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Coincidence hadron production in neutrino-induced reactions

M. A. Ahmed*
Department of Mathematics, University of Durham, Durham, England
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We discuss neutrino-induced reactions leading to a multiparticle hadronic final state from which only a single hadron is detected in coincidence with the outgoing lepton. A peripheral model for such processes is constructed and discussed in detail. Regge and scaling behaviors are studied and expressions for the structure functions and the hadronic density-matrix elements are obtained.

I. INTRODUCTION

The assumption of locality of the weak coupling of lepton pairs in high-energy neutrino reactions endows them with a structure that readily provides testing grounds for the ideas of Regge-pole theory. The simplest reactions, e.g., $\nu + N \to \mu + N + \pi$, can be viewed, in analogy with strong interaction processes, as two-body reactions in which a hadronic weak current simulates a particle with spacelike mass that interacts with the target hadron to generate a two-body hadronic final state. A detailed discussion of high-energy lepton processes in the framework of the Regge-pole model was first made by Pais and Treiman.[1] They exploited the fact that Regge-pole dominance of strong two-body $\to$ two-body reactions implies that all the helicity amplitudes of any given process share a common phase to leading order in energy. This phase is determined by the signature factor of the leading trajectory. This fact then requires the vanishing, to leading order in the energy, of terms in the differential cross-section spectrum that are odd under reversal of all spins and momenta, the so-called quasi-T-violating terms. When carried over to high-energy weak processes the implications of this phase property are numerous, especially for the neutrino-induced reactions due to the presence of parity-violating interactions.

Our interest in this paper is in those neutrino-induced reactions which lead to a multiparticle hadronic state from which a single hadron is detected in coincidence with the outgoing lepton, i.e., processes of the type $\nu + N \to \mu + N + X_1 + \cdots + X_n$ $(n \geq 2)$. Below we shall denote the undetected hadronic system $X_1, \ldots, X_n$ by X for short. In contrast to the fashionable multiperipheral model, Pais and Treiman[1] propose a peripheral model to describe these processes. In the high-energy region one considers Regge trajectories exchanged singly between the vertex connecting the current and the system X and the vertex connecting the initial and final hadrons. In such a picture it then seems reasonable to conjecture that the relative phase properties reflect final-state interactions only among the particles in X. A detailed application of this picture to coincidence electroproduction was recently made by Cheng and Zee.[2] In this paper we consider the more interesting case of neutrino reactions and as mentioned earlier the experimental implications should be richer here. The extra ingredients to the picture described above are as follows: One first considers the exchanges due to spin-J hadrons, (see Fig. 2) and then sums over an infinite number of them.[3] This model is originally due to Van Hove[4] and Durand[5] and is known to generate Regge behavior. Next, one sums over a complete set of states X connected by strong interactions and the correlation function that determines the differential cross section, then acquires a structure in which the commutator between two hadronic currents appears sandwiched between spin-J states. Then one passes to the deep-inelastic region, defined below as a subdomain of the Regge region, and, by the usual argument, the light-cone singularities dominate and provide the main value of the commutator. In Ref. 2 the model of Fritzsch and Gell-Mann[6] for light-cone commutators was used. Here we employ the more general operator-product expansion first proposed by Wilson[7] and applied to the case of weak hadronic currents by Mandula *et al.*[8]

Section II is devoted to some kinematical preliminaries. The kinematics of lepton-hadron scattering processes have been discussed in detail by Muzinich *et al.*[9] and we can afford to be brief. The model is presented in Sec. III, while Sec. IV is devoted to a discussion of the results.

II. PRELIMINARIES

The amplitude for the inelastic reaction $l(q_1) + N(p, \lambda) \to l'(q_2) + X$, Fig. 1, where l and l' are ν or μ and X is an arbitrary hadronic state, is given to lowest order in the weak interactions by

$$M=\frac{G_F}{\sqrt{2}}\,l_\mu(q_2,q_1)\langle X(P_X)|J^\mu(0)|N(p,\lambda)\rangle. \tag{2.1}$$

In the above equation $J^\mu(0)$ is the Cabibbo current

$$\begin{aligned}J_\mu(0)=&\cos\theta_C[V_\mu^{I^+}(0)-A_\mu^{I^+}(0)]\\&+\sin\theta_C[V_\mu^{K^+}(0)-A_\mu^{K^+}(0)],\end{aligned} \tag{2.2}$$

where θ_C is the Cabibbo angle; I^+ and K^+ denote the SU(3) transformation properties. P_X is the 4-momentum of the system X. The matrix element of the leptonic current is

$$l_\mu(q_2,q_1)=\bar u(q_2)\gamma_\mu(1+\gamma_5)u(q_1), \tag{2.3}$$

and $G_F m_p^2\simeq 10^{-5}$ is the dimensionless Fermi constant. We shall work with the approximation of zero lepton masses.

The cross section for unpolarized incident protons and lepton spins summed over can be written in the laboratory frame as

$$\frac{d\sigma}{d\nu\,dq^2d\psi}=\frac{1}{2\pi}\frac{1}{2E^2}\,W d\Phi(X), \tag{2.4}$$

where $q^2=(q_1-q_2)^2<0$, $\nu=p\cdot q/m$, and ψ is defined by Eq. (2.13). In terms of laboratory quantities we have $q^2=-4EE'\sin^2(\frac12\theta)$, with E, E' being the energies of the initial and final leptons, respectively, and θ is the angle between the lepton 3-momenta. W is given by

$$\begin{aligned}W&=\frac12\sum_{\text{lepton spins}}|M|^2\delta^4(P_X-p-q)\\&=\frac12 G_F{}^2\tau_{\mu\nu}W^{\mu\nu},\end{aligned} \tag{2.5}$$

where

$$\tau_{\mu\nu}=(q^2g_{\mu\nu}-q_\mu q_\nu+Q_\mu Q_\nu\pm i\epsilon_{\mu\nu\lambda\sigma}q^\lambda Q^\sigma), \tag{2.6}$$

and

$$\begin{aligned}W_{\mu\nu}=&\frac12\sum_\lambda\langle X(P_X)|J_\mu(0)|N(p,\lambda)\rangle\\&\times\langle X(P_X)|J_\nu(0)|N(p,\lambda)\rangle^*\delta^4(P_X-p-q),\end{aligned} \tag{2.7}$$

with

$$Q=q_1+q_2\,.$$

In Eq. (2.6) the minus sign is for the neutrino reaction and the plus sign is for the antineutrino reaction. The quantity $d\Phi(X)$ is the phase-space volume element for all of the final particles except the final lepton[10]:

$$d\Phi(X)=\prod_j\frac{d^3p_j}{E_j}\,. \tag{2.8}$$

A set of polarization vectors is next introduced with the following properties:

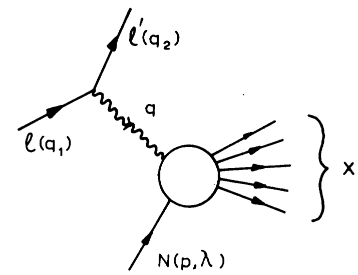

FIG. 1. The lepton-induced multiparticle reaction $l+N\to l'+X$.

