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Radiative corrections in elastic hadron scattering*

Michael R. Sogard†
Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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The radiative correction to elastic hadron scattering is derived and evaluated for a number of processes including scattering from nuclei. The size of the correction is in some instances substantial, and some experimental results, as well as conclusions drawn from them, should probably be reevaluated.

I. INTRODUCTION

The analysis of electron scattering experiments is complicated by radiative effects associated with both the emission and the reabsorption of virtual photons and the emission of real photons. Accordingly radiative corrections programs have been developed to correct for these effects and thus clarify our studies of the structure of the particles bombarded by the electrons.

All charged particles radiate to some extent in a collision, of course, with electrons representing the most extreme case. It does not appear to be well known, however, that radiative effects of a significant magnitude can be present in purely hadronic interactions, where the lightest particle involved is the pion. This point will be illustrated in this paper. We have derived an expression for the radiative correction to any elastic scattering process in which the scattered particle is detected and its momentum measured. We have evaluated this expression numerically over a considerable kinematic range for the elastic reactions $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$. We shall see that these effects have been sizable in kinematic regions covered by past experiments and will become still more important as experiments are pushed to higher energy or precision or both. The corrections do not simply represent a change in cross-section normalization. They depend on the four-momentum transfer t and will therefore modify the observed t (and u) dependence of the cross section. For example, corrections to the π^-p cross section at 20 GeV/c incident momentum can exceed 30% when the pion scatters in the backward direction. Furthermore, owing to terms arising from interference between radiation from the pion and radiation from the proton, the size of the correction for π^-p scattering can be quite different from that for π^+p scattering, thus complicating the interpretation of isotopic spin relations. In addition to the above reactions we will also briefly discuss the size of these corrections in hadron-nucleus scattering. We will then consider how these corrections are to be applied in experimental situations. While our results apply to elastic scattering processes, expressions for analogous corrections to two-body processes such as $\pi^-p\to\pi^0n$ or $K^-p\to\pi^-\Sigma^+$ or photoproduction processes can be easily obtained from our treatment.

Our approach is that developed by Yennie, Frautschi, and Suura[1] and applied to elastic reactions by Tsai[2] and by Meister and Yennie.[3] Figure 1(a) represents the scattering process $1+2\to3+4$. Particle 1 is incident on particle 2, which is at rest in the laboratory. For the moment all four particles are arbitrary. The other diagrams in Fig. 1 represent radiative processes involving the emission and reabsorption of a virtual photon. These cannot be distinguished from the nonradiative elastic process, and their effects must therefore be considered. Diagrams 1(b)–1(e) are known to produce infrared divergences.[4] These are canceled exactly by analogous divergences associated with the diagrams in Fig. 2 which represent the emission of real, soft photons. These diagrams must be included in order to obtain a finite result. Physically, their presence is easy to understand. No experiment has perfect energy resolution; every measured elastic peak includes events in which radiation was emitted during the scattering process. Let us be more precise. Figure 3 represents the momentum spectrum of particle 3 at fixed scattering angle θ_3. For elastic scattering particle 3 is the scattered particle and particle 4 the recoiling target. The momentum resolution is assumed to be of sufficient quality that the width of the peak is a small fraction of the distance between the elastic peak and the threshold for inelastic pion production. We will consider less favorable experimental situations later. The tail on the low-momentum side of the elastic peak arises from real bremsstrahlung. Operationally the measured elastic cross section at fixed angle is defined as follows. At a distance Δp_3 below the elastic peak but above the inelastic threshold a cut is made, and, after appropriate background subtractions are made, the yield above this momentum cut is integrated. Dividing this integrated yield by the spectrometer solid angle and correcting for detection and identification inefficiencies leads to the measured cross section at scattering angle θ_3, $d\sigma_m/d\Omega_3$. Events associated with the emission of real photons which are sufficiently soft that p_3 exceeds the momentum cut are included in the measured cross section. Events in which real bremsstrahlung has reduced p_3 below the cutoff are excluded. Since the shape of the bremsstrahlung spectrum can be calculated without recourse to strong-interaction theory for soft photons, the amount lost from the elastic peak by this mechanism can be determined and a correction factor applied. The radiatively corrected elastic cross section can then be written as

$$\frac{d\sigma}{d\Omega_3}=\frac{d\sigma_m}{d\Omega_3}\,e^{\delta(\Delta p_3)}\,.$$

We will calculate $\delta(\Delta p_3)$ in this paper. The infrared divergences will be handled by assigning a mass λ to the real and virtual photons and letting it go to zero at the end of the calculations. The contribution to the cross section from a radiative diagram can be factored into a part dependent on λ and a second, finite part independent of λ. The former terms will cancel out when all contributions involving both real and virtual photons are considered.[1,2] Therefore only the finite residues need be explicitly calculated. We will only calculate the contributions from real bremsstrahlung in this paper; these represent the dominant part of the radiative correction. The magnitude of the virtual-photon contributions to the radiative correction will be discussed briefly in Sec. V.

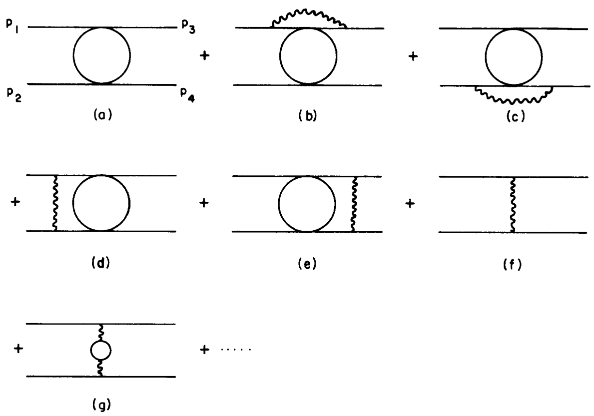

FIG. 1. Feynman diagrams which are associated with elastic scattering. Diagrams (f) and (g) represent Coulomb effects and are important only at very small angles.

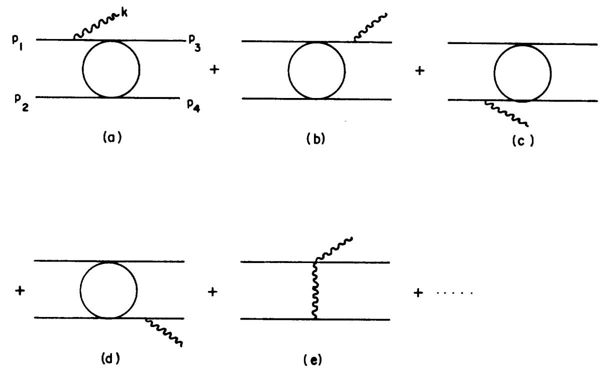

FIG. 2. Feynman diagrams associated with the emission of a real photon. Diagram (e) is associated with radiation from a charged boson and would only be important at very small angles.

