleming.[19]

III. COMMON PROPERTIES FOR ELEMENTARY SYSTEMS

In this section the common properties of elementary systems with spin 0 and $\frac{1}{2}$ (zero and nonzero mass) and spin 1 (zero mass) are considered (these are the systems treated in papers I and II). In what follows, when we write the projection operator Λ (which projects into the space of the allowed wave functions) we refer to the projection operator associated with the corresponding elementary system considered in papers I and II. The explicit form of this projection operator for each case can be found in papers I and II. Notice that this convention is different from the one used in Sec. II, where Λ was used uniquely for electrons. Moreover, when we write an operator, we refer now to the operator associated with the corresponding elementary system considered in papers I and II.

Since we have found uniquely in Sec. II the position operator for electrons, and as for the remaining systems considered in papers I and II the position operator was also unique, we start in this section with the point of view that we know a well-defined position operator. We call the attention of the reader to the difference between the approach of this section and that of Sec. II.

In papers I and II the k component of the position operator at time $t=0$ was written

$$X^k = i\Lambda\partial_k\Lambda + \Lambda R^k\Lambda . \tag{3.1}$$

But from papers I and II, $R^k=0$ for all the particles except the electrons; for the electrons it follows from the present paper, Sec. II, that $G=0$, so that taking into account Eqs. (6.6a), (6.22), and (6.23) in paper I, we know that for this particle $R^k=0$ also. Then for all particles considered in papers I and II, $R^k=0$ so that

$$X^k(t) = i\Lambda\partial_k\Lambda + p^k p_0^{-1} t\Lambda \tag{3.2}$$

(this result was anticipated in the review paper 4 on the localization problem by one of us).

Defining

$$\mathcal{L}^{\mu\nu} = p^\nu X^\mu(t) - p^\mu X^\nu(t) , \tag{3.3}$$

where $X^0(t)=t\Lambda$ and $X^k(t)$ is given by Eq. (3.2), we find that $\mathcal{L}^{\mu\nu}$ is nothing more than the Bargmann-Wigner angular-momentum tensor[7] $L^{\mu\nu}$ [see, Eq. (2.19)] projected into the space of the allowed wave functions; that is,

$$\mathcal{L}^{\mu\nu} = \Lambda L^{\mu\nu}\Lambda . \tag{3.4}$$

Notice that here Eq. (3.3) is the definition of $\mathcal{L}^{\mu\nu}$, but that in Sec. II the definition was given by Eq. (2.18).

$\mathcal{L}^{\mu\nu}$ is an antisymmetric second-rank tensor, as is evident from Eq. (3.4), despite the fact that $X^\mu(t)$ is not a 4-vector.[2,20] Hence $X^\mu(t)$ is a quasi-covariant operator in the sense of Lugarini and Pauri.[21]

For nonzero spin, $L^{\mu\nu}$ does not agree with Eq. (2.2), and hence cannot represent the physical angular-momentum tensor of our theory, which is given by $\mathcal{L}^{\mu\nu}$. $\mathcal{L}^{\mu\nu}$ is a conserved quantity.

Although $X^\mu(t)$ is not a 4-vector operator, it can be extended to a formally covariant operator,[20] in the sense of Fleming.[19]

We now define the spin angular-momentum tensor $\frac{1}{2}\Sigma^{\mu\nu}\Lambda$ as

$$\tfrac{1}{2}\Sigma^{\mu\nu}\Lambda = M^{\mu\nu}\Lambda - \mathcal{L}^{\mu\nu}. \tag{3.5}$$

Notice that here Eq. (3.5) is the definition of $\Sigma^{\mu\nu}$, but that in Sec. II the definition was given by the set of postulates (a), (b), (c), and (d). $\Sigma^{\mu\nu}$ is a constant of the motion. $\frac{1}{2}\Sigma^{\mu\nu}\Lambda$ is nothing more than the Bargmann-Wigner[7] spin angular-momentum tensor $(M^{\mu\nu} - L^{\mu\nu})$ projected into the space of the allowed wave functions as is easily seen from Eqs. (3.4) and (3.5); that is,

$$\tfrac{1}{2}\Sigma^{\mu\nu}\Lambda = \Lambda(M^{\mu\nu} - L^{\mu\nu})\Lambda\,. \tag{3.6}$$

For nonzero spin $(M^{\mu\nu} - L^{\mu\nu})$ does not agree with Eq. (2.2), and hence it cannot represent the physical spin angular-momentum tensor of our theory, which is given by $\frac{1}{2}\Sigma^{\mu\nu}\Lambda$.

The helicity operator associated with the spin $\frac{1}{2}\vec{\Sigma}\Lambda$ is, for each case,

$$\frac{\vec{\Sigma}\Lambda\cdot\vec{p}}{2|\vec{p}|} = \frac{\vec{\sigma}\cdot\vec{p}}{2|\vec{p}|}\Lambda \text{ for electrons,} \tag{3.7}$$

$$\frac{\vec{\Sigma}\Lambda\cdot\vec{p}}{2|\vec{p}|} = \frac{\tilde{\omega}}{2}\Lambda \text{ for neutrinos,} \tag{3.8}$$

$$\frac{\vec{\Sigma}\Lambda\cdot\vec{p}}{2|\vec{p}|} = \tilde{\omega}\Lambda \text{ for photons,} \tag{3.9}$$

where $\tilde{\omega} = \pm 1$ is the helicity and is fixed.[3] The helicities found are the usual.

