Comparison of a dual resonance model with spin with the reactions $\pi^+ p \to \omega \Delta^{++}$, $\pi^- p \to \omega n$, $\pi^- p \to \rho^0 n$, and $\pi^- p \to B^- p$ †

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(Received 24 September 1973)

The model for $\pi N \to \pi\omega N$ scattering constructed in a previous paper is applied to the four quasi-two-body reactions $\pi^+ p \to \omega\Delta^{++}$, $\pi^- p \to \omega n$, $\pi^- p \to \rho^0 n$, and $\pi^- p \to B^- p$. It is found that reasonable agreement with the data is obtained for the features of the model which do not involve the B meson. It is also found that B exchange is too small to account for the unnatural-parity contribution to the two ω-production reactions, indicating that the ρ-ρ cut might be important in these processes.

I. INTRODUCTION

In a previous paper[1] (herein referred to as I) a dual resonance-type model is constructed for the reaction $\pi N \to \pi\omega N$. The main feature of the model is that the spin of the external particles is taken into account. This is done, in the invariant formalism, by combining the analytic structure given by Feynman graphs with the B_5 functions of Bardakci and Ruegg,[2] thus providing a way to analytically continue the amplitude in all kinematical regions. The model contains both normal and abnormal vertices and consequently trajectories of both parities can appear in the same channel [for example, the $(\pi\omega)$ channel contains both the ρ and the B trajectories]. It was also shown that, at least in meson channels, there was no parity doubling. Other properties of the model are the same as those of the generalized Veneziano formula[2]: resonance structure, Regge behavior, duality, and bootstrap consistency (with some restrictions, however, concerning the baryon channels). The only parameters of the model are α', the common slope of the Regge trajectories, and the coupling constants. Relations are derived between them, and making use of the fact that α' and some coupling constants have well-known values, we are left with two free parameters, G_V^ω and G_T^ω, the coupling constants of the ω meson to the $(N\bar{N})$ system.

In this paper the model is compared with the quasi-two-body reactions: $\pi N \to \rho N$, $\pi N \to \omega N$, $\pi N \to \omega\Delta$, and $\pi N \to BN$, which are subprocesses of the reaction $\pi N \to \pi\omega N$. There are several reasons for this choice of reactions: (a) The kinematics is simpler for two-body reactions than for processes with three particles in the final state, and therefore the numerical calculations will be easier; (b) the two-body reactions under consideration will allow us to separate the various dynamical contributions to the amplitude for $\pi N \to \pi\omega N$ and to compare them with experimental data (differential cross sections and the density-matrix elements of the resonances produced); (c) finally, these reactions will test the model in two distinct kinematical regions, baryon-resonance production and meson-resonance production.

The fits to the data are presented in Sec. II, while a discussion of the results concerning the coupling constants is given in Sec. III. Section IV contains the conclusions.

II. THE FITS

A. Generalities

All amplitudes will be calculated in the t-channel baryon-baryon → meson-meson channel and the density-matrix elements will be expressed in the Gottfried-Jackson (G.J.) frame.[3] This is the most natural frame to use in models with t-channel Regge poles since the structure of the G.J. density-matrix elements is closely related to quantum numbers exchanged in the t channel. Since some features are common to several of the reactions under study we mention them here. Let us define

$$f^{\pm}_{\mu 0;\bar{B}B'} = f^{t}_{\mu 0;\bar{B}B'} \pm f^{t}_{-\mu 0;\bar{B}B'} , \tag{2.1}$$

where μ is the helicity of the spinning meson produced (i.e., ρ, ω, or B), $\bar{B}$ is that of the antinucleon, and B' is that of either the nucleon or the Δ. If ρ_{ij} are the G.J. density-matrix elements of the meson produced, we find in all cases that

$$\begin{aligned} \rho_{00}\frac{d\sigma}{dt} &= 2\sum_{B'>0;\bar{B}=\pm 1/2} |f_{00;\bar{B}B'}|^2 , \\ \rho_{10}\frac{d\sigma}{dt} &= \sum_{B'>0;\bar{B}=\pm 1/2} f^{-}_{10;\bar{B}B'} f^{*}_{00;\bar{B}B'} , \\ (\rho_{11}-\rho_{1-1})\frac{d\sigma}{dt} &= \sum_{B'>0;\bar{B}=\pm 1/2} |f^{-}_{10;\bar{B}B'}|^2 , \\ (\rho_{11}+\rho_{1-1})\frac{d\sigma}{dt} &= \sum_{B'>0;\bar{B}=\pm 1/2} |f^{+}_{10;\bar{B}B'}|^2 . \end{aligned} \tag{2.2}$$

From these equations several conclusions can be drawn. Consider first the reactions where a vector meson is produced (i.e., $\pi N \to \rho N$, $\pi N \to \omega N$, $\pi N \to \omega\Delta$). It is well known that, in these processes, natural-parity exchange in the t channel cannot couple to the longitudinally polarized vector meson. Furthermore, at high energy, it can be shown that $f^{-}_{\mu 0;\bar{B}B'}$ is predominantly of unnatural parity, whereas $f^{+}_{\mu 0;\bar{B}B'}$ is dominated by natural-parity exchanges. Consequently,

ρ_{00} is pure unnatural parity at all energies,

ρ_{10} and $\rho_{11}-\rho_{1-1}$ are unnatural parity at high energy (to leading order in s), and (2.3)

$\rho_{11}+\rho_{1-1}$ is natural parity at high energy.

In the case of $\pi N \to \rho N$ and $\pi N \to \omega N$ these conclusions can be refined further if it is assumed that the unnatural-parity contributions are, respectively, pion-Pomeranchon cut and B exchange. In both cases the G parity of the exchange requires that $f^{t}_{\mu 0;1/2\,-1/2}=0$ and the statement (2.3) about $\rho_{11}+\rho_{1-1}$ becomes exact at all energies. It is interesting to note that, due to unequal-mass kinematics,[4] the pure pole natural-parity contribution to all these reactions is suppressed in the forward direction, and hence at the energies studied to date π or B exchanges dominate in this region although their trajectories are much lower than those of the ρ or the ω.

For B production, and more generally for axial-vector production, the conclusions of (2.3) are reversed and ρ_{00}, Reρ_{10}, and $\rho_{11}-\rho_{1-1}$ are dominated by natural parity and $\rho_{11}+\rho_{1-1}$ by unnatural parity.

Now a word of warning: The model constructed in I is a pure pole model and as such will produce dips in differential cross sections when the Regge signature factor goes through zero. One of the well-known problems for Regge theory is the experimental absence of dips predicted by ρ exchange in $\pi N \to \omega N$ and $\pi N \to \omega\Delta$. We assume in this work that the Regge concept is basically correct even if certain details about the shapes of the cross sections are not understood. Consequently, we focus on the over-all magnitudes of various contributions to the cross sections and predicted relations between different reactions; we expect that when (and if) the problem of the missing dips is resolved for t-channel Regge exchanges it will not be difficult to extend the solution to dual models.

In the following, four quasi-two-body reactions, which appear as subprocesses of $\pi N \to \pi\omega N$ scattering, are used to test the model. We recall that the amplitude for $\pi N \to \pi\omega N$ can be defined by the functions A^+ and A^- which are, respectively, the amplitudes for isospin 0 and isospin 1 in the $(N\bar{N})$ channel. The relations between A^+ and A^- (given in Appendix A in I) and the amplitudes for the quasi-two-body reactions are as follows:

For $\pi^- p \to \rho^0 n$,

$$\mathfrak{M}_{\pi^- p\to\rho^0 n} = -\sqrt{2}\,\mathrm{Res}_{\alpha^{\rho}_{12}=1} A^- \;.$$

For $\pi^+ p \to \omega\Delta^{++}$,

$$\mathfrak{M}_{\pi^+ p\to\omega\Delta^{++}} = \mathrm{Res}_{\alpha^{\Delta}_{34}=3/2} A^{3/2} = \mathrm{Res}_{\alpha^{\Delta}_{34}=3/2}(A^+ - A^-) \;.$$

For $\pi^- p \to \omega n$, (2.4)

$$\mathfrak{M}_{\pi^- p\to\omega n} = \tfrac{2}{3}\,\mathrm{Res}_{\alpha^{N}_{34}=1/2} A^{1/2} = \tfrac{2}{3}\,\mathrm{Res}_{\alpha^{N}_{34}=1/2}(A^+ + 2A^-) \;.$$

For $\pi^- p \to B^- p$,

$$\mathfrak{M}_{\pi^- p\to B^- p} = \tfrac{1}{3}\,\mathrm{Res}_{\alpha^{B}_{12}=1}(A^{3/2}+2A^{1/2}) = \mathrm{Res}_{\alpha^{B}_{12}=1}(A^+ + A^-) \;.$$

The dynamical content of the amplitudes under consideration can be decomposed according to the nature of the t-channel exchanges, and for each of these exchanges the model will be compared with experiment:

π exchange. In our model, this is completely determined by the known πNN coupling. We compare with the data for $\rho_{00}\,d\sigma/dt$ in $\pi^- p \to \rho^0 n$ scattering.