$$\begin{aligned}g^{\mu\nu}&=\sum_{m=-1}^{m=1}(-)^m\epsilon_m^\mu(q)[\epsilon_m^\nu(q)]^*+\frac{q^\mu q^\nu}{q^2},\\ \epsilon_m(q)&=(-)\tfrac12(m+1)(2)^{-1/2}(\hat e_x+im\hat e_y),\quad m=\pm1\\ \epsilon_0(q)&=(-q^2)^{-1/2}(|\vec q|,q_0\hat e_z),\end{aligned} \tag{2.9}$$

in a frame where $\hat q=\hat e_z$. The correlation function W is then given as[9]

$$W=\tfrac12 G_F{}^2\sum_{m,m'}\Phi_{mm'}\rho_{mm'}\,, \tag{2.10}$$

where $\rho_{mm'}$ is the helicity density matrix describing the process $J(q,m)+N(p,\lambda)\to X$ and is given by

$$\rho_{mm'}\equiv(-)^{m+m'}(\epsilon_m^\mu)^*W_{\mu\nu}\epsilon_{m'}^\nu\,. \tag{2.11}$$

$\Phi_{mm'}$ is the density matrix for the lepton pair that has been calculated explicitly by the authors of Ref. 9 and shown to be just a finite-dimensional representation of the SO(2,1) group describing the decay of a spin-1 particle with spacelike mass into a lepton pair. In Ref. 9 $\Phi_{mm'}$ is given in a general brick-wall frame R, where

$$\begin{aligned}q^R&=(-q^2)^{1/2}(0,0,0,1),\\ q_1^R&=\tfrac12(-q^2)^{1/2}(\cosh\xi,\ \sinh\xi\cos\psi,\ \sinh\xi\sin\psi,\ 1),\\ q_2^R&=\tfrac12(-q^2)^{1/2}(\cosh\xi,\ \sinh\xi\cos\psi,\ \sinh\xi\sin\psi,\ -1).\end{aligned} \tag{2.12}$$

In Eq. (2.12) ψ is the angle between the normal to the lepton plane and $\hat e_y$ (viz. the normal to the production plane since the 3-momentum vector of the detected hadron can be chosen to lie on the x-z plane), and is given by

$$\cos\psi=(\hat q_1\times\hat q_2)\cdot\hat e_y\,. \tag{2.13}$$

The quantity $\sinh\xi$ is also related to the x,y components of the lepton momenta. Hence the variable ψ and $\sinh\xi$ are invariant under z boosts. Consequently $\sinh\xi$ can be evaluated in any frame related to R by a z boost such as the s-channel center-of-mass frame or the laboratory frame.

In Sec. III we introduce the model that describes the reaction $l + N \to l' + N + X$ when a single hadron in the final state is detected in coincidence with the outgoing lepton.

III. NEUTRINO-INDUCED REACTIONS

For definiteness we consider reactions initiated by neutrinos. As already mentioned in the Introduction the application of Regge-pole ideas to such reactions was first discussed by Pais and Treiman.[1] For the reaction $\nu + p \to \mu' + p' + X$ they propose, in the large-energy limit (i.e., large ν), a peripheral model in contrast to a multiperipheral one. With the momentum-transfer variable $\Delta^2 = (p - p')^2$ held fixed and the invariant mass of the system X also held finite, one considers the exchange of a single trajectory between the hadron vertex, say nucleons, and the vertex connecting the current to the system X. One then conjectures that under such conditions the relative phase properties of the various helicity amplitudes reflect final-state interactions only among the particles in X. In the model we shall use, this picture is implemented by the introduction of an extra ingredient due to Van Hove[4,5]; namely, one simulates the exchange of a Regge trajectory through the exchange of an infinite number of spin-J particles (Fig. 2). In calculating the cross section summed over all systems X apart from a single hadron, which we take to be a nucleon,[3] one then encounters the absorptive part of the scattering amplitude of a current of spacelike mass on a spin-J boson (Fig. 3). In the deep-inelastic limit one is then led to consider the matrix element of the commutator, near the light cone, between the weak current and its adjoint.

The reaction under discussion is described by the six variables ψ, ξ, q^2, $q \cdot P$, $q \cdot \Delta$, Δ^2, where $P = p + p'$. We have seen in Sec. II that the dependence on ψ and ξ factors out explicitly and is contained completely in the leptonic density matrix element $\Phi_{mm'}$. For fixed missing mass we first calculate the two-body–to–two-body amplitude corresponding to Fig. 3. One writes

$$W^{\mu\nu}(J, J') = \mathcal{C}(J)_\beta \mathcal{P}(J)^{\beta;\alpha} W(J, J')^{\mu\nu}_{\alpha;\alpha'} \times \mathcal{P}(J')^{\alpha';\beta'} \mathcal{C}(J')_{\beta'} . \tag{3.1}$$

In this equation β and α stand collectively for the J indices $\beta_1, \ldots, \beta_J$ and $\alpha_1, \ldots, \alpha_J$, respectively, while α' and β' similarly stand for J' indices. $\mathcal{C}(J)_\beta$ represents the coupling of the spin-J boson to the nucleons and is to be calculated from the following effective interaction Lagrangian

$$\mathcal{L} = g(J) V^{\mu_1 \cdots \mu_J} \bar{\psi} \overleftrightarrow{\partial}_{\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_J} \psi . \tag{3.2}$$

$\mathcal{P}(J)^{\alpha;\beta}$ is the propagator of the exchanged off-shell spin-J boson. It has the form[5,11]

$$\mathcal{P}(J)_{\alpha;\beta} = \frac{(-)^J}{\Delta^2 - M^2(J)} i \Gamma^J_{\alpha;\beta}(M^2), \tag{3.3}$$

where

$$\Gamma^J_{\alpha;\beta}(M^2) = \sum_{r=0}^{[J/2]} (-)^r \frac{2^r (2J - 2r)!}{(2J)!\,(J - r)!} \times \{ g_{\alpha_1\beta_1}(M^2) \cdots g_{\alpha_J\beta_J}(M^2) \}^{(J)}_r , \tag{3.4}$$

where $[J/2]$ denotes the maximum integer contained in $\frac{1}{2}J$ and the symbol $\{\cdots\}^{(J)}_r$ is a product of J $g_{\alpha\beta}(M^2)$'s completely symmetrized with respect to either the α's or the β's; in r distinct pairs of the $g_{\alpha\beta}(M^2)$'s the α index of one is interchanged with β index of the other. One easily finds that

$$\mathcal{C}(J)^\beta \Gamma^J_{\beta;\alpha} = (i)^J g(J, \Delta^2) P_{\alpha_1} \cdots P_{\alpha_J} + \cdots , \tag{3.5}$$

where the dots designate terms that would give

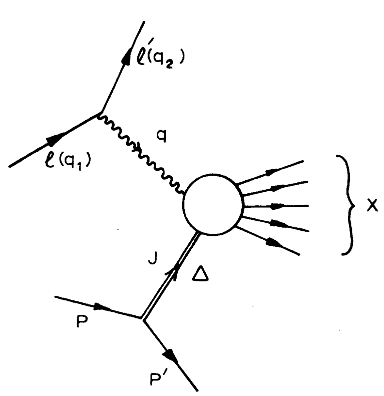

FIG. 2. Spin-J exchange model for the reaction $l + N \to l' + N + X$.

