Pion exchange in the inclusive reaction $p+p \to \Delta^{++}+X$ †

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Absorptive pion exchange in the triple Regge model is shown to provide an adequate description for the inclusive production of Δ^{++}(1238) in proton-proton collisions at an incident energy of 303 GeV.

The recently published measurement of the inclusive Δ^{++}(1238) production at an incident lab momentum of 303 GeV,[1] provides a test of the adequacy of the triple Regge formalism[2] for resonance production in the limit for which its validity has been established, i.e., $s\to\infty$, $M^2\to\infty$, s/M^2 and t fixed. The reaction $p+p\to\Delta^{++}+X$ is particularly suited to this investigation since the Δ^{++} are produced almost entirely at low momentum transfer and large x, hopefully indicating that the Δ production can be explained by a simple exchange mechanism. The data in Ref. 1 are presented as functions of the Feynman variable x, and the transverse momentum squared p_T^2. This alleviates the problem of having to separate phase-space and kinematics effects, a difficulty which one encounters when applying the triple-Regge formalism to fit distributions displayed as functions of the missing mass squared.

A prominent feature of the experimental data (see Fig. 2) is the structure present in the x distribution, which displays a broad maximum, centered about $x=-0.8$. Similar behavior[3] has been previously observed in the charge-exchange spectra for the reaction $p+\text{Be}\to n+X$ at zero mrad. Data for larger angle scattering (50 mrad) does not show this feature. This suggests that the structure is somehow connected with the exchange of the π meson, which is known to dominate the np and Δp vertices in two-body processes.[4]

Kinematics. We consider the reaction

$$a+b\to c+X\,(\text{anything}) \qquad (1)$$

and denote the transverse and longitudinal momentum in the center-of-mass system of particle c by p_T and p_L. The relations between these variables and the invariants are

$$s=(p_a+p_b)^2=(E_a+E_b)^2\,,$$

$$t=(p_a-p_c)^2$$

$$=m_a^2+m_c^2-2p_ap_L-2E_aE_c\,,$$

$$E_c=(m_c^2+p_T^2+p_L^2)^{1/2}\,,$$

$$M_X^2=(p_a+p_b-p_c)^2 \qquad (2)$$

$$=s+m_c^2-2\sqrt{s}\,E_c$$

$$=\text{missing mass squared}\,,$$

$$p_L=xp_L^{\max}\,,$$

when x denotes the Feynman variable and

$$p_L^{\max}=\frac{1}{2\sqrt{s}}\{[s-(m_c+m_X^{\min})^2][s-(m_c-m_X^{\min})^2]\}^{1/2} \qquad (3)$$

($m_X^{\min}$ denotes the minimum mass allowed for X in the reaction).

Another important quantity is the minimum momentum transfer squared:

$$-t_{\min}=(m_a^2-m_c^2)(m_b^2-M_X^2)/s$$
$$+(m_a^2m_b^2-m_c^2M_X^2)(m_a^2m_c^2-m_b^2M_X^2)/s^2\,. \qquad (4)$$

As we will be dealing with resonance production $m_a\neq m_c$, we can use the approximation

$$-t_{\min}\approx(m_a^2-m_c^2)(m_b^2-M_X^2)/s\,. \qquad (5)$$

For any given process m_a, m_b, and m_c are fixed; consequently the missing mass M_X increases with the increasing energy and therefore $t_{\min}\not\to 0$ as $s\to\infty$, unlike the two-body case. Thus, $t_{\min}$ effects can be important and should not be neglected in inclusive processes especially when $m_a\neq m_c$.

The invariants t and M_X^2 are often used to parametrize Regge exchange in inclusive reactions. The danger inherent in trying to fit data displayed in terms of these variables,[5] is the strong mass-dependent effects present due to the kinematic relationship between t and M_X^2; e.g., the kinematic limit for the simple case when $m_c=M_X$ is max $M_X\approx(-st)^{1/4}$.