ρ exchange. This is the only contribution to $(\rho_{11}+\rho_{1-1})\,d\sigma/dt$ in the reactions $\pi^+ p \to \omega\Delta^{++}$ and $\pi^- p \to \omega n$. Since ρ exchange (in πN scattering) is at the origin of the idea of duality, it seems to be the proper place to determine $(G^{\omega}_{V}-G^{\omega}_{T})$ and G^{ω}_{T}, the two parameters in our model.

The following are then predictions of the model: B exchange in both ω production reactions; A_2 exchange in $\pi^- p \to \rho^0 n$ scattering; ω and A_2 exchange in $\pi^- p \to B^- p$.

Before we go into the details of the fits let us list the values of the constants which are the input to the model. The common slope of the Regge trajectories, α', is chosen to be

$$\alpha' = 0.8 \;, \qquad (2.5)$$

a value commonly accepted in Regge phenomenology. The choices

$$\frac{g_{\rho\pi\pi}^{\;2}}{4\pi} = 2 \text{ and } \frac{G_{\pi NN}^{\;2}}{4\pi} = 14.7 \qquad (2.6)$$

need no explanation. From the decay $\Delta \to N\pi$ we obtain

$$\frac{G_{\pi p\Delta^{++}}^{\;2}}{4\pi} = 0.12 \;. \qquad (2.7)$$

The $\rho\omega\pi$ coupling constant can be calculated either by the method of Gell-Mann *et al.*[5] or from the decay $\omega \to \gamma\pi$ using the vector-dominance model. In both cases we find

$$\frac{g_{\rho\omega\pi}{}^2}{4\pi} \simeq 28 \text{ GeV}^{-2} . \tag{2.8}$$

A last constraint comes from the B-meson width, which, together with

$$\frac{g_D}{g_S} = \frac{2M_B{}^2}{M_B{}^2 - M_\omega{}^2 - M_\pi{}^2} = 3.42 \tag{2.9}$$

[cf. Eq. (3.20) in I], determines completely the B-decay parameters.

G_V^ω and G_T^ω, the two coupling constants left for the fitting, are constrained experimentally by[6]

$$5 \leq G_V^\omega/4\pi < 15, \quad |G_T^\omega/G_V^\omega| \text{ small.}$$

B. π exchange in $\pi^- p \to \rho^0 n$

The helicity amplitudes involving π exchange are

$$f^{\pi}_{00;1/2-1/2} = 0 ,$$

$$f^{\pi}_{\pm 10;\bar{B}B} = 0 \quad \forall \bar{B}, B ,$$

$$f^{\pi}_{00;1/21/2} = \frac{\alpha' g_{\rho\pi\pi} g_{\rho\omega\pi} \tau_{\pi\rho} \sqrt{-t}}{2\sqrt{2}\, m_p m_\rho} \times [B_4(-\alpha_t^\pi, \tfrac{5}{2} - \alpha_s) + B_4(-\alpha_t^\pi, \tfrac{5}{2} - \alpha_u)] , \tag{2.10}$$

where

$$\tau_{\pi\rho} = \{[t-(m_\pi + m_\rho)^2][t-(m_\pi - m_\rho)^2]\}^{1/2} . \tag{2.11}$$

It is easy to check that this parameterization has the correct kinematical factors corresponding to evasive solution of the conspiracy relations in the forward direction.[7]

Several features of the model are already apparent. $\rho_{00} d\sigma/dt$ will show the typical $t/(t-m_\pi)^2$ behavior in agreement with experiment. Because π exchange does not contribute to the transversely polarized ρ's (since the π couplings are like the field-theory ones), the statements (2.3) become

$$\rho_{11} = \rho_{1-1} \text{ and } \mathrm{Re}\rho_{10} = 0 .$$

The signature zero is at $t = -1.25$ (GeV/c)^2; however, this zero should not be seen, since, at that value of the momentum transfer, the π contribution is very small. It should be noted that we have not specified what the trajectories in the s and u channels are because they are not leading in these channels; either the N or the Δ trajectory can be chosen and this will not affect the fit since both will produce the same Regge behavior.

The prediction of the model for $\rho_{00} d\sigma/dt$ at 15 GeV/c (Ref. 8b) is shown in Fig. 1. Not surprisingly, the size of $\rho_{00} d\sigma/dt$ compares rather well with experiment, expecially in the forward direction;

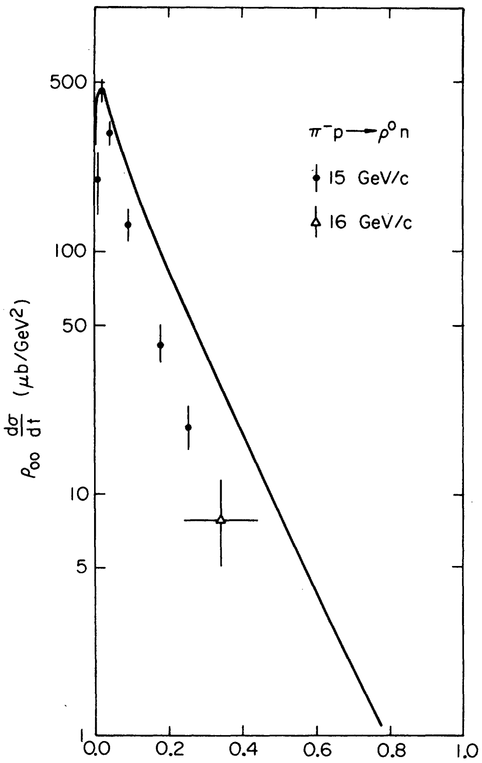

FIG. 1. $\rho_{00} d\sigma/dt$ distribution for $\pi^- p \to \rho^0 n$ at 15 GeV/c (Ref. 8b) and 16 GeV/c (Ref. 8c).

this is so because the π pole, where the coupling constants involved in the calculation are determined, does not lie far from the scattering region. However, the predicted t dependence is not steep enough due to the fact that the vertex functions do not contain the *ad hoc* exponential e^{-at} present in ordinary Regge models. At 7 GeV/c (Ref. 8a), where the model was also applied, we obtained similar agreement with the data.

C. ρ exchange in $\pi^+ p \to \omega\Delta^{++}$ and $\pi^- p \to \omega n$

In $\pi^+ p \to \omega\Delta^{++}$ scattering the contribution of ρ exchange to the invariant amplitude is

$$\mathfrak{M}^{\rho}_{\Delta \bar{N}_5 \to \omega\pi} = -\alpha' \frac{3}{4} \frac{(G_V^\omega - G_T^\omega)}{G_{\pi p \Delta^{++}}} g_{\rho\omega\pi}{}^2 m_\pi \epsilon_{\cdots\mu} \times p_\pi^{\cdot} p_\omega^{\cdot} \omega^{\cdot} \bar{v}_5 i\gamma_5 \left(g^{\mu\nu} - \frac{\gamma^\mu p_5^\nu}{m_\Delta + m_p}\right) \times [B_4(1-\alpha_t^\rho, \tfrac{5}{2} - \alpha_s^\Delta) + B_4(1-\alpha_t^\rho, \tfrac{5}{2} - \alpha_u^N)]\Delta_\nu . \tag{2.12}$$

(The dot notation is defined in I.) Since the individual helicity amplitudes are complicated to write out and do not give any insight on the physics of the reaction, we will not show them here.