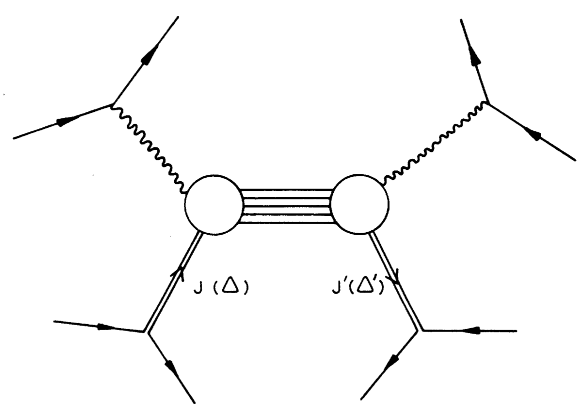

FIG. 3. The function $W^{\mu\nu}(J, J')$ involving the absorptive part of the scattering amplitude of a current on a spin-J target.

rise to lower order contributions in the high-energy limit. The amplitude $W^{\mu\nu}$ is then given by

$$W^{\mu\nu}=\sum_{J,J'} W^{\mu\nu}(J,J')\,. \tag{3.6}$$

$W^{\mu\nu}_{\beta;\alpha}$ is the absorptive part of the scattering amplitude of the weak current with spacelike mass on the boson target. When the spin-J and spin-J' particles are on shell it is defined by

$$\epsilon^{*\alpha_1\cdots\alpha_{J'}}\,W(J,J')^{\mu\nu}_{\alpha_1\cdots\alpha_{J'};\,\alpha'_1\cdots\alpha'_J}\,\epsilon^{\alpha'_1\cdots\alpha'_J}=\tfrac{1}{2}\sum_X \langle\Delta',\epsilon_{J'}|J^\dagger_\nu(0)|X\rangle\langle X|J_\mu(0)|\Delta,\epsilon_J\rangle\delta^4(q+\Delta-P_X)\,. \tag{3.7}$$

In this equation ϵ^α_J denotes the polarization tensor of the spin-J boson and $\Delta^2=M^2(J)$, $\Delta'^2=M^2(J')$. Eventually we shall assume that we can continue to the point $\Delta=\Delta'$, with Δ^2 being small and negative.[12]

It is our intention to study the q^2, $q\cdot P$, and $q\cdot\Delta$ dependences of the invariant amplitudes that appear when $W^{\mu\nu}$ is expanded in a suitable tensor basis. The Regge region which motivated the aforementioned model for $W^{\mu\nu}$ is characterized by

$$q\cdot P\gg -q^2,\; q\cdot\Delta,\; \Delta^2\,. \tag{3.8}$$

Of special interest to us is the behavior of the structure functions in the deep-inelastic Regge region in which $-q^2$ and $q\cdot\Delta$ grow to large values (but still small compared to ν) at fixed ratio, i.e., $\omega=-q^2/(2q\cdot\Delta)$ remaining fixed. The standard argument that the light cone dominates in the deep-inelastic limit can now be used. For the commutator between the weak current and its adjoint we employ the general light-cone (LC) expansion[8]:

$$\begin{aligned}[J^\dagger_\mu(x),J_\nu(0)]=&(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)E_1(x;c_1)Q_1(x,0)-(g_{\mu\nu}\partial_\lambda\partial_\sigma-g_{\mu\lambda}\partial_\nu\partial_\sigma-g_{\nu\sigma}\partial_\mu\partial_\lambda+g_{\mu\lambda}g_{\nu\sigma}\Box)E_2(x;c_2)Q_2^{\lambda\sigma}(x,0)\\&-\epsilon_{\mu\nu\lambda\sigma}\partial^\lambda E_3(x;c_3)Q_3^\sigma(x,0)+\partial_\mu\partial_\nu E_4(x;c_4)Q_4(x,0)-i(g_{\mu\lambda}\partial_\nu+g_{\nu\lambda}\partial_\mu)E_5(x;c_5)Q_5^\lambda(x,0).\end{aligned} \tag{3.9}$$

In writing out Eq. (3.9) we have suppressed the internal-symmetry indices because, as we said earlier, we confine ourselves to definite reactions, namely, those initiated by neutrinos. The functions $E_i(x;c_i)$, $i=1,\ldots,5$, are singular on the light cone. In general, one can have a sum of such functions before each covariant in the expansion (3.9), but we retain only the most singular one in each case. The operators $Q_i(x,0)$ are bilocal and possess an infinte series expansion in terms of local operators.

Next we write down expressions for the matrix elements of the various bilocal operators $Q_i(x,0)$ between states of spins J, J' and momenta Δ and Δ', respectively. First we define the functions $M_{i,\alpha;\beta}$ by

$$\langle\Delta',\epsilon_{J'}|Q_i(x,0)|\Delta,\epsilon_J\rangle=\epsilon^{*\alpha_1\cdots\alpha_{J'}}M_{i,\alpha_1\cdots\alpha'_J;\beta_1\cdots\beta_J}\epsilon^{\beta_1\cdots\beta_J}\,. \tag{3.10}$$

Of course the $Q_i(x,0)$'s can carry Lorentz indices leading to extra Lorentz indices on the $M_{i,\alpha;\beta}$'s. We then write

$$M_{1,\alpha;\beta}=-i\tilde f^{(1)}_{J,J'}x_{\alpha_1}\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots x_{\beta_J}+\sum_{k,l}i\tilde h^{(1)}_{[kl];J,J'}g_{\alpha_k\beta_l}x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{J'}x_{\beta_1}\cdots[x_{\beta_l}]\cdots x_{\beta_J}+\cdots\,. \tag{3.11}$$

The square bracket about a single x, e.g., $[x_{\alpha_l}]$, indicates that the corresponding factor is missing from the product. The dots stand for terms with four or more factors of x missing. Such terms give rise to lower-order contributions in the high-energy limit and need not concern us here. An analogous representation holds in the case of the operator $Q_4(x,0)$ and one only needs to change the label (1) on the scalar functions to read (4), i.e., $\tilde f^{(1)}_{J,J'}$ becomes $\tilde f^{(4)}_{J,J'}$, etc. Next we have

$$\begin{aligned}M^\sigma_{3,\alpha;\beta}=&-i\tilde f^{(3)}_{J,J'}\Delta^\sigma x_{\alpha_1}\cdots_{\alpha_{J'}}x_{\beta_1}\cdots x_{\beta_J}\\&+\tfrac{1}{2}\tilde g^{(3)}_{J,J'}\left[\sum_{k=1}^{J'}g^\sigma_{\alpha_k}x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots x_{\beta_J}+\sum_{k=1}^{J}g^\sigma_{\beta_k}x_{\alpha_1}\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots[x_{\beta_k}]\cdots x_{\beta_J}\right]\\&+i\Delta^\sigma\left[\tilde h^{(3)}_{J,J'}\sum_{k,l}g_{\alpha_k\beta_l}x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots[x_{\beta_l}]\cdots x_{\beta_J}+\cdots\right]+(\Delta\leftrightarrow\Delta')+\cdots\,.\end{aligned} \tag{3.12}$$

