be shown that such a model must have more than one odd-C trajectory, whereupon the nonlinearity of Eqs. (42)–(45) becomes unmanageable. However, we have constructed one such model that is worth mentioning. Consider a system of n even-C trajectories with integer-spaced intercepts, $1, 0, -1, \ldots, 2-n$, and n odd-C trajectories, again with integer-spaced intercepts, $1-\delta, -\delta, \ldots, 2-\delta-n$. If we then require that each trajectory have nonzero double-Regge couplings only to its nearest neighbors, then such a solution is possible *only* for $\delta = \alpha_P - \alpha_\rho = 0.5$, and the resultant value for $\tilde{\gamma}_{PP}^{\pi^-}$ is

$$\tilde{\gamma}_{PP}^{\pi^-} = (2n-2)\tfrac{1}{4}(\alpha_P - \alpha_\rho) = \tfrac{1}{4}(n-1). \quad (51)$$

For $n = 4$ this model is then in agreement with experiment.

It is quite surprising that the requirement of charge conservation is so restrictive upon the double-Regge analysis. In this connection it is worth noting that similar techniques can be applied to the fragmentation region for the incoming particle a, if the ambiguity of what rapidity should be taken to specify a threshold can be resolved. Then, with the double-Regge couplings determined as in this section, the Regge residue ratios x_{ai} for each trajectory i can also be determined from the charge-conservation constraint.

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[1]C. E. DeTar, D. Z. Freedman, and G. Veneziano, Phys. Rev. D 4, 906 (1971).
[2]R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).
[3]S.-Y. Mak and C.-I Tan, Phys. Rev. D 4, 1059 (1972).
[4]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).
[5]R. F. Amann and K. Geer, Nuovo Cimento Lett. 8, 417 (1973).
[6]R. F. Amann and M. L. Blackmon, Phys. Lett. 44B, 266 (1973).
[7]R. F. Amann, Nucl. Phys. B41, 605 (1972).
[8]G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968); Phys. Rev. Lett. 20, 1078 (1968).
[9]C. Schmid, Phys. Rev. Lett. 20, 689 (1968).
[10]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).
[11]Pisa-Stony Brook–ISR Collaboration, as reported by J. C. Sens to the Washington APS Meeting, 1973 (unpublished).
[12]T. Ferbel, Phys. Rev. Lett. 14, 448 (1972).
[13]S. Pinsky, D. Snider, and G. H. Thomas, Argonne Report No. ANL/HEP 7349, 1973 (unpublished).

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Dual resonance model with spin for the reaction $\pi N \to \pi\omega N$ †

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A dual resonance model with spin is constructed for the reaction $\pi N \to \pi\omega N$. The invariant amplitudes are parametrized as sums of five-point (Bardakci-Ruegg) functions in such a way that the scattering amplitude satisfies the following properties: (a) correct spin-parity structure in all meson channels to leading order, (b) correct Regge limits, (c) correct residues at the lowest poles on all trajectories. The only parameters of the model are coupling constants, for which several sets of constraints are derived. A prediction is also made for the D-wave/S-wave ratio in the decay $B \to \pi\omega$.

I. INTRODUCTION

The discovery of the generalized dual resonance model[1] a few years ago made it possible to fit reactions in which 2 particles → 3 particles. In early applications of the model the spin of the particles was never taken into account. This simplified the calculations considerably and nevertheless was enough to give an over-all rough agreement with the data.[2] One drawback of this approach was that the continuation of the amplitude from one kinematical region to another was not reliable and indeed, in actual calculations, the relative size of various quasi-two-body reactions did not agree with the experimental results.[2] Later some attempts were made to introduce the spin of the nucleons in the reaction $PB \to PPB$ (where P stands for a pseudoscalar meson and B for a baryon), but no detailed fits to the data were presented.[3] The method used in the latter case was to parametrize the invariant amplitudes as sums of B_5 functions. Factorization and Regge behavior of the amplitude squared summed over spin were used to reduce the number of parameters.

In this paper we use a slightly different approach. The amplitude is written as a sum of terms of the form[4]

$$\mathfrak{M}=\sum_a K_a(p_i)\,L_a(B_5)\ . \tag{1.1}$$

$K_a(p_i)$ is a Lorentz tensor made up of the momenta of the external particles to be contracted against the wave functions of the external particles. $L_a(B_5)$ is a linear combination of B_5 functions which multiplies $K_a(p_i)$. The sum is over all permutations of external legs and over all possible trajectories in the various channels. It will be shown that the covariants $K_a(p_i)$ determine the normality of the trajectories to be put in the B_5 functions associated with them.[4] Unlike the method used in Ref. 3, this approach ensures the right spin-parity content and consequently the right Regge behavior in all meson channels.

The method is applied here to the reaction $\pi N\to\pi\omega N$. This process was chosen because it contains a vector particle in the final state and the comparison of the measured density matrix elements of the ω with the predictions of the model will provide a test for the treatment of spin. Also, this particular reaction has a very simple isospin decomposition, since, as for $\pi N\to\pi N$ scattering, there are only two independent isospin amplitudes. Finally, data with good statistics are available for this process as well as for the quasi-two-body reactions which can be extracted from it: $\pi N\to\omega\Delta$, $\pi N\to\omega N$, $\pi N\to\rho N$, and $\pi N\to BN$.

The material is organized as follows. Section II deals briefly with the kinematical singularities; in Sec. III the amplitude is constructed in the most general case. Care will be taken to put in the spin factors and to include both normally and abnormally coupled trajectories. Several relations between the coupling constants introduced in the model are derived. Detailed comparison with the data will be presented in a subsequent paper.

II. KINEMATICAL SINGULARITIES

Consider the interaction between two pions (π_1 and π_3), an ω meson (ω_2), a nucleon (N_4), and an antinucleon ($\bar{N}_5$). The scattering amplitude can be conveniently represented by the functions $A_{\alpha\beta}$, where $\alpha,\beta=1,2,3$ refer to the isospin components of the pions and $A_{\alpha\beta}$ is a two-by-two matrix for fixed α and β. The most general form for $A_{\alpha\beta}$ is

$$A_{\alpha\beta}=\delta_{\alpha\beta}A^{+}+\tfrac{1}{2}[\tau_\alpha,\tau_\beta]A^{-}\ , \tag{2.1}$$

where A^+ and A^- correspond, respectively, to the $I=0$ and $I=1$ isospin states in the $(N_4\bar{N}_5)$ channel. As in πN scattering, Bose symmetry will impose some constraint on A^+ and A^-. More precisely, A^+ will be symmetric and A^- will be antisymmetric under the interchange of π_1 and π_3. In the following we are interested in the kinematical structure of A^+ and A^-, and all that is said applies equally well to both. In this section, therefore, we use the general notation A to denote either amplitude.

Experience in calculating amplitudes for reactions with spin has shown that the spin wave functions introduce singularities at thresholds, pseudothresholds, and conspiracy points. Since the wave functions do not change from reaction to reaction, these singularities can be predicted and factored out, leaving behind only amplitudes with dynamical information. Therefore dynamical amplitudes are the appropriate ones to parameterize in any model. Hence one must have an effective way for recognizing and removing the spin complications and singularities. There are two common approaches to this problem: through invariant amplitudes and through helicity amplitudes. Each method has its own set of difficulties in our case.

When there are more than four particles involved in the reaction, the existence of a minimal set of dynamical invariant amplitudes is, in general, no longer guaranteed[5a]; in fact, a basis of invariant amplitudes free of kinematical singularities has been found only in the case of two spin-$\frac{1}{2}$ and three spin-0 particles or of one spin-1 and four spin-0 particles.[5b]

In the helicity formalism, more precise results have been obtained for the five-particle case: The kinematical singularities have been isolated and a set of regularized helicity amplitudes has been constructed.[6] However, this approach is not suitable for our purposes since we want, ultimately, to apply our model to various channels which are related by crossing. Theoretically, it would be possible to check that a given invariant decomposition generates the right kinematical singularities in the helicity amplitudes, but the amount of algebra required to carry out such a program makes this calculation unfeasible.

We will content ourselves with a more primitive method, reminiscent of that used in the early days of Regge theory.[7] We will show that all the covariants which can be constructed out of the momenta, the polarization vector ω, and the γ matrices in the problem can be expanded into a sum of 16 simple covariants, each one being multiplied by an invariant amplitude free of kinematical singularities. We recall that the minimum number, i.e., the number of helicity amplitudes, is only 12. Hence the invariant amplitudes in our decomposition are not a minimal set. This will not, however, cause any difficulty in the end, due to the method of amplitude construction adopted in Sec. III.

The scattering amplitude for the process under consideration must be a scalar; we claim it can be written in the form

$$
\begin{aligned}
A = \bar{v}_5 \{ & A_1 p_1\cdot\omega + A_2(p_4-p_5)\cdot\omega + A_3 p_3\cdot\omega + A_4 \not{\omega} \\
& + \not{p}_1 (B_1 p_1\cdot\omega + B_2(p_4-p_5)\cdot\omega + B_3 p_3\cdot\omega + B_4\not{\omega}) \\
& + \not{p}_3 (C_1 p_1\cdot\omega + C_2(p_4-p_5)\cdot\omega + C_3 p_3\cdot\omega + C_4\not{\omega}) \\
& + [\not{p}_1, \not{p}_3](D_1 p_1\cdot\omega + D_2(p_4-p_5)\cdot\omega \\
& \qquad + D_3 p_3\cdot\omega + D_4\not{\omega})\} u_4 ,
\end{aligned} \tag{2.2}
$$

where ω is the polarization vector of the meson ω. The A_j's, ..., D_j's are functions of the $s_{i,i+1}$ defined by

$$s_{i,i+1} = (p_i + p_{i+1})^2, \quad i = \{1, 2, 3, 4, 5\}. \tag{2.3}$$

We have assumed here that all particles are incoming. The A_i, ... are free of kinematical singularities and they carry the dynamics of the model. Although there are four independent momenta, $\not{p}_4$ and $\not{p}_5$ do not appear because they can be reduced to terms already included by making use of the Dirac equation, and $(p_4+p_5)\cdot\omega$ goes away because of the condition $p_2\cdot\omega = 0$.

Calculation of an arbitrary Feynman graph for $\pi N \to \pi\omega N$ may, of course, produce covariants which look different from the ones appearing in Eq. (2.2), but experience has shown that all covariants can be reduced to the ones appearing above. For example, instead of using a basis made up of four vectors we could equally well use one constructed from four independent axial vectors of the type $\epsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_3^\beta p_4^\gamma$. Because of parity, only an even number of ϵ, ... can appear in each term of the amplitude, and they can always be reduced, without introducing kinematical singularities, to a combination of the terms in Eq. (2.2) with the help of

$$\epsilon_{\alpha\beta\gamma\delta}\,\epsilon_{\lambda\mu\nu\rho} = -\det\begin{vmatrix} g_{\alpha\lambda} & g_{\alpha\mu} & g_{\alpha\nu} & g_{\alpha\rho} \\ g_{\beta\lambda} & g_{\beta\mu} & g_{\beta\nu} & g_{\beta\rho} \\ g_{\gamma\lambda} & g_{\gamma\mu} & g_{\gamma\nu} & g_{\gamma\rho} \\ g_{\delta\lambda} & g_{\delta\mu} & g_{\delta\nu} & g_{\delta\rho} \end{vmatrix} \tag{2.4}$$

and of

$$\epsilon_{\alpha\beta\gamma\delta}\,\epsilon^{\alpha}{}_{\mu\nu\rho} = \det\begin{vmatrix} g_{\beta\mu} & g_{\beta\nu} & g_{\beta\rho} \\ g_{\gamma\mu} & g_{\gamma\nu} & g_{\gamma\rho} \\ g_{\delta\mu} & g_{\delta\nu} & g_{\delta\rho} \end{vmatrix} . \tag{2.5}$$

Other allowed terms in the amplitude are $\gamma_5\epsilon_{\mu\nu\sigma\tau}$, $\gamma_5\epsilon_{\mu\nu\sigma\tau}\gamma^\tau$, $\gamma_5\epsilon_{\mu\nu\sigma\tau}\gamma^\sigma\gamma^\tau$, etc, where the free indices are contracted with the momenta of the problem and the polarization vector of the ω. They can be reduced to simpler terms using the identities

$$
\begin{aligned}
\epsilon_{\mu\nu\sigma\tau}\gamma_5 = -i\,(&\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\tau - g_{\mu\nu}\gamma_\sigma\gamma_\tau - g_{\nu\sigma}\gamma_\mu\gamma_\tau \\
& - g_{\sigma\tau}\gamma_\mu\gamma_\nu - g_{\tau\mu}\gamma_\nu\gamma_\sigma + g_{\nu\tau}\gamma_\mu\gamma_\sigma \\
& + g_{\mu\sigma}\gamma_\nu\gamma_\tau + g_{\mu\nu}g_{\sigma\tau} + g_{\nu\sigma}g_{\mu\tau} - g_{\mu\sigma}g_{\nu\tau}),
\end{aligned} \tag{2.6}
$$

$$
\begin{aligned}
&\epsilon_{\mu\nu\sigma\tau}\gamma^\tau\gamma_5 = -i\,(\gamma_\mu\gamma_\nu\gamma_\sigma - g_{\mu\nu}\gamma_\sigma - g_{\nu\sigma}\gamma_\mu + g_{\mu\sigma}\gamma_\nu), \\
&\epsilon_{\mu\nu\sigma\tau}\gamma^\sigma\gamma^\tau\gamma_5 = i[\gamma_\mu, \gamma_\nu] = -2i\sigma_{\mu\nu}, \\
&\epsilon_{\mu\nu\sigma\tau}\gamma^\nu\gamma^\sigma\gamma^\tau\gamma_5 = 6i\gamma_\mu, \\
&\epsilon_{\mu\nu\sigma\tau}\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\tau\gamma_5 = -24i.
\end{aligned}
$$

In summary, any covariant which can enter the amplitude is decomposable into a linear combination of the terms in Eq. (2.2), each term being multiplied by a function of the invariants in Eq. (2.3), which is free of kinematical singularities.

