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Neutron-proton form factors. II

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In a recent letter we suggested that the momentum dependence of the exact meson propagator may be responsible for the dipole-like behavior of the nucleon electromagnetic form factors over the limited range of data available. In this paper we extend the analysis to include more than one vector-meson propagator, and, by assuming a simplified form for the imaginary part of the level shift, we calculate the form factors explicitly. The nucleon electromagnetic form-factor data are fitted for momentum transfer to 2 GeV/c assuming just one isovector-vector meson and two isoscalar-vector mesons. The resulting fit is much better than that obtained using the dipole formula, and minχ^2 occurs for values of the masses and widths of the vector mesons close to the experimental values of the ρ, ω, and φ resonances. The six independent meson-nucleon coupling constants are determined. Their values are consistent with the assumption that the nucleon charge and the magnetic moment distributions are extended, that is, that there are no hard-core contributions to the form factors. This assumption completely determines the isovector coupling constants and reduces the number of independent coupling constants to two. The pion and kaon charge form factors, as deduced from the nucleon charge form factors assuming universality, are compared with the available data. The pion form factors fit the data in both the spacelike and the timelike regions, including the resonance shape for the ρ contribution to the pion form factor. The kaon form-factor data disagree with the assumption of universality in both the spacelike and timelike regions.

I. INTRODUCTION

In recent years the vector-dominance model (VDM) has lost favor with theorists as a prescription for calculating the nucleon electromagnetic form factors because the model predicts a single-pole form whereas observations show a double-pole or dipole form. Because the isoscalar part of the nucleon form factors, from the VDM point of view, should depend upon a linear combination of the ω and φ meson propagators, it is possible to generate, over a limited range of data, a dipole-like dependence by a careful selection of the parameters. However, for relatively low momentum transfers, the isovector part of the form factors should be dominated by the ρ-meson contribution alone, little influence being made by more massive mesons such as the ρ'. Consequently, VDM would predict a single-pole behavior for the isovector contribution, whereas observations show a dipole form.

In a recent letter[1] we suggested that the single-pole form for the isovector-vector meson is too severe an approximation to the exact propagator, that the cut contribution cannot be ignored. This observation can be based upon the fact that the general expression for a single-particle propagator is a function of three parameters: the mass and width of the resonance and the decay threshold value. The single-pole form is extracted from the general expression only if both the width and threshold value are small compared to the mass. These conditions are not well satisfied for either the ρ, ω, or φ meson parameters.

In an attempt to elaborate this point in a model-independent way,[1] we expanded an expression for the exact propagator in powers of the momentum squared (k^2) to see if the parameters could be chosen in such a way as to approximate a double-pole behavior to order k^4. This expression drew criticism from two sources.[2]

The most direct way to establish our point is to calculate the vector-meson propagator and show that the resulting form factors are a significant improvement over either the single-pole approximation or the dipole form for an extended range of the momentum-transfer variable. We have done this for values $0 \leq k^2 \leq 4$ $(\text{GeV}/c)^2$, a range which includes all the existing data for the neutron form factors and the proton electric form factor, yet is sufficiently low enough that the ρ meson should be the dominant contribution to the isovector part of the form factors. The resulting minχ^2 values are shown in Table I. We find that the theory is sensitive (see Fig. 3) to the values of the masses and widths of the vector mesons and that those values, as shown in Table II, are close to the experimental masses of the ρ, ω, and φ mesons. Most importantly, the meson-nucleon coupling constants, determined by our fit, satisfy sum rules which imply that there is no direct coupling (hard-core terms) between the photon and the nucleon. On this basis the nucleon has only an "apparent" charge and an "apparent" magnetic moment since the photon must turn into a vector meson which in turn interacts strongly with the nucleon.

We find that our fit to the isovector part of the nucleon form factor also fits the ρ contribution to the pion charge form factor for both timelike and spacelike values of k^2. However, a fit to the K^+ charge form-factor data is possible only if the ρ, K coupling constant is taken to be zero.

II. THE NUCLEON FORM FACTORS

The Dirac charge and Pauli magnetic form factors, as obtained in our previous paper,[1] are

TABLE I. Comparison of minχ^2 for n data points.

Form factor	n	Monopole [a,d]	Dipole [b,c]	Exact propagator [a]
	(A) Momentum transfer 0–1 GeV/c			
$\mathcal{G}_E^p$	26	16	21	22
$\mathcal{G}_M^p$	13	26	40	10
$\mathcal{G}_E^n$	36	37	274	24
$\mathcal{G}_M^n$	15	171	19	28
Total	90	250	354	84
	(B) Momentum transfer 0–2 GeV/c			
$\mathcal{G}_E^p$	31	22	28	31
$\mathcal{G}_M^p$	27	74	131	52
$\mathcal{G}_E^n$	36	36	274	24
$\mathcal{G}_M^n$	15	151	19	30
Total	109	283	452	137

[a] Six adjustable parameters.

[b] Two adjustable parameters.

[c] The large minχ^2 which occurs for the dipole is due to the breakdown of the "scaling law."

[d] The large minχ^2 for the monopole reflects the inadequacy to fit both the isovector and isoscalar parts of the form factors.

$$f_1^{e,N} = \delta_{N,p} - i \sum_M f_1^{N,M} G_M(k^2),$$
$$f_2^{e,N} = (\mu_N/2M_N) - i \sum_M f_2^{N,M} G_M(k^2), \qquad (1)$$

where M is an index in one-to-one correspondence with the ρ, ω, and φ mesons. The letter N indicates the neutron (n) or proton (p) form factors and $\delta_{N,p}$ is one for $N=p$ and zero otherwise. Isotopic-spin invariance requires

$$f_i^{n,\rho} = -f_i^{p,\rho}, \quad f_i^{n,\omega} = f_i^{p,\omega}, \quad f_i^{n,\varphi} = f_i^{p,\varphi}, \qquad (2)$$

so that our theory has six independent coupling constants. In our notation the Sachs form factors are

$$\mathcal{G}_E^N = f_1^{e,N} - (k^2/2m_N) f_2^{e,N},$$
$$\mathcal{G}_M^N = f_1^{e,N} + 2m_N f_2^{e,N}. \qquad (3)$$