Again a similar expansion holds for the matrix element of $Q_5^\lambda(x,0)$ with the obvious change, $\sigma\to\lambda$ and the replacement of the label (3) on the scalar functions by (5). Finally, we have

$$M^{\lambda\sigma}_{2,\alpha;\beta}=-i\tilde{f}^{(2)}_{J,J'}\Delta^\lambda\Delta^\sigma x_{\alpha_1}\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots x_{\beta_J}+i\tilde{l}^{(2)}_{J,J'}g^{\lambda\sigma}x_{\alpha_1}\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots x_{\beta_J}$$
$$+\tfrac{1}{2}\tilde{g}^{(2)}_{J,J'}\Big[\sum_{k=1}^{J'}(\Delta^\lambda g^\sigma_{\alpha_k}+\Delta^\sigma g^\lambda_{\alpha_k})x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots x_{\beta_J}$$
$$+\sum_{l=1}^{J}(\Delta^\lambda g^\sigma_{\beta_l}+\Delta^\sigma g^\lambda_{\beta_l})x_{\alpha_1}\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots[x_{\beta_l}]\cdots x_{\beta_J}\Big]$$
$$+\tfrac{1}{4}i\tilde{h}^{(2)}_{J,J'}\Big[\sum_{k\neq l}g^\lambda_{\alpha_k}g^\sigma_{\alpha_l}x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots[x_{\alpha_l}]\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots x_{\beta_J}$$
$$+\sum_{k,l}(g^\lambda_{\alpha_k}g^\sigma_{\beta_l}+g^\sigma_{\alpha_k}g^\lambda_{\beta_l})x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots[x_{\beta_l}]\cdots x_{\beta_J}$$
$$+\sum_{k\neq l}g^\lambda_{\beta_k}g^\sigma_{\beta_l}x_{\alpha_1}\cdots x_{\alpha_{J'}}x_{\beta_1}\cdots[x_{\beta_k}]\cdots[x_{\beta_l}]\cdots x_{\beta_J}\Big]+(\Delta\leftrightarrow\Delta')+\cdots\,. \tag{3.13}$$

Now $W_{\mu\nu}$ can be written as a sum of five terms corresponding to the five terms that occur in the LC expansion, Eq. (3.9). We write

$$W_{\mu\nu}=\sum_{i=1}^{5}W^{(i)}_{\mu\nu}\,. \tag{3.14}$$

As an illustration we write the expression for $W^{(2)}_{\mu\nu}$:

$$W^{(2)}_{\mu\nu}=\sum_{J,J'}\frac{(-i)^J(i)^{J'}}{2(2\pi)^4}\int d^4x\,e^{iq\cdot x}\frac{g(J,\Delta^2)g(J',\Delta'^2)}{[M^2(J)-\Delta^2][M^2(J')-\Delta'^2]}(g_{\mu\nu}\partial_\lambda\partial_\sigma-g_{\mu\lambda}\partial_\nu\partial_\sigma-g_{\nu\sigma}\partial_\mu\partial_\lambda+g_{\mu\lambda}g_{\nu\sigma}\Box)E_2(x;c_2)$$
$$\times\{-i\tilde{f}^{(2)}_{J,J'}\Delta^\lambda\Delta^\sigma(x\cdot P)^{J+J'}+i\tilde{l}^{(2)}_{J,J'}g^{\lambda\sigma}(x\cdot P)^{J+J'}+\tfrac{1}{2}(J+J')\tilde{g}^{(2)}_{J,J'}[\Delta^\lambda P^\sigma+\Delta^\sigma P^\lambda](x\cdot P)^{J+J'-1}$$
$$+\tfrac{1}{4}i(J+J')(J+J'-1)\tilde{h}^{(2)}_{J,J'}P^\lambda P^\sigma(x\cdot P)^{J+J'-2}+(\Delta\leftrightarrow\Delta')+\cdots\}\,. \tag{3.15}$$

In evaluating (3.15) and similar expressions we retain only the most leading LC singularity. We shall standardize the functions $E(x;c)$ by the definition[13]:

$$iE(x;c)=\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot x}\left[\left(\frac{1}{-k^2-i\epsilon k_0}\right)^{2-(c/2)}-\left(\frac{1}{-k^2+i\epsilon k_0}\right)^{2-(c/2)}\right]$$
$$=\frac{-i}{2^{4-c}\pi^2}\frac{\Gamma(c/2)}{\Gamma(2-(c/2))}\times[(-x^2-i\epsilon x_0)^{-c/2}-(-x^2+i\epsilon x_0)^{-c/2}]\,. \tag{3.16}$$

Next we observe that on the right-hand side of Eq. (3.11), etc., there occur scalar functions that depend in general on the five variables $x\cdot\Delta$, $x\cdot\Delta'$, Δ^2, Δ'^2, $\Delta\cdot\Delta'$. Denoting a prototype of such functions by $\tilde{\phi}_{J,J'}(x\cdot\Delta,x\cdot\Delta',\ldots)$ we define the two-dimensional Fourier transform $\phi_{JJ'}(\xi,\eta,\ldots)$ by

$$\tilde{\phi}_{J,J'}(x\cdot\Delta,x\cdot\Delta',\ldots)=\int\int d\xi\,d\eta\,e^{i(\xi\Delta+\eta\Delta')\cdot x}\times\phi_{J,J'}(\xi,\eta,\ldots)\,. \tag{3.17}$$

By using Eqs. (3.16) and (3.17) one can evaluate the $W^{(i)}_{\mu\nu}$'s and hence $W_{\mu\nu}$. The J and J' summations are performed in the usual manner by the Sommerfeld-Watson transformation.[14] The calculations are quite lengthy but straightforward and will not be reproduced here. The resultant tensor structure of $W_{\mu\nu}$ reads

$$W_{\mu\nu}=g_{\mu\nu}W_1+\Delta_\mu\Delta_\nu W_2+P_\mu P_\nu W_3+\tfrac{1}{2}(P_\mu\Delta_\nu+P_\nu\Delta_\mu)W_4$$
$$+i\epsilon_{\mu\nu\lambda\sigma}q^\lambda\Delta^\sigma W_5+i\epsilon_{\mu\nu\lambda\sigma}q^\lambda P^\sigma W_6$$
$$+i\epsilon_{\mu\nu\lambda\sigma}\Delta^\lambda P^\sigma W_7+\cdots\,. \tag{3.18}$$

The dots in Eq. (3.18) stand for terms that give zero upon contraction with the lepton tensor $\tau^{\mu\nu}$. Leaving these aside, the most general tensor decomposition of $W_{\mu\nu}$ involves nine invariant amplitudes. We see that our model gives two of those to be identically zero.

