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Multiperipheral multifireball model and the inclusive cross section

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A multiperipheral multifireball model with a rising total cross section in a certain energy region is described, and the single-fireball contribution to the single-particle inclusive cross section in the pionization region is studied. The total cross section in this model rises like $\ln s$ near the single-fireball threshold, changes to a $\ln(\ln s)$ behavior until it reaches a maximum at an extremely high energy, and eventually falls to zero. The magnitude of the possible rise is proportional to the triple-Pomeron coupling. For the single-particle inclusive cross section in the pionization region, the contribution from the term proportional to a Pomeron-Reggeon-Reggeon coupling is shown to be as important as the term proportional to the triple-Pomeron coupling. The contribution from the Pomeron-Reggeon-Reggeon term is estimated by extrapolating the required couplings from the scattering data in the intermediate energy region, and it contributes to a rise of about 16 mb. The rise of the inclusive cross section due to the triple-Pomeron term is about 7 mb when the rise of the total cross section is assumed to be 4 mb at CERN Intersecting Storage Rings energy. The over-all rise of the inclusive cross section in this model is thus about 23 mb.

I. INTRODUCTION

The results of the recent ISR (CERN Intersecting Storage Rings) high-energy production experiments, e.g., the rising proton-proton total cross section, the rising single-particle inclusive cross section in the pionization region, and so on, make the predictions of pure multiperipheral models with only short-range correlations look rather unrealistic. One way to modify the pure multiperipheral picture is to introduce a two-component picture,[1] and various works have been published along this direction.[2] There is a kind of detailed model within the spirit of the two-component picture which uses the multi-Regge exchange model[3] as its basis. This kind of model assumes that the multiple exchange of the Pomeranchuk singularities will generate long-range correlations, and that the production mechanism besides the Pomeranchuk singularity exchange is purely multiperipheral, so we call it a "multiperipheral multifireball model." The origin of this kind of model can be dated back to the Chew-Pignotti model[4] and to a series of papers[5,6] utilizing the ABFST model[7] as the responsible purely multiperipheral mechanism.

The aim of this article is to construct a multiperipheral multifireball model with a rising total cross section in a certain energy region, and investigate in detail the single-fireball effect of this model to the rising behavior of the single-particle inclusive cross section at the pionization region. We assume that the Pomeranchuk singularity consists of a set of complicated Regge singularities, in which there is a Regge-pole component with the intercept slightly less than unity (hereafter we call it "the Pomeranchuk pole"). We further assume that the Pomeranchuk pole is generated by the purely multiperipheral component, and that only the multiple exchange of this Pomeranchuk pole is as important as the purely multiperipheral process for the high-energy production amplitude. The approach which tries to impose consistency between the input and the output Pomeranchuk singularity, e.g., the perturbative approach toward the renormalization of the Pomeron,[8] is not pursued in this article. The rising behavior of the total cross section is generated in our model by the introduction of a switch-on factor for a Pomeranchuk-pole propagator. This switch-on factor becomes one in the Regge asymptotic region, but damps down the contribution of the Pomeranchuk-pole exchange when it is extrapolated to the non-Regge region according to the duality argument. This switch-on factor makes our model resemble to some extent the Chew and Snider complex-pole model.[9] Since we are interested only in the effect of this model on the rising behavior of cross sections over a certain energy region, no effort is made to relate such rising behavior to some specific kinds of Regge singularities, e.g., complex poles.[10]

Our model indicates that the rising of the total cross section, as expected, is due to the term linearly proportional to the triple-Pomeron coupling (PPP). The total cross section in this model will rise to a maximum and then fall to zero. The energy to reach the maximum is very high (an order-of-magnitude estimate gives $\sqrt{s} \approx 10^3$ GeV). The total cross section starts to rise from the single-fireball threshold with a $\ln s$ behavior, which then changes to a $\ln(\ln s)$ behavior before it reaches the maximum. For the single-particle inclusive cross section the term proportional to the Pomeron-Reggeon-Reggeon coupling (*PRR*) is as important as that proportional to the triple-Pomeron coupling. The threshold of the triple-Pomeron term in the inclusive cross section is much higher than that in the total cross section in our model, so the rise of the inclusive cross section in the pionization region will be more marked over a wider energy region than the rise of the total cross section. The rising behavior of the inclusive cross section in the pionization region is due to the existence of the switch-on factors and the exponential damping of the Regge-pole exchange amplitude when the absolute value of the momentum transfer increases.