Since Λ is the identity for scalar mesons,[2,3] by Eq. (3.4) we see that $L^{\mu\nu}$ is the orbital angular-momentum tensor for scalar mesons. Hence Eq. (3.4) tells us that the orbital angular-momentum tensor corresponding to any of the elementary systems considered, is nothing more than the scalar mesons' angular-momentum tensor $L^{\mu\nu}$ projected into the space of the allowed wave functions. Then, from the definition of spin [see Eq. (3.5)] we recover a notion of spin similar to the nonrelativistic one.

We know[2,3] that the components of the position operator do not commute for the electrons, the neutrinos, or the photons. But from Eq. (3.2) written in standard units, it is evident (since Λ does not depend on $\hbar$) that[4]

$$\lim_{\hbar\to 0} \hbar^{-1}[X^k(t), X^l(t)] = 0\,. \tag{3.10}$$

This is an expected result for the classical limit of the position operator, where the Poisson brackets vanish.

IV. CONCLUSIONS

With four postulates we found in Sec. II that the spin of the electrons is the Hilgevoord-Wouthuysen[8] spin. Then we found the value of a parameter of the electron's position operator of paper I so that this operator is unique. In Sec. III we started from the point of view that we know the position operator for the particles considerated in papers I and II; then we wrote them as shown in Eq. (3.2). We define the angular-momentum tensor $\mathcal{L}^{\mu\nu}$ through Eq. (3.3) which is also given by Eq. (3.4). The spin angular-momentum tensor is defined by Eq. (3.5), which is also given by Eq. (3.6). Both $\mathcal{L}^{\mu\nu}$ and $\Sigma^{\mu\nu}$ are constants of motion. The helicity associated with $\Sigma^{\mu\nu}$ is the usual [see Eqs. (3.7), (3.8), and (3.9)]. Finally the position operator is consistent with the correct nonrelativistic limit [see Eq. (3.10)].

APPENDIX

We give here for reference a list of the projected 16 matrices of Dirac theory with the Λ given by Eq. (2.1):

$$(1)\ \Lambda\gamma^\mu\Lambda = m^{-1}p^\mu\Lambda\,, \tag{A1}$$

$$(2)\ \Lambda\sigma^{\mu\nu}\Lambda \equiv \Lambda(\tfrac{1}{2}i)[\gamma^\mu, \gamma^\nu]\Lambda = \sigma^{\mu\nu}\Lambda + im^{-1}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)\Lambda\,, \tag{A2}$$

$$(3)\ \Lambda\gamma^5\Lambda = 0\,, \tag{A3}$$

$$(4)\ \Lambda\gamma^5\gamma^\nu\Lambda = i(2m)^{-1}p_\mu\epsilon^{\mu\nu\alpha\beta}\Lambda\sigma_{\alpha\beta}\Lambda, \tag{A4}$$

where $\epsilon^{0123} = 1$, or also

$$(5)\ \Lambda\gamma^5\gamma^\nu\Lambda = \gamma^5\gamma^\nu\Lambda - m^{-1}\gamma^5 p^\nu\Lambda\,. \tag{A5}$$

For reference, also, we give the following relation:

$$\Lambda\sigma^{\mu\nu}p_\nu\Lambda = 0\,. \tag{A6}$$

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Equivalence of Yang-Feldman and action-principle quantization in pathological field theories*

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The quantization of pathological field theories, suffering noncausal propagation properties and the Johnson-Sudarshan effect, is investigated from both the Yang-Feldman and action-principle points of view. For a particularly simple system of this kind, the two quantization approaches are shown to be exactly equivalent. It is argued that the equivalence also holds for theories of interacting spin-3/2 fields.

I. INTRODUCTION

The pathological nature of theories with interacting spin-$\frac{3}{2}$ fields can be classified as type I, the noncausal propagation of disturbances, and type II, the Johnson-Sudarshan effect. The pathology of type I has been discussed by Velo and Zwanziger[1] for minimal couplings to the electromagnetic field and by Singh[2] for the interaction with pions and nucleons suggested by Nath *et al.*[3] The pathology of type II, discovered by Johnson and Sudarshan,[4] considering the electromagnetic interaction, refers to their result that the spinor anticommutator, positive-definite by form, in fact is indefinite if Schwinger's action principle[5] is used for quantization. That the pathology of type II also occurs for spin-$\frac{3}{2}$ fields in interaction with a pion and nucleon was shown by Hagen.[6]

Although there has been little controversy regarding the pathology of type I, the question of the quantization-method independence of the pathology of type II has received some attention, originating with the work of Gupta and Repko.[7] These authors noted that the simplest choice of canonical variables corresponding to the commutation relations of Johnson and Sudarshan, who considered the electromagnetic field as external, failed to satisfy Heisenberg's equation of motion in the fully quantized theory. Consistency required a transformation in the canonical variables, leading Gupta and Repko to suggest that the Johnson-Sudarshan anticommutator should be modified and thus that the pathology of type II may be in question. Later Kimel and Nath,[8] using a generalized Yang-Feldman[9] approach, verified that the canonical variable transformation of Ref. 7 was required but also showed that the Johnson-Sudarshan anticommutator was *invariant* under this transformation.[10] The origin of this invariance remained a puzzle which will be dealt with in the present paper. Similarly, the equivalence of the commutation relations obtained from the action principle and the Yang-Feldman approach for the interaction of a spin-$\frac{3}{2}$ field with a nucleon and a pion has been established as well.[11] Since perturbation techniques were applied in Refs. 8 and 11, the assertion that the two quantization methods yield equivalent results has been proved there only to second order in the coupling constant. Recently, however, Soo[12] has extended the methods of Ref. 8 to fourth order and