The decision as to which terms survive in the high-energy region depends on the values of the parameters c_i that characterize the singular functions $E_i(x;c_i)$. We now turn to a discussion of these parameters. One makes the assumption that the local fields that appear in the expansion of the $Q_i(x,0)$'s have leading scale dimension $J+2$ when the dimension of the symmetry-breaking Hamiltonian density, $\mathcal{H}_B$, is greater than two, but that the fields contributing to $Q_4(x, 0)$ and $Q_5^\lambda(x, 0)$ have leading dimension $J+d$, where $d=\dim \mathcal{H}_B$, when d is less than two.[8] We take the leading dimension of the weak current J_μ to be three. Then, allowing for the possibility that those parts of J_μ with leading dimension might commute on the light cone, one obtains for the case $d \geq 2$ (Ref. 8)

$$2 \geq c_k, \quad k=1, 3, 4, 5$$
$$0 \geq c_2 . \tag{3.19}$$

When $d<2$ we have instead

$$4 \geq c_{4,5}+d, \tag{3.20}$$

with the inequalities for the remaining c_i's being the same as in Eq. (3.19). Equations (3.19) and (3.20) are obtained by performing an infinitesimal dilatation transformation on both sides of Eq. (3.9). Now since $\partial^\mu J_\mu = i[\mathcal{H}_B, \int d^3x\, J_0(x)]$ it follows that the divergence $\partial^\mu J_\mu$ belongs to the same SU(3) $\times$SU(3) multiplet as $\mathcal{H}_B$ and hence must possess the same dimension. Then taking the divergence of Eq. (3.9) we get in the same manner as above

$$d-2 \geq c_{4,5} . \tag{3.21}$$

This result holds in the case $d>2$, while in the case $d<2$ we have

$$0 \geq c_{4,5} . \tag{3.22}$$

Note that if the bounds expressed by Eqs. (3.19) and (3.21) are saturated corresponding to maximal singularity structure of the LC commutator, then $\mathcal{H}_B$ will have parts with dimension 4. So the inequalities cannot both be maximally satisfied as equalities simultaneously if one is not willing to tolerate terms in $\mathcal{H}_B$ with as high a dimension as those of the symmetric Hamiltonian.[15]

Taking note of Eqs. (3.19) and (3.20) and keeping only the leading contributions arising from each singularity in the LC expansion we arrive at the following expressions for W_i, $i=1, \ldots, 7$, in the deep-inelastic region characterized earlier:

$$W_1 \simeq -S(\alpha, \Delta^2)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}\left\{(2q\cdot\Delta)^{-1+(c_1/2)}[E_1+2\alpha E_2]+\tfrac{1}{2}(2q\cdot\Delta)^{c_2/2}E_3\right\}, \tag{3.23}$$

$$W_2 \simeq -S(\alpha, \Delta^2)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}\left\{(2q\cdot\Delta)^{-2+(c_1/2)}F_1+(2q\cdot\Delta)^{-2+(c_4/2)}F_2 +4(2q\cdot\Delta)^{-2+(c_5/2)}F_3+2(2q\cdot\Delta)^{-1+(c_2/2)}[F_4-2\alpha F_5]\right\}, \tag{3.24}$$

$$W_3 \simeq S(\alpha, \Delta^2)2\alpha(2\alpha-1)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha-2}\left\{-(2q\cdot\Delta)^{-2+(c_1/2)}G_1-2(2q\cdot\Delta)^{-2+(c_4/2)}G_2 +2(2q\cdot\Delta)^{-2+(c_5/2)}G_3-(2q\cdot\Delta)^{-1+(c_2/2)}G_4\right\}, \tag{3.25}$$

$$W_4 \simeq S(\alpha, \Delta^2)4\alpha\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha-1}\left\{(2q\cdot\Delta)^{-2+(c_1/2)}H_1+(2q\cdot\Delta)^{-2+(c_4/2)}H_2+2(2q\cdot\Delta)^{-2+(c_5/2)}H_3+(2q\cdot\Delta)^{-1+(c_2/2)}H_4\right\}, \tag{3.26}$$

$$W_5 \simeq -2S(\alpha, \Delta^2)(q\cdot P/q\cdot\Delta)^{2\alpha}(2q\cdot\Delta)^{-2+(c_3/2)}L_1, \tag{3.27}$$

$$W_6 \simeq -2S(\alpha, \Delta^2)\alpha(q\cdot P/q\cdot\Delta)^{2\alpha-1}(2q\cdot\Delta)^{-2+(c_3/2)}L_2, \tag{3.28}$$

$$W_7 \simeq -2S(\alpha, \Delta^2)\alpha(q\cdot P/q\cdot\Delta)^{2\alpha-1}(2q\cdot\Delta)^{-2+(c_3/2)}L_3 . \tag{3.29}$$

In these equations $\alpha=\alpha(\Delta^2)$ is the largest root of the equation

$$M(\alpha(\Delta^2))-\Delta^2=0, \tag{3.30}$$

and $S(\alpha, \Delta^2)$ is given by

$$S(\alpha, \Delta^2)=\frac{\pi^2 g^2(\alpha(\Delta^2), \Delta^2)}{2\sin^2\pi\,\alpha(\Delta^2)}\left(\frac{d\,\alpha(\Delta^2)}{d\Delta^2}\right)^2 . \tag{3.31}$$

The functions E_i, F_i, G_i, H_i, and L_i are functions of ω, Δ^2, and $\alpha(\Delta^2)$. They are given by double integrals involving the Fourier transforms ϕ_i of the functions $\tilde{\phi}_i$ introduced earlier in Eqs. (3.11), etc. They are listed in the Appendix.

Using Eqs. (2.9) and (3.23)–(3.29) one can now calculate the elements of the hadronic density matrix $\rho_{mm'}$, defined by Eq. (2.11), in the laboratory frame (or in any frame in which q has the form $q=(q_0, 0, 0, |\vec{q}|)$. We have (with $\pm$ denoting ± 1):

$$\rho_{++} \simeq S(\alpha, \Delta^2)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}\left\{(2q\cdot\Delta)^{-1+(c_1/2)}[E_1+2\alpha E_2]+\tfrac{1}{2}(2q\cdot\Delta)^{c_2/2}E_3+2(|\vec{q}|\Delta_0+q_0\Delta_3)(2q\cdot\Delta)^{-2+(c_3/2)}L_1\right\}, \tag{3.32}$$

$$\rho_{--} \simeq S(\alpha, \Delta^2)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}\left\{(2q\cdot\Delta)^{-1+(c_1/2)}[E_1+2\alpha E_2]+\tfrac{1}{2}(2q\cdot\Delta)^{c_2/2}E_3-2(|\vec{q}|\Delta_0+q_0\Delta_3)(2q+\Delta)^{-2+(c_3/2)}L_1\right\}, \quad (3.33)$$

$$\rho_{+-}=\rho_{-+}\simeq \tfrac{1}{2}S(\alpha, \Delta^2)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}\Delta_1{}^2\left\{(2q\cdot\Delta)^{-2+(c_1/2)}F_1+(2q\cdot\Delta)^{-2+(c_4/2)}F_2+4(2q\cdot\Delta)^{-2+(c_5/2)}F_3\right\}. \quad (3.34)$$