The contribution of the *PRR* term to the rise of the inclusive cross section in the pionization region is estimated by extrapolating the required couplings from the scattering data at the intermediate energy region, where the single-fireball effect can be neglected. The over-all rise due to the *PRR* term is of the order of 16 mb. The rise due to the *PPP* term may overlap with that from the neglected multifireball events, but the estimate of the over-all rise due to the term *PPP* gives a lower limit to the possible rise of the inclusive cross section at higher energy. The order of the rise from the *PPP* term is about 7 mb when the rise of the total cross section at ISR energy is taken to be about 4 mb.

II. TOTAL CROSS SECTION AND ITS RISING BEHAVIOR

We consider the scattering of only one kind of scalar meson with mass μ throughout this article, and all the effects of the total angular momentum of a fireball are neglected. As discussed in Sec. I, we consider only the multiple exchange of a Pomeranchuk pole with the intercept slightly less than unity as well as the purely multiperipheral component in a production process. From this simplified assumption, the forward absorptive part of an elastic scattering amplitude can be decomposed into multifireball terms.[12] We write it as

$$\mathrm{Im}A(s) = B_0(s) + B_1(s) + B_r(s),$$

where $B_0(s)$, $B_1(s)$, and $B_r(s)$ are expressed graphically in Figs. 1(a), 1(b), and 1(c), respectively. The bubble denoted by M in the figures implies a purely multiperipheral amplitude with only short-range correlations, and the wavy line denoted by P implies the Pomeranchuk pole. We note that the purely multiperipheral production amplitude does not contain any Pomeranchuk-pole exchange, according to our basic assumption mentioned in Sec. I. The term $B_0(s)$ is the combination of the