The quantity $G_M(k^2)$ is defined as before[3]:

$$G_M(k^2) = I_M'(0) + k^{-2}[I_M(0) - I_M(k^2)], \qquad (4a)$$

$$I_M(k^2) = \frac{m_{0M}{}^2}{2\pi i} \int_{\mathcal{C}} dm' \mathfrak{D}^{-1}(m', m_M, m_M^*) \times (k^2 + m'^2 - i\epsilon)^{-1}, \qquad (4b)$$

where $\mathfrak{D}(m', m_M, m_M^*)$ satisfies the dispersion relation

$$\mathfrak{D}(m', m_M, m_M^*) = (m' - m_M)(m' - m_M^*)\pi^{-1} \times \int_{\mathcal{C}''} dm'' \frac{I^M(m'')}{(m'' - m')(m'' - m_M)(m'' - m_M^*)}. \qquad (5)$$

The contours $\mathcal{C}$ and $\mathcal{C}''$ are defined in Fig. 1. The positions of the conjugate poles in the unphysical sheet are defined by

$$m_M \equiv m_{0M} - i(\Gamma_M/2), \qquad (6)$$

where m_{0M} and Γ_M correspond to the mass and width of the resonance. The poles in the integrand of Eq. (4b) at $m' = \pm ik$ are located on the physical

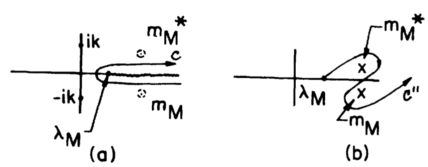

FIG. 1. The m' plane showing the singularities and the contours in the definitions of the functions (a) $I_M(k^2)$ and (b) $\mathfrak{D}^{-1}(m', m_M, m_M^*)$.

sheet outside of $\mathfrak{C}$. The quantity $I^M(m'')$ is the imaginary part of the level shift $R^M(m'')$:

$$R^M(m'') = D^M(m'') - iI^M(m''),$$
$$\mathfrak{D}(m'', m_M, m_M^*) = m'' - m_{0M} - R^M(m''). \tag{7}$$

We assume that the significant contribution to $\mathfrak{D}$ occurs for values of m'' well above threshold so that to a good approximation

$$I^M(m'') = \alpha_M m'', \quad m'' \geq \lambda_M,$$
$$=0, \quad m'' < \lambda_M, \tag{8}$$

where λ_M is the decay threshold—$2m_\pi$ for the ρ, $3m_\pi$ for the ω, and $2m_K$ for the φ. The strength α_M can be estimated from the decay rate

$$I^M(m_{0M}) = \alpha_M m_{0M} \approx (\Gamma_M/2). \tag{9}$$

A straightforward calculation gives

$$\mathfrak{D}(m', m_M, m_M^*) = -2m_{0M}\left\{\frac{\Gamma_M m'}{8\pi m_{0M}^2}\ln\left[\frac{(m'-\lambda_M)^2}{(m_M-\lambda_M)(m_M^*-\lambda_M)}\right] + \left[1 - \frac{m'}{m_{0M}} + \left(\frac{\Gamma_M}{2m_{0M}}\right)^2\right]\left[1 - \frac{i}{4\pi}\ln\left(\frac{m_M^*-\lambda_M}{m_M-\lambda_M}\right)\right]\right\}, \tag{10}$$

where the phase of $(m'-\lambda_M)$ is zero on the physical sheet above the cut, which can be taken from threshold to ∞ along the real axis, and Eq. (9) has been used to eliminate α_M. The points m_M and m_M^* in the logarithmic terms are on the physical sheet. This function has a logarithmic branch point at $m' = \lambda_M$ and conjugate zeros at $m' = m_M$ and $m' = m_M^*$ on the second or unphysical sheet.[4] It is easy to verify that $\mathfrak{D}$ reduces to the single-pole approximation

$$\mathfrak{D}(m', m_M, m_M^*) \approx m' - m_{0M} \tag{11a}$$

only if the logarithmic term containing m' is small and

$$(\Gamma_M/m_{0M}) \ll 1, \quad (\lambda_M/m_{0M}) \ll 1. \tag{11b}$$

The function I_M is now easily calculated using the Cauchy integral theorem since the integrand of Eq. (4b) vanishes sufficiently rapidly on an infinite circle in the m' plane. Consequently,

$$I_M(k^2) = (m_{0M}^2/2ik)[\mathfrak{D}^{-1}(ik, m_M, m_M^*) - \mathfrak{D}^{-1}(-ik, m_M, m_M^*)]. \tag{12}$$

Again for comparison purposes, Eq. (11) can be used to show that

$$I_M(k^2) \approx -m_{0M}^2(k^2 + m_{0M}^2)^{-1}, \tag{13}$$

the usual pole form for the single-particle propagator.

It should also be pointed out that $I_M(k^2)$ tends to zero as $(1/k^2)$ and consequently $G_M(k^2)$ tends to $I_M'(0)$ for large k^2. The large-k^2 behavior of the propagator is therefore consistent with the single-pole approximation so that there are no "hidden" factors of $(1/k^2)$ in Eq. (4) which could account for the dipole-like behavior. Our point therefore is that the "dipole" law is an empirical fit, not particularly good from a χ^2 point of view, which applies over a limited range of data. The propagator proposed in this paper will fit the data over this limited range but eventually, for very large momentum transfers, must follow the single-pole behavior.