II. CALCULATION OF THE RADIATIVE CORRECTION

We must calculate the cross section associated with the diagrams of Fig. 2. Our formalism is similar to that of Tsai.[2] For low-energy photons we can ignore magnetic-moment effects and regard the radiation as coming solely from the convection currents. (For spinless particles such as the π or K, of course, no approximation is involved.) Depending on the experimental situation we may not always be able to restrict ourselves to sufficiently low-energy, or sufficiently soft, photons. We will consider such situations in Sec. V. We also regard the hadronic reaction as being undisturbed by the radiation,[5] i.e., we take $T(p_1, p_3)$, defined below, to be evaluated on the mass shell at the values of p_1 and p_3 which would obtain in the absence of radiation. We can then write the matrix element for scattering with bremsstrahlung as

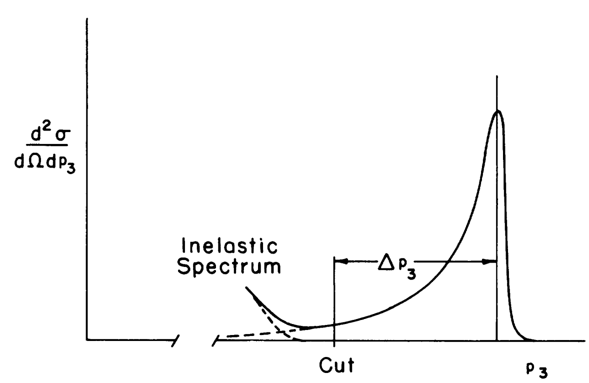

FIG. 3. The elastic peak at fixed scattering angle together with the tail arising from real bremsstrahlung and the onset of the inelastic threshold. Effects of experimental resolution are also shown.

$$A = e\left(z_3\frac{p_3\cdot\epsilon}{p_3\cdot k} - z_1\frac{p_1\cdot\epsilon}{p_1\cdot k} + z_4\frac{p_4\cdot\epsilon}{p_4\cdot k} - z_2\frac{p_2\cdot\epsilon}{p_2\cdot k}\right)T(p_1,p_3), \qquad (2.1)$$

where ez_1, ez_2, ez_3, and ez_4 are the charges of the four particles and T is the matrix element for the hadronic reaction in the absence of radiation; ϵ is the polarization of the emitted photon. Taking the modulus squared of A and averaging and summing over initial and final states leads to the differential cross section,

$$d\sigma_i = -\frac{e^2}{V}\frac{N_1N_2N_3N_4}{E_1m_2}\left(z_3\frac{p_3}{p_3\cdot k} - z_1\frac{p_1}{p_1\cdot k} + z_4\frac{p_4}{p_4\cdot k} - z_2\frac{p_2}{p_2\cdot k}\right)^2 \times\sum|T|^2(2\pi)^4\delta^4(p_1+p_2-p_3-p_4-k)\frac{1}{(2\pi)^3}\frac{d^3p_3}{E_3}\frac{1}{(2\pi)^3}\frac{d^3p_4}{E_4}\frac{1}{(2\pi)^3}\frac{d^3k}{2\omega}. \qquad (2.2)$$

The photon has been given a finite mass λ, i.e., $\omega=(k^2+\lambda^2)^{1/2}$. The N_i are normalization constants; $N_i=\frac{1}{2}$ if particle i is a boson, and $N_i=m_i$ if i is a fermion. The incident flux is given by $V=p_1/E_1$. The measured cross section involves an integration over a region of phase space determined by the experimental situation. The integration is of the form

$$X=\int\frac{d^3p_3}{E_3}\frac{d^3k}{2\omega}\frac{d^3p_4}{E_4}\delta^4(p_1+p_2-p_3-p_4-k) \times\chi^2\sum|T|^2, \qquad (2.3)$$

where

$$\chi^2 = -\left(z_3\frac{p_3}{p_3\cdot k} - z_1\frac{p_1}{p_1\cdot k} + z_4\frac{p_4}{p_4\cdot k} - z_2\frac{p_2}{p_2\cdot k}\right)^2 \equiv \sum_{i,j=1}^{4}s_{ij}z_iz_j\frac{(p_i\cdot p_j)}{(p_i\cdot k)(p_j\cdot k)} \qquad (2.4)$$

and s_{ij} is ±1. Our remarks so far have been general. From this point on, however, the details of our procedure are determined by the particular experimental conditions specified in the Introduction, namely detecting the momentum of the scattered particle within a limited angular range. Figure 4 illustrates the kinematic region of interest. The line AD describes the position of the elastic peak as a function of the scattering angle. The line BC represents the momentum cutoff.

We begin by integrating over p_4. This leads to

$$X=\int\frac{d^3p_3}{E_3}\frac{d^3k}{\omega}\delta((r-k)^2-m_4^2)\theta(E_4)\chi^2\sum|T|^2, \qquad (2.5)$$

where we have introduced

$$r=p_4+k=p_1+p_2-p_3$$

and

$$\theta(y)=\begin{cases}1, & y>0\\ 0, & y<0.\end{cases}$$

The energy of a photon which leads to $\vec{p}_3=\vec{p}_{3\,\text{min}}$ is a strong function of its direction of emission. By transforming to a coordinate system in which $\vec{p}_4+\vec{k}=0$, i.e., $r=(r_0,0)$, we can simplify the photon integration, because the δ function is then independent of the direction of the photon. Letting a tilde represent the variables in this special frame we have

$$\frac{\tilde{k}^2d\tilde{k}}{\tilde{\omega}}=\tilde{k}d\tilde{\omega}=(\tilde{\omega}^2-\lambda^2)^{1/2}d\tilde{\omega}$$

and (2.6)

$$(r-k)^2-m_4^2=r^2-2r_0\tilde{\omega}+\lambda^2-m_4^2.$$

Therefore

$$\int(\tilde{\omega}^2-\lambda^2)^{1/2}\delta((r-k)^2-m_4^2)d\tilde{\omega}=\frac{(\tilde{\omega}^2-\lambda^2)^{1/2}}{2r_0} = \frac{[(k\cdot r)^2-\lambda^2r^2]^{1/2}}{2r^2},$$

so that

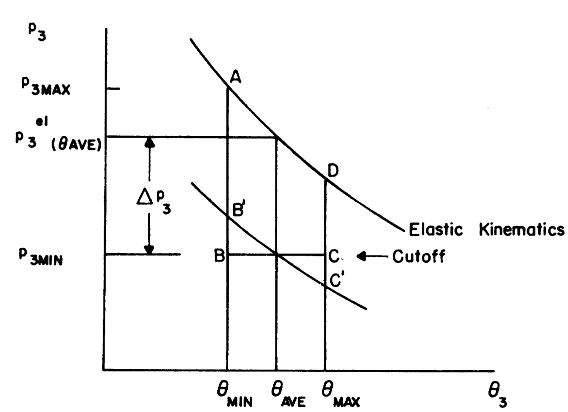

FIG. 4. A spectrometer angle bin extends from θ_{min} to θ_{max}. Elastic scattering occurs along the line AD. The cutoff occurs along BC. The average angle θ_{av} depends on the angular dependence of the elastic cross section.

$$X=\int d\Omega_3 \sum |T|^2 \int_{E_{3\,\min}}^{E_{3\,\max}} p_3 dE_3 \frac{[(k\cdot r)^2-\lambda^2 r^2]^{1/2}}{2r^2}\theta(r^2-r_{\min}{}^2)\int \chi^2 d\tilde{\Omega}, \tag{2.7}$$

where

$$r_{\min}{}^2 = m_4{}^2+2m_4\lambda+\lambda^2 \sim m_4{}^2+2m_4\lambda .$$

We now do the p_3 integration. We define x as

$$\begin{aligned} x &= r^2 - m_4{}^2 \\ &= m_1{}^2+m_2{}^2+m_3{}^2-m_4{}^2+2m_2(E_1-E_3) \\ &\quad -2E_1E_3+2p_1p_3\cos\theta_3 . \end{aligned}$$