For the ωn final state, we obtain for the invariant amplitude

$$\mathfrak{M}^{\rho}_{N_5 N\to\omega\pi}=\frac{\alpha' g_{\rho\omega\pi}{}^2}{2\sqrt{2}\,G_{\pi NN}}\epsilon\ldots{}^{\mu}p^{\cdot}_{\pi}p^{\cdot}_{\omega}\omega^{\cdot}\bar{v}_5\left(2\frac{G^{\omega}_T}{m_p}[(m_p{}^2-\tfrac{1}{4}t)\gamma_\mu+m_p p_{5\mu}]-\alpha' m_p(G^{\omega}_V-G^{\omega}_T)[(m_p{}^2-\tfrac{1}{2}t)\gamma_\mu+m_p p_{5\mu}]\right)$$
$$\times[B_4(1-\alpha^{\rho}_t,\tfrac{5}{2}-\alpha^N_s)+B_4(1-\alpha^{\rho}_t,\tfrac{5}{2}-\alpha^N_u)]\,u\,, \qquad (2.13)$$

and for the helicity amplitudes

$$f^{\rho}_{\pm10;1/21/2}=-\frac{\alpha'^2 g_{\rho\omega\pi}{}^2(G^{\omega}_V-G^{\omega}_T)}{16G_{\pi NN}}m_p\frac{\sqrt{\phi}\,t}{(4m_p{}^2-t)^{1/2}}\mathfrak{R}_\rho\,,$$
$$f^{\rho}_{\pm10;\frac{1}{2}-\frac{1}{2}}=-\frac{\alpha' g_{\rho\omega\pi}{}^2}{8G_{\pi NN}m_p}\left(\frac{t(s-u)}{(4m_p{}^2-t)^{1/2}}+\sqrt{-t}\,\tau_{\pi\omega}\right)\left(\alpha' m_p(G^{\omega}_V-G^{\omega}_T)(m_p{}^2-\tfrac{1}{2}t)-2\frac{G^{\omega}_T}{m_p}(m_p{}^2-\tfrac{1}{4}t)\right)\mathfrak{R}_\rho\,, \qquad (2.14)$$
$$f^{\rho}_{00;\bar{B}B}=0\quad\forall\,\bar{B},B\,,$$

where

$$\tau_{\pi\omega}=\{[t-(m_\omega+m_\pi)^2][t-(m_\omega-m_\pi)^2]\}^{1/2}\,, \qquad (2.15)$$

$$\mathfrak{R}_\rho=B_4(1-\alpha^{\rho}_t,\tfrac{5}{2}-\alpha^N_s)+B_4(1-\alpha^{\rho}_t,\tfrac{5}{2}-\alpha^N_u)\,, \qquad (2.16)$$

and ϕ, the Kibble function, is defined by

$$\phi=-[st(s+t-2m_p{}^2-m_\pi{}^2-m_\omega{}^2)+t(m_p{}^2-m_\pi{}^2)(m_p{}^2-m_\omega{}^2)+m_p{}^2(m_\omega{}^2-m_\pi{}^2)^2]\,. \qquad (2.17)$$

A few words of comment about this parameterization: First, in order to reduce the number of B_4 functions and to be able to extract the factor $[B_4(t,s)+B_4(t,u)]$ we have modified the baryon trajectories so that the arguments $\frac{5}{2}-\alpha^N_s$ and $\frac{5}{2}-\alpha^N_u$ appear in the s and u channels, respectively (this simplification will make no difference as far as the Regge behavior is concerned). Second, it can be checked that the amplitudes possess the correct kinematical singularities for an evasive ρ.[7] Finally, the extra t factor in $f^{\rho}_{\pm10;1/21/2}$ does not have a kinematical origin but comes from the $\rho\bar{N}N$ coupling in the model.

The signature factor $\mathfrak{R}_\rho$ is of the form

$$\Gamma(1-\alpha^{\rho}_t)(1-e^{-i\alpha^{\rho}(t)})(\alpha' s)^{\alpha^{\rho}(t)-1}=\frac{\pi\alpha^{\rho}(t)}{\Gamma(1+\alpha^{\rho}(t))}\frac{(1-e^{-i\pi\alpha^{\rho}(t)})}{\sin\pi\alpha^{\rho}(t)}(\alpha' s)^{\alpha^{\rho}(t)-1}\,.$$

This term, which contains the $\alpha_\rho(t)$ dependence of the amplitudes, shows that the ρ trajectory chooses the nonsense (Gell-Mann) mechanism; this is so because the invariant functions B_5 (and consequently also the B_4's obtained by taking the residues) contain the argument $1-\alpha^{\rho}(t)$ necessary to eliminate the ghost at $\alpha^{\rho}(t)=0$. This choice of coupling mechanism will create dips in all ρ-exchange amplitudes at $t=-0.6$ (GeV/c)^2. No such dips are observed in $(\rho_{11}+\rho_{1-1})d\sigma/dt$ and the (s,u) terms are unable to fill the dips created by the (s,t) and (u,t) terms. Thus, either a secondary trajectory is present, or another ghost-eliminating mechanism might be necessary (generated by terms like $[B_4(-\alpha^{\rho}_t,\frac{5}{2}-\alpha^N_s)-B_4(-\alpha^{\rho}_t,\frac{5}{2}-\alpha^N_u)]$).

As shown by Eq. (2.12), $(G^{\omega}_V-G^{\omega}_T)$ is the only free parameter in the natural-parity contribution to the reaction $\pi^+p\to\omega\Delta^{++}$ and it will therefore be determined by normalizing the prediction of the model for $(\rho_{11}+\rho_{1-1})d\sigma/dt$ to the experimentally measured distribution at 11.7 GeV/c (Ref. 9b). We find

$$G^{\omega}_V-G^{\omega}_T=9\,. \qquad (2.18)$$

Using this value we can then, in the same fashion, determine G^{ω}_T from $\pi^-p\to\omega^0 n$ scattering at 6.95 GeV/c (Ref. 10b), and we find

$$G^{\omega}_T=-2.2\,. \qquad (2.19)$$

The distributions for $(\rho_{11}+\rho_{1-1})d\sigma/dt$ are not shown because of their poor experimental statistics in both reactions. As a further test for the model we computed the ratio $|f^{\rho}_{10;1/21/2}/f^{\rho}_{10;\frac{1}{2}-\frac{1}{2}}|$; it was found that over a wide range in t the ratio was approximately equal to 0.17, except in the forward direction, where it was much smaller, and this is in good agreement with several analyses of $\pi^-p\to\pi^0 n$ scattering.[6]

It should be emphasized that, in the model constructed, the only freedom we have is in fixing the sizes of the ρ-exchange contribution to the ω-production reactions considered above, but we have no way to vary the shapes of the t distributions.

The following sections deal with other predictions of the model.

D. B exchange in $\pi^+p\to\omega\Delta^{++}$ and $\pi^-p\to\omega n$

The contribution of B exchange to $\pi^+p\to\omega\Delta^{++}$ scattering is given by

$$\mathfrak{M}^{B}{}_{\Delta\bar{N}_5\to\omega\pi}=-\alpha'\,\frac{3}{4}\,\frac{(G_V^{\omega}-G_T^{\omega})}{G_{\pi p\Delta^{++}}}\,m_\pi\bar{v}_5\left[\frac{m_\Delta-m_p}{m_\Delta+m_p}\,\frac{g_D}{m_B}\left(g_S'm_B\omega^\mu+\frac{g_D}{m_B}\,p_\pi\cdot\omega p_\pi^\mu\right)g_{\mu\nu}\right.$$
$$\left.-\tfrac{1}{2}g_{\rho\omega\pi}{}^2\left(\tfrac{1}{2}(t-m_\omega{}^2-m_\pi{}^2)\omega^\mu+p_\pi\cdot\,\omega p_\pi^\mu\right)\frac{\gamma_\mu p_{5\nu}}{m_\Delta+m_p}\right]$$
$$\times\left[B_4(1-\alpha_t^B,\tfrac{5}{2}-\alpha_s^\Delta)+B_4(1-\alpha_t^B,\tfrac{5}{2}-\alpha_u^N)\right]\Delta^\nu\ , \qquad (2.20)$$

where

$$g_S'=g_S+\tfrac{1}{4}\frac{g_{\rho\omega\pi}{}^2}{g_D}\,(m_B{}^2-t)\ ,$$

and the contribution to $\pi^-p\to\omega^0n$ scattering is given by

$$\mathfrak{M}^{B}_{\bar{N}_5N\to\omega\pi}=\frac{\alpha'^2G_S{}^2(G_V^\omega-G_T^\omega)}{2\sqrt{2}\,G_{\pi NN}}\,\bar{v}_5i\gamma_5\left(\omega^\mu+\frac{G_D}{G_S}p_\pi\cdot\omega p_\pi^\mu\right)p_{5\mu}\,\left[B_4(1-\alpha_t^B,\tfrac{5}{2}-\alpha_s^N)+B_4(1-\alpha_t^B,\tfrac{5}{2}-\alpha_u^N)\right]u\ . \qquad (2.21)$$