Equations (1), (2), (4), (12), and (13) completely determine the form factors in terms of the six independent coupling constants $f_i^{N,M}$ for both the "exact" vector-meson propagator and the single-pole approximation. The data[5] for the neutron and proton form factors were fitted using a standard routine[6] which minimized the total χ^2. Fits to the data were obtained for the exact propagator, for the pole approximation, and for the dipole form, with the scaling law

$$\mathcal{G}_{DE}^p = (1 + k^2/0.71)^{-2}, \quad \mathcal{G}_{DE}^n = 0,$$
$$\mathcal{G}_{DM}^p = \mathcal{G}_{DE}^p(1+\mu_p), \quad \mathcal{G}_{DM}^n = \mathcal{G}_{DE}^p(\mu_n). \tag{14}$$

The results summarized in Table I and shown graphically in Fig. 2 were obtained assuming the masses and widths of the ρ, ω, and φ mesons fixed at their best experimental values.[7] As can be seen, the minχ^2 values for the exact propagator represent a considerable improvement over either the dipole or single-pole form. It might be argued that the dipole form with scaling is still more striking since it has only two adjustable parameters,[8] whereas the exact propagator formulation has six adjustable parameters. However, it is clear from Table I that the exact propagator fits the isovector part of the form factors

$$\mathcal{G}_{E,M}^V = \tfrac{1}{2}(\mathcal{G}_{E,M}^p - \mathcal{G}_{E,M}^n) \tag{15}$$

considerably better than the other two formulations. This part of the exact propagator also has only two adjustable parameters corresponding to $f_1^{p,\rho}$ and $f_2^{p,\rho}$, and as will be shown later, these can also be fixed by the assumption that the charge and magnetic distributions are extended.

From these results we conclude that the vector-dominance model provides an adequate description for the nucleon electromagnetic form factors at least for momentum transfers up to 2 GeV/c. In

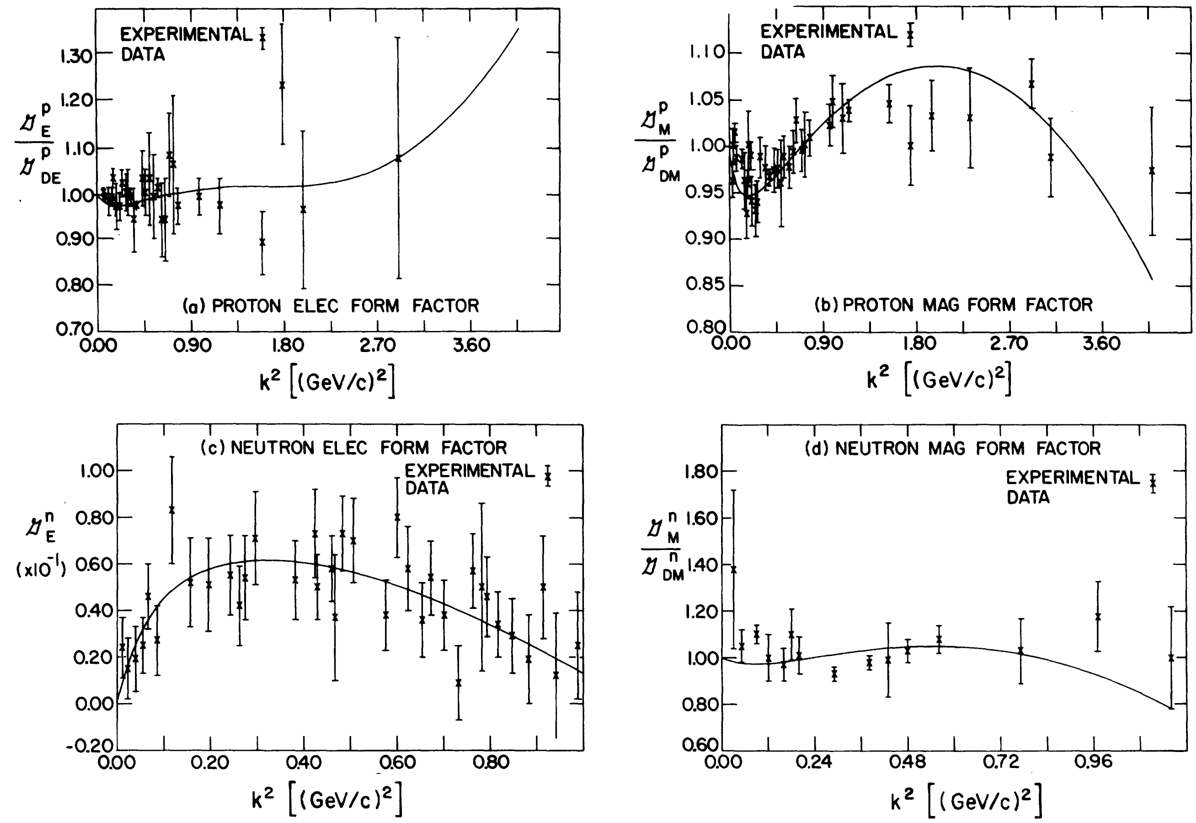

FIG. 2. The Sachs nucleon electromagnetic form factors vs momentum transfer squared (k^2). Note that the dipole dependence has been factored. The solid curve is our fit to the data. (a) Proton electric form factor; (b) proton magnetic form factor; (c) neutron electric form factor; (d) neutron magnetic form factor.

addition we conclude that the single-pole approximation is too severe when describing unstable-particle propagators over a large range of momentum transfer.

Since the vector-dominance model with the exact propagator does describe the data, it is reasonable to ask how critical to the minχ^2 value are the mass and width of the vector mesons? In other words, to what extent can the form-factor data be used to determine the parameters corresponding to the masses and widths of the mesons? To test this, the number of adjustable parameters was increased to include the masses and widths of all three mesons, thereby making a total of 12 adjustable parameters. The total minχ^2 decreased from 137.1 to a new value 123.7. A mapping of minχ^2 as a function of each adjustable parameter was obtained by fixing that parameter at a series of different values and, for each value, minimizing χ^2 with respect to the 11 remaining parameters. The results for the isovector meson parameters are shown in Fig. 3. The results for the two isoscalar meson parameters are similar with the one exception mentioned below. Experience showed that the minχ^2 surfaces are sufficiently "rough" so that local minima cause the computer program to stick at values of χ^2 above the minimum value. Changes in min$\chi^2 < 2$ in many cases did not seem meaningful, and, in one case, Γ_φ vs minχ^2, it was not possible to find a smooth curve such as shown in Fig. 3. The error reported in Table II is our best estimate obtained from many different computer runs. The other errors reported in Table II correspond to a change in value of the parameter necessary to cause minχ^2 to increase 12 units to the value of χ^2 expected for a fit with no adjustable parameters. We want to emphasize that these errors are not meant to be interpreted as statistical errors, but are given primarily to show the degree of sensitivity of our theory to the various parameters in fitting the experimental data.