This discussion does not constitute a proof that the decomposition (2.2) is free of kinematical singularities but, rather, makes this assumption plausible.

In Sec. III we will never make use of the form (2.2), but we will express the amplitude as a sum of terms which make the dynamics of the reaction more apparent. Using Eqs. (2.4)–(2.6) we can always go back to the standard form (2.2).

III. CONSTRUCTION OF THE AMPLITUDE

A. General features of $\pi N \to \pi\omega N$; choice of the trajectories

Before we start constructing the amplitude we look at the experimental data to get some clues about the dynamics of the reaction. The most striking fact about the reaction $\pi^+ p \to \pi^+ \omega p$ is that it is dominated by the quasi-two-body processes $\pi^+ p \to B^+ p$ and $\pi^+ p \to \omega\Delta^{++}$, which at 8 GeV/$c$ (Ref. 8e) account for half of the cross section. Besides the B peak the $(\omega\pi)$ mass distribution also shows a bump around 1700 MeV (Refs. 8f–8h; 9f, 9h) which can be ascribed to the $g(1680)$, a 3^- resonance lying on the ρ trajectory. All this suggests that in the $(\omega\pi)$ channels the important trajectories will be that of the ρ and of the B, whereas the Δ_δ trajectories will dominate the (πN) channels. Although the production of the N^*_{1688} does not seem to be a dominant feature of the reaction $\pi N \to \pi\omega N$, we will nevertheless include for completeness the N_α-N_γ exchange-degenerate trajectory in the (πN) and (ωN) channels.

A study of the density matrix elements of the ω

in the reaction $\pi N \to \omega\Delta$ (Ref. 10) and $\pi N \to \omega N$ (Ref. 11) shows that, besides the ρ exchange in the t channel, there must be a strong unnatural contribution to account for the nonvanishing value of ρ_{00}. This can be explained either by a ρ-ρ cut or by B exchange. We choose here the second possibility since we do not know (in present dual resonance models) whether the Reggeon-Reggeon cut should be dual to the background, the resonance part, or both.[12] So here again, but this time in the momentum-transfer region, we see that the ρ and B trajectories are the dominant ones in the $(\omega\pi)$ channels.

For the reaction $\pi^{\pm}p \to B^{\pm}p$ the situation is not as clear since the density matrix elements of the B have never been measured with precision. The ω trajectory will certainly be present in the $(B\pi)$ channel since $\omega\pi$ is the only known decay mode of the B meson. Cohen *et al.*[13] studied $\pi^- d \to B^-\Delta^0 p_{\text{spectator}}$ at 7 GeV/c; this reaction proceeds through $I=1$ exchange. The exchanged trajectory is most probably the A_2, as suggested by the rather flat t-distribution, and on the basis of the large cross section observed (25 μb vs 40 μb for $\pi p \to Bp$ at the same energy) we can safely conclude that the $\pi A_2 B$ vertex is large.

There is no compelling need for an unnatural-parity-exchange piece in $\pi^{\pm}p \to B^{\pm}p$. Indeed, a sharp forward peak is observed in the t distribution[8e,9f,9g]; if unnatural-parity exchange were important we might expect a forward dip (since the unnatural-parity exchanges will couple only to helicity $\lambda = \pm 1$ for the B in the Gottfried-Jackson frame). Furthermore, at 11 GeV/c (Ref. 9h), the only energy at which the density matrix elements of the B meson are known, $\rho_{11}^{\text{GJ}} = 0.25 \pm 0.05$ and $\rho_{1-1}^{\text{GJ}} = 0.0 \pm 0.3$ and this is compatible with having only the ω and A_2 trajectories in the t channel (the prediction in the latter case is $\rho_{11}^{\text{GJ}} \simeq -\rho_{1-1}^{\text{GJ}}$). Note that there is no obvious candidate for unnatural-parity exchange; the $B\pi\pi$ vertex does not exist because of the spin-parity of the B, and A_1, not being a resonance,[14] is out of the question.

In conclusion, we will put only ω and A_2 trajectories in the $N\bar{N}$ channel when coupling to B in the $(\omega\pi)$ channels, but, of course, π, ω, and A_2 trajectories will be necessary in conjunction with ρ in the $(\omega\pi)$ channels.

Armenise *et al.*[9g] present a careful study of $\pi^- p \to \pi^-\omega p$ at 9.1 GeV/c after subtracting the events corresponding to the reaction $\pi^- p \to B^- p$. They show that the two multiperipheral graphs with either the pion or the ω at the external meson vertex will dominate the nonresonant part of the reaction and that baryonic exchanges can be neglected. An examination of the Dalitz plot also leads to the same conclusions.[8f,9f]

B. Properties of the scattering amplitude

The amplitude will be constructed in the spirit of the dual resonance model and therefore should describe resonance production as well as the background term. It will obey the following constraints:

(1) Crossing symmetry should hold.

(2) It will be required to reduce the Feynman amplitudes for all graphs obtained by taking the residues at the lowest poles on the trajectories. However, for simplicity, daughterlike terms (i.e., those terms corresponding to particles of spin less than the one under study) will be neglected.

Also, in meson channels, the resonances lying on a trajectory will have the right spin-parity assignments given by their positions along the trajectory. For baryon channels, correct parity cannot be realized in the usual B_5 formalism (fixed cuts and fixed poles are probably needed),[15] and we will content ourselves with parity doublets of the right spin. Only the N and Δ poles will be forced to have the correct residues. As a consequence, we will see that the amplitude has the correct asymptotic behavior in the single- and the double-Regge region when only meson trajectories are exchanged; for the baryon exchanges all we can say is that the amplitude squared summed over spins will have the correct Regge behavior, but the individual helicity amplitudes will not necessarily have this property. This fact, however, is not of great annoyance in the reaction under study since it was seen experimentally that the baryonic exchanges were negligible. Also there is some evidence for parity doubling along the exchange-degenerate N_α-N_γ trajectory except at the $N_\alpha(938)$ and $N_\gamma(1526)$ poles.

All of the above will be enforced only at the leading level (i.e., for the highest power in s). This, in principle, takes care of duality and factorization.

(3) Bootstrap consistency should be valid. This is just a pompous way of saying that the five-point amplitude should reduce, when one takes the residue at a pole on a leading trajectory, to the corresponding four-point function. For example, the residue at the π (or ω) pole in the $N\bar{N}$ channel should give the known amplitude for $\pi\pi \to \pi\omega$ (Ref. 16) (or $\pi\omega \to \pi\omega$) (Ref. 17) scattering.

(4) Trajectories should have signature, except the N_α-N_γ one. This will be achieved, as in the original B_5 model, by adding terms with different ordering of the external legs.

(5) Bose symmetry and isospin will, of course, be put into the model.

(6) A somewhat less quantitative constraint is that of "maximum duality," which says that the

terms in the amplitude should be leading in as many channels as possible.

(7) Finally, we will try to use as few terms as possible in the construction of the amplitude.

The only parameters in the model are coupling constants, which will be determined by the requirements above and by fits to the data. By way of construction, since we work in the invariant formalism, the amplitude has all the required analyticity properties and therefore could be used to fit the annihilation channel $p\bar{p}\to\pi^+\pi^-\omega$. In view of past experience, good agreement with the data is in no way guaranteed for that channel, since requirements (6) and (7) tend to somewhat different approximations in the scattering and annihilation regions.

C. The method

As said in the Introduction we build the amplitude as a sum of terms, each term being the product of a linear combination of B_5's by a polynomial covariant constructed out of the momenta, polarization vectors, and γ matrices.[4] These terms do not have to be the same as those of the invariant decomposition of Sec. II and their number is not limited. As was seen before, we can always get back to Eq. (2.2) without introducing unwanted kinematical singularities.

In our case, the covariants used are the Feynman amplitudes corresponding to two-particle exchanges. We then have to find what quantum numbers these structures imply in crossed channels (those channels without our exchanged Feynman poles). The trajectories in the B_5 functions will then be chosen to have these quantum numbers. If a meson channel appears to contain a parity doublet, the corresponding trajectory will be depressed so that it does not contribute to the leading order. The covariants will introduce in a simple way trajectories with abnormal-parity coupling to external particles (essentially the $\pi\rho\omega$ vertex); this coupling is absent in the standard B_5 functions. In summary, the basic structure of the amplitude will come from the covariants, whereas their invariant coefficients will provide a way to analytically continue the amplitude to all kinematical regions.

After briefly reviewing the resonance structure of the B_5 we will analyze the meson channels and then the baryon channels. The method will be applied to some meson scattering amplitudes, which have been calculated by other means, before we tackle the five-body problem.

D. Spins and tensors

A particle of momentum p and integer spin J can be represented as a symmetric tensor of rank J denoted $S_{\mu_1\cdots\mu_J}$, such that all tensors of rank $J-2$ obtained by contracting two indices vanish and, furthermore, satisfying the condition[18]

$$p^{\mu_1}\cdot S_{\mu_1\cdots\mu_J}=0\ .$$

In the rest frame of particle J, $S_{\mu_1\cdots\mu_J}$ reduces to a 3-tensor which transforms under the irreducible representation $\mathfrak{D}(J)$ of the rotation group. If, under the Lorentz transformations of negative determinants, $S_{\mu_1\cdots\mu_J}$ picks up an extra minus sign, it is said to have unnatural parity; otherwise it has natural parity. If the two subsidiary conditions were relaxed, $S_{\mu_1\cdots\mu_J}$ would represent a sequence of particles of spin J, $J-1,\ldots,0$.

It can be shown that the residue of $B_n(\ldots,-\alpha_{im},\ldots)$ at the pole $\alpha_{im}=J$ can be written as a product of two symmetric tensors of rank J such as

$$T^{\mu_1\cdots\mu_J}(p_i,\ldots,p_m)\,T_{\mu_1\cdots\mu_J}(p_{m+1},\ldots,p_{i-1}),$$

one of them made up of the momenta $p_i, p_{i+1},\ldots,p_m$, and the other one of $p_{m+1},\ldots,p_{i-1}$; the rank of the tensors is independent of the number of external legs and they always have positive parity.[19] Using the definition above, this represents, in the channel α_{im}, a particle of spin J together with all its daughters, normally coupled to particles $i,\ldots,m$. Hopkinson and Chan Hong-Mo[19] showed also that, provided all trajectories have the same slope, the first daughter has only natural parity with respect to the external particles.

We can use a similar technique to find the spin-parity content of the covariant. To analyze the structure in the (i,m) channel we split the covariant into the product of two tensors, one of them containing quantities (i.e., momenta, polarization tensors, γ matrices) relative to particles $i,\ldots,m$ and the other one containing all the rest. A typical decomposition would be of the form

$$K^{\nu_1\cdots\nu_l}(p_i,p_{i+1},\ldots,p_m)K_{\nu_1\cdots\nu_l}(p_{m+1},\ldots,p_{i-1}).$$

From $K^{\nu_1\cdots\nu_l}(p_i,\ldots,p_m)$ one can extract the symmetric tensor of highest rank, say, J, and then study its behavior under parity to determine its "naturality," n_J, defined as parity $\times(-1)^J$. In the B_5 functions associated with such a structure we will put the arguments $J-\alpha_{im}$, where α_{im} represents a trajectory of the right naturality. We can try now to take the residue at $\alpha_{im}=N>J$ and by combining the tensors $K^{\nu_1\cdots\nu_l}\,T^{\mu_1\cdots\mu_{N-J}}$ we can extract a symmetric tensor of rank N of the same naturality as $K^{\nu_1\cdots\nu_l}$; this shows that if we enforce the right spin-parity at the lowest pole it will also hold true all along the trajectory. Note that $K^{\nu_1\cdots\nu_l}$ can contain tensors of both naturalities (e.g., the antisymmetric tensor of rank 2) and unless we take special action it will give parity doubling along the trajectory.

If we now go to the corresponding Regge limit, that is, if all

$$s_{pq} = (p_p + p_q)^2$$

become large and of order s, whereas all $s_{p'p''}/s$ and all $s_{q'q''}/s$ go to zero, where

$$i \leq \begin{Bmatrix} p \\ p' \\ p'' \end{Bmatrix} \leq m \text{ and } m+1 \leq \begin{Bmatrix} q \\ q' \\ q'' \end{Bmatrix} \leq i-1 ,$$

then $B_5 \sim s^{\alpha_{im}-J}$. In the meantime, the covariant goes at most like s^J (the exact power depends on the helicity states of the external particles) and the product $B_5 \times$(covariant) will exhibit the right Regge behavior.