We follow the established procedure[2] and write the Lorentz-invariant differential cross section,

$$\rho(p_c) = E_c \frac{d^3\sigma}{d^3p}$$
$$= \frac{E_c}{\pi} \frac{d^2\sigma}{dp_L dp_T^2}$$
$$= \frac{1}{\pi}\left(\frac{E_c}{p^*_{\max}}\right)\frac{d^2\sigma}{dx\, dp_T^2}, \tag{6}$$

in the triple-Regge limit; when $M_X^2/s \approx 1-x$, we have

$$\rho(p_c) = \frac{s}{\pi}\frac{d^2\sigma}{dt\, dM^2}. \tag{7}$$

In the kinematic region, t fixed, and both s/M^2 and $M^2 \to \infty$ (see Fig. 1), the inclusive cross section is dominated by the leading Regge exchanges in the $p\bar{\Delta}$ channel; therefore

$$\frac{d^2\sigma}{dt\, dM^2} = \sum_{ij} \beta^i_{p\bar{\Delta}}(t)\beta^j_{p\bar{\Delta}}(t)\xi_i\xi_j^* s^{\alpha_i(t)+\alpha_j(t)-2} \times \text{Disc} f_{ij}(M^2, t). \tag{8}$$

Here β denotes the Regge residue and ξ the signature factor, these are discussed in detail below. According to the generalized optical theorem the inclusive cross section is proportional to a discontinuity in M_X^2 of the forward $ab\bar{c} \to ab\bar{c}$ amplitude.[6] Hence $f_{ij}(M^2, t)$ represents the amplitude for Reggeon-particle forward scattering.

The $M_X^2 \to \infty$ limit of $f_{ij}(M^2, t)$ is given by the triple-Regge formula

$$\text{Disc} f_{ij}(M^2, t) \xrightarrow[M^2\to\infty]{} \sum_K g^K_{ij}\beta^K_{b\bar{b}}(0) \times (M^2)^{\alpha_K(0)-\alpha_i(t)-\alpha_j(t)}. \tag{9}$$

At an incident energy of 300 GeV it is reasonable to assume that Pomeron exchange dominates the Reggeon-particle scattering and so take $\alpha_K(0) = 1$. The value of the spherical harmonic moment $\langle Y_2^0 \rangle = 0.092$ suggests that π exchange dominates the Δp vertex, so we assume

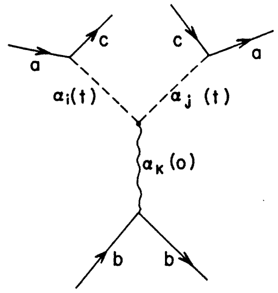

FIG. 1. Diagrammatic expression for the inclusive cross section in the triple-Regge approximation.

$$\alpha_i(t) = \alpha_j(t) = \alpha_\pi(t) = (t - \mu^2). \tag{10}$$

With these substitutions the inclusive differential cross section has the form

$$\frac{d^2\sigma}{dt\, dM^2} \approx K|\beta^\pi_{p\bar{\Delta}}(t)|^2 |\xi_\pi|^2 \left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)-1}, \tag{11}$$

where K denotes a factor related to the strength of the $\pi\pi$-Pomeron coupling and in this analysis is assumed to be a constant; the signature factor

$$\xi_\pi = \Gamma(-\alpha_\pi(t))(1 + e^{-i\pi\alpha_\pi(t)}). \tag{12}$$

It is well known from two-body reactions that the shape of the differential cross section reflects the structure of the different helicity amplitudes. We therefore feel that it is important to include the exact form of the residue function for the different values of the helicities of particles a and c, in our case the proton and Δ. These vertices have been calculated in the Born approximation and are given in Eq. (33) of Ref. 7. In addition, absorptive effects to the pion exchange are introduced following the recipe suggested by Williams.[8] The essence of this procedure is to replace factors of t that occur in the residue and that are of dynamical origin by m_π^2.

The factorizable amplitude $f_{ac\to ca}(s, t) = |\beta^\pi_{ac}|^2$ is then of the form $(-t)^\lambda P(\lambda, s, m_\pi^2)$ where $\lambda = \lambda_a - \lambda_c$ denotes the s-channel helicity flip at the ac vertex, and $P(\lambda, s, m_\pi^2)$ is a polynomial in $\cos\theta$. The modification of the Williams method has been discussed in detail in Ref. 4.

In any absorptive model the two-body pion-exchange amplitude peaks when $n = 0$, where n is the net s-channel helicity flip, and dips if $n \neq 0$. For the unpolarized reaction $ac \to ca$, $n = 0$ or 2. Consequently, contrary to the standard nonabsorptive treatment,[9] we anticipate a *forward peak* in $d\sigma/dt$ or $d\sigma/dp_T^2$ as $x \to 1$.