As can be seen from Table II and Fig. 3, the theory is highly sensitive to the parameters which correspond to the masses and widths of the mesons and the best values compare favorably to the known experimental masses and widths for the ρ, ω and φ mesons. The vector-dominance model with the "exact" propagators therefore seems to give a realistic picture for electron-nucleon elastic scattering.

TABLE II. Masses and widths of vector mesons (MeV) and meson-nucleon coupling constants as determined from the range of data 0–2 GeV/c (momentum transfer). See text for an explanation of the errors.

	$\gamma_\rho = 2.6 \pm 0.2$				$\gamma_\omega = 7.7 \pm 0.8$				$\gamma_\varphi = 6.2 \pm 0.9$		
$g_1^{n,\rho}$	$g_2^{n,\rho}$	$g_1^{p,\rho}$	$g_2^{p,\rho}$	$g_1^{n,\omega}$	$g_2^{n,\omega}$	$g_1^{p,\omega}$	$g_2^{p,\omega}$	$g_1^{n,\varphi}$	$g_2^{n,\varphi}$	$g_1^{p,\varphi}$	$g_2^{p,\varphi}$
$-2.6^{+0.3}_{-0.1}$	6.25 ± 0.13	$2.6^{+0.1}_{-0.3}$	6.67 ± 0.13	$16.0^{+0.04}_{-1.2}$	-9.5 ± 0.2	$16.0^{+0.04}_{-1.2}$	10.1 ± 0.2	-6.43 ± 0.26	11.4 ± 0.1	-6.43 ± 0.26	-12.2 ± 0.1
	$m_{0\rho} = 767 \pm 4$				$m_{0\omega} = 797^{+4}_{-15}$				$m_{0\varphi} = 1066 \pm 50$		
	$\Gamma_\rho = 115^{+23}_{-2}$				$\Gamma_\omega = 10 \pm 1$				$\Gamma_\varphi = 5 \pm 1$		

The dimensionless meson-nucleon coupling constant, $g_i^{N,M}$, reported in Table II are obtained from $f_i^{N,M}$ according to the relationships

$$if_1^{N,M} = m_{0M}^2(2\gamma_M)^{-1} g_1^{N,M},$$
$$if_2^{N,M} = m_{0M}^2(\mu_N/2m_N)(2\gamma_M)^{-1} g_2^{N,M}, \tag{16}$$

where γ_M is the usual[9] meson-photon coupling constant. The values shown in Table II for γ_M are obtained from Ref. 9. Note that there are only six independent $g_i^{N,M}$ because of Eq. (5). The errors quoted for $g_i^{N,M}$ do not include the experimental error in γ_M.

III. PARAMETER SENSITIVITY

For the case of n independent parameters, highly correlated by the data, the only true way to determine the sensitivity is to determine the χ^2 surface in an orthogonal coordinate system of $n+1$ degrees of freedom formed by χ^2 and the parameters. A

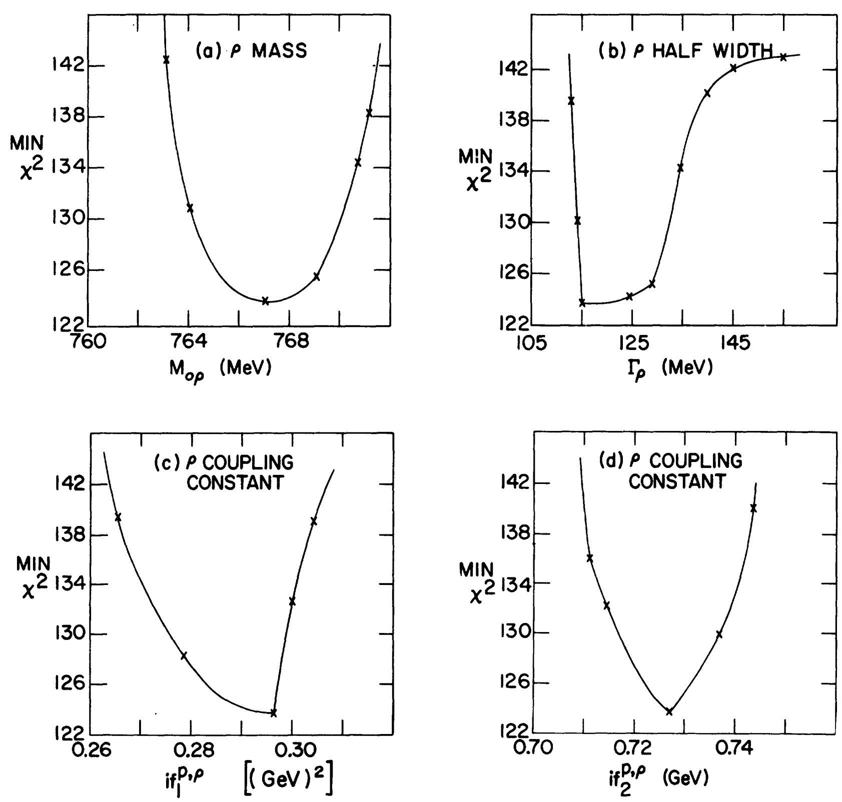

FIG. 3. Minχ^2 vs various isovector meson parameters. (a) Mass parameter; (b) half-width parameter; (c) $if_1^{p,\rho}$, the isovector meson-proton charge coupling constant; (d) $if_2^{p,\rho}$, the isovector meson-proton magnetic coupling constant.

plane perpendicular to the χ^2 axis, at some selected χ^2 point, such as 12 units above $\min\chi^2$ as in our case, will then intersect the χ^2 surface determining a closed surface in a space formed by the n parameters alone. This surface encloses the values of the parameters which determine the $\min\chi^2$ point. The points on this surface represent the range over which the parameters can vary. Since this surface is impossible to construct for a multiparameter system, one usually defines the maximum range for a particular parameter as the values where this surface intersects the particular parameter axis, the origin of the coordinate system being taken as the $\min\chi^2$ point. This corresponds to the procedure described in Sec. II, which, for example, led to the results displayed in Fig. 3. This procedure therefore never determines the full range that the parameters may vary unless the extremums of the surface happen to coincide with the coordinate axis. This point should be kept well in mind when studying any data analysis.