Our integral then becomes[6]

$$X=\int d\Omega_3 \sum |T|^2 \frac{p_3}{4m_2\eta'}\int_{x_{\min}}^{x_{\max}} \frac{x dx}{2(x+m_4{}^2)} \times \int \chi^2 d\tilde{\Omega}, \tag{2.8}$$

where

$$\eta' = 1+\frac{1}{m_2}\left(E_1 - p_1\frac{E_3}{p_3}\cos\theta_3\right).$$

In the limit of high-energy scattering where $m_1 \ll E_1$ and $m_3 \ll E_3$, η' reduces to the familiar compaction factor encountered frequently in electron scattering,

$$\eta = E_1/E_3 = 1+(E_1/m_2)(1-\cos\theta_3).$$

The lower integration limit is $x_{\min}=2m_4\lambda$, which corresponds to the value of x along the elastic contour AD in Fig. 4. The curve BC determines the value of $x_{\max}$, which is a function of θ_3. Owing to the $1/\omega$ form of the bremsstrahlung spectrum, most scattered particles lie near AD, while relatively few are near BC. We therefore replace BC by the angle-independent line $B'C'$, with little error, which leads to a value for $x_{\max}$ of

$$\begin{aligned} x_{\max} &= m_1{}^2+m_2{}^2+m_3{}^2-m_4{}^2+2m_2(E_1-E_{3\,\min}) \\ &\quad -2(E_1E_{3\,\min}-p_1p_{3\,\min}\cos\theta_3). \end{aligned}$$

We will also approximate the momenta p_i in χ^2 by their elastic peak values. The most appropriate value for the average angle θ_{av} will depend on the angular dependence of $\sum|T|^2$. Defining $\Delta p_3 = p_3^{el}(\theta_{av}) - p_{3\,\min}$, where $p_3^{el}(\theta_{av})$ is the momentum at the elastic peak at $\theta_3=\theta_{av}$, we finally get

$$x_{\max} = 2m_2\eta\Delta E', \tag{2.9}$$

where

$$\Delta E' = \frac{\eta'}{\eta}\Delta E_3 = \frac{\eta'}{\eta}\frac{p_3}{E_3}\Delta p_3 . \tag{2.10}$$

Our final expression for X is now

$$X = d\Omega_3 \sum |T(\theta_{av})|^2 \frac{p_3}{4m_2\eta'} \times \int_{2m_4\lambda}^{2m_2\eta\Delta E'} \frac{x dx}{2(x+m_4{}^2)} \int \chi^2 d\tilde{\Omega}. \tag{2.11}$$

The elastic cross section at angle θ_{av}, in the absence of bremsstrahlung, is given by

$$d\sigma = \frac{N_1N_2N_3N_4}{(2\pi)^2 m_2}\frac{p_3}{p_1}\sum |T(\theta_{av})|^2 \frac{d\Omega_3}{m_2\eta'} . \tag{2.12}$$

Our inelastic cross section $d\sigma_i/d\Omega_3$ is therefore related to the elastic cross section by

$$\frac{d\sigma_i}{d\Omega_3} = \frac{d\sigma}{d\Omega_3}\frac{\alpha}{8\pi^2}\int_{2m_4\lambda}^{2m_2\eta\Delta E'} \frac{x dx}{2(x+m_4{}^2)}\int \chi^2 d\tilde{\Omega}, \tag{2.13}$$

where $\alpha = e^2/4\pi$.

In performing the photon angular integration in the special frame it is useful to express the kinematic variables in covariant forms: $\tilde{\omega} = k\cdot r/(r^2)^{1/2}$, $\tilde{E}_i = p_i\cdot r/(r^2)^{1/2}$; also $k\cdot r = \frac{1}{2}(x+\lambda^2)$. Expressed in terms of laboratory quantities the quantities $p_i\cdot r$ have the following forms:

$$\begin{aligned} p_1\cdot r &= \tfrac{1}{2}(m_1{}^2+m_4{}^2-m_2{}^2-m_3{}^2)+m_2E_3, \\ p_2\cdot r &= m_2E_4 \\ p_3\cdot r &= \tfrac{1}{2}(m_1{}^2+m_4{}^2-m_2{}^2-m_3{}^2)+m_2E_1, \\ p_4\cdot r &= m_4{}^2 . \end{aligned} \tag{2.14}$$

We then have

$$\begin{aligned} \int d\tilde{\Omega}\frac{(p_i\cdot p_j)}{(p_i\cdot k)(p_j\cdot k)} &= (p_i\cdot p_j)\int_0^1 dy \int \frac{d\tilde{\Omega}}{(p_y\cdot k)^2} \\ &= 4\pi(p_i\cdot p_j)\int_0^1 \frac{dy}{|\tilde{\vec{k}}|^2 p_y{}^2+\lambda^2\tilde{E}_y{}^2} \\ &= 4\pi(p_i\cdot p_j)\int_0^1 \frac{dy}{[(k\cdot r)^2-\lambda^2 r^2]p_y{}^2+\lambda^2(p_y\cdot r)^2} \\ &= 16\pi(p_i\cdot p_j)\int_0^1 \frac{(x+m_4{}^2)dy}{(x^2-4\lambda^2m_4{}^2)p_y{}^2+4\lambda^2(p_y\cdot r)^2}, \end{aligned} \tag{2.15}$$

where $p_y = p_i y + p_j(1-y)$, and we have neglected all but leading terms involving λ^2. The integration over x leads to

$$\int_{2m_4\lambda}^{2m_2\eta\Delta E'} \frac{x\,dx}{2(x+m_4{}^2)} \int \chi^2 d\bar{\Omega}$$

$$= 4\pi s_{ij} z_i z_j (p_i\cdot p_j) \int_0^1 \frac{dy}{p_y{}^2}\left\{ \ln\frac{p_y{}^2}{\lambda^2} - 2\ln\frac{p_j\cdot r}{m_2\eta\Delta E'} - 2\ln\left[1+\frac{(p_i\cdot r - p_j\cdot r)}{p_j\cdot r}y\right]\right\}$$

$$= 4\pi s_{ij} z_i z_j (p_i\cdot p_j) \left\{\int_0^1 \frac{dy}{p_y{}^2} \ln\frac{p_y{}^2}{\lambda^2} - 2\ln\frac{p_j\cdot r}{m_2\eta\Delta E'}\int_0^1\frac{dy}{p_y{}^2} - 2\int_0^1 \ln\left[1+\frac{(p_i\cdot r - p_j\cdot r)}{p_j\cdot r}y\right]\frac{dy}{p_y{}^2}\right\}$$

$$= 4\pi s_{ij} z_i z_j (p_i\cdot p_j)\int_0^1 \frac{dy}{p_y{}^2}\ln\frac{p_y{}^2}{\lambda^2} - 8\pi s_{ij} z_i z_j I_{ij}\,, \tag{2.16}$$

where

$$I_{ij} = (p_i\cdot p_j)\ln\frac{p_j\cdot r}{m_2\eta\Delta E'}\int_0^1\frac{dy}{p_y{}^2} + (p_i\cdot p_j)\int_0^1 \ln\left[1+\frac{(p_i\cdot r - p_j\cdot r)}{p_j\cdot r}y\right]\frac{dy}{p_y{}^2}\,. \tag{2.17}$$

The quantity I_{ij} represents the finite residue associated with the bremsstrahlung. The integral involving λ^2 contains the infrared divergence and will cancel an analogous divergent expression obtained from a diagram in Fig. 1 associated with the emission and reabsorption of a virtual photon between lines i and j.