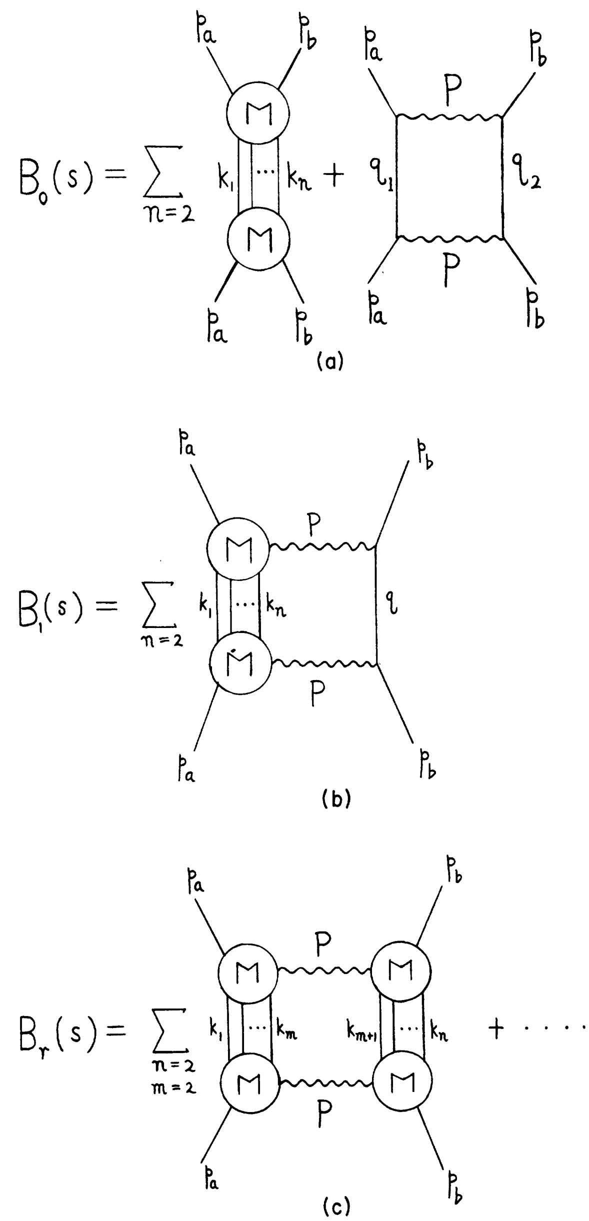

FIG. 1. (a) The pure-multiperipheral component and the simplest two-Pomeranchuk–pole branch-point contribution to the forward absorptive part of an elastic amplitude. The bubble M denotes the pure-multiperipheral amplitude and the wavy line P denotes the Pomeranchuk-pole propagator. (b) One-fireball contribution to the forward absorptive part of the elastic amplitude. (c) The multifireball contributions to the forward absorptive part of the elastic amplitude.

purely multiperipheral component and the simplest two-Pomeron branch-point component. The term $B_1(s)$ is the single-fireball contribution, and the term $B_r(s)$ includes all the remaining multifireball contributions. All the possible interference diagrams are neglected. We first consider the contribution of $B_1(s)$ in this section.

The energy region of interest to us in this article is so high that the elastic scattering amplitude is completely in its Regge asymptotic region, and an exchanged Reggeon propagator of the elastic amplitude takes its ordinary form $s^{\alpha(t)}$. But for the single-fireball term $B_1(s)$ such a high energy still may not assure the Regge asymptotic region for the exchanged Pomeranchuk pole, since the Pomeranchuk-pole propagator depends on s/s_1, where s is the square of the incoming energy in the center-of-mass frame and s_1 is the mass squared of the fireball. This implies that in the process of calculating $B_1(s)$, we must extrapolate the Pomeranchuk-pole exchange amplitude, as in Fig. 1(b), out of its Regge asymptotic region. Therefore we insert in the Pomeranchuk-pole propagator a switch-on factor, which becomes one in the Regge asymptotic region and becomes zero at the lowest kinematical threshold. Some modifications of a pure Regge-pole propagator are necessary when we try to express the low-energy amplitude also in a Reggeon-exchange form,[13] and the introduction of such a switch-on factor is a phenomenologically simple representation of the modification used in some analyses of finite-energy sum rules.[14] Unfortunately the determination of such a switch-on factor from the low-energy data, e.g., through the finite-energy sum rules, is somewhat ambiguous, so we consider the introduction of such a switch-on factor in our model as a purely theoretical device to generate a kind of threshold effect for the fireball events.

The term $B_1(s)$ can be written, using the factorizability of the Pomeranchuk pole, as

$$B_1(s)=\sum_{n=2}\int d^4q\,\delta^+(q^2-\mu^2)\int\prod_{j=1}^{n}d^4k_j\,\delta^+(k_j{}^2-\mu^2)\delta^4\Big(p_a+p_b-q-\sum_{j=1}^{n}k_j\Big)$$
$$\times|A_M(p_a;k_1,k_2,\ldots,k_n;Q)|^2|f_P(s,s_1,t)|^2\beta_P{}^2(t), \tag{2.1}$$

where

$$s=(p_a+p_b)^2,$$
$$s_1=(k_1+k_2+\cdots+k_n)^2,$$
$$Q=p_b-q,\quad t=Q^2.$$

The amplitude A_M stands for the purely multiperipheral production amplitude of the process $a+P\to 1+2+\cdots+n$, the function f_P is the Pomeranchuk-pole propagator, and $\beta_P(t)$ is the Pomeron-particle-particle coupling. We define