This disclaimer having been made, we believe that the sensitivity of the fits to the various parameters, determined as described in Sec. II, is due to the highly constrained nature of this analysis. Note that there are four separate mathematical expressions related by the VDM hypothesis, which correspond to the four Sachs form factors, each with a different momentum dependence, although each contains the same propagators which are identical in mathematical form. There are only six independent coupling constants (two if the extended distribution hypothesis is also made) and six constant mass and width parameters. These expressions are overdetermined by 109 data points with rather tight error flags. (Note that the gross functional dependence has been normalized out in Fig. 2.) The system is therefore highly constrained, much more constrained than if we had to fit a single smoothly varying function such as $\mathcal{G}_M^p(k^2)$ with an equivalent number of data points.

The sensitivity of the coupling constants to the number of form factors fitted was tested in the early phases of the analysis. With the masses and widths of the vector mesons fixed at their experimental values, the data for two of the four form factors were fitted and the six independent coupling constants determined. These values of the coupling constants were then used to predict the other two form factors. Although the $\min\chi^2$ values for these fits were excellent, in every case we found the predicted curves bore little relationship to the experimental data of the remaining two form factors. The values of the coupling constants varied widely, some even changed sign, depending upon which pair of form factors were fitted.

We have also tested the sensitivity of the ρ mass parameter to the number of data points. The number of data points was reduced from 109 to 90, thereby taking into account all the neutron data but keeping the proton data to 1 GeV/c. The ρ mass was found to vary from its best value of 765 MeV at a $\min\chi^2$ value of 84 (see Table IA for comparison of $\min\chi^2$ values for other fits) to values of 795 MeV and 700 MeV for an increase in χ^2 of 12, a considerably larger spread for this mass than that shown in Table II for the 2-GeV/c range of data. This behavior seems reasonable to us, since if the model is physically correct, the inclusion of more data must tend to fix the parameters more precisely.

Finally, we would like to comment on the sensitivity of the fits to the various width parameters. As seen from Table I, the "exact" propagator fits the data considerably better, from a χ^2 point of view, than the monopole fit, which already accounts for the gross features of the form factors. The exact propagator fit therefore represents a fine tuning to the data by adjusting those parameters which it contains that are different from those of the monopole. If an expansion of $I_M(k^2)$ is made (k^2 spacelike), neglecting only terms of order $[\Gamma_M/(m_{0M}-\lambda_M)]^2$ which are not directly multiplied by a function of k^2, then

$$I_M(k^2) = -m_{0M}{}^2 D^{-1}(k^2)\left\{1+\frac{\Gamma_M}{2\pi(m_{0M}-\lambda_M)} - \frac{\Gamma_M}{4\pi m_{0M}}\ln\left[\frac{k^2+\lambda_M{}^2}{(m_{0M}-\lambda_M)^2}\right]\right\}, \tag{17a}$$

where

$$D(k^2) = k^2\left\{1+\frac{\Gamma_M}{2\pi(m_{0M}-\lambda_M)} - \frac{\Gamma_M}{4\pi m_{0M}}\ln\left[\frac{k^2+\lambda_M{}^2}{(m_{0M}-\lambda_M)^2}\right]\right\}^2 + m_{0M}{}^2\left[1+\frac{\Gamma_M}{2\pi(m_{0M}-\lambda_M)} + \frac{\Gamma_M k}{2\pi m_{0M}{}^2}\arctan\left(\frac{k}{\lambda_M}\right)\right]^2. \tag{17b}$$

As can be seen from these equations, if compared to Eq. (13), those terms responsible for the difference between the k^2 behavior of the exact and monopole propagators, are directly proportional to the width parameters. Consequently, since the $\min\chi^2$ values for the exact propagator reduce substantially over those obtained for the monopole propagator, the allowable values for the widths must be highly constrained by the data points.

IV. SUM RULES

The fits leading to the values shown in Table II were based upon an analysis that implicitly assumed the possibility of pointlike contributions to both the Dirac charge and the Pauli magnetic form factors. If both of these distributions are extended, then $f_i^{e,N}(k^2)$, $i=1,2$, as well as their isovector and isoscalar counter parts $f_{iS}(k^2)$, $f_{iV}(k^2)$ must tend to zero for large k^2. This imposes the following constraints on the coupling constants as deduced from Eqs. (1), (4), (15), and (16) and the fact that to a very good approximation $m_{0M}^2 I_M'(0) = 1$:

$$g_1^{p,\rho} = \gamma_\rho\,, \tag{18a}$$

$$g_1^{p,\omega}\gamma_\omega^{-1} + g_1^{p,\varphi}\gamma_\varphi^{-1} = 1\,, \tag{18b}$$

$$\sum_M g_2^{N,M}(2\gamma_M)^{-1} = 1\,. \tag{18c}$$

As can be seen from Table II, Eqs. (18a) and (18b) are satisfied to within a few percent, well inside the tolerances listed in Table II, whereas the sum in Eq. (18c) is 1.5 ± 0.2 for the neutron and 1.0 ± 0.2 for the proton. It should also be pointed out that Eqs. (18) can be interpreted as being equivalent to the statement that the apparent nucleon charge and magnetic moments are entirely due to the vector mesons vis-à-vis the vector-dominance model. This is most easily seen by using the sum rules to combine the terms in Eq. (1). Thus $f_i^{e,N}$ are homogeneous functions of the nucleon-meson coupling constants. Note that because of the normalization used to define $g_2^{N,M}$, Eq. (16), the correct sign for the proton and neutron anomalous moment is also obtained.