Our measured cross section is then the sum of the elastic and inelastic cross sections:

$$\frac{d\sigma_m}{d\Omega} = \frac{d\sigma}{d\Omega} + \frac{d\sigma_i}{d\Omega} = \frac{d\sigma}{d\Omega}(1-\delta)\,, \tag{2.18}$$

where

$$\delta = \frac{\alpha}{\pi}\sum_{ij} s_{ij} z_i z_j I_{ij}\,. \tag{2.19}$$

It is worth commenting that δ is inherently positive and differs by a sign from the definition of Tsai.[2] Note that if either particle i or particle j is neutral there will be no contribution to δ.

We have calculated the correction to the elastic cross section associated with the emission of a single photon (and, implicitly, with the emission and reabsorption of a single virtual photon). In fact many photons can be emitted in the scattering process (indeed an infinite number of infrared photons can be emitted), and these higher-order effects cannot be neglected. Because we are dealing with soft photons which do not affect the scattering process, the emission and/or the reabsorption are independent events, and the over-all behavior is described by a Poisson distribution. It can then be shown that for the general case of multiple photon emission Eq. (2.18) should be replaced by[1]

$$\frac{d\sigma_m}{d\Omega} = \frac{d\sigma}{d\Omega}e^{-\delta}\,. \tag{2.20}$$

This is our final result.

We now evaluate I_{ij} in the general case. We get that

$$\begin{aligned} I_{ij}+I_{ji} = \frac{p_i\cdot p_j}{2Q_{ij}}\Bigg\{ &\ln\left[\frac{Q_{ij}-p_i\cdot(p_i-p_j)}{Q_{ij}+p_i\cdot(p_i-p_j)}\,\frac{Q_{ij}+p_j\cdot(p_i-p_j)}{Q_{ij}-p_j\cdot(p_i-p_j)}\right]\left(\ln\frac{p_i\cdot r}{m_2\eta\Delta E'}+\ln\frac{p_j\cdot r}{m_2\eta\Delta E'}\right) \\ &+2\ln\left(\frac{y_1-1}{y_1}\right)\ln|1+c_{ij}y_1| - 2\ln\left(\frac{y_2-1}{y_2}\right)\ln|1+c_{ij}y_2| \\ &+\ln\left(\frac{r\cdot p_j}{r\cdot p_i}\right)\left[\ln\left(\frac{y_1-1}{y_1}\right)-\ln\left(\frac{y_2-1}{y_2}\right)\right] \\ &-2\Phi\left(\frac{y_1-1}{y_1+1/c_{ij}}\right)+2\Phi\left(\frac{y_1}{y_1+1/c_{ij}}\right)+2\Phi\left(\frac{y_2-1}{y_2+1/c_{ij}}\right)-2\Phi\left(\frac{y_2}{y_2+1/c_{ij}}\right)\Bigg\}\,, \end{aligned} \tag{2.21}$$

where

$$Q_{ij} = [(p_i\cdot p_j)^2 - m_i{}^2 m_j{}^2]^{1/2}\,,\quad y_1 = \frac{m_j{}^2 - p_i\cdot p_j + Q_{ij}}{m_i{}^2+m_j{}^2-2p_i\cdot p_j}\,,\quad y_2 = \frac{m_j{}^2 - p_i\cdot p_j - Q_{ij}}{m_i{}^2+m_j{}^2-2p_i\cdot p_j}\,,$$

and

$$c_{ij} = \frac{r\cdot(p_i-p_j)}{r\cdot p_j}\,.$$

$\Phi(x)$ is the Spence function[7]

$$\Phi(x)=\int_0^x -\ln|1-y|\,\frac{dy}{y}\,.$$

The following integral was of great utility:

$$\int_0^1 \frac{\ln(1+dy)dy}{ay^2+by+c}=\int_0^1 \frac{\ln(1+dy)dy}{a(y-y_1)(y-y_2)}$$

$$=\frac{1}{a}\frac{1}{y_1-y_2}\left[\ln|1+dy_1|\ln\left(\frac{y_1-1}{y_1}\right)-\Phi\left(\frac{y_1-1}{y_1+1/d}\right)+\Phi\left(\frac{y_1}{y_1+1/d}\right)\right.$$

$$\left.-\ln|1+dy_2|\ln\left(\frac{y_2-1}{y_2}\right)+\Phi\left(\frac{y_2-1}{y_2+1/d}\right)-\Phi\left(\frac{y_2}{y_2+1/d}\right)\right].$$

The quantities y_1 and y_2 are the roots of the denominator of the integrand.

This expression can be applied to any two-body process in which the particles are relatively stable. By that we mean that the particle widths Γ are much smaller than the maximum energy which can be carried off by a photon in the rest frame of the decaying particle. Then for photon energies ranging from $k\sim\Gamma$ to $k\sim\Delta E$, evaluated in the rest frame, the initial and final states are two-particle states, and we can use Eq. (2.20). For $k<\Gamma$ we have to worry about the details of the decay of the unstable particle. If $\Gamma\ll\Delta E$ we can use Eq. (2.20) over the entire photon energy range with negligible error. Therefore this formalism will work for reactions involving particles which decay weakly or electromagnetically, such as $K^-p\to\pi^-\Sigma^+$. It will not work for reactions like $\pi^-p\to\rho^-p$.

III. THE CORRECTION FOR ELASTIC SCATTERING

We will now obtain an explicit expression for the radiative correction to elastic scattering. We let $m_1=m_3=\mu$ and $m_2=m_4=M$. Also $z_1=z_3=1$ and $z_2=z_4=Z$. Furthermore, Z contains the relative sign of the charges of the beam and target particles, i.e., $Z=-|Z|$ if the particles have like charges, and $Z=|Z|$ if they have charges of opposite sign. After a great deal of simplification we arrive at the following expression for δ:

$$\delta=\frac{\alpha}{\pi}\left\{\left[\frac{2\mu^2-t}{Q}\ln\left(\frac{Q-t}{Q+t}\right)-1+Z\left(\frac{1}{\beta_1}\ln Q_1-\frac{1}{\beta_3}\ln Q_3\right)\right]\left(2\ln\frac{E_1}{\Delta E'}-3\ln\eta\right)\right.$$

$$-\frac{2\mu^2-t}{Q}\left[\Phi\left(-\frac{(E_1-E_3)(Q-t)}{(E_1+E_3)t-(E_1-E_3)Q}\right)+\Phi\left(\frac{(E_1-E_3)(Q-t)}{(E_1+E_3)t+(E_1-E_3)Q}\right)\right.$$

$$-\Phi\left(-\frac{(E_1-E_3)(Q+t)}{(E_1+E_3)t-(E_1-E_3)Q}\right)-\Phi\left(\frac{(E_1-E_3)(Q+t)}{(E_1+E_3)t+(E_1-E_3)Q}\right)$$

$$\left.-\ln\left(\frac{Q-t}{Q+t}\right)\ln\left(1+\frac{(E_1-E_3)^2\mu^2}{E_1E_3t}\right)\right]$$