If we denote the quantity appearing inside the curly bracket in Eq. (3.34) by Ψ we can then write

$$\rho_{+0}=\rho_{0+}\simeq S(\alpha, \Delta^2)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}\frac{1}{2(\omega q\cdot\Delta)^{1/2}}\left\{\Delta_1(|\vec{q}|\Delta_0+q_0\Delta_3)\Psi+\Delta_1(2q\cdot\Delta)^{-1+(c_3/2)}[\alpha L_3-2\omega L_1]\right\}, \quad (3.35)$$

$$\rho_{-0}=\rho_{0-}\simeq -S(\alpha, \Delta^2)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha}\frac{1}{2(\omega q\cdot\Delta)^{1/2}}\left\{\Delta_1(|\vec{q}|\Delta_0+q_0\Delta_3)\Psi-\Delta_1(2q\cdot\Delta)^{-1+(c_3/2)}[\alpha L_3-2\omega L_1]\right\}. \quad (3.36)$$

Finally,

$$\begin{aligned}\rho_{00}\simeq -S(\alpha, \Delta^2)\left(\frac{q\cdot P}{q\cdot\Delta}\right)^{2\alpha} & (2q\cdot\Delta)^{-1+(c_1/2)}\left[E_1+2\alpha E_2+\frac{1}{4\omega}F_1+\frac{1}{2\omega}\alpha(\alpha-1)G_1-H_1\right] \\ & +(2q\cdot\Delta)^{c_2/2}\left[\tfrac{1}{2}E_3+\frac{1}{2\omega}F_4-\frac{\alpha}{\omega}F_5+\frac{1}{2\omega}\alpha(2\alpha-1)G_4-H_4\right] \\ & +(2q\cdot\Delta)^{-1+(c_4/2)}\left[\frac{1}{4\omega}F_2+\frac{1}{\omega}\alpha(2\alpha-1)G_2-H_2\right] \\ & +(2q\cdot\Delta)^{-1+(c_5/2)}\left[\frac{1}{\omega}F_3-\frac{1}{\omega}\alpha(2\alpha-1)G_3-2H_3\right]-(2q\cdot\Delta)^{-2+(c_3/2)}4(|\vec{q}|\Delta_0+q_0\Delta_3)L_1 .\end{aligned} \quad (3.37)$$

The correlation function $W=\tau^{\mu\nu}W_{\mu\nu}$ that determines the cross section can be written down easily in terms of the elements $\rho_{mm'}$. Reading the values of $\Phi_{mm'}$ from Ref. 9 one finds that

$$\begin{aligned}W=\tfrac{1}{2}G_F{}^2q^2\{&-2\sinh^2\xi\,\mathrm{Re}\rho_{00}^{+}-\sqrt{2}\cos\psi\sinh 2\xi\,\mathrm{Re}\rho_{+0}^{+}-(1+\cosh^2\xi)\,\mathrm{Re}\rho_{++}^{+} \\ &+\cos 2\psi\sin^3\xi\,\mathrm{Re}\rho_{+-}^{+}+\sin 2\psi\sinh^2\xi\,\mathrm{Im}\rho_{+-}^{-} \\ &-\sqrt{2}\sin\psi\sinh 2\xi\,\mathrm{Im}\rho_{+0}^{-}-2^{3/2}\sin\psi\sinh\xi\,\mathrm{Im}\rho_{+0}^{+}+2\cosh\xi\,\mathrm{Re}\rho_{++}^{-}-2^{3/2}\cos\psi\sinh\xi\,\mathrm{Re}\rho_{+0}^{-}\}.\end{aligned} \quad (3.38)$$

The superscripts ($\pm$) on the elements of the hadronic-density matrix signify parity nonconservation at the hadron vertex. Thus $\rho_{mm'}^{+}$ is given by the VV and the AA parts of the hadronic weak current and $\rho_{mm'}^{-}$ is given by the VA part. These can be easily read from Eqs. (3.32) to (3.37). The boost variable ξ is given in terms of laboratory quantities by

$$\cosh\xi=\frac{E+E'}{|\vec{q}|} . \quad (3.39)$$

IV. DISCUSSION

In Sec. III we have obtained expressions for the structure functions in the deep-inelastic Regge region envisaged as the subdomain of the Regge region (3.8) in which the mass of the current and the missing hadronic mass become large but are still small compared to ν. We have retained only the leading contributions arising from each singularity that occurs in the LC expansion. Each of the structure structure functions W_i is found to be some power of $(q\cdot P/q\cdot\Delta)$ times another function, say, f_i. [See Eqs. (3.23)–(3.29).] The scaling properties of the f_i's are of special interest. We first note that scaling obtains for $W_1, \ldots, W_4$ if the following condition holds:

$$c_1=c_4=c_5=c_2+2 . \quad (4.1)$$

It is evident that W_5, W_6, W_7 are already in the scaling form in the above sense. If we now assume that the inequalities (3.19) and (3.21) of Sec. III for the case $d>2$ are satisfied maximally as equalities, then we are led to conclude that $d=4$. Now the form of the singular functions $E_i(x, c_i)$ given in Eq. (3.16) as homogeneous functions of degree c_i in x is suggested by scale invariance at short distances. If d has parts with as high a dimension as 4, then scale invariance breaks down at short distances and one can no longer derive the form stated earlier for $E(x, c_i)$ but must accept it as a separate postulate. Now, of course, the situation expressed by Eq. (4.1) is not the only one that leads to scaling. Indeed, if we assume that $2\leq d<4$, then Eq. (3.21) implies that $c_{4,5}<2$, and if either the parameter c_1 or c_2 attains its maximum value of two or zero, respectively, then scaling obviously obtains.[16] In any case, Eq. (4.1) can hold without the condition constraining the parameters c_1, c_2, c_4, c_5 to their maximum values holding and we would still obtain scaling. For $d>2$ a particularly attractive situation is the one in which the inequality (3.19) is satisfied as an equality for $k=1, 2, 3$ but not for $k=4, 5$. For the latter values it is the inequality (3.21) that is maximally satisfied. In this case one has

$$c_1 = c_3 = 2, \quad c_2 = 0, \tag{4.2}$$

$$c_4 = c_5 = d-2\,. \tag{4.3}$$

Condition (4.2) would then give a structure for the first three terms in the LC commutator of two weak currents identical to that of the LC commutator of two electromagnetic currents.[17] The structure of the latter commutator has, of course, been sanctioned by the scaling observed in deep-inelastic scattering. Condition (4.3) gives a maximum singularity structure for the LC of the weak current and its divergence. The scaling behavior that follows from it for the structure functions of inclusive neutrino scattering has been studied by the authors of Ref. 8. Now, if one accepts Eqs. (4.2) and (4.3), then one can in principle determine d from experiments on coincidence hadron production in neutrino-induced reactions. In fact, by extracting the term $(q\cdot P/q\cdot\Delta)^{2\alpha}$ one can study the behavior of the residual functions in the cross section $q\cdot\Delta$. Since, e.g., the residual W_1 would have the structure

$$a_1 + b_1(2q\cdot\Delta)^{d-2}+\cdots,$$

where a_1, b_1 are independent of $q\cdot\Delta$, a determination of d in this way is possible in principle but the experimental demands may be rather strong.