We specialize now to the system $\pi\pi\omega N\bar{N}$ and analyze some simple tensors which will be useful later.

1. $\pi_1 \pi_3$ channel

(i) Structures of the form p_1^μ, p_3^μ or $(p_1 - p_3)^\mu$ will be associated with a 1^- particle, namely, the ρ. They are all equivalent up to 0^+ satellites since they differ by $(p_1 + p_3)^\mu$, which transforms as a scalar under the rotation group in the $(\pi_1\pi_3)$ rest frame.

(ii) $p_1^\mu p_1^\nu$, $p_1^\mu p_3^\nu$, $p_3^\mu p_3^\nu$ are easily seen to correspond to a 2^+ particle—the f meson in this case. Similarly terms like $p_1^\mu p_1^\nu p_1^\rho$ would represent the g meson plus satellites.

2. $\pi_1\omega_2$ channel

Each structure has to contain the polarization vector of the ω once.

(i) ω^μ, the polarization vector of the ω, is a 1^- tensor, but it is associated with a 1^+ particle, the B meson in this case, since the pion has unnatural parity. The $B\omega\pi$ vertex thus defined is mostly S-wave.

(ii) $p_1 \cdot \omega p_1^\mu$ is also a B coupled to $\pi\omega$ in a mixture of S and D waves. These two structures exhaust all the $B\omega\pi$ couplings.

(iii) $\epsilon_{\mu\nu\rho\sigma} p_1^\nu p_2^\rho \omega^\sigma$ (to simplify the writing we use the notation $\epsilon_\mu \ldots p_1^\cdot p_2^\cdot \omega^\cdot$) is an axial vector with which we associate the ρ (remember that π has odd parity). It is a P-wave coupling and the only possible $\omega\rho\pi$ vertex.

(iv) $p_1^\mu \omega^\nu - p_1^\nu \omega^\mu = [p_1^\mu, \omega^\nu]$ is an antisymmetric tensor of rank 2; it has six independent components. To find its spin-parity content it is convenient to go to the rest frame of the $(\pi_1\omega_2)$ system. Introduce the notation

$$V^\mu = v^\mu - \frac{(p_1+p_2)\cdot v(p_1+p_2)^\mu}{m_{12}^2} , \quad (3.1)$$

where $m_{12}^2 = (p_1+p_2)^2$. In the rest frame of particle $(\pi_1\omega_2)$, V^μ reduces to a 3-vector $V^i = v^i$. Using this here we get

$$[p_2^\mu, \omega^\nu] = [P_2^\mu, W^\nu] + \frac{1}{m_{12}^2}\{(p_1+p_2)\cdot p_2[(p_1+p_2)^\mu, W^\nu] + (p_1+p_2)\cdot\omega[P_2^\mu, (p_1+p_2^\nu)]\}. \quad (3.2)$$

In the rest frame of (12) the term in $1/m_{12}^2$ becomes

$$X^i = \frac{(p_1+p_2)^0}{m_{12}^2}[(p_1+p_2)\cdot p_2 W^i - (p_1+p_2)\cdot\omega P_2^i] \quad (3.3)$$

and all its other components are 0; it is a vector and therefore is associated with a 1^+ particle. The only nonvanishing components of $[P_2^\mu, W^\nu]$ in the rest frame of $(\pi_1\omega_2)$ are $[P_2^i, W^j]$, where $i, j = 1, 2, 3$. Define

$$A^l = \epsilon_{lij} P_2^i W^j ; \quad (3.4)$$

equivalently

$$[P_2^i, W^j] = \epsilon_{lij} A^l , \quad (3.5)$$

which shows that $[P_2^\mu, W^\nu]$ represents a 1^- particle.

In conclusion $[p_2^\mu, \omega^\nu]$ is a parity doublet of spin 1. Note that $[p_2^\mu, \omega^\nu]$ is similar to the tensor $F^{\mu\nu}$ in electromagnetism which contains a vector (the electric field $\vec{E}$) and an axial vector (the magnetic field $\vec{B}$).

(v) $p_1^\mu[p_2^\nu, \omega^\rho]$; using the same technique it can be shown that this structure contains states of spin 0, 1, 2 of both parities.

(vi) $\omega^\mu[p_1^\nu, p_2^\rho]$ is a 2^- particle with its spin-1 daughters of both parities.

3. $N_4\bar{N}_5$ channel

The following tensors, being well known, will be listed without explanations.

(i) $\bar{v}\gamma_\mu u$ and $\bar{v}(p_5 - p_4)_\mu u$ correspond to the two possible couplings of a vector particle. The first one is an S-wave coupling to the $S = 1$ state of the $N_4\bar{N}_5$ system; the second one is a P-wave coupling to the $S = 0$ state.

(ii) $\bar{v}\{\gamma_\mu, (p_4 - p_5)_\nu\}u = \bar{v}[\gamma_\mu(p_4-p_5)_\nu + \gamma_\nu(p_4-p_5)_\mu]u$ and $\bar{v}(p_4-p_5)_\mu(p_4-p_5)_\nu u$ correspond to a 2^+ particle and describe its most general coupling to an $N\bar{N}$ system.

(iii) $\bar{v}\gamma_5 u$ is of course associated with the π trajectory.

(iv) $\bar{v}\gamma_\mu\gamma_5 u$ and $\bar{v}(p_4-p_5)_\mu\gamma_5 u$ are characteristic of a 1^+ trajectory, but these two terms have opposite G parity, the first one having that of the A_1 and the second one having that of the B meson.

4. $\pi_3 N_4$ channel

$\pi_3 N_4$ resonate in a $\frac{1}{2}^+$ state through $(\not p_3+\not p_4+m_R)\times\gamma_5 u_4$, where a sum over the polarization states of the resonance m_R has been performed. If this resonance is a nucleon then the vertex reduces to $-\gamma_5\not p_3 u_4$. Such a term will appear in the amplitude multiplied by an invariant like $B_5(\ldots,\frac{1}{2}-\alpha_{34},\ldots)$.

A πN resonance of spin-parity $\frac{3}{2}^+$ is described by $\Lambda^{\mu\nu}p_{3\nu}u_4$, where $\Lambda^{\mu\nu}$, the spin-projection operator of a spin-$\frac{3}{2}$ particle, is

$$\Lambda^{\mu\nu}=\left(g^{\mu\nu}-\tfrac{1}{3}\gamma^\mu\gamma^\nu-\frac{2}{3}\frac{p^\mu p^\nu}{m_R{}^2}-\frac{p^\mu\gamma^\nu-p^\nu\gamma^\mu}{3m_R}\right)\times(\not p+m_R) \qquad (3.6)$$

and $p=p_3+p_4$. Ignoring the presence of spin-$\frac{1}{2}$ daughters we can replace $\Lambda^{\mu\nu}$ by $g^{\mu\nu}(\not p+m_R)$. Therefore, if in the amplitude there is a term of type $p_3^\mu(\not p_3+\not p_4+m_R)u_4$, it will indicate the presence of a spin-$\frac{3}{2}^+$ particle. Likewise spin $\frac{3}{2}^-$ is represented by $-p_3^\mu\gamma_5(\not p_3+\not p_4-m_R)u_4$.

In general a πN resonance of spin $J+\frac{1}{2}$ and parity $(-1)^{J+1}$ is described by $\Lambda^{\mu_1\cdots\mu_J}(\not p_\pi+\not p_p+m_J)u$, where m_J is the mass of the resonance and $\Lambda^{\mu_1\cdots\mu_J}$ is a totally symmetric tensor of rank J; a resonance of opposite parity is

$$\Lambda^{\mu_1\cdots\mu_J}(\not p_\pi+\not p_p+m_J)\gamma_5 u=-\Lambda^{\mu_1\cdots\mu_J}\gamma_5(\not p_\pi+\not p_p-m_J)u\,.$$

In the amplitude such a term would be multiplied by $B_5(\ldots,J+\frac{1}{2}-\alpha_{\pi N},\ldots)$ if it is to be leading in this πN channel. Taking now the residue at $\alpha_{\pi N}=J+\frac{3}{2}$ we will find an expression of the form

$$\Lambda^{\mu_1\cdots\mu_{J+1}}(\not p_\pi+\not p_p+m_J)u=\Lambda^{\mu_1\cdots\mu_{J+1}}[(\not p_\pi+\not p_p+m_{J+1})u+(m_J-m_{J+1})u]\,,$$

but

$$(m_J-m_{J+1})u=\tfrac{1}{2}[(\not p_\pi+\not p_p+m_{J+1})+(m_{J+1}-\not p_\pi-\not p_p)]\times(m_J-m_{J+1})u\,,$$

which contributes to both parities of spin $J+\frac{3}{2}$.

5. $\omega_2 N_4$ channel

(i) Terms like $(\not p_2+\not p_4+m_p)\not\omega u_4$ or $(\not p_2+\not p_4+m_p)\times p_4\cdot\omega u_4$ correspond to the two possible couplings of ω to a nucleon trajectory in the $(\omega_2 N_4)$ channel.

(ii) $(\not p_2+\not p_4+m_N^*)\omega^\mu u_4$, $(\not p_2+\not p_4+m_N^*)p_4\cdot\omega p_2^\mu u_4$, and $(\not p_2+\not p_4+m_N^*)\not\omega p_2^\mu u_4$ are the only couplings for the ωNN^* vertex.

E. Modification of the invariant functions[3]

In order to get the right spin-parity assignment at the lowest poles on the baryon trajectories, we have to slightly modify the definition of the B_5 functions. In this work we consider only graphs which have adjacent nucleon legs, therefore there are only two baryon trajectories in the B_5's. Assume there exists in the amplitude a covariant term Γ which carries spin $\frac{3}{2}$ in both (34) and (51) channels. If we want to associate this term with Δ resonances, the residue at $\alpha_{34}^\Delta=\alpha_{51}^\Delta=\frac{3}{2}$ should look like

$$\bar v_5(\not p_1+\not p_5-m_\Delta)\Gamma(\not p_3+\not p_4+m_\Delta)u_4$$

or (3.7)

$$\bar v_5(\not p_1-m_p-m_\Delta)\Gamma(\not p_3+m_p+m_\Delta)u_4\,.$$

When we go away from the double pole each term in (3.7) will pick up a B_5 function, the arguments of which are chosen according to the rules described in Sec. III D. For instance, the invariant attached to the term $-(m_p+m_\Delta)^2\bar v_5\Gamma u_4$ will be completely determined from the structure of $\bar v_5\Gamma u_4$; assuming it has the form

$$B_5(l-\alpha_{12},m-\alpha_{23},\tfrac{3}{2}-\alpha_{34}^\Delta,p-\alpha_{45}^\omega,\tfrac{3}{2}-\alpha_{51}^\Delta),$$

the covariant $(m_p+m_\Delta)\bar v_5\not p_1\Gamma u_4$ will have the following properties:

(i) In channels (23) and (34) there is no change.

(ii) In channel (12) the momentum p_1^μ will bring one more unit of orbital angular momentum and the corresponding argument in the B_5 function will be $l+1-\alpha_{12}$.

(iii) In the (45) channel the matrix γ_μ will carry one unit of spin if the structure Γ is a tensor made up only of the momenta and the ω polarization vector (or $\gamma_5\times$tensor); if Γ already contains γ_μ, then we will have a parity doublet (recall that $\bar v_5\gamma_\mu\gamma_\nu u_4$ is a parity doublet of spin 1); in any case the (45) trajectory should be depressed by one unit.

(iv) In the (51) channel it will combine with the above structure to give the correct spin-parity at the pole $\alpha_{51}^\Delta=\frac{3}{2}$.

Applying the same technique to the remaining terms in (3.7) the complete expression looks like

$$\bar v_5\Big[\Gamma B_5(l-\alpha_{12},m-\alpha_{23},\tfrac{3}{2}-\alpha_{34}^\Delta,p-\alpha_{45},\tfrac{3}{2}-\alpha_{51}^\Delta)+\frac{\Gamma\not p_3}{m_\Delta+m_p}B_5(l-\alpha_{12},m+1-\alpha_{23},\tfrac{3}{2}-\alpha_{34}^\Delta,p+1-\alpha_{45},\tfrac{3}{2}-\alpha_{51}^\Delta)$$
$$-\frac{\not p_1\Gamma}{m_\Delta+m_p}B_5(l+1-\alpha_{12},m-\alpha_{23},\tfrac{3}{2}-\alpha_{34}^\Delta,p+1-\alpha_{45},\tfrac{3}{2}-\alpha_{51}^\Delta)$$
$$-\frac{\not p_1\Gamma\not p_3}{(m_\Delta+m_p)^2}B_5(l+1-\alpha_{12},m+1-\alpha_{23},\tfrac{3}{2}-\alpha_{34}^\Delta,p+1-\alpha_{45},\tfrac{3}{2}-\alpha_{51}^\Delta)\Big]u_4\,. \qquad (3.8)$$

To simplify the writing we will denote such a term by

$$\bar{v}_5\Gamma\mathcal{C}(l-\alpha_{12}, m-\alpha_{23}, \tfrac{3}{2}-\alpha^{\Delta}_{34}, p-\alpha_{45}, \tfrac{3}{2}-\alpha^{\Delta}_{51})u_4 .$$

Similar modifications can be made, as needed, for terms which contain an N and an N^* trajectory (we were not able to find a term with a nucleon pole in both baryon channels).