Note that, in contrast with the case of $\omega\Delta$ production, the B-meson contribution to the above reaction comes entirely from that part of the five-point amplitude where the B is dual to itself. The various helicity amplitudes for ωn production are

$$f^{B}_{00;\,1/21/2}=-\frac{\alpha'^2G_S{}^2(G_V^\omega-G_T^\omega)}{8\sqrt{2}\,G_{\pi NN}m_\omega m_p}\,\frac{\sqrt{-t}}{\tau_{\pi\omega}}$$
$$\times\left[(s-u)(t+m_\omega{}^2-m_\pi{}^2)+\tau_{\pi\omega}{}^2(m_p{}^2+m_\pi{}^2-s)\frac{G_D}{G_S}\right]\mathfrak{R}_B\ , \qquad (2.22)$$
$$f^{B}_{\pm10;1/21/2}=\pm\frac{\alpha'^2G_S(G_V^\omega-G_T^\omega)}{4G_{\pi NN}m_p}\,\frac{\sqrt{\phi}\sqrt{-t}}{\tau_{\pi\omega}}\,\mathfrak{R}_B\ ,$$
$$f^{B}_{\mu0;1/2-1/2}=0\quad \mu=\pm1,0\,.$$

$\mathfrak{R}_B$ is defined in the same way as $\mathfrak{R}_\rho$ in Eq. (2.16).

As is the case for the ρ trajectory, the B chooses an evasive solution to the forward conspiracy relations. Since the $B\bar{N}N$ coupling is in a pure spin $S=0$ state, the amplitudes with spin flip at the nucleon vertex vanish. Without going into the details of the fit we can already make predictions about the density-matrix elements ρ_{00} and ρ_{10}, which are dominated by B exchange. The element ρ_{10} contains the expression

$$t\sqrt{\phi}\left[(s-u)(t+m_\omega{}^2-m_\pi{}^2)+\frac{G_D}{G_S}\,\tau_{\pi\omega}{}^2(m_p{}^2+m_\pi{}^2-s)\right]$$
$$\sim -st\sqrt{\phi}\,\frac{G_D}{G_S}\left[t^2-2t\left(m_\omega{}^2+\frac{G_S}{G_D}\right)+m_\omega{}^2\left(m_\omega{}^2-2\,\frac{G_S}{G_D}\right)\right] \qquad (2.23)$$

if we keep only the terms leading in s and neglect $m_\pi{}^2$ compared with $m_\omega{}^2$. The remaining t dependence from the kinematical factors is smooth. Besides the factor $t\sqrt{\phi}$ which vanishes in the forward direction, the main t dependence and the sign of $\mathrm{Re}\rho_{10}$ are those of

$$\frac{G_D}{G_S}\left[t^2-2t\left(m_\omega{}^2+\frac{G_S}{G_D}\right)+m_\omega{}^2\left(m_\omega{}^2-2\,\frac{G_S}{G_D}\right)\right]\,.$$

Studying the position of the roots of this quadratic

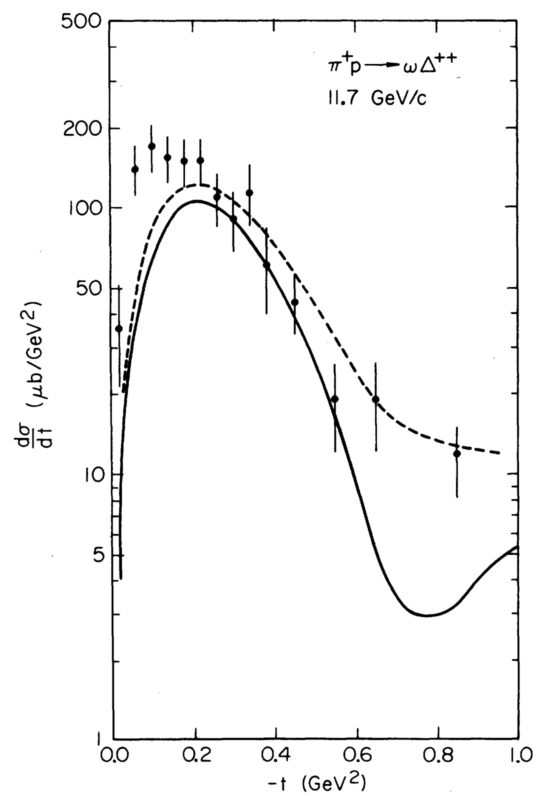

FIG. 2. Differential cross section for $\pi^+p\to\omega\Delta^{++}$ at 11.7 GeV/c (Ref. 9b). The solid curve is the prediction of the model; the dashed curve is the prediction when the B-exchange amplitude is arbitrarily multiplied by a factor 4.

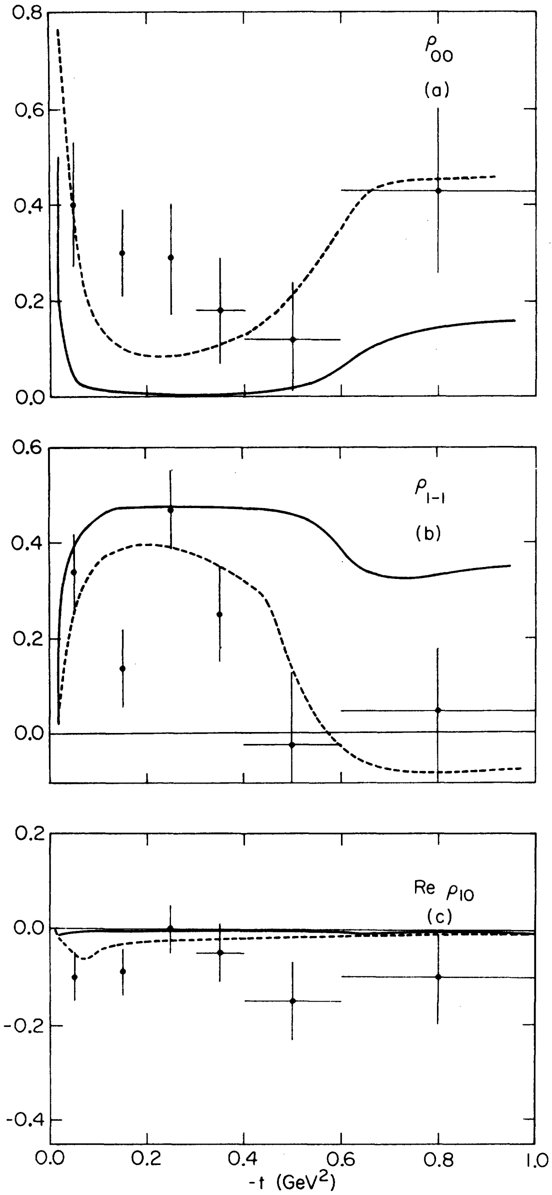

FIG. 3. Density-matrix elements in the Gottfried-Jackson frame for the ω produced in $\pi^+p \to \omega\Delta^{++}$ at 11.7 GeV/c. The definition of the solid and dashed lines is the same as for Fig. 2.

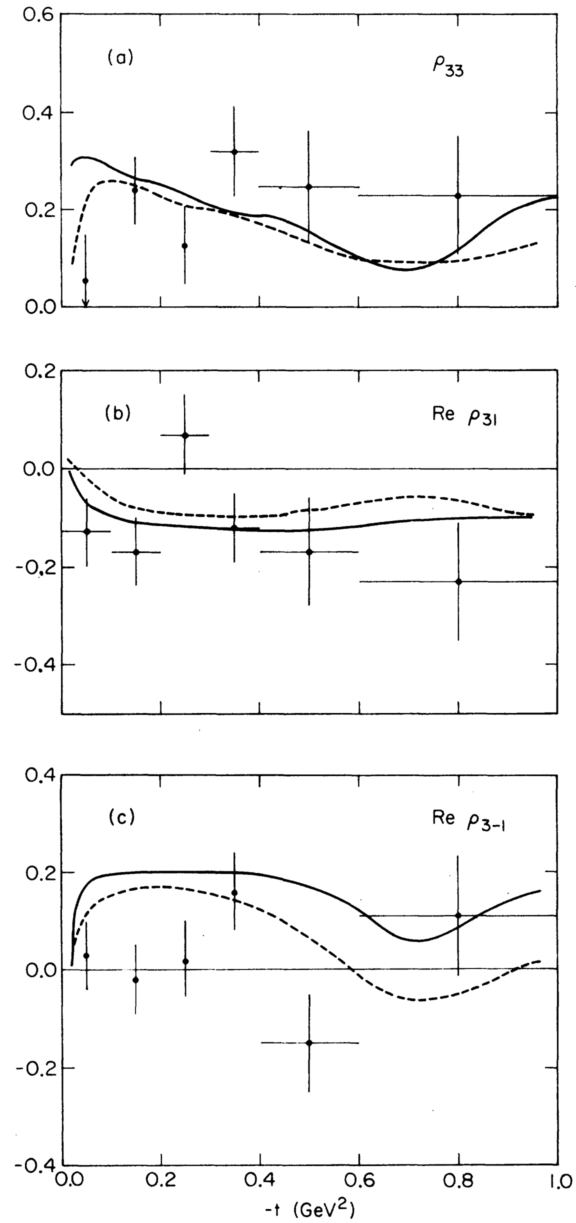

FIG. 4. Same as Fig. 3 for the Δ.