For the case $d<2$ (Ref. 18) one can still assume the validity of Eq. (4.2). However, we recall that (3.20) and (3.22) cannot obviously be simultaneously satisfied as equalities. If $c_4 = c_5 = 4-d$, then the contributions arising from the symmetry-breaking interactions to the structure functions W_i would be the leading ones as far as the $q\cdot\Delta$ dependences go. If, on the other hand, $c_4=c_5=0$, then there is no trace of the symmetry-breaking interactions in the asymptotic behavior. These two situations can again be distinguished, in principle, experimentally by examining the behavior of the residual functions in $q\cdot\Delta$.

Next we look into the differential cross-section spectrum that the model gives. Now the implication of Regge-pole dominance for strong two-body $\to$ two-body reactions is that all helicity amplitudes for a given process possess a common phase to leading order in energy. This phase is determined by the signature factor of the dominant Regge trajectory. This then implies the vanishing, to leading order in energy, of odd correlation terms in the differential cross section spectrum, i.e., correlations that are odd under reversal of all spins and momenta. For weak transitions the presence of parity-violating interactions would then give rise to several such odd correlation terms. Pais and Treiman[1] have pointed out that this phase property of Regge-pole theory might transcend its other more detailed features and the vanishing of such odd correlation terms would provide a critical test of the idea of single-trajectory dominance. In our peripheral model for coincidence nucleon production in neutrino-initiated reactions the nucleon was regarded as uninteracting and separated from the final state. It is natural to suppose that the relative phase properties reflect final-state interactions among the particles in the system X. Moreover, in the coincidence production cross section we are summing over a complete set of channels X connected by strong interactions. We then expect that the odd correlation terms in W that arise from the parity-violating interaction [those containing $\rho^-_{mm'}$ in Eq. (3.38)] and the odd correlation term proportional to $\sin\phi$ in the remaining part of the spectrum should vanish to leading order. This expectation is based on the fact that the Van Hove model that we have used to construct W generates Regge asymptotic behavior. Now with the condition (4.2) it is readily seen that terms proportional to ρ^-_{++} in the differential cross-section spectrum would give a vanishing contribution to leading order if the following condition is met:

$$L_1(\alpha,\omega,\Delta^2)=0\,. \tag{4.4}$$

From the integral representation of L_1 given by Eq. (A18) of the appendix we deduce the sum rule

$$\int d\xi\,d\eta\left[\frac{1}{(\omega-\xi-\eta-i\epsilon)^{2\alpha+1}} - \frac{1}{(\omega-\xi-\eta+i\epsilon)^{2\alpha+1}}\right] f^{(3)}(\xi,\eta,\Delta^2,\alpha(\Delta^2))=0. \tag{4.5}$$

This condition arises from the term ρ^-_{++}. Now the dominant terms in W come from ρ^+_{++} and ρ^+_{00}. Relative to those ρ^-_{+0} is suppressed. In fact

$$\frac{\rho^-_{+0}}{\rho^+_{++}} \sim \frac{1}{(2q\cdot\Delta)^{1/2}},$$

and hence this odd correlation term gives a vanishing contribution relative to ρ^+_{++} and ρ^+_{00} in accordance with expectations. It is also seen from Eqs. (3.32)–(3.36) that the coefficient of the $\sin\psi$ term in the remaining part of the spectrum vanishes identically. The sum rule (4.5) involves what may be called the "Reggeon structure function." Finally we note that the sum rule (4.5) can be avoided if $c_3 < c_1$. In fact, when this is so we have

$$\frac{\rho^-_{++}}{\rho^+_{++}} \sim \frac{1}{[2q \cdot \Delta]^{(c_1 - c_3)/2}}$$

and the odd correlation term ρ^-_{++} is then automatically suppressed relative to the leading terms in the spectrum.

Note added in proof. Since this paper was submitted, some papers have appeared which deal with the topic of one-particle inclusive production in lepton-induced reactions in the deep-inelastic region. We have listed them in Ref. 19.

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APPENDIX

Here we give expressions for the functions E_i, F_i, G_i, H_i and L_i that occur in Eqs. (3.23)–(3.29). Defining the function $Y(\omega, \xi, \eta; c, \alpha)$ by

$$Y(\omega, \xi, \eta; c, \alpha) = \frac{1}{(\omega - \xi - \eta - i\epsilon)^{-(c/2)+2+2\alpha}} - \frac{1}{(\omega - \xi - \eta + i\epsilon)^{-(c/2)+2+2\alpha}}, \quad \text{(A1)}$$

we then have

$$E_1 = \alpha(2\alpha + 1 - c_1)\int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_1 + 2,\ \alpha) f^{(1)}, \quad \text{(A2)}$$

$$E_2 = -\tfrac{1}{2}(2\alpha - 1)(2\alpha + 2 - c_1) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_1,\ \alpha) f^{(1)}, \quad \text{(A3)}$$

$$E_3 = \alpha(2\alpha + 3 - c_2)\int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_2,\ \alpha) f^{(2)}, \quad \text{(A4)}$$

$$F_1 = \alpha(2\alpha + 3 - c_1)\int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_1,\ \alpha) \times (\xi + \eta)^2 f^{(1)}, \quad \text{(A5)}$$

$$F_2 = \alpha(2\alpha + 3 - c_4)\int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_4,\ \alpha) \times (\xi + \eta)^2 f^{(4)}, \quad \text{(A6)}$$

$$F_3 = \alpha(2\alpha + 3 - c_5)\int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_5,\ \alpha) \times (\xi + \eta) f^{(5)}, \quad \text{(A7)}$$

$$F_4 = \alpha \int d\xi\, d\eta [(2\alpha + 3 - c_2) Y(\omega, \xi, \eta;\ c_2,\ \alpha)(\xi + \eta) - (2\alpha + 1 - c_2) Y(\omega, \xi, \eta;\ c_2 + 2,\ \alpha)] f^{(2)}, \quad \text{(A8)}$$

$$F_5 = -\tfrac{1}{2}(2\alpha - 1)(2\alpha + 2 - c_2) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_2,\ \alpha) f^{(2)}, \quad \text{(A9)}$$

$$G_1 = (\alpha - 1)(2 + 1 - c_1)\int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_1,\ \alpha) f^{(1)}, \quad \text{(A10)}$$

$$G_2 = (\alpha - 1)(2\alpha + 1 - c_4) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_4,\ \alpha) f^{(4)}, \quad \text{(A11)}$$

$$G_3 = (\alpha - 1)(2\alpha + 1 - c_5) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_5,\ \alpha) g^{(5)}, \quad \text{(A12)}$$

$$G_4 = (\alpha - 1)\int d\xi\, d\eta [(2\alpha + 1 - c_2) Y(\omega, \xi, \eta;\ c_2,\ \alpha) g^{(2)} - \tfrac{1}{4}(2\alpha - 1 - c_2) \times Y(\omega, \xi, \eta;\ c_2 + 2,\ \alpha) h^{(2)}], \quad \text{(A13)}$$