F. Application: meson-meson scattering

In the following, ω will be treated as a stable particle (a reasonable approximation in view of its small width). As a guide to the construction of the five-point function we start with the amplitudes for $\pi\pi\to\pi\omega$ and $\pi\omega\to\pi\omega$ scattering, which should appear as residues of the $\pi N\to\pi\omega N$ amplitude according to the requirement of bootstrap consistency.

1. $\pi\pi\to\pi\omega$ scattering[16]

This is just the case discussed by Veneziano. It provides a trivial illustration of our method. There is one isospin amplitude A^-, and ρ is the only possible trajectory since we decided to throw away the $B\pi\pi$ coupling. The Feynman term for ρ in the (π_1, ω_2) channel reads

$$\mathfrak{M}_{\pi\pi\to\pi\omega} = g_{\rho\omega\pi} g_{\rho\pi\pi}\epsilon\ldots{}^{\mu} p_1^{\cdot} p_2^{\cdot}\omega^{\cdot} i(-g_{\mu\nu})(p_{45}-p_3)^{\nu}$$
$$= -2ig_{\rho\omega\pi}g_{\rho\pi\pi}\epsilon\ldots. p_1^{\cdot}p_2^{\cdot}p_3^{\cdot}\omega^{\cdot}, \qquad (3.9)$$

where it appears that this structure contains ρ in all channels. The full amplitude is then

$$A^-_{\pi\pi\to\pi\omega} = \alpha' i g_{\rho\omega\pi} g_{\rho\pi\pi}\epsilon\ldots. p_1^{\cdot} p_2^{\cdot} p_3^{\cdot}\omega^{\cdot}$$
$$\times[B(1-\alpha^{\rho}_{12}, 1-\alpha^{\rho}_{23}) + B(1-\alpha^{\rho}_{12}, 1-\alpha^{\rho}_{13})$$
$$+B(1-\alpha^{\rho}_{23}, 1-\alpha^{\rho}_{13})], \qquad (3.10)$$

where α' is the slope of the ρ trajectory.

2. $\pi\omega\to\pi\omega$ scattering[17]

Here both ρ and B trajectories are allowed in $(\pi\omega)$ channels and f in $(\pi\pi)$ channel. Consider first the structure given by B in the (12) channel [see Fig. 1(a)]:

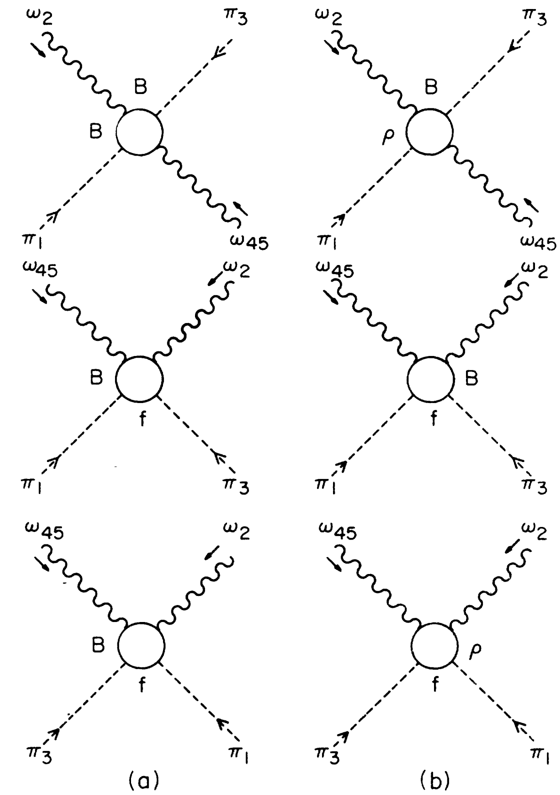

FIG. 1. (a) The three diagrams in $\pi\omega\to\pi\omega$ scattering which do not contain the ρ trajectory. (b) The two diagrams which contain the ρ trajectory in the $(\pi_1\omega_2)$ channel and the third one obtained by permutation of the external legs.

$$\mathfrak{M}^{B\,12}_{\pi\omega\to\pi\omega} = -i(g_S m_B)^2\omega\cdot\omega_{45} + g_S g_D p_1\cdot\omega p_1\cdot\omega_{45}$$
$$+g_S g_D p_3\cdot\omega p_3\cdot\omega_{45} + \left(\frac{g_D}{m_B}\right)^2 p_1\cdot p_3\, p_1\cdot\omega p_3\cdot\omega_{45} \qquad (3.11)$$

if we neglect daughterlike terms coming from the $(p_1+p_2)_\mu(p_1+p_2)_\nu$ part of the B propagator. According to the rules detailed in Sec. III E it appears that the terms of Eq. (3.11) contain a B in the $(\pi_3\omega_2)$ channel with both S- and D-wave couplings and an f in $(\pi_1\pi_3)$ with S wave only at the $f\omega\omega$ vertex.

The amplitude follows easily provided we symmetrize $\mathfrak{M}^{B\,12}$ under π_1 and π_3 interchange [see Fig. 1(a)]:

$$A^B_{\pi\omega\to\pi\omega} = \tfrac{1}{2}i\alpha'\omega_\mu\omega_{45\nu}\{(G_S{}^2 g^{\mu\nu} + G_S G_D p_1^\mu p_1^\nu + G_S G_D p_3^\mu p_3^\nu)[B(1-\alpha^B_{12}, 1-\alpha^B_{23}) + B(1-\alpha^B_{12}, 2-\alpha^f_{13}) + B(1-\alpha^B_{23}, 2-\alpha^f_{13})]$$
$$+G_D{}^2 p_1\cdot p_3(p_1^\mu p_3^\nu[B(1-\alpha^B_{12}, 3-\alpha^B_{23}) + B(1-\alpha^B_{12}, 2-\alpha^f_{13}) + B(3-\alpha^B_{23}, 2-\alpha^f_{13})]$$
$$+p_1^\nu p_3^\mu[B(3-\alpha^B_{12}, 1-\alpha^B_{23}) + B(3-\alpha^B_{12}, 2-\alpha^f_{13}) + B(1-\alpha^B_{23}, 2-\alpha^f_{13})])\}. \qquad (3.12)$$

The presence of all three permutations of external momenta gives signature in all channels. Here, G_S and G_D are not the $B\omega\pi$ coupling constants but just parameters since there is still the possibility that ρ exchange in one $(\pi\omega)$ channel contributes to B trajectory in the crossed one.

Consider now the Born term with ρ exchange in the $(\pi_1\omega_2)$ channel [Fig. 1(b)]:

$$\mathfrak{M}^{\rho\,12}_{\pi\omega\to\pi\omega}=ig_{\rho\omega\pi}{}^2\epsilon_{\ldots\mu}p_1^{\cdot}p_2^{\cdot}\,\omega^{\cdot}\epsilon^{\mu}\ldots p_3^{\cdot}p_{45}^{\cdot}\omega_{45}^{\cdot}\,. \tag{3.13}$$

The spin-parity content in the $(\pi_1\pi_3)$ channel is given by

$$p_1^{\rho}p_3^{\sigma}\epsilon_{\rho\ldots\mu}\,p_2^{\cdot}\,\omega^{\cdot}\,\epsilon^{\mu}{}_{\sigma\ldots}\,p_{45}^{\cdot}\omega_{45}^{\cdot}\,,$$

which is a 2^+ particle, i.e., an f. The $f\omega\omega$ coupling here is rather complicated and contains both S and D waves; since nothing is known experimentally about it we will not pursue it further.

To find out what the quantum number content is in the $(\pi_3\omega_2)$ channel, expand the covariant and get

$$\begin{aligned}\epsilon_{\ldots\mu}p_1^{\cdot}p_2^{\cdot}\,\omega^{\cdot}\epsilon^{\mu}\ldots p_3^{\cdot}p_{45}^{\cdot}\omega_{45}^{\cdot}=&-(p_2\cdot p_3\omega_\nu-p_3\cdot\omega p_{2\nu})\\&\times(p_1\cdot p_{45}\omega_{45}^{\nu}-p_1\cdot\omega_{45}p_{45}^{\nu})\\&+p_{3\nu}(p_{2\rho}\,\omega_\sigma-p_{2\sigma}\,\omega_\rho)p_1^{\nu}p_{45}^{\rho}\omega_{45}^{\sigma}\,.\end{aligned} \tag{3.14}$$

The first term is obviously a B particle with a well-defined D/S ratio at the $B\omega\pi$ vertex. The second one contains a piece, namely $\frac{1}{2}(p_2\cdot p_3\omega^\mu+p_3\cdot\omega p_3^\mu)(p_1\cdot p_{45}\omega_{45_\mu}+p_1\cdot\omega_{45}p_{1_\mu})$, of spin-parity 1^+ in the $(\pi_3\omega_2)$ channel. The remaining terms in Eq. (3.14) are of spin 2 and should be made nonleading in the amplitude since there is no known meson of quantum numbers $I^G=1^+$ and $J=2$ which couples to $(\omega\pi)$.

There are several possible ways to attach the covariants to the structure (3.14), the simplest one being

$$\begin{aligned}&\epsilon_{\ldots\mu}p_1^{\cdot}p_2^{\cdot}\,\omega^{\cdot}\epsilon^{\mu}\ldots p_3^{\cdot}p_{45}^{\cdot}\omega_{45}^{\cdot}B(1-\alpha_{12}^{\rho},3-\alpha_{23}^{B})\\&\quad-\tfrac{1}{2}(p_3\cdot p_2\omega^\mu+p_3\cdot\omega p_3^\mu)(p_1\cdot p_{45}\omega_{45_\mu}+p_1\cdot\omega_{45}p_{1_\mu})\\&\quad\times B(3-\alpha_{12}^{\rho},1-\alpha_{23}^{B})\,.\end{aligned} \tag{3.15}$$

It has all the required properties as far as the pole structure and the Regge behavior are concerned.

To simplify the writing we denote the term (3.15) by

$$\epsilon_{\ldots\mu}p_1^{\cdot}p_2^{\cdot}\,\omega^{\cdot}\epsilon^{\mu}\ldots p_3^{\cdot}p_{45}^{\cdot}\omega_{45}^{\cdot}B'(1-\alpha_{12}^{\rho},1-\alpha_{23}^{B})\,. \tag{3.16}$$

We emphasize that no B_4 function appears with both the ρ and the B poles. However, since the covariant $\epsilon_{\ldots\mu}p_1^{\cdot}p_2^{\cdot}\,\omega^{\cdot}\epsilon^{\mu}\ldots p_3^{\cdot}p_{45}^{\cdot}\omega_{45}^{\cdot}$ was seen to contain both ρ in the $(\pi_1\omega_2)$ channel and B in the $(\pi_3\omega_2)$ channel, we feel justified in using this condensed notation.

In conclusion, putting everything together, we find that the contribution of ρ exchange to $\pi\omega$ scattering is, after taking into account Bose symmetry,

$$\begin{aligned}A^{\rho}_{\pi\omega\to\pi\omega}=i\tfrac{1}{2}\alpha' g_{\rho\omega\pi}{}^2\{&\epsilon_{\ldots\mu}p_1^{\cdot}p_2^{\cdot}\omega^{\cdot}\epsilon^{\mu}\ldots p_3^{\cdot}p_{45}^{\cdot}\omega_{45}^{\cdot}[B'(1-\alpha_{12}^{\rho},1-\alpha_{23}^{B})+B(1-\alpha_{12}^{\rho},2-\alpha_{13}^{f})+B'(1-\alpha_{23}^{B},2-\alpha_{13}^{f})]\\&+\text{terms with }\pi_1\text{ and }\pi_3\text{ interchanged}\}.\end{aligned} \tag{3.17}$$

Although the term $B'(1-\alpha_{23}^{B},2-\alpha_{13}^{f})$ does not contain the ρ pole, it is necessary here to ensure that all the trajectories have the right signature; note that it multiplies only that part of the $\epsilon_{\ldots\mu}\epsilon^{\mu}\ldots$ covariant which has the B_{23} pole.