V. PION AND KAON CHARGE FORM FACTORS

An analysis similar to that of Ref. 1 can also be made for the pion and kaon charge form factors. This results in the expressions

$$f^{e,\pi}(k^2) = 1 - m_{0\rho}^2(2\gamma_\rho)^{-1} g^{\rho,\pi} G_\rho(k^2) \tag{19}$$

for the pion form factor and

$$2f_S^{e,K} = 1 - m_{0\omega}^2(2\gamma_\omega)^{-1} 2g^{\omega,K} G_\omega(k^2) - m_{0\varphi}^2(2\gamma_\varphi)^{-1} 2g^{\varphi,K} G_\varphi(k^2)\,, \tag{20a}$$

$$2f_V^{e,K} = 1 - m_{0\rho}^2(2\gamma_\rho)^{-1} 2g^{\rho,K} G_\rho(k^2)\,, \tag{20b}$$

for the isoscalar (S) and isovector (V) parts of the kaon form factors. With these definitions the charge form factors for the K^+ and K^0 are

$$f^{e,K^+}(k^2) = f_V^{e,K}(k^2) + f_S^{e,K}(k^2)\,,\qquad f^{e,K^0}(k^2) = f_S^{e,K}(k^2) - f_V^{e}(k^2)\,. \tag{21}$$

The assumptions of universality,[10] within the vector-dominance model, means a given vector meson couples the same way to all hadrons. This implies there can be only one isoscalar and one isovector form factor. Comparison of Eqs. (19), (20), and (21) to the Dirac charge form factor defined by the first of Eqs. (1) gives the following constraints imposed by this assumption on the coupling constants:

$$2g_1^{\rho,p} = g^{\rho,\pi} = 2g^{\rho,K}\,, \tag{22a}$$

$$g_1^{\omega,p} = g^{\omega,K}\,,\qquad g_1^{\varphi,p} = g^{\varphi,K}\,. \tag{22b}$$

These constraints require twice the isovector part of the Dirac charge form factor $2f_{1V}(k^2)$ to be the same as the pion form factor and they require the Dirac charge form factor of the proton $f_1^{e,p}(k^2)$ to be the same as the K^+ charge form factor.

Note that if the pion and kaon charge distributions are assumed to be extended, then the requirement that $f^{e,\pi}(k^2)$, $f_S^{e,K}(k^2)$, and $f_V^{e,K}(k^2)$ tend to zero for large k^2 also imposes the constraint Eq. (22a), quite apart from the assumption of universality, and also imposes the constraint parallel to Eq. (18b):

$$g^{\omega,K}\gamma_\omega^{-1} + g^{\varphi,K}\gamma_\varphi^{-1} = 1\,,$$

which is consistent with Eq. (22b).

In Fig. 4 we have compared our fit to the nucleon form factors to the pion and K^+ form factor data available in the literature[11] for spacelike values of the momentum transfer k^2. Since the data do not represent direct measurements of the meson form factors, but are in fact dependent upon the theory used to extract the data, biases are difficult to estimate. We simply take the data at face value.

In Fig. 4(a) we have compared our fit with the isovector part of the nucleon charge form factor $2f_{1V}$ to the pion data. The upper and lower curves, which bound the crosshatched region correspond to the upper and lower values of the nucleon coupling constants as obtained from Table II. The fit to the data seems acceptable. For comparison purposes, the vector part of the nucleon form factor as obtained from the dipole rule with scaling as defined by Eq. (14) is shown as the curve labeled F_V.

However, as shown in Fig. 4(b), the proton charge form factor $f_1^{e,p}(k^2)$ does not fit the K^+ data. Since $f_1^{e,p}(k^2)$ is a good fit to the proton data, it follows that the proton data and the kaon data are inconsistent with the usual assumption of universality. We do find that $f^{e,K^+}(k^2)$ gives the closest fit to the data when Eq. (22b) is satisfied but with $g^{\rho,K} = 0$. This implies that, in contrast with the isovector part of the nucleon form factor, $f_V^{e,K}(k^2)$ arises entirely from the point interaction between