equation as a function of g_D/g_S [$=m_B^2G_D/G_S$ by Eq. (3.18) in I], we find that for $g_D/g_S>5$, ρ_{10} is always positive; for $5>g_D/g_S>0$, ρ_{10} starts out with negative values and becomes positive for some value of t between 0 and -0.6 (correspondingly, ρ_{00} vanishes in the same interval); and for $g_D/g_S<0$, ρ_{10} is negative and neither ρ_{10} nor ρ_{00} vanishes in the measured momentum transfer range.

In the data, ρ_{10} is negative for all measured values of t($|t|<1$ (GeV/c)^2). Hence we need g_D/g_S negative. However, our model in its present form predicts just the opposite [see Eq. (2.9)]. To change this we could either try to put in the part of the five-point amplitude where the B meson is dual to the ρ or else add satellites, but neither will be done here.

In the $\omega\Delta^{++}$ production, where a similar discussion can be made, the combination of the terms where the B is dual to the ρ and those which contain the B in both ($\omega\pi$) channels produces a negative g_D/g_S ratio for B in the momentum-transfer region, and we expect to get the right qualitative

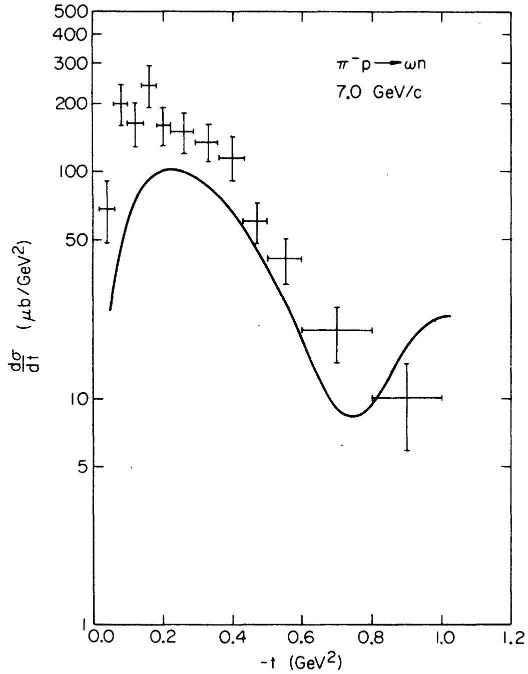

FIG. 5. Differential cross section for $\pi^- p \to \omega n$ scattering at 6.95 GeV/c (Ref. 10b).

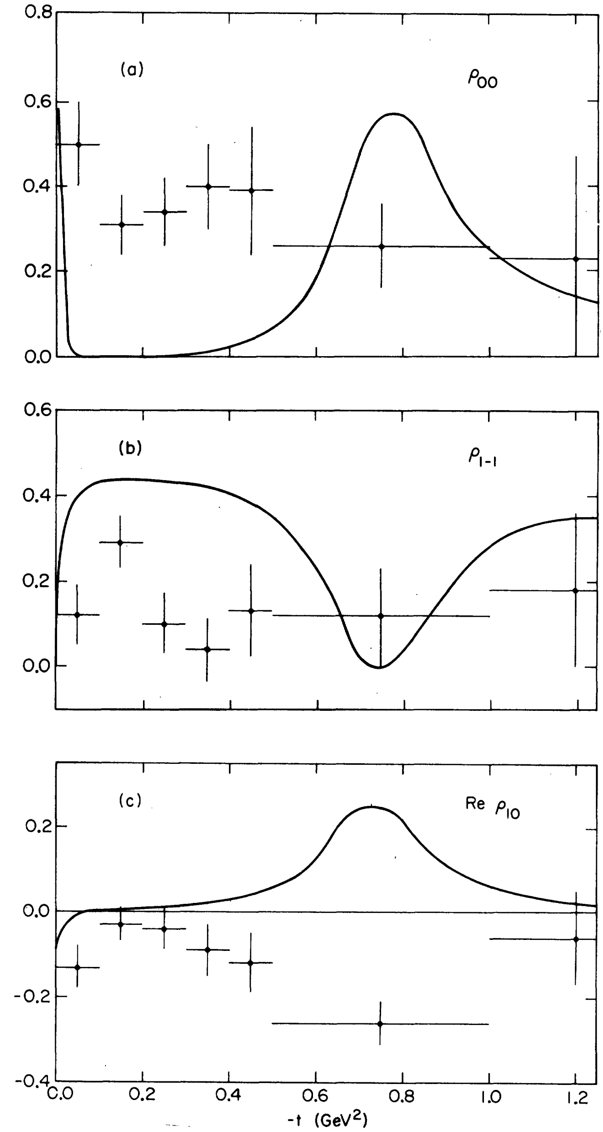

FIG. 6. Same as Fig. 3 for the ω produced in $\pi^- p \to \omega n$ at 6.95 GeV/c (Ref. 10b).

features for ρ_{10}.

An interesting feature of the data is the broad dip at $t = -0.2$ (GeV/c)^2 observed in ρ_{00} in both $\omega\Delta$ and ωn scattering.[9,10] This dip has been much talked about in the literature and was explained, in pure pole models, by a wrong-signature nonsense zero of the B trajectory[11]; the trajectory slope required to produce a zero at this point is $\alpha_B' = 0.58$ (GeV/c)$^{-2}$. This solution is not very attractive for two reasons: First, this value of α_B' is much smaller than that of the well-established Regge trajectories; second, it would give $\alpha_B(0) = 0.11$, in contradiction with the observations of Holloway *et al.*[10a] In fact, in the newer data, this dip is too broad to be associated with a signature zero. It is probably a consequence of unequal-mass kinematics. Indeed $(\rho_{11} + \rho_{1-1}) d\sigma/dt$, which represents the natural-parity contribution to the cross section, is suppressed in the forward direction due to the presence of half-angle factors in the helicity amplitudes and peaks at about $t = -0.2$ (GeV/c)^2. At this value of t, it dominates over $\rho_{00} d\sigma/dt$ [and $(\rho_{11} - \rho_{1-1}) d\sigma/dt$], which peaks at smaller t or even $t = 0$. The net effect is a wide dip in ρ_{00}. The same argument can also be used to account for the smallness of ρ_{10} at $t = -0.2$ (GeV/c)^2; $\rho_{10} d\sigma/dt$ is pure unnatural-parity [cf. Eq. (2.2)] and no structure is expected there, whereas $d\sigma/dt$ shows a maximum due to the ρ contribution.

The predictions of the model for $\pi^+ p \to \omega\Delta^{++}$ scattering at 11.7 GeV/c (Ref. 9b) are represented by the solid line in Figs. 2–4, where the differential cross section and the Gottfried-Jackson density-matrix elements for both the ω and Δ are displayed. In Figs. 5 and 6, we show the results for the reaction $\pi^- p \to \omega n$ at 6.95 GeV/c.[10b] In both reactions it appears that the B contribution is too small, as evidenced by the almost vanishing values predicted for ρ_{00} between $t = -0.05$ and $t = -0.4$ GeV/c. The structure at $t = -0.7$ GeV/c, especially visible on all three ω density-matrix elements in Fig. 6, comes from the signature factor of the ρ trajectory. The Δ density-matrix ele-

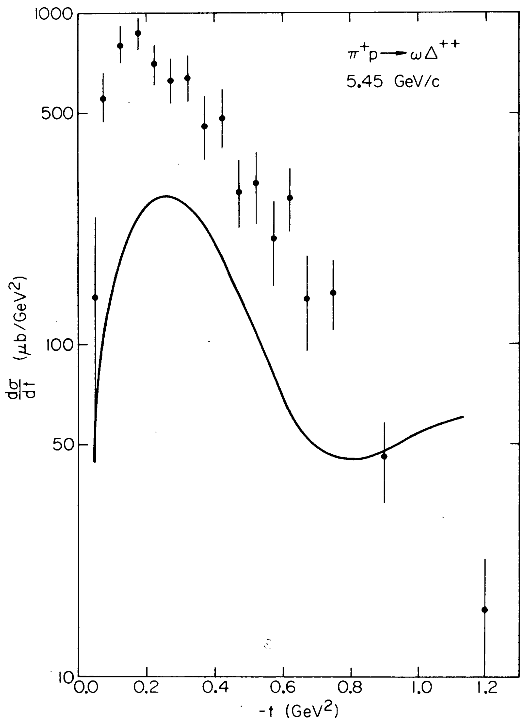

FIG. 7. Same as Fig. 2 at 5.45 GeV/c (Ref. 9a).