$$-Z^2\left[\ln\frac{E_4}{M}-2\ln\frac{M}{\eta\Delta E'}\left(\frac{1}{\beta_4}\ln B_4-1\right)-\frac{1}{\beta_4}\left(\ln B_4\ln\frac{E_4+M}{2M}-\Phi(-R_4B_4)+\Phi(R_4/B_4)-\Phi(R_4)+\Phi(-R_4)\right)\right]$$

$$+\frac{Z}{\beta_3}\left[\ln\left|\frac{Mb_3^-}{E_3a_3}\frac{c_{14}}{E_1a_3}\right|\ln R_3^-+\Phi\left(\frac{E_3-M}{p_3}\right)-\Phi\left(-\frac{E_3-M}{p_3}\right)+\Phi\left(-\frac{E_3-M}{p_3R_3^-}\right)-\Phi\left(\frac{E_3-M}{p_3R_3^+}\right)\right.$$

$$+\Phi\left(\frac{M(M-E_3^+)(E_1-E_4)}{c_{11}}\right)-\Phi\left(\frac{M(M-E_3^-)(E_1-E_4)}{c_{14}}\right)+\Phi\left(\frac{(ME_3^+-\mu^2)(E_1-E_4)}{c_{14}}\right)$$

$$\left.-\Phi\left(\frac{(ME_3^--\mu^2)(E_1-E_4)}{c_{11}}\right)-\ln\left|\frac{Mb_3^+}{E_3a_3}\frac{c_{11}}{E_1a_3}\right|\ln R_3^+\right]$$

$$\left.-\frac{Z}{\beta_1}[\text{same expression with }1\to 3]\right\}. \qquad (3.1)$$

TABLE I. The fractional change in the cross section arising from radiative effects, $e^{\delta}-1$, for $x^{+}p$ and $x^{-}p$ scattering for the three projectiles $x=\pi, K, p$. The quantities t and u are in units of $(\mathrm{GeV}/c)^2$. The value of t occurring at a c.m. angle of 90° is marked by an asterisk. The cut has been placed midway between the elastic peak and the threshold for pion production.

$\pi p \to \pi p$

$-t$	$-u$	$e^{\delta^+}-1$	$e^{\delta^-}-1$
	$p_1=2$ GeV/c		
0.100		0.016	0.017
1.000		0.041	0.059
1.511*		0.041	0.073
2.000	0.862	0.039	0.084
3.021	−0.159	0.021	0.110
	$p_1=5$ GeV/c		
0.100		0.021	0.021
0.500		0.047	0.051
1.000		0.061	0.069
2.000		0.073	0.091
4.277*		0.078	0.127
5.000		0.076	0.137
7.486	1.000	0.053	0.177
7.986	0.500	0.043	0.187
8.558	−0.072	0.027	0.202
	$p_1=10$ GeV/c		
0.100		0.024	0.025
0.500		0.056	0.058
1.000		0.073	0.078
5.000		0.113	0.142
8.952*		0.118	0.180
10.00		0.116	0.190
16.87	1.000	0.060	0.272
17.37	0.500	0.048	0.283
17.91	−0.037	0.031	0.298
	$p_1=20$ GeV/c		
0.100		0.028	0.028
0.500		0.065	0.066
1.000		0.085	0.087
5.000		0.138	0.153
18.33*		0.169	0.247
35.13	1.500	0.077	0.382
35.63	1.000	0.067	0.393
36.65	−0.019	0.035	0.423
	$p_1=100$ GeV/c		
0.100		0.036	0.036
0.500		0.084	0.084
1.000		0.110	0.111
2.000		0.140	0.141
10.00		0.225	0.232
93.38*		0.340	0.461
185.2	1.500	0.094	0.795
186.3	0.500	0.065	0.829
186.8	−0.004	0.043	0.856
	$p_1=200$ GeV/c		
0.100		0.039	0.039
0.500		0.092	0.092
1.000		0.121	0.121
2.000		0.154	0.154
10.00		0.248	0.252
373.4	1.000	0.086	1.071
374.4	−0.002	0.047	1.128

$Kp \to Kp$

$-t$	$-u$	$e^{\delta^+}-1$	$e^{\delta^-}-1$
	$p_1=2$ GeV/c		
1.000		0.010	0.026
1.411*		0.010	0.035
2.000		0.010	0.047
2.823		0.008	0.061
	$p_1=5$ GeV/c		
1.000		0.018	0.026
2.000		0.027	0.044
4.171*		0.032	0.075
5.000		0.031	0.087
7.304	1.000	0.017	0.123
7.804	0.500	0.012	0.132
8.342	−0.038	0.007	0.140
	$p_1=10$ GeV/c		
1.000		0.023	0.027
5.000		0.054	0.081
8.842*		0.060	0.118
16.66	1.000	0.021	0.207
17.16	0.500	0.014	0.216
17.68	−0.020	0.007	0.225
	$p_1=100$ GeV/c		
1.000		0.027	0.029
5.000		0.069	0.083
18.21*		0.099	0.171
35.42	1.000	0.024	0.315
36.43	−0.010	0.007	0.336
	$p_1=100$ GeV/c		
1.000		0.034	0.035
2.000		0.055	0.056
5.000		0.092	0.095
185.5	1.000	0.029	0.690
186.5	−0.002	0.008	0.721
	$p_1=200$ GeV/c		
1.000		0.038	0.038
2.000		0.060	0.061
5.000		0.100	0.102
373.1	1.000	0.032	0.925
374.2	−0.001	0.008	0.963

TABLE I (*Continued*)

$-t$	$e^{\delta^+}-1$	$e^{\delta^-}-1$	$-t$	$e^{\delta^+}-1$	$e^{\delta^-}-1$
		$pp \to pp$			
	$p_1=10$ GeV/c			$p_1=200$ GeV/c	
1.000	0.010	0.014	1.000	0.015	0.016
5.000	0.031	0.056	5.000	0.055	0.058
8.540*	0.035	0.087	10.00	0.084	0.091
	$p_1=20$ GeV/c		20.00	0.120	0.134
			92.95*	0.186	0.292
1.000	0.012	0.014			
5.000	0.041	0.054		$p_1=100$ GeV/c	
10.00	0.058	0.089	1.000	0.016	0.017
17.91*	0.067	0.134	10.00	0.092	0.095
			20.00	0.132	0.139
			186.8*	0.260	0.385

The quantity t is the four-momentum transfer squared, $t=(p_1-p_3)^2$; also,

$$\eta = E_1/E_3\,,$$

$$Q=(t^2-4\mu^2 t)^{1/2}\,,$$

$$B_4=\left(\frac{1+\beta_4}{1-\beta_4}\right)^{1/2},$$

$$R_4=\left(\frac{E_4-M}{E_4+M}\right)^{1/2},$$

$$\beta_4=p_4/E_4\,,$$

and

$$\beta_i=p_i/E_i\,,$$

$$Q_i=\frac{p_i+E_i-\mu^2/M}{p_i-E_i+\mu^2/M}\,\frac{p_i+E_i-M}{p_i-E_i+M}\,,$$

$$a_i=M^2+\mu^2-2ME_i\,,$$

$$b_i^{\pm}=p_i(E_i^{\pm}-M)\,,$$

$$R_i^{\pm}=\frac{M(M-E_i^{\pm})}{ME_i^{\mp}-\mu^2}\,,$$

$$E_i^{\pm}=E_i\pm p_i\,,$$

where $i=1, 3$. Also

$$c_{ii}=M^2E_i-ME_iE_j^{+}+\mu^2E_4-E_4ME_j^{-}\,,$$

$$c_{i4}=M^2E_i-ME_4E_j^{+}+\mu^2E_4-E_iME_j^{-}\,,$$

where $j=3, 1$ when $i=1, 3$.