$$\tilde g(p_a{}^2,K^2,Q^2)=\sum_{n=2}\int\prod_{j=1}^{n}d^4k_j\,\delta^+(k_j{}^2-\mu^2)\delta^4\Big(K-\sum_{j=1}^{n}k_j\Big)|A_M(p_a;k_1,\ldots,k_n;Q)|^2,$$

where the four-momentum K is defined as

$$K=\sum_{j=1}^{n}k_j=p_a+p_b-q.$$

The arguments of $\tilde g$ are chosen from the observation that $\tilde g$ is the forward absorptive part of the elastic scattering amplitude of the process $a+P\to a+P$. Using this $\tilde g$, Eq. (2.1) can be written as

$$B_1(s)=\int ds_1\int d^4q\,\delta^+(q^2-\mu^2)\delta\big((p_a+p_b-q)^2-s_1\big)|f_P(s,s_1,t)|^2\beta_P{}^2(t)\tilde g(\mu^2,s_1,t). \tag{2.2}$$

We parameterize the four-momenta p_a, p_b, and q as

$$p_a=(\tfrac{1}{2}\sqrt{s},0,0,k),$$
$$p_b=(\tfrac{1}{2}\sqrt{s},0,0,-k),$$
$$q=\big((\mu^2+q_L{}^2)^{1/2},\ q_L\sin\theta\cos\varphi,\ q_L\sin\theta\sin\varphi,\ q_L\cos\theta\big),$$

where

$$K=\tfrac{1}{2}(s-4\mu^2)^{1/2}.$$

Since the integrand of Eq. (2.2) is independent of φ, we carry that integration out, and then carry out the q_L integration with the help of the δ functions to obtain

$$B_1(s) = \frac{\pi\gamma_P}{4k\sqrt{s}} \int ds_1 \rho_P(s, s_1) \times \int_{t_{\max}}^{t_{\min}} dt\, \tilde{g}(\mu^2, s_1, t)\, e^{c_0 t} \left(\frac{s}{s_1}\right)^{2\alpha_P(t)}, \tag{2.3}$$

where the $\cos\theta$ integration is transformed into a t integration by the relations

$$t = (q - p_b)^2 = 2\mu^2 - \tfrac{1}{2}(s + \mu^2 - s_1) - 2kq_L \cos\theta,$$

$$q_L = \frac{[(s - (\sqrt{s_1} + \mu)^2)(s - (\sqrt{s_1} - \mu)^2)]^{1/2}}{4s},$$

and

$$t_{\min} = 2\mu^2 - \tfrac{1}{2}(s + \mu^2 - s_1) + 2kq_L, \quad t_{\max} = 2\mu^2 - \tfrac{1}{2}(s + \mu^2 - s_1) - 2kq_L. \tag{2.4}$$

The function $\beta_P(t)$ in Eq. (2.2) is assumed to be

$$\beta_P^2(t) = \gamma_P^2 e^{c_0 t},$$

and the Pomeranchuk-pole propagator f_P is

$$f_P(s, s_1, t) = \rho_P(s, s_1)\left(\frac{s}{s_1}\right)^{\alpha_P(t)}, \tag{2.5}$$

where ρ_P is the switch-on factor, and $\alpha_P(t)$ is the Pomeranchuk-pole trajectory with the linear form $\alpha_P(t) = a_P + b_P(t)$.