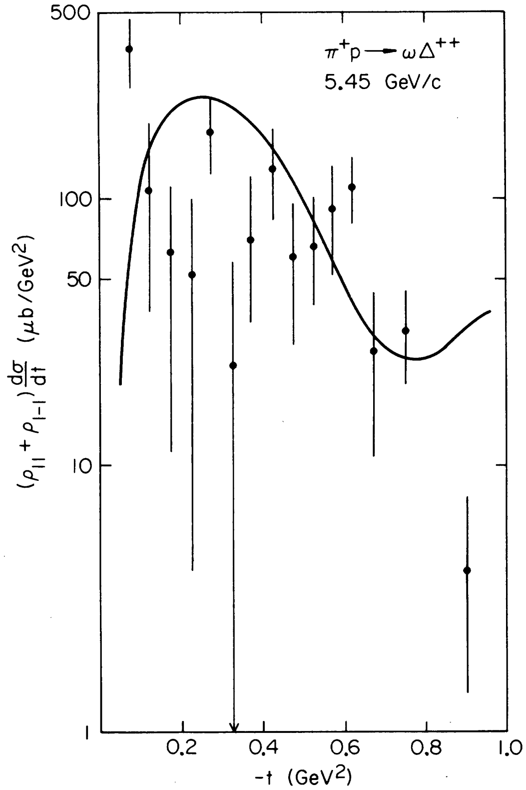

FIG. 8. Natural-parity contribution to the differential cross section for $\pi^+p \rightarrow \omega\Delta^{++}$ at 5.45 GeV/c.

ments predicted by the model (see Fig. 4) agree fairly well with the data due to the fact that, in these quantities, the B- and the ρ-exchange contribution are not separated.

To check the energy dependence, we show, in Figs. 7 and 8, $d\sigma/dt$ and $(\rho_{11}+\rho_{1-1})d\sigma/dt$ for $\pi^+p \rightarrow \omega\Delta^{++}$ at 5.45 GeV/c.[9a] Figure 8 shows that the ρ-exchange part of the amplitude has the right magnitude, but a comparison between Fig. 2 and Fig. 7 shows that the over-all cross section is better predicted at larger energies. This is expected because the magnitude of the unnatural-parity contribution relative to the natural-parity contribution decreases with energy and therefore the contribution of the improperly predicted B exchange becomes smaller. At 5.45 GeV/c, where it is important in the data, the predicted cross section is too small by a factor of about 3.

E. A_2 exchange in $\pi^-p \rightarrow \rho^0 n$

The helicity amplitudes we are concerned with are

$$
\begin{aligned}
f^{A_2}_{\pm 10;1/2\,1/2} = -\frac{\alpha' g_{\rho\omega\pi}}{4}\,\frac{\sqrt{\phi}}{4m_p^{\,2}-t}\Big\{ & (G_V^{\omega}-G_T^{\omega})[B_4(1-\alpha_t^{A_2},\tfrac{3}{2}-\alpha_s^{\Delta})-B_4(1-\alpha_t^{A_2},\tfrac{3}{2}-\alpha_u^{\Delta})] \\
& +2G_T^{\omega}\Big(\frac{t}{4m_p^{\,2}}-1\Big)\,[B_4(1-\alpha_t^{A_2},\tfrac{5}{2}-\alpha_s^{N})-B_4(1-\alpha_t^{A_2},\tfrac{5}{2}-\alpha_u^{N})] \\
& +\alpha'(G_V^{\omega}-G_T^{\omega})\Big(\frac{s-u}{2}\Big)\,[B_4(2-\alpha_t^{A_2},\tfrac{5}{2}-\alpha_s^{N})+B_4(2-\alpha_t^{A_2},\tfrac{5}{2}-\alpha_u^{N})]\Big\}\,, \\
f^{A_2}_{\pm 10;1/2\,-1/2} = -\frac{\alpha' g_{\rho\omega\pi}(G_V^{\omega}-G_T^{\omega})}{8m_p(4m_p^{\,2}-t)^{1/2}}\,\{ & [t(s-u)\pm\tau_{\pi\rho}\sqrt{-t}\,(4m_p^{\,2}-t)][B_4(1-\alpha_t^{A_2},\tfrac{5}{2}-\alpha_s^{N})-B_4(1-\alpha_t^{A_2},\tfrac{5}{2}-\alpha_u^{N})] \\
& -\alpha'\phi[B_4(2-\alpha_t^{A_2},\tfrac{5}{2}-\alpha_s^{N})+B_4(2-\alpha_t^{A_2},\tfrac{5}{2}-\alpha_u^{N})]\}\,,
\end{aligned}
\tag{2.24}
$$

where $\tau_{\pi\rho}$ is defined in Eq. (2.11) and ϕ is obtained from Eq. (2.17) by replacing m_ω by m_ρ.

It should be recalled that, since we did not include cuts, we are unable to reproduce the sharp forward peak in $\rho_{11}^{H} d\sigma/dt$ (the H superscript stands for helicity frame) which is well accounted for by destructive interference between the π-pole contribution and its associated π-Pomeranchon cut.[12,13] We regard this cut as an accepted refinement of the Regge picture which can be easily added to our calculation. Its effect is important in only a small part of the scattering region, and the comparison with the data outside this region will not be affected by its omission. Hence we feel justified in neglecting the cut at this time.

The predictions of the model for the differential cross section and the density-matrix elements for the ρ are shown in Figs. 9 and 10, respectively. The predicted t dependence is less steep than that of the data; this may be attributed to a number of facts. First, $\rho_{00} d\sigma/dt$ has the same property, indicating that the Regge residue functions given by the model may not be exactly correct. Second, an amplitude analysis of the recent CERN-Munich data at 17.2 GeV/c (Ref. 8d) showed that the A_2 contribution interferes destructively with the π-Pomeranchon cut (neglected in our model), thereby making $d\sigma/dt$ much sharper.[14] Finally, the $\omega\bar{N}N$ coupling constants (and consequently the $A_2\bar{N}N$ coupling constants which are proportional to them [see Eqs. (3.28) in I]) might have been overestimated because of the dip problem in $\pi^+p\to\omega\Delta^{++}$ and $\pi^-p\to\omega n$. We doubt that the overestimation is very great, however, due to the reasonable values of the other coupling constants predicted (see Sec. III below).

The predicted values for the (Gottfried-Jackson) density-matrix elements are in very good agreement with the new data at 17.2 GeV/c (Ref. 8d) in which it was found that, for $-t > 0.45$ (GeV/c)^2,

$$(\rho_{00}-\rho_{11}) \sim -0.40 \, ,$$

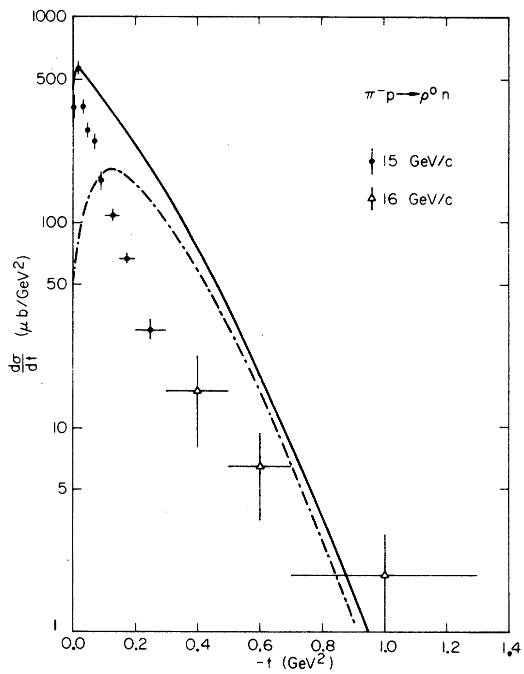

FIG. 9. Differential cross section for $\pi^-p\to\rho^0 n$ at 15 GeV/c (Ref. 8b) and 16 GeV/c (Ref. 8c). The dash-dot line represents the A_2 contribution.

$$\rho_{1\,-1} \sim 0.40 \, ,$$

$$\mathrm{Re}\rho_{10} \sim 0 \, .$$

We also applied the model to data at 7 GeV/c (Ref. 8a) and we found a similar agreement with the data.