$$H_1 = -\tfrac{1}{2}(2\alpha - 1)(2\alpha + 2 - c_1) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_1,\ \alpha)(\xi + \eta) f^{(1)}, \quad \text{(A14)}$$

$$H_2 = -\tfrac{1}{2}(2\alpha - 1)(2\alpha + 2 - c_4) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_4,\ \alpha)(\xi + \eta) f^{(4)}, \quad \text{(A15)}$$

$$H_3 = -\tfrac{1}{2}(2\alpha - 1)(2\alpha + 2 - c_5) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\ c_5,\ \alpha)[f^{(5)} - (\xi + \eta) g^{(5)}], \quad \text{(A16)}$$

$$\begin{aligned} H_4 = -\tfrac{1}{2}(2\alpha - 1)\int d\xi\, d\eta \times [&(2\alpha - c_2) Y(\omega, \xi, \eta;\ c_2 + 2,\ \alpha) g^{(2)} \\ &+ (2\alpha + 2 - c_2) Y(\omega, \xi, \eta;\ c_2,\ \alpha) \xi g^{(2)} \\ &+ (2\alpha + 2 - c_2)(2\alpha - 1) Y(\omega, \xi, \eta;\ c_2,\ \alpha) g^{(2)} \\ &- (2\alpha + 2 - c_2) Y(\omega, \xi, \eta;\ c_2,\ \alpha) f^{(2)}], \end{aligned} \quad \text{(A17)}$$

$$L_1 = \alpha(2\alpha + 3 - c_3)\int d\xi\, d\eta\, Y(\omega, \xi, \eta;\, c_3, \alpha) f^{(3)}, \tag{A18}$$

$$L_2 = -\tfrac{1}{2}(2\alpha - 1)(2\alpha + 2 - c_3) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\, c_3, \alpha) g^{(3)}, \tag{A19}$$

$$L_3 = -\tfrac{1}{2}(2\alpha - 1)(2\alpha + 2 - c_3) \times \int d\xi\, d\eta\, Y(\omega, \xi, \eta;\, c_3, \alpha)[(\xi + \eta) g^{(3)} + 2 f^{(3)}]. \tag{A20}$$

In these equations the functions ϕ appearing in the integrand stand for $[\phi_{J,J'}(\xi, \eta, \Delta^2)]_{J=J'=\alpha}$, where the $\phi_{J,J'}$ are the Fourier transforms [see Eq. (3.17)] of the functions $\tilde{\phi}_{J,J'}$ that occur in Eq. (3.11), etc. [see Eq. (3.17)]. In deriving Eqs. (A2)–(A20) one makes the assumption that the functions $\phi_{J,J'}$ vanish sufficiently rapidly as the variables ξ and η approach infinity. One also assumes that as functions of J and J' they are smooth and vanish sufficiently rapidly for infinite J and J', so that after the Sommerfeld-Watson transform operation is performed one simply replaces J and J' by α to obtain the ϕ's.

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*On leave of absence from the Physics Department, University of Khartoum, Khartoum, Sudan.

[1]A. Pais and S. B. Treiman, Phys. Rev. D 1, 907 (1970).

[2]T. P. Cheng and A. Zee, Phys. Rev. D 6, 885 (1972).

[3]We shall be mainly concerned in this paper with the most important situation in which the initial and detected hadrons are both nucleons. Some people may have qualms over viewing the leading Pomeranchuk trajectory in the manner of the Van Hove model. These can be put to rest if, as the evidence suggests, the Pomeranchuk trajectory communicates with the hadrons via the f and f' trajectories. In this connection see Ref. 2 as well as R. Carlitz, M. B. Green, and A. Zee [Phys. Rev. Lett. 26, 1515 (1971); Phys. Rev. D 4, 3439 (1971)].

[4]L. Van Hove, Phys. Lett. 24B, 183 (1967).

[5]L. Durand III, Phys. Rev. 154, 1537 (1967).

[6]H. Fritzsch and M. Gell-Mann, in *Coral Gables Conference on Fundamental Interactions at High Energy II*, edited by A. Perlmutter, G. J. Iverson, and R. M. Williams (Gordon and Breach, New York, 1970).

[7]K. Wilson, Phys. Rev. 179, 1499 (1969).

[8]J. Mandula, A. Schwimmer, J. Weyers, and G. Zweig, Phys. Lett. 37B, 109 (1971).

[9]I. J. Muzinich, Ling-Lie Wang, and Jiunn-Ming Wang, Phys. Rev. D 2, 1985 (1970); see also T. P. Cheng and Wu-Ki Tung, *ibid.* 3, 733 (1971).

[10]We use the following conventions: The states are normalized covariantly: $\langle p'\lambda'|p\lambda\rangle = E\delta^3(\vec{p} - \vec{p}\,')\delta_{\lambda\lambda'}$; the lepton spinors satisfy $\bar{u}u = m$, $\bar{v}v = -m$; the metric tensor is $g_{\mu\nu} = (1, -1, -1, -1)$.

[11]For a summary of the properties of high-spin boson propagators see the Appendix of D. Steele, Phys. Rev. D 2, 1610 (1970).

[12]In the differential cross section what enters is the absorptive part of the forward scattering amplitude of the current on the spin-J particle. However, we perform the calculation with nonforward geometry and later pass to the limit of the forward direction.

[13]Y. Frishman, Phys. Rev. Lett. 25, 966 (1970).

[14]We assume that the functions $\phi_{J,J'}$ have a smooth behavior in J and J' and vanish sufficiently rapidly for $J, J' \to \infty$.

[15]In Ref. 8 it was assumed that (3.21) and (3.22) are maximally obeyed as equalities. Our point here is that if this happens with (3.21), then it cannot happen for (3.19) with $k = 4, 5$. For $d < 2$ it is obvious that (3.20) and (3.22) cannot simultaneously hold as equalities. Thus, the authors of Ref. 8 assume a maximal singularity structure for the commutator of the weak current and its divergence for $4 > d \geq 2$, with $c_4 = c_5 = d - 2$ and consequently $2 > c_{4,5}$. If it turns out that it is the commutator between the *currents* that is maximally singular, then the sum rules for the structure functions of inclusive neutrino reactions of Ref. 8 will have to be correspondingly modified.

[16]This, of course, holds provided that the appropriate scaling functions that appear inside the curly brackets in Eq. (3.23) – (3.26) do not trivially vanish.

[17]R. Brandt and G. Preparata, Nucl. Phys. B27, 547 (1971).

[18]S. G. Brown in a recent paper [Phys. Lett. 39B, 399 (1972)] shows that under certain assumptions the case $d < 2$ can be excluded.

[19]H. Fritzsch and P. Minkowski, Nucl Phys. B55, 63 (1973); R. Gatto and G. Preparata, DESY Report No. DESY 73/17, 1973 (unpublished). See also the talk of G. Altarelli in Rome University Nota Interna No. 459, 1973 (unpublished).