Comparison with experimental results. (i) The distribution in x: The experimental data and the model predictions are illustrated in Fig. 2. The data are quoted for a selection such that $p_T^2 \leq 0.1$ GeV^2. To compare with the data, we have integrated the invariant cross section over the interval $0 \leq p_T^2 \leq 0.1$ GeV^2. We see that the absorbed pion exchange provides a reasonable fit over the interval $0.8 \leq |x| \leq 1$, but is deficient at lower values of x where the triple-Regge formalism is in any case not applicable. This is in keeping with our experience in two-body scattering, where the pion contribution is known to be negligible for large values of momentum transfer, which in inclusive reactions correspond to small x for fixed p_T^2. In an attempt to improve the fits we added the exchange of a ρ trajectory, but to no avail for, if the ρ contribution

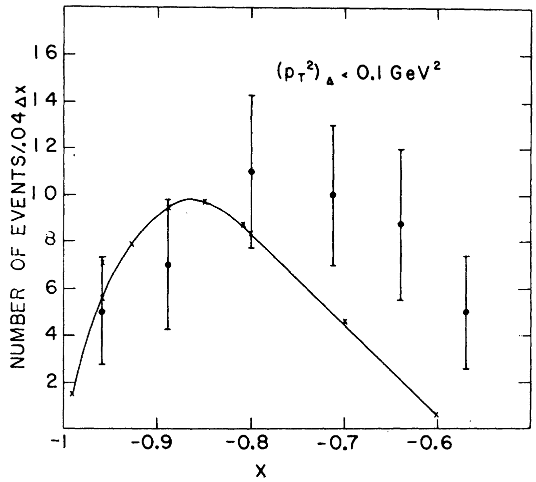

FIG. 2. Longitudinal momentum distribution of Δ^{++} for $(p_T^2)_\Delta < 0.1$ GeV^2, presented in terms of the Feynman variable x. The data points are from Ref. 1 and the solid line is the prediction of absorbed pion exchange (normalization is arbitrary).

is adjusted to fill in part of the distribution for $x < -0.8$, it then badly overshoots the data for x near -1.

In Fig. 3 we plot the structure function

$$\frac{1}{\pi}\int_0^{0.1} \frac{E_c}{p^*_{\max}} \frac{d^2\sigma}{dx\, dp_T^2} dp_T^2 \qquad (13)$$

as a function of x for various energies. It remains to be seen, when experimental data are available

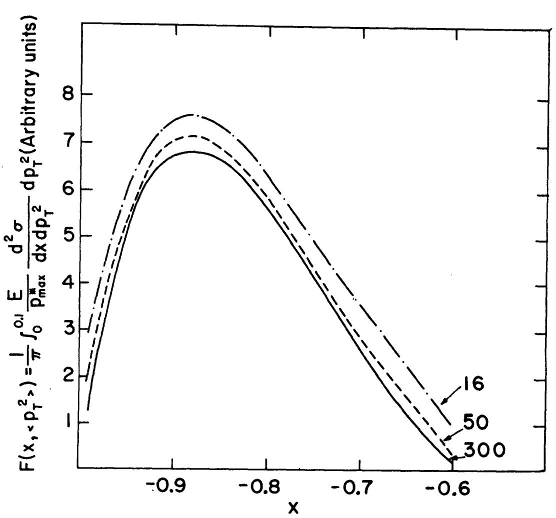

FIG. 3. The structure function

$$F(x, \langle p_T\rangle^2) = \frac{1}{\pi}\int_0^{0.1} \frac{E_c}{p^*_{\max}} \frac{d^2\sigma}{dx\, dp_T^2} dp_T^2$$

as a function of x for various incident energies.

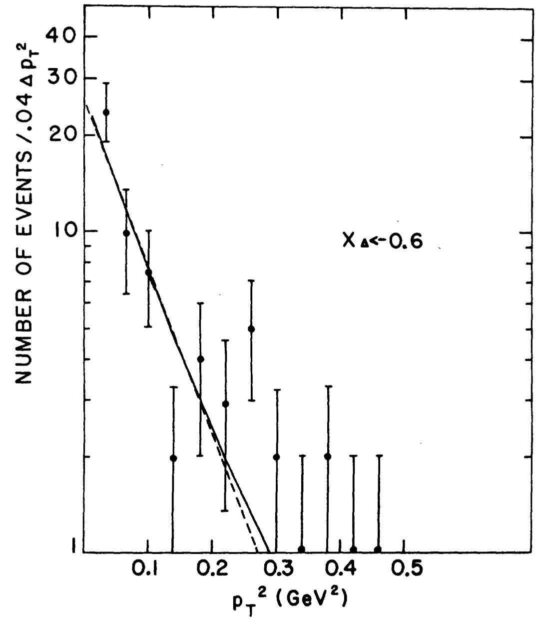

FIG. 4. Transverse momentum distribution of Δ^{++} for $x_\Delta < -0.6$. The data points are from Ref. 1, and the solid line is the prediction of absorbed pion exchange (normalization same as in Fig. 2). The dashed line is a plot of the function $\exp(-12\, p_T^2)$.