F. ω and A_2 exchange in $\pi^-p\to B^-p$

The invariant amplitude in this case is given by

$$\begin{aligned}\mathfrak{M}_{\bar{N}_5 N_4\to B^-\pi^+} = -\tfrac{1}{4}\alpha'\left(g_S'' m_B B^\mu + \frac{g_D}{m_B} p_\pi\cdot B p_\pi^\mu\right)\bar{v}_5\Big\{&(G_V^\omega - G_T^\omega)\gamma_\mu[B_4(1-\alpha_t^\omega, \tfrac{5}{2}-\alpha_s^\Delta)+3B_4(1-\alpha_t^\omega, \tfrac{5}{2}-\alpha_u^\Delta)]\\ &+2\frac{G_T^\omega}{m_p}(p_4-p_5)_\mu B_4(1-\alpha_t^\omega, \tfrac{5}{2}-\alpha_s^N)\\ &+4\alpha'(G_V^\omega - G_T^\omega)\not{p}_\pi p_{5\mu} B_4(2-\alpha_t^\omega, \tfrac{5}{2}-\alpha_s^N)\Big\}u_4\, , \end{aligned} \quad (2.25)$$

where

$$g_S'' = g_S - \tfrac{1}{4}\frac{g_{\rho\omega\pi}^{\;2}}{g_D}(m_\omega^{\;2}-t) \, . \quad (2.26)$$

In the kinematical region of interest the ratio g_S''/g_D is always negative.

We compare the model with the data for $\pi^-p\to B^-p$ at 9.1 GeV/c.[15] The computed value of the cross section is about 10 times too large (440 μb vs 45 μb) (see Fig. 11). The contribution of $\rho_{00} d\sigma/dt$ alone accounts for this fact. That is, the model seems to enhance the amplitudes for longitudinally polarized B's over those required by the data. Note, however that the forward peak in the differ-

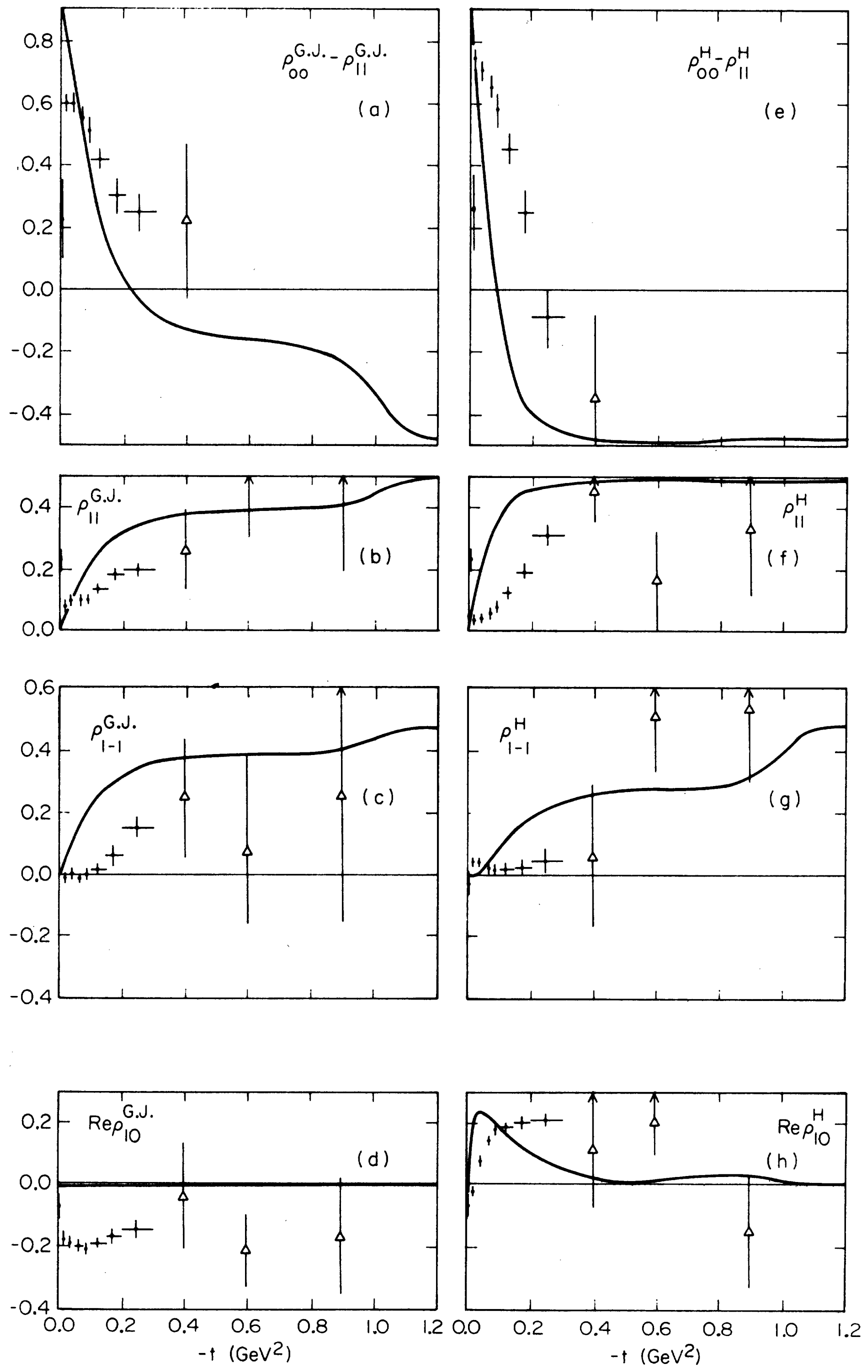

FIG. 10. (a)–(d) Density-matrix elements in the Gottfried-Jackson frame for the ρ produced in $\pi^- p \to \rho^0 n$ at 15/16 GeV/c. (e)–(h) Same as above in the helicity frame. The definition of the dots and of the open triangles is the same as that of Fig. 9. Data points corresponding to unphysical values have not been shown.

ential cross section is correctly reproduced. A similar effect was obtained by Fox and Hey in a model which extrapolated the $B\pi$ "Regge ω" coupling from the $B\to\omega\pi$ decay, and they were also unable to fit the data properly.[16]

In order to get a better agreement with the data, we tried a solution in which the B trajectory was exchange-degenerate (this is possible, although no meson with the quantum numbers $J^P=0^-$, $I^G=1^+$ has been discovered, because the 0^- member of

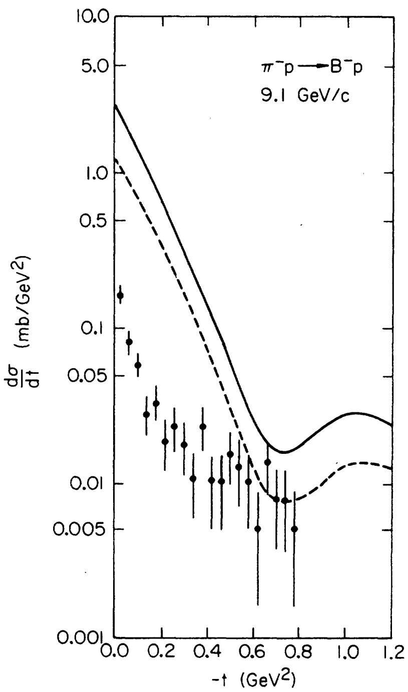

FIG. 11. Differential cross section for $\pi^-p \to B^-p$ scattering at 9.1 GeV/c (Ref. 15). The dashed line represents the prediction of the model when the B trajectory is exchange-degenerate.

the B trajectory would have a mass below the 4π threshold). The prediction under such an hypothesis is shown by the dash-dot curve in Fig. 11, and it is still too large by a factor 5.

III. DISCUSSION OF THE COUPLING CONSTANTS

In this section we look at the predictions of the model for the various coupling constants of the mesons ρ, ω, B, and A_2 which enter the model. Since some of them depend on the momentum transfer, to allow comparison with other theoretical or experimental predictions we use the values of the coupling constants for all the particles on the mass shell.