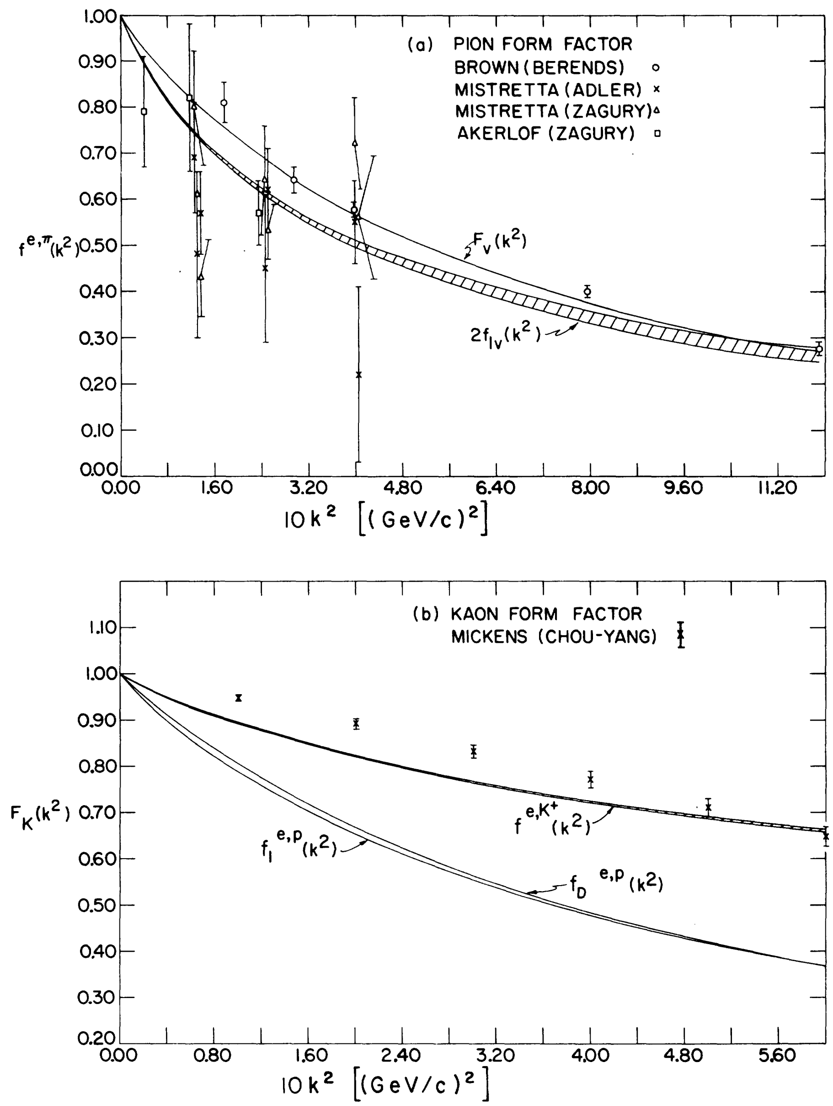

FIG. 4. (a) The pion charge form factor $f^{e,\pi}(k^2)$ vs momentum transfer squared (k^2). The two curves labeled $2f_{1V}(k^2)$ represent the upper and lower limits of our fit to the isovector part of the nucleon Dirac form factor. The curve labeled $F_V(k^2)$ is the isovector part of the nucleon Dirac form factor derived from the dipole law with scaling. (b) The K^+ charge form factor $F_K(k^2)$ vs momentum transfer squared (k^2). The two curves labeled $f^{e,K^+}(k^2)$ represent the upper and lower limits of our fit with $g^{\rho,K}=0$ and $g_1^{\omega,p}=g^{\omega,K}$, $g^{\varphi,p}=g^{\varphi,K}$. The curves labeled $f_1^{e,p}(k^2)$ and $f_D^{e,p}(k^2)$ are, respectively, our fit and the dipole fit with scaling to the Dirac form factor of the proton.

the photon and kaon, whereas $f_S^{e,K}(k^2)$ represents the same extended distribution as the isoscalar nucleon form factor. In addition, this result seems to imply a contradiction to unitary symmetry, since on the basis of unitary symmetry, one would expect a nonvanishing value for $g^{\rho,K}$. Consequently, the most likely candidate for the discrepancy shown in Fig. 4(b) between $f_1^{e,p}(k^2)$ and the data points is the theoretical interpretations needed to extract the kaon form factor from the original data.

There are also data available[12,13] for the pion and kaon charge form factors for timelike values of k^2. The data points for the pion charge form factor are shown in Fig. (5a) along with our upper and lower bounds to $|2f_{1V}(k^2)|^2$ analytically continued from the spacelike values for k^2 [Fig. (4a)] to timelike values. The agreement is excellent when account is taken of the $\omega\rightarrow 2\pi$ contribution to the pion form factor. This result, for the case of our upper bound, is shown in Fig. 5(b). The solid curve is the generalization of Eq. (19) to include the ω contribution,

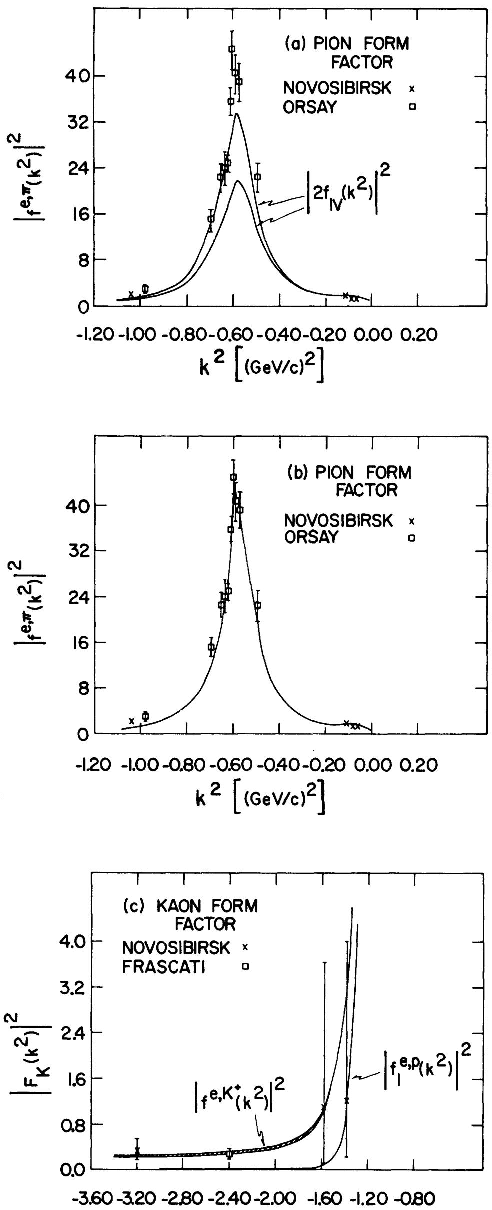

FIG. 5. (a) The pion charge form factor $f^{e,\pi}(k^2)$ vs momentum transfer for the timelike region ($k^2<0$). The two curves labeled $2f_{1V}(k^2)$ represent the upper and lower limits of our fit to the isovector part of the nucleon Dirac form factor. (b) The pion charge form factor $f^{e,\pi}(k^2)$ vs momentum transfer squared for the timelike region ($k^2<0$). The solid curve represents a typical fit to the data when the $\omega\to 2\pi$ contribution is included according to Eq. (23). (c) The K^+ charge form factor $F_K(k^2)$ vs momentum transfer squared for the timelike region ($k^2<0$). The two curves labeled $f^{e,K^+}(k^2)$ represent the upper and lower limits of our fit with $g^{\rho,K}=0$ and $g^{\omega,p}=g^{\omega,K}$, $g^{\varphi,p}=g^{\varphi,K}$. The curve labeled $f_1^{e,p}(k^2)$ is our fit to the Dirac form factor of the proton analytically continued to the timelike region.