The terms proportional to Z are interference terms arising from the product of matrix elements for radiation from the two particles. The Z^2 terms represent radiation purely from the target particle. For relatively forward scattering angles the interference terms essentially cancel out, and the Z^2 terms are small, because the momentum transfer and therefore the recoil energy are relatively small. The Spence functions are negligible. At backward angles the interference term multiplied by Z/β_3 becomes quite small, leaving the other Z term (which is intrinsically negative) to contribute significantly to the size of δ. Furthermore, the momentum transfer is now relatively large, and the recoil-associated Z^2 terms become important. The Spence functions cannot be neglected here. In situations where both particles 1 and 3 are relativistic the terms following the Spence functions can be neglected. Indeed, in the limit that $\beta_1=\beta_3=1$ and $-t \gg \mu^2$ our expression for δ reduces exactly to the one derived by Tsai[2] for elastic electron-proton scattering, save for the first two terms in the latter expression. These represent corrections arising from vacuum polarization and a vertex correction to the electron. Such virtual-photon effects will be discussed in Sec. V.

The quantity $e^{\delta}-1$ represents the fractional change in the measured cross section arising from radiative effects. We have calculated this quantity for $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ elastic scattering in Table I, for a range of laboratory scattering angles covering nearly 180° (except for the proton) and incident momenta of 2, 5, 10, 20, 100, and 200 GeV/c. Our cut has been placed midway between the elastic peak and the inelastic pion threshold. The kinematic variables t and $u=(p_2-p_3)^2$ are also shown. We see that the radiative effects grow with energy and are quite large for backward-angle scattering. Moreover, for relatively small-angle scattering the effects are essentially independent of the sign of the charge of the incident particle, while in the backward hemisphere the effects decrease rapidly with increasing angle for positively charged incident particles, but continue to increase for incident particles of negative charge. We can qualitatively understand this behavior from classical considerations. The field of a relativistic charged particle is essentially compressed into a plane transverse to the direction of motion. When the particle scatters, it changes direction in a time short compared to that required to transmit the information of scattering to its field. Consequently a fraction of the field continues in the original direction, resulting in radiation. Backward scattering represents the most extreme change of direction, of course. If, however, the beam and target particles are of the same charge, that fraction of the field of the beam particle which continues forward can attach itself to the recoiling target particle, which is traveling in nearly the same direction; then little radiation ensues.

The effects of radiation are most serious for πp scattering, both because of the absolute size of the effects and because of the rich structure in the backward scattering peaks, where the effects are largest. Because of the considerably larger corrections to $\pi^- p$ scattering than to $\pi^+ p$ scattering the presence of the radiation can significantly influence isotopic spin relations. The effects are naturally smaller for Kp and pp scattering.

The radiative correction is zero at $t=0$ and increases with $-t$. This causes the slope of the forward diffraction peak to appear steeper than it actually is. While this error is not large, amounting to only a few percent for πp scattering at 100 GeV/c, it represents an effect which does not disappear asymptotically. Indeed, for small scattering angles and $M^2 \gg -t \gg \mu^2$, δ can be approximated by

$$\delta \simeq \frac{2\alpha}{\pi}\left[\ln\left(\frac{-t}{\mu^2}\right)-1\right]\ln\frac{E_1}{\Delta E'} \equiv T(t)\ln\frac{E_1}{\Delta E'}\,.$$

Our measured cross section is then

$$\frac{d\sigma_m}{d\Omega}=\frac{d\sigma}{d\Omega}\,e^{-T\ln(E_1/\Delta E')}\,.$$

At high energies $s \simeq 2ME_1$, so that

$$\frac{d\sigma_m}{d\Omega}=\frac{d\sigma}{d\Omega}\,e^{-T\ln(s/2M\Delta E')}$$

$$=\frac{d\sigma}{d\Omega}\left(\frac{s}{2M\Delta E'}\right)^{-T}$$

Thus the radiative effects introduce a weak s dependence into the cross section at high energy, which causes the diffraction peak to shrink.

At large scattering angles, but away from the backward peak region, the radiative corrections are large for beam particles of either charge. Predictions have been made of the s dependence of $d\sigma/dt$ at fixed center-of-mass angle based on a composite model of hadrons.[8] At fixed center-of-mass angle the t-dependent radiative effects will appear as an additional s dependence, amounting to changes in the cross section of about 17% for $\pi^+ p$ scattering and 25% for $\pi^- p$ scattering at 20 GeV/c and an angle of 90° in the c.m. system. Values of t corresponding to scattering at 90° in the c.m. system are denoted by an asterisk in Table I.

So far we have only considered reactions in which the target particle was a proton. Radiative corrections will of course also play a role in reactions involving nuclei. Elastic $\pi^- d$ scattering, for example, has been measured at 9 GeV/c incident momentum out to momentum transfers of 2 (GeV/c)^2.[9] Radiative corrections in the neighborhood of 5–10% might be expected in this region. While corrections of this size cannot reconcile the present discrepancies between theory and experiment in the double-scattering region, they do act to reduce the difference, and they certainly must be included in future work involving more precise experiments and theories.

Similar comments can be made about heavier nuclei. The range of momentum transfers over which the elastic scattering cross section is appreciable is not large. Momentum transfers of this size would not in themselves lead to significant radiative corrections. While Z is greater than unity, the recoiling mass is much heavier now, so it is not clear whether the Z and Z^2 terms are any more important than before. However, in order to ensure the exclusion of inelastic events we must now choose ΔE to be proportionately much smaller than before. For example, the first excited state of C^{12} is at an excitation of 4.43 MeV. If a $\pi^- C^{12}$ scattering experiment were performed with sufficient resolution to permit separation of the elastic peak from the inelastic contributions, the radiative correction would amount to about 7% for an incident momentum of 1 GeV/c and $t=-0.5$ GeV/c. The same correction for $\pi^+ C^{12}$ scattering would be about 5.5%. In a similar experiment for πHe^4 scattering for incident momentum 1 GeV/c and $t=-1$ (GeV/c)^2 (ΔE now corresponds to an excitation energy of 20 MeV) the radiative corrections are 6.9% for $\pi^- He^4$ scattering and 4.6% for $\pi^+ He^4$ scattering.

IV. EXPERIMENTAL CONSIDERATIONS

It is perhaps worth emphasizing once more that the radiative correction calculation in Sec. III applies to a particular experimental situation, namely detecting the elastically scattered particle at fixed angle and measuring its momentum. It does not apply when the recoil particle is detected instead, although an appropriate expression can be derived from our formalism by defining particle

3 rather than particle 4 as the recoil particle. Although Eq. (3.1) is no longer applicable, everything derived up to that point is. This situation is discussed further in Sec. V. Coincidence experiments represent another situation again. Owing to kinematic correlations the phase-space integrations become quite complicated, and a general expression representing the correction for a coincidence experiment cannot be obtained.