We can calculate now the residue of the full $\pi\omega$ scattering amplitude at the B pole in the (12) channel, for example, and comparing with Eq. (3.11) we find

$$\frac{g_D}{g_S}=\frac{m_B{}^2}{p_1\cdot p_2}\,, \tag{3.18}$$

$$G_S=[(g_S m_B)^2+\tfrac{1}{2}(p_1\cdot p_2)^2 g_{\rho\omega\pi}{}^2]^{1/2}\,,$$

$$\frac{G_D}{G_S}=\frac{1}{p_1\cdot p_2}\,, \tag{3.19}$$

where

$$p_1\cdot p_2=\tfrac{1}{2}(m_B{}^2-m_\omega{}^2-m_\pi{}^2)\,.$$

Note that the part of the amplitude where the B is dual to the ρ has a ghostlike contribution to the B pole (square of the coupling constants being negative). However, the over-all amplitude is ghost-free. Also it is the presence of the ρ in the dual channel which forces a well-defined value for g_D/g_S which corresponds to a fixed ratio A_D/A_S of D-wave/S-wave. We can then determine the decay parameters of the B meson:

$$\frac{g_D}{g_S}=\frac{2m_B{}^2}{m_B{}^2-m_\omega{}^2-m_\pi{}^2}=3.42\,, \tag{3.20}$$

$$\left|\frac{A_D}{A_S}\right|^2=0.032,\quad |F_0|^2=0.18\,. \tag{3.21}$$

A_S and A_D are the S-wave and D-wave amplitudes of the decay $B\to\omega\pi$, whereas F_0 and F_1 are the helicity amplitudes normalized to 1, i.e., $|F_0|^2+2|F_1|^2=1$.

These results compare rather well with those of a recent bootstrap calculation[20] as well as with the experimental data.[9] However, they are in disagreement with the predictions of the quark model.[21]

G. Application: $\pi N \to \pi\omega N$ scattering

The amplitude will be constructed as a sum over resonances in the $(\pi_1\omega_2)$ and $(N_4\overline{N}_5)$ channels. Following the discussion of Sec. III A, we consider only the six graphs where the N_4 and $\overline{N}_5$ momenta are adjacent (Fig. 2). Some of the other permutations which can become important in certain kinematical regions [e.g., backward scattering or (πN) subenergy in the resonance region] will be considered later. To get the contribution of π or ω exchange in the annihilation channel we just replace π_{45} or ω_{45} in the expressions derived previously by the corresponding Feynman propagators multiplied by the wave functions of N_4 and $\overline{N}_5$ and the beta functions by B_5's, taking into account all six permutations we kept. Thus Eq. (3.10) will be multiplied by $\overline{V}_5 i\gamma_5 u_4$, whereas ω_{45} in Eq. (3.12) and Eq. (3.17) will be replaced by

$$\bar{v}_5\left[(G_V^\omega - G_T^\omega)\gamma^\mu + \frac{G_T^\omega}{2m_p}(p_4 - p_5)^\mu\right]u_4 \,.$$

The selection between Δ and N^* trajectories will be determined by isospin and coupling constant requirements.

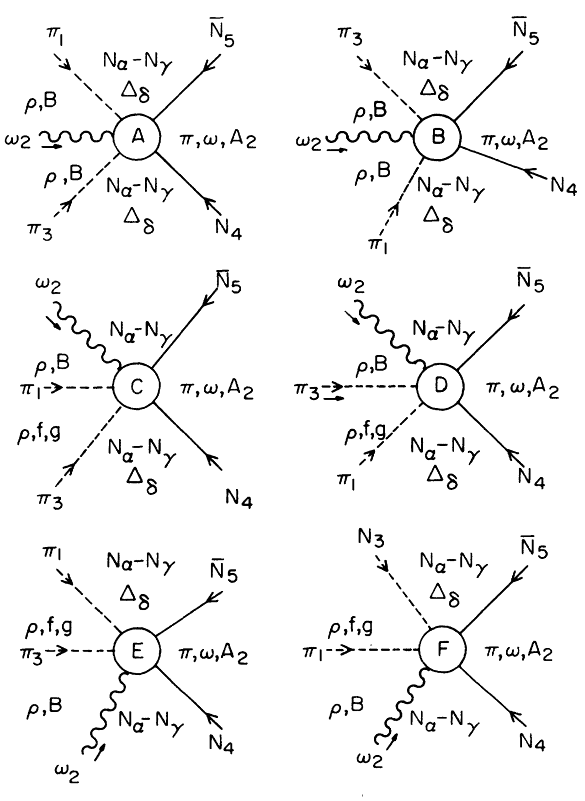

FIG. 2. The six diagrams, with adjacent nucleon legs, used in the construction of Eqs. (3.23) and (3.24). The various trajectories used in the model are also shown.

The relative coefficients of the B_5's associated with the same covariant will be ± 1, the sign being determined by Bose symmetry and the signature of the meson trajectories (the baryon trajectories are exchange-degenerate at present because of the graphs we left out). As an example,

$$[B_5(l-\alpha_{12}, m-\alpha_{23}, n-\alpha_{34}, p-\alpha_{45}, q-\alpha_{51}) \pm B_5(1 \leftrightarrow 2 \text{ interchanged})] \quad (3.22)$$

gives in the (12) channel a trajectory starting at $\alpha_{12}=1$ (or $\alpha_{12}=l+1$) with signature $(-1)^l$ (or $(-1)^{l+1}$). See Table I for the signature of meson trajectories in terms having ω exchange in $(N_4\overline{N}_5)$ channel. Recall finally that ω exchange terms will contribute to the $I=0$ amplitude A^+. A typical term in A^+ will be

$$(G_V^\omega - G_T^\omega)\bar{v}_5\epsilon\ldots{}_\mu p_1^{\boldsymbol\cdot} p_2^{\boldsymbol\cdot}\omega^{\boldsymbol\cdot}\epsilon^\mu\ldots p_3^{\boldsymbol\cdot}(p_4+p_5)^{\boldsymbol\cdot} \times \gamma^{\boldsymbol\cdot} u_4(A+B+C+D+E+F)\,,$$

where $A, B, \ldots$ refer to the graphs of Fig. 2.

It is easy now to introduce the contribution of the A_2 trajectory in the amplitude A^-. This amplitude is antisymmetric in the interchange of π_1 and π_3; therefore the combinations $(A\text{-}B)$, $(C\text{-}D)$, and $(E\text{-}F)$ should appear as the coefficients of the same kinematical structure. Care should also be taken to include the signature of the A_2 trajectory; this can be achieved in the following way: Start from the ω exchange amplitude in A^+ and change the relative signs of the invariants to get the pairs $(A\text{-}B)$, $(C\text{-}F)$, and $(D\text{-}E)$. According to Eq. (3.22) this will be a trajectory of positive signature starting at spin 2, which is exactly what we want. Next we fix the sign of $(A\text{-}B)$ with respect to the other permutations; this is done by making sure that the (12) trajectories, i.e., ρ and B, have negative signature and this requires the pair $(A+C)$. It follows that all other meson trajectories have the right signature properties as shown in Table II.

TABLE I. The symbols A, B, ... refer to the graphs of Fig. 2 and to the corresponding invariants in Eq. (3.23). This table shows what combinations of invariants, containing the argument $(1-\alpha_{45}^\omega)$, are necessary to give the proper signature factors to the various meson trajectories.

Channel	$(\pi_1\omega_2)$	$(\pi_3\omega_2)$	$(\pi_1\pi_3)$	$(N_4\overline{N}_5)$
Invariants	$A+C$	$A+E$	$C+D$	$A+B$
Invariants	$B+F$	$B+D$	$E+F$	$C+F$
Invariants				$D+E$
Trajectories	ρ, B	ρ, B	f	ω

TABLE II. This is the same as Table I for the terms of Eq. (3.24) which contain invariants with the argument $(1-\alpha_{45}^{A_2})$.

Channel	$(\pi_1\omega_2)$	$(\pi_3\omega_2)$	$(\pi_1\pi_3)$	$(N_4\bar{N}_5)$
Invariants	$A+C$	$A+E$	$C-D$	$A-B$
Invariants	$-B-F$	$-B-D$	$E-F$	$C-F$
Invariants				$-D+E$
Trajectories	ρ, B	ρ, B	g	A_2

This construction yields, of course, an A_2 trajectory strongly exchange-degenerate with that of the ω (i.e., identical trajectories and related coupling constants). This is characteristic of dual models where ω and A_2 appear together in channels with an odd number of pions.

This method, however, will only partially give the coupling $A_2N\bar{N}$. To get the full A_2 contribution we have to add other terms so that when we go to the A_2 pole the propagator has the right symmetry property.

The explicit form of the amplitudes A^+ and A^- is shown in Appendix A. Each amplitude can be decomposed as a sum of terms. Thus we write

$$A^+ = A_\rho^+ + A_B^+ , \tag{3.23}$$

where A_ρ^+ and A_B^+ reduce, respectively, to $A^\rho_{\pi\omega\to\pi\omega}$ [see Eq. (3.17)] and $A^B_{\pi\omega\to\pi\omega}$ [see Eq. (3.12)], when the residue is taken at the ω pole in the channel $(N_4\bar{N}_5)$. Likewise A^- is written as

$$A^- = A_\rho^- + A_B^- + A_\pi^- , \tag{3.24}$$

where A_ρ^- and A_B^- are obtained from A_ρ^+ and A_B^+ as explained above (and therefore contain the A_2 trajectory in the nucleon-antinucleon channel); A_π^- contains the π trajectory in the channel $(N_4\bar{N}_5)$ and reduces to Eq. (3.10) by taking the residue at the π pole.

The choice of the baryon trajectories is explained in the following subsection.

H. Isospin constraints and baryon trajectories

The relative phase of the amplitudes A^+ and A^- is chosen so that

$$A^{3/2} = A^+ - A^- ,$$
$$A^{1/2} = A^+ + 2A^- \tag{3.25}$$

represent the isospin-$\frac{3}{2}$ and -$\frac{1}{2}$ amplitudes, respectively, for the (34) [equivalently (51)] channel. The isospin constraints in those channels are as follows: No Δ trajectory should appear in $A^{1/2}$ and no N-N^* should be present in $A^{3/2}$. We also want to calculate the residues at the N and Δ poles; however, in this kinematical region the graphs which have been neglected up to now can become important. Indeed, the arguments which were used to disregard the "baryon-exchange" graphs were valid as long as the subenergies in (34) and (51) channels were large (Regge region), which is not the case at the baryon poles of interest.

Since our main interest is in the determination of the coupling constants, the effect of the baryon-exchange graphs can be analyzed in the simpler case when one of the $(\pi\omega)$ channels is on the mass shell. Consider, first, the case when $\alpha_{12}^\rho = 1$ and construct the amplitude for $\pi N \to \rho N$. It will be obtained by taking the residue in the amplitudes A^+ and A^- and adding the covariant functions corresponding to graphs where the nucleon lines are separated by the pion line (see Fig. 3) which also defines the notations. The most general form using only terms leading in all channels is

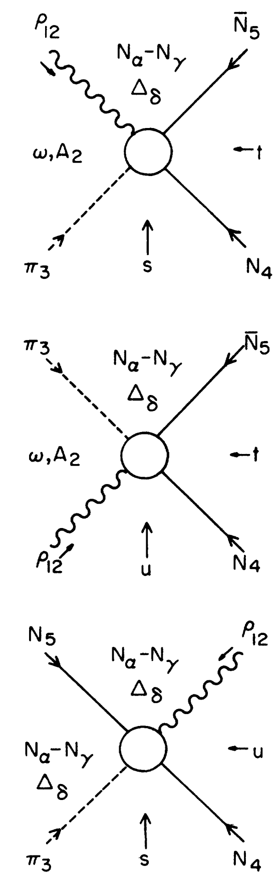

FIG. 3. The three diagrams in $\pi N \to \rho N$ scattering and the definition of the s, t, and u channels.

$$A^+ = -\tfrac{1}{2}\alpha'\bar{v}_5\Big\{g_{\rho\omega\pi}(G_V^\omega - G_T^\omega)\epsilon\ldots p_3^\cdot \rho_{12}^\cdot p_\rho^\cdot \gamma^\cdot\,[a_\Delta(\mathcal{C}(1-\alpha_t^\omega, \tfrac{3}{2}-\alpha_s^\Delta) + \mathcal{C}(1-\alpha_t^\omega, \tfrac{3}{2}-\alpha_u^\Delta)) + a_N(\mathcal{C}(1-\alpha_t^\omega, \tfrac{3}{2}-\alpha_s^N) + \mathcal{C}(1-\alpha_t^\omega, \tfrac{3}{2}-\alpha_u^N)) + b_\Delta\mathcal{C}(\tfrac{3}{2}-\alpha_s^\Delta, \tfrac{3}{2}-\alpha_u^\Delta) + b_N\mathcal{C}(\tfrac{3}{2}-\alpha_s^N, \tfrac{3}{2}-\alpha_u^N) + c\,\mathcal{C}(\tfrac{3}{2}-\alpha_s^N, \tfrac{3}{2}-\alpha_u^\Delta) + d\,\mathcal{C}(\tfrac{3}{2}-\alpha_s^\Delta, \tfrac{3}{2}-\alpha_u^N)]$$
$$+ g_{\rho\omega\pi}\frac{G_T^\omega}{2m_p}\epsilon\ldots p_3^\cdot \rho_{12}^\cdot p_\rho^\cdot (p_4-p_5)^\cdot\,[a'_\Delta(\mathcal{C}(1-\alpha_t^\omega, \tfrac{3}{2}-\alpha_s^\Delta) + \mathcal{C}(1-\alpha_t^\omega, \tfrac{3}{2}-\alpha_u^\Delta)) + \cdots]$$
$$+ g_{\rho A_2\pi}\frac{G_1^{A_2}}{m_p}\epsilon\ldots p_3^\cdot \rho_{12}^\cdot p_\rho^\cdot (p_4-p_5)^\cdot \not{p}_3[a''_\Delta(\mathcal{C}(2-\alpha_t^\omega, \tfrac{3}{2}-\alpha_s^\Delta) - \mathcal{C}(2-\alpha_t^\omega, \tfrac{3}{2}-\alpha_u^\Delta)) + \cdots]\Big\}u_4 \quad (3.26)$$

(and a similar equation for A^-), where $\rho_{12}^\mu = \epsilon^\mu\ldots p_1^\cdot p_2^\cdot \omega^\cdot$.