$$f^{e,\pi}(k^2)=2f_{1V}(k^2)-m_{0\omega}{}^2(2\gamma_\omega)^{-1}g^{\omega,\pi}G_\omega(k^2). \tag{23}$$

The only free parameters in this equation are $g^{\omega,\pi}$, considered complex to be consistent with the Orsay analysis,[12] $m_{0\omega}$, and the total width Γ_ω. The minχ^2 value for the fit is 8. The results can be summarized as follows:

$$m_{0\omega}=781\text{ MeV},\quad \Gamma_\omega=10.7\text{ MeV}$$

and for

$$g^{\omega,\pi}=|g^{\omega,\pi}|e^{i\alpha},\quad |g^{\omega,\pi}|=0.25,\quad \alpha=76°.$$

The ratio

$$|g^{\omega,\pi}/g^{\rho,\pi}|=4.8\times10^{-2}, \tag{24}$$

obtained from the fit, can then be used to deduce a branching ratio

$$B_{\omega,2\pi}=(\Gamma_{\omega,2\pi}/\Gamma_\omega)=3\%. \tag{25}$$

This branching ratio and the value for the phase angle α are consistent with the Orsay results.[12]

It should be pointed out that the spread of curves, represented by the upper and lower bounds shown in Fig. 5(a), is due to the changes of the various ρ parameters when these parameters are correlated by the nucleon form factor data. If we release this correlation by also allowing $g^{\rho,\pi}$, $m_{0\rho}$, and Γ_ρ to vary independently within the bounds given in Table II as well as the ω parameters, then the fit to the data using Eq. (23) is considerably improved. In this way minχ^2 values as low as 0.45 have been achieved. In this case we find $|g^{\omega,\pi}|=0.31$, $\alpha=99°$, and $B_{\omega,2\pi}=5\%$.

Finally the value of the ρ,π coupling constant

$$g^{\rho,\pi}=2g_1^{e,p}=2\gamma_\rho$$
$$=5.2^{+0.2}_{-0.6}$$

or

$$(g^{\rho,\pi})^2/4\pi=2.2^{+0.2}_{-0.4} \tag{26}$$

is in excellent agreement with the experimental result obtained from the decay rate.[14]

The data for the kaon charge form factor in the timelike region are shown in Fig. 5(c) along with the solid curves which represent our expressions for $|f_1^{e,p}(k^2)|^2$ and $|f^{e,K^+}(k^2)|^2$ (with $g^{\rho,K}=0$) analytically continued from the spacelike region [Fig. 4(b)]. Again the data imply $g^{\rho,K}=0$.

Parallel to the result given by Eq. (26), $g^{\varphi,K}$ should be the same as that obtained from the decay formula. Our result which includes the uncertainty in γ_φ is (assuming universality)

$$g^{\varphi,K}=g_1^{p,\varphi}=-6.4\pm0.3$$

as compared with the experimental result[14]

$$|g_{\rm exp}^{\varphi,K}| = 4.4 \pm 0.2 .$$

VI. DEVIATIONS FROM THE FIT

Two final remarks: The initial slope of the electric form factor of the neutron has been measured as[15] $(d\mathcal{G}_E^n/dk^2) = 0.50 \pm 0.01$ GeV^{-2}. The slope as determined by our fit to the nucleon data is $1.16^{+0.10}_{-0.04}$ GeV^{-2}. Thus the $\mathcal{G}_E^n$ data appear not to be consistent with the slope measurement. This inconsistency has also been noted by other authors.[16]

The data for the magnetic form factor of the proton go to a momentum transfer of 5 GeV/c. Our fits, as can be seen from Table I, deteriorate slightly in going from 1 to 4 (GeV/c)^2. The deterioration rapidly increases when the full 25-(GeV/c)^2 range of the $\mathcal{G}_M^p$ data is used.

It is possible that these two discrepancies are due to the following approximations which may not be justified:

1. The 3π decay channel of the φ meson and the π, γ decay channel for the ω meson were ignored in the derivation of the meson propagators as well as a cut at the $2K$ threshold for the ω and ρ mesons.
2. Higher-mass resonances of the ρ, such as the ρ', were ignored.
3. The assumption was made that no experimental bias exists between different sets of data.

These possibilities will be included in a future analysis.

ACKNOWLEDGMENTS

We would like to acknowledge the many discussions of this work with our colleagues, Professor R. A. Leacock, Professor Bing-lin Young, and Professor N. W. Dean and the hospitality of the Aspen Center of Physics, where we (C.L.H.) had the opportunity to discuss these results with Professor S. Gasiorowicz of the University of Minnesota and Professor M. A. B. Bég of Rockefeller University.

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[2] C. J. Goebel, Phys. Rev. Lett. 29, 1042 (1972); C. R. Hagen and E. C. G. Sudarshan, *ibid.* 29, 1044 (1972).

[3] It is easy to show that if the gauge condition $k_\mu \,\mathrm{Tr}[\Gamma_{\mu\rho}\Delta_F^{(1)}] = 0$ is imposed, then Eq. (15) of Ref. 1 is exact with $\bar{f}^{\gamma,M} = 0$.

[4] Note, with this choice of phase,

$$m_1 m_1^* = |m_1|^2 \exp(i2\pi).$$

[5] For the proton electromagnetic form factors and the neutron magnetic form factor, the source is E. Lohrmann, in *Proceedings of the Fifth International Conference on Elementary Particles, Lund, 1969,* edited by G. von Dardel (Berlingska Boktryckeriet, Lund, Sweden, Sweden, 1970); in *High Energy Physics,* proceedings of the Fifteenth International Conference on High Energy Physics, Kiev, 1970, edited by V. Shelest (Naukova Dumka, Kiev, U. S. S. R., 1972).

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