Another point worth emphasizing is that the corrections calculated here are only illustrative of those applied in a real experiment. This is because of the ambiguity associated with the location of the momentum cut and the effects of finite resolution, which vary from one experiment to another and which we have so far ignored. In Sec. II we assumed the beam and spectrometer momentum resolution to be of sufficiently high quality that the observed width of the elastic peak could be attributed mainly to the presence of a radiative tail. This is rarely a realistic assumption. With such an assumption it was meaningful to speak of a unique separation Δp_3 between the elastic peak and the momentum cut. This is no longer possible, if the peak width becomes a substantial fraction of the nominal separation. We can still calculate a radiative correction in such a situation if the data are distributed in momentum bins whose width is small compared with the nominal separation. Each bin can then be treated as an elastic peak with a radiative correction, calculated from (2.20), whose size is determined by the width of the bin. The yield in each bin is depleted by the loss of events to lower-energy bins through radiation and augmented by events lost by the same process from higher-energy bins. One can then radiatively correct the spectrum iteratively by beginning at the highest-momentum bin and working down. This procedure is commonly used in electron scattering, and the reader is referred to the literature for details.[10] Notice that we are still assuming that the resolution of the beam-spectrometer system is good enough that there is no significant overlap of the elastic peak and inelastic continuum. An overlap situation would be difficult to analyze.

It is probably unnecessary to add that the expressions derived here cannot be applied to colliding-beam experiments. While a considerable amount of work has been done for leptonic reactions, nothing, to the author's knowledge, exists for ISR-type hadronic reactions.

V. VIRTUAL-PHOTON EFFECTS AND OTHER CORRECTIONS

We have already included the effects of radiative diagrams of the form illustrated in Fig. 1 so far as the cancellation of the infrared divergences is concerned, but we have not included any contributions to δ from these diagrams. We will discuss here the errors associated with such omissions.

To begin with, we cannot really calculate these contributions, because the particles have structure, and we do not know their form factors everywhere. We must therefore fall back on rough estimates based on neglecting this internal structure and treating the hadrons like point particles. For $-t \gg \mu^2$ the vertex correction for a spinless particle of mass μ leads to an additional term in δ of the form[11,12]

$$\delta_{\text{vertex}} = -\frac{2\alpha}{\pi}\left[\ln\left(-\frac{t}{\mu^2}\right) - 1\right].$$

For a pion the vertex correction is only about 3.5%, even at $-t = 100\ (\text{GeV}/c)^2$. It is of course even smaller for heavier particles. The exchange of virtual photons between the incoming or outgoing hadrons and vacuum polarization effects represent Coulomb corrections to the scattering process. These may be significant at very small angles, where Coulomb-nuclear interference effects become important.[13]

We have made a number of assumptions in deriving (2.21), and we must now consider their validity and limitations. These approximations are discussed by Meister and Yennie.[3] In performing the integration in (2.8) we assigned the elastic peak values to p_3 and p_4 appearing in χ^2. This approximation is good, so long as the energy of particle 4, the recoil particle, is nonrelativistic in the special reference frame in which the photon angular distribution is isotropic. Using the notation of Meister and Yennie in which p_3 and p_4 represent the four-momenta at the elastic peak, while p_3' and p_4' are the four-momenta when a photon is emitted, we can write

$$x = (p_4' + k)^2 - M^2 = 2(p_3 - p_3')\cdot(p_1 + p_2)\,. \tag{5.1}$$

In our type of experiment $\vec{p}_3$ and $\vec{p}_3'$ are parallel. Therefore, to good enough accuracy,

$$x = 2M\eta'\delta E_3 \approx 2M\eta\delta p_3\,. \tag{5.2}$$

It can then be shown that the energy of the photon in the special reference frame is given by[3]

$$\tilde{\omega} = \frac{\frac{1}{2}x}{(M^2 + x)^{1/2}}\,. \tag{5.3}$$

The energy of particle 4 is $\tilde{E}_4 = (\tilde{\omega}^2 + M^2)^{1/2}$. Let x_1 be the maximum value of x. If $x_1 \ll M^2$, particle 4 is nonrelativistic and our approximation is valid. If $x_1 \gg M^2$ our approximation is poor, and we cannot neglect the dependence of p_3 and p_4 on k in χ^2.

This leads to a correction term to δ arising from the energy dependence of the recoil particle[3,14]:

$$\delta_4 = \frac{\alpha}{2\pi} Z^2 \left[\ln\left(1 + \frac{x_1}{M^2}\right)\right]^2 . \tag{5.4}$$

This term is never significant in our calculations. For the case where the recoil particle is detected, and the scattered particle is light, this term should be considered. It then takes the form[3]

$$\delta_3 = \frac{\alpha}{2\pi} \left[\ln\left(1 + \frac{x_2}{\mu^2}\right)\right]^2 , \tag{5.5}$$

where x_2 is the maximum value of

$$x = (p_3' + k)^2 - \mu^2$$
$$= \frac{2p_4}{E_4} \frac{E_1 + M}{E_4 + M} M \delta p_4 . \tag{5.6}$$

This term is still quite small, amounting to ~1% in πp scattering. However, it is important for the case of elastic electron-proton scattering when the proton is detected. In that case μ in (5.5) is replaced by the electron mass, while x_2 remains essentially unchanged.

We next consider our neglect of spin effects. The spin terms introduce extra powers of k in the numerator of the amplitude and so may lead to significant effects in the case of a relativistic undetected final-state fermion. In this situation a photon emitted parallel to the undetected particle could carry off most of the unobserved energy. (Another effect is the dependence of the final-state fermion projection operator on k. This is included in the correction term below.) This effect is again important in electron-proton scattering where the proton is detected. In our situation, since the π and K are spinless, we need only worry about this effect in connection with the recoil proton. The correction term is given by[3]

$$\delta_{\text{spin}} = -\frac{\alpha Z^2}{4\pi} \ln\left(1 + \frac{x_1}{M^2}\right) . \tag{5.7}$$

This term is even smaller than the previous correction.

Another effect we have not considered is the variation of the basic hadronic reaction with k. If the elastic cross section depends strongly on t or s, for example, the k dependence should be kept inside the integral, and t and s should be evaluated at $(p_1 - p_3' - k)^2$ and $(p_1 + p_2 - k)^2$, respectively. This effect appears to be quite small everywhere except in the backward direction. The backward peak is a strong function of $u = (p_2 - p_3')^2$, and the range of u in going from the elastic peak down to the cutoff in p_3 used in Table I causes a variation in the πp cross section of the order of 40%. Because of the $1/\omega$ form of the integrand, this variation probably does not affect the correction much. Furthermore, if the elastic peak momentum spectrum is differentially corrected as discussed in Sec. IV and the elastic cross section in each momentum bin is evaluated at a value of u appropriate to that bin, the error will be negligible. Again details are to be found in electron-scattering papers.[10]

More violent variations in the cross section will at some point begin to have an effect on the form of the radiative corrections, of course. This is not an academic point. Rapid variations (Ericson fluctuations[15]) in the backward πp elastic cross sections at 5 GeV/c have been reported recently.[16] These variations presumably arise from statistical fluctuations in the direct-channel resonance level spacings in situations where a considerable overlapping of neighboring resonances occurs. Considerable interaction between radiative effects and these fluctuations is possible. On the one hand, a rapidly varying cross section invalidates the approximations made in evaluating the integral in Eq. (2.7). This problem can be avoided by bringing $E_{3\,\text{min}}$ sufficiently close to $E_{3\,\text{max}}$ that the cross section is essentially constant. A more subtle problem is that radiation in the initial state will degrade the energy resolution of the incident beam, which will tend to reduce the size of the fluctuations (since more resonances will then be averaged over). The fact that these fluctuations are observed indicates that this problem is not acute. However, it may partially account for the considerably smaller fluctuations seen in the $\pi^- p$ channel (where radiative effects are relatively large) as opposed to the $\pi^+ p$ channel.