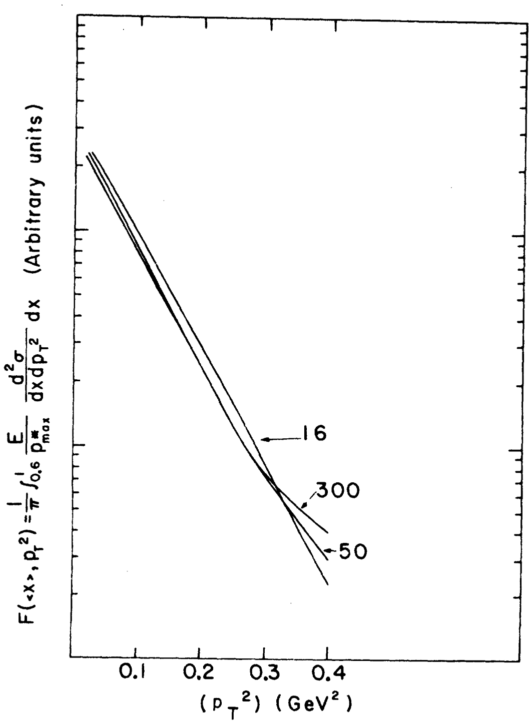

FIG. 5. The structure function

$$F(\langle x\rangle, p_T^2) = \frac{1}{\pi}\int_{0.6}^{1} \frac{E_c}{p^*_{\max}} \frac{d^2\sigma}{dx\, dp_T^2} dx$$

as a function of p_T^2 for various incident energies.

at lower energies, whether the data scale as well as predicted by the model.

(ii) The distribution in p_T^2: In Fig. 4 we display the p_T^2 distribution for $x < -0.6$. The experimental data fall off with a slope of about 12 GeV^2, as indicated by the dashed line. The model prediction (solid line in Fig. 4) has a falloff of $\exp(-12p_T^2)$. The factor $(s/M^2)^{2\alpha_\pi(t)-1}$ behaves like $\exp(-3p_T^2)$ for fixed x; the additional damping in the transverse momentum is a consequence of the explicit momentum-transfer dependence of the signature factor and residue functions. In Fig. 5 we sketch predicted transverse-momentum behavior of the function

$$F(\langle x\rangle, p_T^2) = \frac{1}{\pi}\int_{0.6}^{1} \frac{E_c}{p^*_{\max}} \frac{d^2\sigma}{dx\, dp_T^2}\, dx$$

at various energies.

For small center-of-mass production angles the variable $t' = t - t_{\min}$ is related to p_T^2 by $-t' = p_T^2/x$, and since $\langle x\rangle \approx 0.75$ we have that the inclusive distribution behaves like $\exp(9t')$. This behavior is very similar to that in the related two-body process $p + p \to \Delta^{++} + n$, where the differential cross section falls off like $\exp(10t)$.

(iii) Spin alignment of Δ^{++}: In an inclusive reaction it is theoretically more convenient to consider the spin alignment of the resonance in the well-defined t channel, i.e., $p\bar{\Delta} \to \bar{p}X$, for in the alternate s-channel description, the Δ is coupled with the varying missing mass M_X. In the quasi-two-body process $p + p \to \Delta^{++} + n$, pure pion exchange predicts $\rho^J_{11} = 0.5$, $\mathrm{Re}\rho^J_{31} = 0$, and $\mathrm{Re}\rho^J_{3,-1} = 0$, while absorptive pion exchange yields values $\rho^J_{11} = 0.38$, $\mathrm{Re}\rho^J_{31} = 0.06$, and $\mathrm{Re}\rho^J_{3,-1} = 0.03$.

The t-channel values quoted by Dao *et al.*[1] for the spherical harmonics $\langle Y_0^2\rangle = 0.092 \pm 0.028$, $\langle Y_1^2\rangle = 0.03 \pm 0.02$, and $\langle Y_2^2\rangle = 0.02 \pm 0.02$ correspond to the following spin-density matrices in the Gottfried-Jackson frame: $\rho^J_{11} = 0.432 \pm 0.058$, $\mathrm{Re}\rho^J_{31} = -0.08 \pm 0.056$, and $\mathrm{Re}\rho^J_{3,-1} = -0.056 \pm 0.056$.

The closeness of the measured values of the spin-density matrix elements to what one would expect from absorbed pion exchange, together with reasonable fits to the data shown in Figs. 2 and 4, lends credence to our assumption that pion exchange plays a dominant role in the reaction $p + p \to \Delta^{++} + X$.

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