The π-exchange term does not have leading contribution in N and/or Δ in baryon channels because, otherwise, the isospin constraints in Eq. (3.26) cannot be satisfied. Hence, it does not appear in Eq. (3.26), where only terms leading in all channels are kept. The parameters $a_\Delta, \ldots$ are determined by requiring that no Δ appear in the s channel in $A^{1/2}$ and no N-N^* in $A^{3/2}$, that the Δ trajectory be signatured, and that the correct residues be obtained at the lowest poles in all channels.

Among the latter type of constraints, an important one is that imposed by the special form of the $\rho\Delta\bar{N}$ vertex. The most general vector meson-nucleon-Δ coupling is given by

$$\rho^\mu\bar{v}_5 i\gamma_5\left[G_1 g_{\mu\nu} + G_2\frac{\gamma_\mu p_{5\nu}}{m_\Delta + m_p} + G_3\frac{p_{5\mu}p_{5\nu}}{(m_\Delta+m_p)^2}\right]\Delta^\nu , \quad (3.27)$$

but since the $\rho\Delta\bar{N}$ coupling is mainly magnetic ($M1$) the choice $G_1/G_2 = -1$ and $G_3 = 0$ is necessary. To calculate now the contribution of the amplitude to the $\rho_{12}\Delta_{34}\bar{N}_5$ vertex, we just replace, as suggested in Sec. III D, the term $p_3^\mu u_4$ by the Δ wave function in all three covariants of Eq. (3.26). Then, making use of Eq. (2.6) we find that only the first kinematical structure has the desired form for a $\rho\Delta\bar{N}$ coupling. Since the three covariants are independent, terms leading in Δ in the last two of them can appear neither in $A^{1/2}$ nor in $A^{3/2}$. It follows, then, that only $\bar{v}_5\epsilon\ldots p_3^\cdot p_{12}^\cdot p_\rho^\cdot\gamma^\cdot u_4$ contains the Δ trajectory. If we assume, for simplicity, that it does not contain the N-N^* trajectory, we get the following set of relations:

$$g_{\rho A_2\pi}\frac{G_2^{A_2}}{m_p^2} = \alpha' g_{\rho\omega\pi}\frac{G_T^\omega}{2m_p},$$
$$\frac{G_1^{A_2}}{G_2^{A_2}} = -\frac{G_V^\omega - G_T^\omega}{G_T^\omega}, \quad (3.28)$$
$$G_{\rho p\Delta^{++}} = \tfrac{3}{2}g_{\rho\omega\pi}\frac{G_V^\omega - G_T^\omega}{G_{\pi p\Delta^{++}}}m_\pi .$$

Since the B-meson couplings are related to those of ρ by the mechanism described in Sec. III F, its coupling constants to the Δ pole are completely determined. We find

$$G_P^{BA_2\pi}\frac{G_1^{A_2}}{m_p} = -\tfrac{1}{2}\alpha' G_S(G_V^\omega - G_T^\omega),$$
$$\frac{G_P^{BA_2\pi}}{G_L^{BA_2\pi}} = \frac{G_S}{G_D},$$
$$\frac{G_L^{BA_2\pi}}{G_D} = \frac{g_{\rho A_2\pi}}{g_{\rho\omega\pi}}, \quad (3.29)$$
$$G^1_{Bp\Delta^{++}} = \frac{3}{2}\frac{g_D}{m_B}\frac{G_V^\omega - G_T^\omega}{G_{\pi p\Delta^{++}}}\frac{m_\Delta - m_p}{m_\Delta + m_p}m_\pi ,$$
$$\frac{G^2_{Bp\Delta^{++}}}{G^1_{Bp\Delta^{++}}} = -\frac{1}{4}\frac{g_{\rho\omega\pi}{}^2(m_B{}^2 - m_\omega{}^2 - m_\pi{}^2)}{g_S g_D}\frac{m_\Delta + m_p}{m_\Delta - m_p},$$
$$G^3_{Bp\Delta^{++}} = 0 .$$

Until now we have always avoided talking about $\rho N\bar{N}$ and $BN\bar{N}$ coupling. This is because most of the terms which contribute to the N-N^* trajectory start at $J=\frac{3}{2}$. However, the model can be refined to include the nucleon pole. Look at the ρ-exchange graph. The expansion of the $\epsilon\ldots$ covariants, using Eq. (2.6), contains terms with spin-parity $\frac{1}{2}^+$ in the baryon channels. Indeed, we find

$$\bar{v}_5\epsilon\ldots\rho^\cdot p_3^\cdot p_5^\cdot p_4^\cdot u_4 = \rho^\mu\bar{v}_5[(m_p{}^2 - p_5\cdot(p_3+p_4))\gamma_\mu - m_p(p_4-p_5)_\mu] \times i\gamma_5\not{p}_3 u_4 \quad (3.30)$$

plus other terms of spin $\frac{3}{2}$. The terms explicitly written out correspond to a $\frac{1}{2}^+$ particle in the (34) channel coupled to a ρ meson in the (12) channel and the arguments of the B_5 functions attached to them will be $\frac{1}{2}-\alpha_{34}^N$, the other covariant we have

$$\bar{v}_5\epsilon\ldots\rho^\cdot p_3^\cdot p_5^\cdot p_4^\cdot\not{p}_3 u_4 = \rho^\mu\bar{v}_5[-p_5\cdot(p_3+p_4)\gamma_\mu - \tfrac{1}{2}m_p(p_4-p_5)_\mu] \times i\gamma_5\not{p}_3 u_4 \quad (3.31)$$

plus other terms of spin $\frac{3}{2}$. Putting everything together we obtain the $\rho N\bar{N}$ coupling in this model:

$$G_T^\rho = \tfrac{1}{2}\frac{g_{\rho\omega\pi}}{G_{\pi NN}} m_p[\alpha' m_p{}^2(G_V^\omega - G_T^\omega) - 2G_T^\omega] \,,$$

$$\frac{G_V^\rho}{G_T^\rho} = \frac{m_\rho{}^2}{2m_p{}^2}\left[1+\frac{1}{\alpha' m_p{}^2[(G_V^\omega - G_T^\omega)/G_T^\omega]-2}\right] \qquad (3.32)$$

at the ρ pole.

It should be emphasized that the values "predicted" above are not the unique solution in this model but they seem to give reasonable agreement with the data.

Doing the same for the $BN\bar{N}$ coupling, we find

$$G_V^B - G_T^B = 0 \text{ as required by } G \text{ parity},$$

$$G_T^B = \alpha' m_p \frac{G_S{}^2}{g_S m_B}\,\frac{G_V^\omega - G_T^\omega}{2G_{\pi NN}} \,. \qquad (3.33)$$

I. Summary of the results

How well are the properties defined in Sec. III B satisfied? The amplitude can certainly be continued to any kinematical region. Also, Bose symmetry has been explicitly put in. We are able to obtain the double-pole residues pictured in Figs. 4(a)–4(h); only the ρ-A_2 exchange graph shown in Fig. 4(i) did not fit in our model. One feature, however, is that the π-exchange term is not leading in the baryon channels; this is so because otherwise we could not enforce the isospin constraints. While this is unfortunate in light of requirement (6) it is not too bad experimentally since a measure of the strength of this graph compared with the other ones is given by the ratio $g_{\rho\pi\pi}/g_{\rho\omega\pi} \sim 0.3$. Whether the requirements of "maximal duality" and "minimum number of B_5's" are verified is largely a matter of taste. Using various constraints we can obtain relations between the coupling constants introduced in the model. The independent parameters are chosen to be

$$g_{\rho\pi\pi},\ g_{\rho\omega\pi},\ g_S,\ G_{\pi NN},\ G_{\pi N\Delta},\ G_V^\omega,\ G_T^\omega, G_1^{A_2}, \alpha',$$

but all of them except G_T^ω, G_T^ω, $G_1^{A_2}$ are fairly well determined from other theories or experiments and $G_1^{A_2}$ will always enter in the model in known combinations with other coupling constants [see Eq. (3.28)]. We are left with G_V^ω and G_T^ω only.

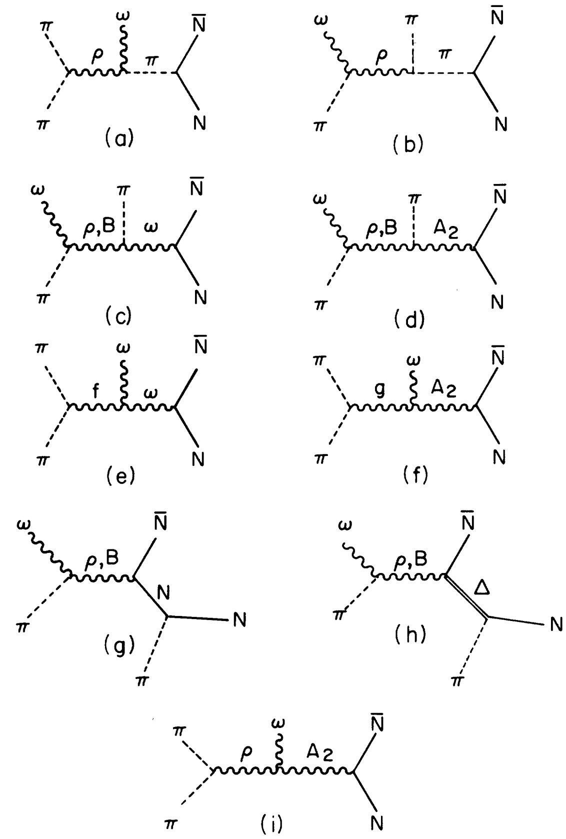

FIG. 4. (a)–(h) The double-pole graphs included in the model. (i) The double-pole graph not included in the model.

In an accompanying paper, this model is compared with the data for the quasi-two-body reactions which can be extracted from the reaction $\pi N \to \pi\omega N$. Fits to the differential cross sections and the density matrix elements are presented.

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I wish to thank Professor L. M. Jones for her patient guidance and constant encouragement during the course of this work.

APPENDIX A: THE AMPLITUDE FOR THE REACTION $\pi N \to \pi\omega N$

In the following,

$$\epsilon_{\ldots\mu} p_1^{\cdot} p_2^{\cdot}\, \omega^{\cdot}\, \epsilon^{\mu}\ldots p_3^{\cdot}(p_4+p_5)^{\cdot}\gamma^{\cdot}\, \mathcal{C}_A'(1-\alpha_{12}^\rho, 1-\alpha_{23}^\rho, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{51}^\Delta)$$

is a shorthand notation for

$$\epsilon_{\ldots\mu} p_1^{\cdot} p_2^{\cdot}\, \omega^{\cdot}\, \epsilon^{\mu}\ldots p_3^{\cdot}(p_4+p_5)^{\cdot}\gamma^{\cdot}\, \mathcal{C}_A(1-\alpha_{12}^\rho, 3-\alpha_{23}^B, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{51}^\Delta)$$
$$-\tfrac{1}{2}(p_3\cdot p_2\omega^\mu + p_3\cdot\omega p_3^\mu)[p_1\cdot(p_4+p_5)\gamma_\mu + p_{1\mu}\not{p}_1]\,\mathcal{C}_A(3-\alpha_{12}^\rho, 1-\alpha_{23}^B, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{51}^\Delta)$$

which follows from Eq. (3.15). When instead of $1-\alpha_{12}^\rho$ in $\mathcal{C}'$ there is $3-\alpha_{12}^\rho$ we just drop the first term. We have the following equations:

$$A_\rho^+ = \alpha'^2 \frac{g_{\rho\omega\pi}}{4}\epsilon_{\dots\mu} p_1^\cdot p_2^\cdot \omega^\cdot \bar{v}_5$$

$$\times\Big\{ g_{\rho\omega\pi}(G_V^\omega - G_T^\omega)\epsilon^\mu{}_{\dots} p_3^\cdot (p_4+p_5)^\cdot \gamma^\cdot$$

$$\times[\mathcal{C}'_A(1-\alpha_{12}^\rho, 1-\alpha_{23}^B, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{51}^\Delta) + \mathcal{C}'_B(1-\alpha_{23}^B, 1-\alpha_{12}^\rho, \tfrac{3}{2}-\alpha_{41}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{53}^\Delta)$$

$$+\mathcal{C}_C(1-\alpha_{12}^\rho, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{52}^N) + \mathcal{C}_D(1-\alpha_{23}^B, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{41}^\Delta, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{52}^N)$$