A. Natural-parity meson coupling constants

From Eqs. (3.32) and (3.28) in I we obtain the coupling of the ρ meson to the $(\bar{N}N)$ and $(\Delta\bar{N})$ systems. Assuming that $g_{\rho A_2 \pi}$ is known from the decay $A_2 \to \rho\pi$,

$$g_{\rho A_2 \pi}{}^2/4\pi = 41 \text{ GeV}^{-4}\,, \tag{3.1}$$

we can readily calculate the $A_2N\bar{N}$ coupling constants [see Eq. (3.28) in I]. A summary of our results concerning the coupling constants is given in Table I (see Ref. 17). We recall that G_V^ω and G_T^ω are determined from fitting the natural-parity exchange in the reactions $\pi^+p \to \omega\Delta^{++}$ and $\pi^-p \to \omega n$, whereas the other coupling constants are consequences of the model after $g_{\rho\pi\pi}$, $g_{\rho\omega\pi}$, $G_{\pi NN}$, $G_{\pi N\Delta}$, and $G_{\rho A_2\pi}$ are chosen according to Eqs. (2.6), (2.7), (2.8), and (3.1). Whenever comparison with other calculations is possible, the agreement is rather good. This is not surprising since the model reproduces rather well the sizes of the various natural-parity exchanges except those in the production of the B meson.

B. The B-meson coupling constants

In I we found that the ratio g_D/g_S, which is a measure of the D-wave/S-wave ratio in the decay $B \to \omega\pi$, has the value $g_D/g_S = 3.42$. This is compatible with a recent bootstrap calculation[18] which yields for the same ratio the value 2.56. As shown in Table II, the decay parameters calculated in our model compare rather well with the measured values,[19] but are in complete disagreement with the quark model,[20] which predicts $|F_0|^2 = 1$. It is interesting to remark that the quantities displayed in Table II depend only on the ratio g_D/g_S, but not on the size of g_S or g_D. Note also that a small $|F_0|^2$ in the $B \to \omega\pi$ decay requires a positive g_D/g_S ratio, but we have seen that in order to reproduce the qualitative features of the ω-production reactions we need a negative value (i.e., g_D/g_S must be less than 0 when the B meson is in the momentum-transfer region).

TABLE I. G_V^ω and G_T^ω are actually fitted and the other coupling constants are predictions using as input to the model the values of $g_{\rho\pi\pi}$ and $G_{\pi NN}$, $G_{\pi p\Delta^{++}}$, $g_{\rho\omega\pi}$, $g_{\rho A_2\pi}$ given in Eqs. (2.6), (2.7), (2.8), and (3.1), respectively. The "other calculation" value for $G_{\rho p\Delta^{++}}$ is obtained from the reaction $\gamma N \to \Delta \to \pi N$ (Ref. 17) using the vector-dominance model. All the other coupling constants used for comparison are taken from Ref. 6.

	G_T^ω/G_V^ω	G_V^ω	G_T^ρ/G_V^ρ	G_V^ρ	$G_1^{A_2}$	$G_2^{A_2}$	$G_{\rho p\Delta^{++}}$
Calculated or fitted	−0.30	6.9	4	1.95	−16.6	−8.3	22.5
Other calculations	0.1 ± 0.2	4.5 ± 2.5	5 ± 1	3.2 ± 0.7	Not given	Not given	18 ± 2

TABLE II. The decay parameters of the B meson: predictions of this model and comparison with other theoretical and experimental values.

References	g_D/g_S	$\lvert D/S\rvert^2$	$\lvert F_0\rvert^2$
19a		0.4 ± 0.2	0.06 ± 0.10
19b		0.03 to 3	0.10 ± 0.08
19c		Not given	0.184 ± 0.052
19d		$0.00^{+0.06}_{-0.00}$	0.33 ± 0.06
19e		0.04 to 3	0.09 ± 0.07
Quark model (Ref. 20)		0.5	1
Bootstrap model (Ref. 18)	2.56	0.033	Not given
This model	3.42	0.032	0.18

Making use now of the experimental width of the $B \to \omega\pi$ decay, we can predict more coupling constants involving the B meson. In particular, taking the residue of A^- (see Appendix A in I) at $\alpha_{12}^{B} = 1$ and $\alpha_{45}^{A_2} = 2$, we find for the coupling constants at the $BA_2\pi$ vertex

$$\frac{g_L}{g_P} = \left[\frac{g_S}{g_D} + \frac{1}{4}\,\frac{g_{\rho\omega\pi}{}^2}{g_D{}^2}\,(m_{A_2}{}^2 - m_\omega{}^2)\right]^{-1}$$

and (3.2)

$$g_L = m_B \frac{g_{\rho A_2\pi}}{g_{\rho\omega\pi}}\, g_D$$

(see Appendix B in I for the definition of g_L, g_P), and with the help of Eq. (3.33) in I the couplings of the B meson to the $(N\bar{N})$ and $(\Delta\bar{N})$ systems can be obtained. The results are summarized in Table III.

Unfortunately, none of the coupling constants (except g_S and g_D) listed in this table have been measured experimentally. The numbers do show that the B-meson couplings to the baryon vertices are of reasonable size (comparable to those of the more well-known mesons), but the B trajectory is unable to fill the large unnatural-parity contribution in the ω-production reactions due to its low trajectory. In other words, if we insist that the B meson alone contributes to these reactions, it should have abnormally large coupling constants at the baryon vertices; this possibility is not very attractive since the backward production of the B meson is not important. As an illustration of this, we relax the constraints imposed by the model on the B-exchange amplitude in the ω-production reactions and we allow the unnatural-parity amplitude in $\pi^+ p \to \omega\Delta^{++}$ reaction, for instance, to be multiplied by a factor 4. The corresponding prediction is shown by the dashed line in Figs. 2–4. We find that, indeed, the fits to the differential cross section and all density-matrix elements are considerably improved, but for this we pay the price of having large coupling constants for the B meson. In this respect, this model is not different from other pure pole (or pure pole + weak cut) models (see, for example, Ref. 11).

IV. CONCLUSIONS

The model for $\pi N \to \pi\omega N$ scattering constructed in I is applied to the four quasi-two-body reactions: $\pi^+ p \to \omega\Delta^{++}$, $\pi^- p \to \omega n$, $\pi^- p \to \rho^0 n$, and $\pi^- p \to B^- p$. When the two parameters of the model are determined by fitting the size of the ρ-exchange contribution to the reactions $\pi^+ p \to \omega\Delta^{++}$ and $\pi^- p \to \omega n$, the model has the following properties:

It relates coupling constants to each other and in this respect is rather successful.

It relates coupling constants (i.e., behavior at poles) to behavior in the scattering region. For π exchange and for ρ exchange the model agrees well with data (however, we still have the usual problem at the nonsense wrong-signature point of the ρ trajectory). This good agreement for the ρ trajectory is not surprising in view of Randa's work[21] on ρ-exchange amplitudes.

It relates the sizes of various amplitudes in several reactions; in general the natural-parity exchanges were adequately reproduced. However, the model failed in its attempts to fit the B production and also in its attempt to explain the large unnatural-parity exchange in the ω-production reactions by B exchange. In the latter case, it is plausible to assume that the ρ-ρ cut plays an important role. In the former case, it seems that the data show a suppression of the cross section which we cannot explain.

In conclusion, it appears that the attention paid to spin theory in I allowed us to obtain a set of mutually consistent coupling constants which agree with those obtained in other analyses. We also have reason to believe that this kind of model might provide a proper way to continue the amplitude from the pole region to the scattering region

TABLE III. The B-meson coupling constants calculated in this model. The notation is defined in Appendix B in I.

g_D/g_S	g_S	g_L/g_P	g_P	G_T^B	$G^1_{Bp\Delta^{++}}$	$G^2_{Bp\Delta^{++}}/G^1_{Bp\Delta^{++}}$	$G^3_{Bp\Delta^{++}}/G^1_{Bp\Delta^{++}}$
3.42	2.99	0.84	1.23	3.5	1.75	16.2	0

(in particular for the π and the ρ). However, the inclusion of duality into the Regge model has not solved the problems associated with the B meson in the usual Regge language. This probably indicates that the ρ-ρ cut is important in ω production.

ACKNOWLEDGMENT

I wish to thank Professor L. M. Jones for many helpful discussions and numerous suggestions throughout the course of this work.

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†Work supported in part by the National Science Foundation under Grant No. GP25303.

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