The complications mentioned here are probably not too serious for the πp case, because the direct-channel resonances which produce the fluctuations represent only a small contribution to the differential cross section. Furthermore, fluctuations should be absent in exotic reactions such as $K^+ p \to K^+ p$ or $pp \to pp$. They would presumably be largest in reactions which are nonexotic in the s channel but exotic in the u channel, such as $K^- p \to K^- p$ and $\bar{p}p \to \bar{p}p$.[17]

VI. DISCUSSION

We have seen that radiative corrections, usually considered to be significant only in electron scattering, are by no means negligible in hadronic processes. Indeed, for the case of backward $\pi^- p$ scattering the corrections can exceed 30% for incident momenta in the range of 10–20 GeV/c, a kinematic region which has already been explored experimentally. While the slope of the backward peak will not be much affected by radiative effects,[18] the absolute cross section is strongly affected, and the size of the effect is reaction-dependent. Thus radiative effects will strongly influence isotopic spin relations between $\pi^- p \to p\pi^-$, $\pi^+ p \to p\pi^+$, and $\pi^- p \to n\pi^0$. Another example of a situation where radiative effects may play a significant role is the SU(3) relation at backward angles,

$$\frac{d\sigma}{d\Omega}(K^- p \to \Sigma^+ \pi^-) = \frac{d\sigma}{d\Omega}(\pi^- p \to p\pi^-),$$

where both reactions are expected to proceed through Δ_δ exchange. At lower energies, where direct-channel resonances are dominant, the radiative correction is smaller but still not negligible, and, because of its angular dependence, it may significantly affect partial-wave analyses above 1–2 GeV. At forward angles the radiative corrections are smaller but are still not negligible. Furthermore, they grow logarithmically with energy. Radiative corrections may be important in hadron-nucleus scattering and are probably already needed in the analysis of πd scattering at large momentum transfers.

As has been mentioned before, the formalism developed here can be easily extended to cover many two-body processes. One application worth mentioning, since it figures in the isotopic spin relations mentioned above, is $\pi^- p \to \pi^0 n$. Besides the effects discussed in this paper, the measured cross section may be influenced in another way. If the γ rays from the decaying π^0 are detected, analysis of the event may be complicated by the presence of other photons radiated by the incident π^- or p.[19]

Radiative corrections in inelastic reactions are much more difficult to deal with, yet we know that they are quite important in deep-inelastic ep scattering.[20] If radiative corrections are not made in inelastic electron-nucleon scattering experiments, the measured inelastic structure functions can never approach the Bjorken scaling limit. In principle the same line of reasoning applies to hadronic inclusive reactions. Whether radiative processes will have an observable effect on the approach to scaling of inclusive reactions at NAL energies is at the moment an open question.

We have seen that sizable radiative effects exist in elastic hadronic interactions. They are of sufficient magnitude in some cases to probably require reanalysis and reinterpretation of the experimental results. At higher accelerator energies radiative effects will become still larger.

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†Present address: Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850.

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[16]F. H. Schmidt, C. Baglin, P. J. Carlson, A. Eide, V. Gracco, E. Johansson, and A. Lundy, Phys. Lett. 45B, 157 (1973).
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[18]Suppose the backward peak is described by the simple Regge expression $d\sigma/du = K(u)s^{2[\alpha(u)-1]}$. While $K(u)$ is directly affected by the radiative correction, the slope $\alpha(u)$ is related only logarithmically to $d\sigma/du$ and is therefore rather insensitive to radiative effects. Furthermore, the range of u over which the backward peak is prominent corresponds in general to a relatively small fractional change in t; the radiative correction is therefore relatively constant over this peak.
[19]L. Rosenson and D. Barton, private communiaation.
[20]Current radiative correction programs for this reaction include radiation only from the electron and ignore that from hadrons. Very crude estimates suggest that over the kinematic range presently studied hadronic radiative effects of several percent might exist.

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Decay $E^0 \to \nu\gamma$ in the Georgi-Glashow model*

So Young Pi and Jack Smith
The Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790
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We compute the decay rate $\Gamma(E^0 \to \nu_e\gamma)$ in the Georgi-Glashow model where E^0 is a neutral heavy electron. The calculation is done in the renormalizable gauge where $\xi = 1$ and the diagrams have only logarithmic singularities. Analogous results hold for the decay $\Gamma(M^0 \to \nu_\mu\gamma)$. We find that $\Gamma(E^0 \to \nu_e\gamma)/[\Gamma(E^0 \to e^-e^+\nu_e) + \Gamma(E^0 \to \mu^+e^-\nu_\mu)]$ ranges from roughly $5\alpha/\pi$ to $15\alpha/\pi$ depending upon the mass of the neutral lepton and charged boson.

I. INTRODUCTION

Recent experimental results from CERN[1] and NAL[2] have indicated that there is a new phenomenon in weak-interaction theory which does not produce events with charged muons. Although there has always been motivation to search for new effects in weak-interaction theory, especially strangeness-changing neutral currents, the recent intensified searches are a direct consequence of new theoretical advances. Due to the work of Weinberg[3] and Salam,[4] we now have the possibility of unifying the theories of weak and electromagnetic interactions. In general two main approaches have been studied. One predicts the existence of new neutral currents, with $\Delta S = 0$, which interact with hadrons and leptons, leading to muonless events. The other approach follows from work by Georgi and Glashow,[5] who suggest that there exist heavy (charged and neutral) leptons which probably decay rapidly into hadrons. If their decay branching ratios into muons are very small their experimental signature is also the absence of muons among the final particles. Some general comments on their possible decay modes have been given by Bjorken and Llewellyn Smith.[6]

In this paper we present a calculation of one possible decay mode of the neutral heavy electron, which we call E^0. The decay mode under consideration is $E^0 \to \nu_e\gamma$. There are several reasons for having an accurate evaluation of this decay rate. First, there is the obvious reason that it could be important experimentally. The branching ratio for $\Gamma(E^0 \to \nu_e\gamma)/\Gamma(E^0 \to e^-e^+\nu_e)$ can be calculated exactly and gives us a model-dependent lower bound on the experimental rate for events without leptons to that for events with leptons. Second, it is of importance when looking at the inverse problem, namely production by neutrinos of heavy neutral leptons in the Coulomb field of a nucleus. This problem is considerably more involved because of complications with hadrons and will be discussed in a later publication. Third, the problem involves the usual complications of higher-order calculations in unified field theories. We have investigated some of the proposed regularization schemes to become more familiar with their pitfalls and will give some general remarks later.

Although we have chosen to discuss the decay $E^0 \to \nu_e\gamma$ in one specific model, namely the Georgi-Glashow O(3) model, it should be pointed out that there exist alternative models, for instance in the