$$+\mathcal{C}'_E(1-\alpha_{23}^B, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{51}^\Delta, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{42}^N) + \mathcal{C}_F(1-\alpha_{12}^\rho, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{53}^\Delta, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{42}^N)]$$

$$+ g_{\rho\omega\pi}\frac{G_T^\omega}{m_p}\epsilon^\mu{}_{\dots} p_3^\cdot p_5^\cdot p_4^\cdot$$

$$\times[\mathcal{C}'_A(1-\alpha_{12}^\rho, 1-\alpha_{23}^B, \tfrac{3}{2}-\alpha_{34}^N, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{51}^N) + \mathcal{C}'_B(1-\alpha_{23}^B, 1-\alpha_{12}^\rho, \tfrac{5}{2}-\alpha_{41}^N, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{53}^N)$$

$$+\mathcal{C}_C(1-\alpha_{12}^\rho, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{34}^N, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{52}^N) + \mathcal{C}_D(1-\alpha_{23}^B, 2-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{41}^N, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{52}^N)$$

$$+\mathcal{C}'_E(1-\alpha_{23}^B, 2-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{51}^N, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{42}^N) + \mathcal{C}_F(1-\alpha_{12}^\rho, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{53}^N, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{42}^N)]$$

$$+2g_{\rho A_2\pi}\frac{G_1^{A_2}}{m_p}\epsilon^\mu{}_{\dots} p_3^\cdot p_5^\cdot p_4^\cdot$$

$$\times\{\not{p}_3[\mathcal{C}_A(1-\alpha_{12}^\rho, 3-\alpha_{23}^B, \tfrac{3}{2}-\alpha_{34}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{51}^N) - \mathcal{C}_B(3-\alpha_{23}^B, 1-\alpha_{12}^\rho, \tfrac{5}{2}-\alpha_{41}^N, 2-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{53}^N)$$

$$+\mathcal{C}_C(1-\alpha_{12}^\rho, 3-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{34}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{52}^N) - \mathcal{C}_D(3-\alpha_{23}^B, 3-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{41}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{52}^N)$$

$$+\mathcal{C}_E(3-\alpha_{23}^B, 3-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{51}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{41}^N) - \mathcal{C}_F(1-\alpha_{12}^\rho, 3-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{53}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{42}^N)]$$

$$-\not{p}_1[\mathcal{C}_A(3-\alpha_{12}^\rho, 1-\alpha_{23}^B, \tfrac{3}{2}-\alpha_{34}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{41}^N) - \mathcal{C}_B(1-\alpha_{23}^B, 3-\alpha_{12}^\rho, \tfrac{5}{2}-\alpha_{41}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{53}^N)$$

$$+\mathcal{C}_C(3-\alpha_{12}^\rho, 3-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{34}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{52}^N) - \mathcal{C}_D(1-\alpha_{23}^B, 3-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{41}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{52}^N)$$

$$+\mathcal{C}_E(1-\alpha_{23}^B, 3-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{51}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{41}^N) - \mathcal{C}_F(3-\alpha_{12}^\rho, 3-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{53}^N, 2-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{42}^N)]\}\Big\} u_4$$

+terms with π_1 and π_3 interchanged,

$$A_B^+ = \alpha'^2 \frac{\bar{v}_5}{4}$$

$$\times\Big\{(G_V^\omega - G_T^\omega)(G_S{}^2\omega^\mu + G_S G_D p_1\cdot\omega p_1^\mu + G_D G_S p_3\cdot\omega p_3^\mu)\gamma_\mu$$

$$\times[\mathcal{C}_A(1-\alpha_{12}^B, 1-\alpha_{23}^B, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{51}^\Delta) + \mathcal{C}_C(1-\alpha_{12}^B, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{52}^N)$$

$$+\mathcal{C}_E(1-\alpha_{23}^B, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{51}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{42}^N) + \pi_1 \leftrightarrow \pi_3]$$

$$+(G_V^\omega - G_T^\omega)G_D{}^2 p_1\cdot p_3\gamma_\mu$$

$$\times\{p_1\cdot\omega p_3^\mu[\mathcal{C}_A(1-\alpha_{12}^B, 3-\alpha_{23}^B, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{51}^\Delta) + \mathcal{C}_B(3-\alpha_{23}^B, 2-\alpha_{12}^B, \tfrac{5}{2}-\alpha_{41}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{53}^\Delta)$$

$$+\mathcal{C}_C(1-\alpha_{12}^B, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{34}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{52}^N) + \mathcal{C}_D(3-\alpha_{23}^B, 2-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{41}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{52}^N)$$

$$+\mathcal{C}_E(3-\alpha_{23}^B, 2-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{51}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{42}^\Delta) + \mathcal{C}_F(1-\alpha_{12}^B, 2-\alpha_{13}^f, \tfrac{3}{2}-\alpha_{53}^\Delta, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{42}^N)] + \pi_1 \leftrightarrow \pi_3\}$$

$$+\frac{G_T^\omega}{2m_p}(G_S{}^2\omega^\mu + G_S G_D p_1\cdot\omega p_1^\mu + G_D G_S p_3\cdot\omega p_3^\mu)(p_4-p_5)_\mu$$

$$\times[\mathcal{C}_A(1-\alpha_{12}^B, 1-\alpha_{23}^B, \tfrac{5}{2}-\alpha_{34}^N, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{51}^N) + \mathcal{C}_C(1-\alpha_{12}^B, 2-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{41}^N, 1-\alpha_{45}^\omega, \tfrac{5}{2}-\alpha_{52}^N)$$

$$+\mathcal{C}_E(1-\alpha_{23}^B, 2-\alpha_{13}^f, \tfrac{5}{2}-\alpha_{51}^N, 1-\alpha_{45}^\omega, \tfrac{3}{2}-\alpha_{42}^N) + \pi_1 \leftrightarrow \pi_3]$$

$$+\frac{G_T^\omega}{2m_p}G_D{}^2p_1\cdot p_3(p_4-p_5)_\mu$$

$$\times\{p_1\cdot\omega p_3^\mu[\mathcal{C}_A(1-\alpha_{12}^B,3-\alpha_{23}^B,\tfrac{5}{2}-\alpha_{34}^N,1-\alpha_{45}^\omega,\tfrac{7}{2}-\alpha_{51}^N)+\mathcal{C}_B(3-\alpha_{23}^B,1-\alpha_{12}^B,\tfrac{7}{2}-\alpha_{41}^N,1-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{53}^N)$$

$$+\mathcal{C}_C(1-\alpha_{12}^B,2-\alpha_{13}^f,\tfrac{5}{2}-\alpha_{34}^N,1-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{52}^N)+\mathcal{C}_D(2-\alpha_{23}^B,2-\alpha_{13}^f,\tfrac{7}{2}-\alpha_{41}^N,1-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{52}^N)$$

$$+\mathcal{C}_E(3-\alpha_{23}^B,2-\alpha_{13}^f,\tfrac{7}{2}-\alpha_{51}^N,1-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{42}^N)+\mathcal{C}_F(1-\alpha_{12}^B,2-\alpha_{13}^f,\tfrac{5}{2}-\alpha_{53}^N,1-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{42}^N)]$$

$$+\pi_1\leftrightarrow\pi_3\}$$

$$-2\Big[\frac{G_1^{A_2}}{m_p}(G_S\omega^\mu+G_Dp_1\cdot\omega p_1^\mu)(G_P^{BA_2\pi}\not{p}_3\{p_{5\mu}[\mathcal{C}_A(1-\alpha_{12}^B,3-\alpha_{23}^B,\tfrac{1}{2}-\alpha_{34}^N,2-\alpha_{45}^\omega,\tfrac{3}{2}-\alpha_{51}^N)$$

$$+\mathcal{C}_C(1-\alpha_{12}^B,3-\alpha_{13}^f,\tfrac{1}{2}-\alpha_{34}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{52}^N)$$

$$-\mathcal{C}_D(3-\alpha_{23}^B,3-\alpha_{13}^f,\tfrac{5}{2}-\alpha_{41}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{52}^N)]$$

$$+p_{4\mu}[\mathcal{C}_B(3-\alpha_{23}^B,1-\alpha_{12}^B,\tfrac{3}{2}-\alpha_{41}^N,2-\alpha_{45}^\omega,\tfrac{1}{2}-\alpha_{53}^N)$$

$$+\mathcal{C}_F(3-\alpha_{13}^f,1-\alpha_{12}^B,\tfrac{1}{2}-\alpha_{53}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{42}^N)$$

$$-\mathcal{C}_E(3-\alpha_{23}^B,2-\alpha_{13}^f,\tfrac{5}{2}-\alpha_{51}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{42}^N)]\}$$

$$+G_L^{BA_2\pi}\not{p}_3p_{3\mu}\{p_3\cdot p_5[\mathcal{C}_A(1-\alpha_{12}^B,5-\alpha_{23}^B,\tfrac{5}{2}-\alpha_{34}^N,2-\alpha_{45}^\omega,\tfrac{7}{2}-\alpha_{51}^N)$$

$$+\mathcal{C}_C(1-\alpha_{12}^B,3-\alpha_{13}^f,\tfrac{5}{2}-\alpha_{34}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{52}^N)$$

$$-\mathcal{C}_D(5-\alpha_{23}^B,3-\alpha_{13}^f,\tfrac{5}{2}-\alpha_{41}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{52}^N)]$$

$$+p_3\cdot p_4[\mathcal{C}_B(5-\alpha_{23}^B,1-\alpha_{12}^B,\tfrac{7}{2}-\alpha_{41}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{53}^N)$$

$$+\mathcal{C}_F(3-\alpha_{13}^f,1-\alpha_{12}^B,\tfrac{5}{2}-\alpha_{53}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{42}^N)$$

$$-\mathcal{C}_E(5-\alpha_{23}^B,3-\alpha_{13}^f,\tfrac{5}{2}-\alpha_{51}^N,2-\alpha_{45}^\omega,\tfrac{5}{2}-\alpha_{42}^N)]\})$$

$$+\text{terms with }\pi_1\text{ and }\pi_3\text{ interchanged}\Big]\Big\}u_4,$$

$$A_p^-=\alpha'^2\bar{v}_5\Big\{\frac{g_{\rho\omega\pi}}{4}\Big[\epsilon_{\cdots\mu}p_1^\cdot p_2^\cdot\omega^\cdot\Big(g_{\rho A_2\pi}\frac{G_1^{A_2}}{\alpha' m_p}\epsilon^\mu\ldots p_3^\cdot(p_4+p_5)^\cdot\gamma^\cdot(A-B+C-D+E-F)$$

$$+g_{\rho A_2\pi}\frac{2G_2^{A_2}}{\alpha' m_p{}^2}\epsilon^\mu\ldots p_3^\cdot p_5^\cdot p_4^\cdot(A-B+C-D+E-F)$$

$$+g_{\rho A_2\pi}\frac{2G_1^{A_2}}{m_p}\epsilon^\mu\ldots p_3^\cdot p_5^\cdot p_4^\cdot[\not{p}_3(A+B+C+D+E+F)-\not{p}_1(A+B+C+D+E+F)]\Big)$$

$$+\pi_1\leftrightarrow\pi_3\Big]\Big\}u_4,$$

$$A_B^-=\alpha'^2\frac{\bar{v}_5}{4}\Big\{\frac{G_1^{A_2}}{\alpha' m_p}(G_SG_P^{BA_2\pi}\omega^\mu+G_DG_P^{BA_2\pi}p_1\cdot\omega p_1^\mu+G_SG_L^{BA_2\pi}p_3\cdot\omega p_3^\mu)\gamma_\mu(A-B+C-D+E-F)$$

$$+\frac{G_1^{A_2}}{\alpha' m_p}G_DG_L^{BA_2\pi}p_1\cdot p_3\gamma^\mu[p_1\cdot\omega p_{3_\mu}(A-B+C-D+E-F)+\pi_1\leftrightarrow\pi_3]$$

$$+\frac{G_2^{A_2}}{\alpha' m_p{}^2}(G_SG_P^{BA_2\pi}\omega^\mu+G_DG_P^{BA_2\pi}p_1\cdot\omega p_1^\mu+G_SG_L^{BA_2\pi}p_3\cdot\omega p_3^\mu)(p_4-p_5)_\mu(A-B+C-D+E-F)$$

$$+\frac{G_2^{A_2}}{\alpha' m_p{}^2}G_DG_L^{BA_2\pi}p_1\cdot p_3(p_4-p_5)^\mu[p_1\cdot\omega p_{3_\mu}(A-B+C-D+E-F)+\pi_1\leftrightarrow\pi_3]$$

$$+\frac{2G_1^{A_2}}{m_p}((G_S\omega^\mu+G_Dp_1\cdot\omega p_1^\mu)\{G_P^{BA_2\pi}\not{p}_3[p_{5_\mu}(A+C+D)-p_{4_\mu}(B+E+F)]\}$$

$$+G_L^{BA_2\pi}p_{3_\mu}\not{p}_3[p_3\cdot p_5(A+C+D)-p_3\cdot p_4(B+E+F)]-\pi_1\leftrightarrow\pi_3)\Big\}u_4,$$

$$A_\pi^- = \alpha'^2 \tfrac{1}{2} g_{\rho\pi\pi} g_{\rho\omega\pi} G_{\pi NN} \bar{v}_5 \{\epsilon \ldots . p_1^{\cdot} p_2^{\cdot} p_3^{\cdot} \omega^{\cdot} i \gamma_5 [\mathcal{C}_A(1-\alpha_{12}^\rho, 1-\alpha_{23}^\rho, \tfrac{5}{2}-\alpha_{34}, -\alpha_{45}^\pi, \tfrac{5}{2}-\alpha_{51}) + \mathcal{C}_C(1-\alpha_{12}^\rho, 1-\alpha_{13}^\rho, \tfrac{5}{2}-\alpha_{34}, -\alpha_{45}^\pi, \tfrac{5}{2}-\alpha_{52}^N) + \mathcal{C}_B(1-\alpha_{23}^\rho, 1-\alpha_{13}^\rho, \tfrac{5}{2}-\alpha_{51}, -\alpha_{45}^\pi, \tfrac{5}{2}-\alpha_{42}^N) + \pi_1 \longleftrightarrow \pi_3] + \pi_1 \longleftrightarrow \pi_3\} u_4 .$$

$A, B, \ldots$ in A_ρ^- (or A_B^-) stand for the corresponding $\mathcal{C}_A$, $\mathcal{C}_B, \ldots$ in A_ρ^+ (or A_B^+).

APPENDIX B: DICTIONARY OF THREE-POINT FUNCTIONS

All momenta are incoming. The quantities on the right of the $\otimes$ symbol refer to the isospin.

$\rho\pi\pi$ vertex

$$\mathfrak{M} = g_{\rho\pi\pi} \rho_\mu^*(p_1 - p_3)^\mu \otimes \vec{\rho}^* \cdot (\vec{\pi}_1 \times \vec{\pi}_3) .$$

$f\pi\pi$ vertex

$$\mathfrak{M} = \frac{g_{f\pi\pi}}{m_f} f_{\mu\nu}^* p_1^\mu p_3^\nu \otimes \vec{\pi}_1 \cdot \vec{\pi}_3 .$$

$\rho\omega\pi$ vertex

$$\mathfrak{M} = g_{\rho\omega\pi} \epsilon_{\alpha\beta\gamma\delta} p_\pi^\alpha p_\omega^\beta \omega^\gamma \rho^{*\delta} \otimes \vec{\rho}^* \cdot \vec{\pi} .$$

$\rho A_2 \pi$ vertex

$$\mathfrak{M} = g_{\rho A_2 \pi} \epsilon_{\alpha\beta\gamma\delta} p_\pi^\alpha p_\rho^\beta \rho^{*\gamma} A_2^{\delta\epsilon} p_{\rho\epsilon} \otimes \vec{\rho}^* \cdot (\vec{\pi} \times \vec{A}_2) .$$

$B\omega\pi$ vertex

$$\mathfrak{M} = \left(g_S m_B B^* \cdot \omega + \frac{g_D}{m_B} p_\pi \cdot \omega p_\pi \cdot B^* \right) \otimes \vec{B}^* \cdot \vec{\pi} .$$

$BA_2\pi$ vertex

$$\mathfrak{M} = \left(g_P A_2^{\alpha\beta} p_{\pi\alpha} B_\beta^* + \frac{g_L}{m_B^2} A_2^{\alpha\beta} p_{\pi\alpha} p_{\pi\beta} B^* \cdot p_\pi \right) \otimes \vec{B}^* \cdot (\vec{\pi} \times \vec{A}_2) .$$

$\pi N\bar{N}$ vertex

$$\mathfrak{M} = G_{\pi NN} \bar{v}_5 i \gamma_5 u_4 \otimes \chi^\dagger(\bar{N}_5) i\tau_2 \vec{\tau} \chi(N_4) \cdot \vec{\pi} .$$

$\rho N\bar{N}$ vertex

$$\mathfrak{M} = \bar{v}_5 \left[(G_v^\rho - G_T^\rho)\gamma_\mu + \frac{G_T^\rho}{2m_p}(p_4 - p_5)_\mu \right] \rho^\mu u_4 \otimes \chi^\dagger(\bar{N}_5) i\tau_2 \vec{\tau} \chi(N_4) \cdot \vec{\rho} .$$

$\omega N\bar{N}$ vertex

$$\mathfrak{M} = \bar{v}_5 \left[(G_V^\omega - G_T^\omega)\gamma_\mu + \frac{G_T^\omega}{2m_p}(p_4 - p_5)_\mu \right] \omega^\mu u_4 \otimes \chi^\dagger(\bar{N}_5) i\tau_2 \chi(N_4) .$$

$BN\bar{N}$ vertex

$$\mathfrak{M} = \bar{v}_5 i \gamma_5 \frac{G_T^B}{2m_p}(p_4 - p_5)_\mu \otimes \chi^\dagger(\bar{N}_5) i\tau_2 \vec{\tau} \chi(N_4) \cdot \vec{B} .$$

$A_2 N\bar{N}$ vertex

$$\mathfrak{M} = \bar{v}_5 \left(\frac{G_1^{A_2}}{m_p} [\gamma_\mu(p_4 - p_5)_\nu + \gamma_\nu(p_4 - p_5)_\mu] + \frac{G_2^{A_2}}{m_p^2}(p_4 - p_5)_\mu (p_4 - p_5)_\nu A_2^{\mu\nu} \right) u_4 \otimes \chi^\dagger(\bar{N}_5) i\tau_2 \vec{\tau} \chi(N_4) \cdot \vec{A}_2 .$$

$\pi\bar{N}\Delta^{++}$ vertex

$$\mathfrak{M} = \frac{G_{\pi p \Delta^{++}}}{m_\pi} \bar{v}_5 p_\pi^\mu \Delta_\mu .$$

$\rho\bar{N}\Delta^{++}$ vertex

$$\mathfrak{M} = G_{\rho p \Delta^{++}} \bar{v}_5 i \gamma_5 \left(g_{\mu\nu} - \frac{\gamma_\mu p_{5\nu}}{m_\Delta + m_p} \right) \rho^\mu \Delta^\nu .$$

$BN\Delta^{++}$ vertex

$$\mathfrak{M} = \bar{v}_5 \left(G_{BN\Delta^{++}}^1 g_{\mu\nu} + \frac{G_{BN\Delta^{++}}^2}{m_\Delta + m_p} \gamma_\mu p_{5\nu} + \frac{G_{BN\Delta^{++}}^3}{(m_\Delta + m_p)^2} p_{5\mu} p_{5\nu} \right) B^\mu \Delta^\nu .$$

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[1]K. Bardakci and H. Ruegg, Phys. Lett. 28B, 342 (1968).

[2]For example, the $\pi\bar{K}N\bar{N}$ system has been extensively studied in the dual resonance model and both the 3-body and quasi-2-body final states have been fitted. See Chan Hong-Mo *et al.*, Nucl. Phys. B19, 173 (1969); B. Petersson and G. H. Thomas, *ibid.* B20, 451 (1970); R. O. Raitio, *ibid.* B21, 427 (1970); K. Kajantie and S. Papageorgiou, *ibid.* B22, 31 (1970); J. Bartsch *et al.*, *ibid.* B23, 1 (1970).

[3]I. Bender *et al.*, Z. Phys. 237, 107 (1970); A. C. Hirsh-

feld and M. G. Schmidt, Nuovo Cimento 6A, 639 (1971); G. Benfatto *et al.*, *ibid.* 4A, 209 (1971); M. B. Green and R. L. Heimann, Phys. Lett. 30B, 642 (1969).
[4]G. P. Canning and M. A. Jacobs, Phys. Rev. D 3, 891 (1971).
[5a]K. Hepp, Helv. Phys. Acta 37, 55 (1964).
[5b]K. Hepp, Helv. Phys. Acta 36, 365 (1963).
[6]H. Navelet and E. Pittet, Nuovo Cimento 7A, 185 (1972).
[7]H. F. Jones and M. D. Scadron, Nucl. Phys. B4, 267 (1968).
[8]The papers listed here are concerned with the cross section of the reaction $\pi N \to BN$:
[8a]4 GeV/c: M. Aderholz *et al.*, Phys. Rev. 138, B897 (1965).
[8b]5 GeV/c: C. L. Pols *et al.*, Nucl. Phys. B25, 109 (1970).
[8c]6.7 GeV/c: S. Miyashita *et al.*, Phys. Rev. D 1, 771 (1970).
[8d]7.0 GeV/c: P. Slattery *et al.*, Nuovo Cimento 50A, 377 (1967).
[8e]8.0 GeV/c: M. Aderholz *et al.*, Nucl. Phys. B8, 45 (1968).
[8f]8.0 GeV/c: N. M. Cason *et al.*, Phys. Rev. D 1, 851 (1970).
[8g]11 GeV/c: F. Conte *et al.*, Phys. Lett. 22, 702 (1966).
[8h]11.2 GeV/c: C. Caso *et al.*, Nuovo Cimento Lett. 3, 707 (1970).
[8i]16.0 GeV/c: J. Ballam *et al.*, Phys. Rev. D 3, 2606 (1971).
[8j]18.5 GeV/c: M. H. Hones *et al.*, Phys. Rev. D 2, 827 (1970).
[9]Here we refer only to papers which deal with the spin-parity analysis of the B meson and the determination of the B-decay parameters:
[9a]3 and 5 GeV/c: A. Werbrouck *et al.*, Nuovo Cimento Lett. 4, 1267 (1970).
[9b]3.25 GeV/c: I. A. Erofeev *et al.*, Yad. Fiz. 11, 805 (1970) [Sov. J. Nucl. Phys. 11, 450 (1970)].
[9c]3.9 GeV/c: M. Abramovich *et al.*, CERN Report No. CERN/D.Ph.II/PHYS 70-17 (unpublished).
[9d]5 GeV/c: G. Ascoli *et al.*, Phys. Rev. Lett. 20, 1411 (1968).
[9e]5 and 7.5 GeV/c: G. Ascoli *et al.*, in High Energy Physics, proceedings of the Fifteenth International Conference on High Energy Physics, Kiev, 1970, edited by V. Shelest (Naukova Dumka, Kiev, U.S.S.R., 1972).
[9f]7.1 GeV/c: R. Ott, Ph.D. thesis, LBL, 1972 (unpublished).
[9g]9.1 GeV/c: N. Armenise *et al.*, paper submitted to the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972 (unpublished).
[9h]11 GeV/c: M. Afzal *et al.*, Nuovo Cimento 15A, 61 (1973).
[10]3.7 GeV/c: G. S. Abrams *et al.*, Phys. Rev. Lett. 25, 619 (1970); 3.7 GeV/c: K. W. J. Barnham *et al.*, Phys. Rev. D 7, 1384 (1973); 5 GeV/c: C. L. Pols *et al.*, Nucl. Phys. B25, 109 (1970); 5.45 GeV/c: I. J. Bloodworth *et al.*, *ibid.* B35, 79 (1971); 8 GeV/c: M. Aderholz *et al.*, *ibid.* B8, 45 (1968); 11.7 GeV/c: D. Evans *et al.*, *ibid.* B51, 205 (1973); 18.5 GeV/c: N. N. Biswas *et al.*, Phys. Rev. D 2, 2529 (1970).
[11]3.7–5.5 GeV/c: L. E. Holloway *et al.*, Phys. Rev. Lett. 27, 1671 (1971); 4.2 GeV/c: G. S. Abrams *et al.*, *ibid.* 23, 673 (1971); 5.1 GeV/c: N. Armenise *et al.*, Nuovo Cimento 65A, 637 (1970); 6.95 GeV/c: J. A. J. Matthews *et al.*, Phys. Rev. Lett. 26, 400 (1971); 9.0 GeV/c: N. Armenise *et al.*, in *Proceedings of the Fifth International Conference On Elementary Particles, Lund, 1969,* edited by G. von Dardel (Berlingska Boktryckeriet, Lund, Sweden, 1970); 10 GeV/c: E. Shibata and M. Wahling, Phys. Lett. 22, 354 (1966).
[12]H. Harari, Phys. Rev. Lett. 26, 1079 (1971).
[13]D. Cohen *et al.*, Phys. Rev. D 8, 23 (1973).
[14]G. Ascoli *et al.*, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 1, p. 3.
[15]See, for instance, J. N. J. White and R. K. P. Zia, Nuovo Cimento 6A, 203 (1971).
[16]G. Veneziano, Nuovo Cimento 57A, 190 (1968).
[17]A. Capella *et al.*, Nuovo Cimento 64A, 361 (1969).
[18]E. M. Corson, *Introduction to Spinors, Tensors and Relativistic Wave Equations,* (Blackie, London, 1953).
[19]J. F. L. Hopkinson and Chan Hong-Mo, Nucl. Phys. B10, 399 (1969).
[20]Pu Shen and Kyungsik Kang, Phys. Rev. D 7, 269 (1973).
[21]H. Lipkin, Phys. Rev. 159, 1303 (1967).