We assume that the purely multiperipheral component generates the Pomeranchuk pole and an ordinary-meson pole with the intercept $\frac{1}{2}$, which we subsequently denote as R. Since $\tilde{g}$ is the forward absorptive part of the purely multiperipheral amplitude of the process $a + P \to a + P$, it can be written as

$$\tilde{g}(\mu^2, s_1, t) = \tilde{g}_{PPP}(\mu^2, s_1, t) + \tilde{g}_{RPP}(\mu^2, s_1, t),$$

where

$$\tilde{g}_{PPP}(\mu^2, s_1, t) = \pi\gamma_P \epsilon_{PPP} e^{c_1 t} \left(\frac{s_1}{\mu^2}\right)^{\alpha_P(0)} \rho_P(s_1, \mu^2), \tag{2.6}$$

$$g_{RPP}(\mu^2, s_1, t) = \pi\gamma_R \epsilon_{RPP} e^{c_2 t} \left(\frac{s_1}{\mu^2}\right)^{\alpha_R(0)} \rho_R(s_1, \mu^2).$$

The constants ϵ_{PPP} and ϵ_{RPP} are the triple-Pomeron coupling and the Reggeon-Pomeron-Pomeron coupling, respectively. The function ρ_R is the switch-on factor for the ordinary-meson trajectory.

The switch-on factor $\rho(x, y)$ approaches unity when x/y is large, and approaches zero when x/y is small. The half-value point of $\rho(x, y)$ is denoted by $x/y = D$. We assume that in our model a Pomeranchuk pole and an ordinary Regge pole differ such that

$$D_P \gg D_R. \tag{2.7}$$

The value D_R for an ordinary meson Regge pole should be of the order of the kinematical threshold, i.e., $D_R \approx 4$, from the implication of the semilocal duality of the finite-energy sum rule.[15] From the Harari and Freund two-component duality,[16] the Pomeranchuk pole is dual to the background, and the ordinary Regge poles are dual to the resonance component. Equation (2.7) implies that we are assuming the smallness of the background in the low- and intermediate-energy regions. In order to simplify our model further, we assume that

$$\rho(x, y) = \begin{cases} 0 & \text{for } x/y < D \\ 1 & \text{for } x/y \geq D. \end{cases} \tag{2.8}$$

We only consider this step-function–type switch-on factor throughout the remaining part of this article. Though this kind of step-function–type switch-on factor is not realistic, it is enough to give an intuitive picture to our qualitative investigation, and the introduction of a more realistic switch-on factor will make the final solution quantitatively more complicated, but will not alter the general character.

The term $B_1(s)$ of Eq. (2.3) can then be separated into two parts, $B_1^{PPP}(s)$ and $B_1^{RPP}(s)$, by substituting $\tilde{g}$ of Eq. (2.6) into Eq. (2.3). We obtain

$$B_1^{PPP}(s) \approx \frac{\pi^2 \epsilon_{PPP} \gamma_P^3}{4k\sqrt{s}} \int_{D_P \mu^2}^{s/D_P} ds_1 s_1^{\alpha_P(0)} e^{(c_0 + c_1)t_{\min}} \left(\frac{s}{s_1}\right)^{2\alpha_P(t_{\min})} \frac{1}{c_0 + c_1 + 2b_P \ln(s/s_1)} \tag{2.9}$$

and

$$B_1^{RPP}(s) \approx \frac{\pi^2 \epsilon_{RPP} \gamma_R \gamma_P^2}{4k\sqrt{s}} \int_{4\mu^2}^{s/D_P} ds_1 s_1^{\alpha_R(0)} e^{(c_0 + c_2)t_{\min}} \left(\frac{s}{s_1}\right)^{2\alpha_P(t_{\min})} \frac{1}{c_0 + c_2 + 2b_P \ln(s/s_1)}, \tag{2.10}$$

where we have used Eq. (2.8) and put $D_R = 4$. Also we have used the property $|t_{\min}| < |t_{\max}|$, and the terms depending on $t_{\max}$ are neglected due to the exponential damping of the integrand as $|t|$ increases. The above expressions already show that B_1^{PPP} vanishes if $s < D_P^2\mu^2$, and B_1^{RPP} vanishes if $s < 4D_P\mu^2$.

The variable $t_{\min}$ is defined in Eq. (2.4) as a function of s_1. The main contributions of B_1^{PPP} and B_1^{RPP} come from the region of the s_1 integration where $t_{\min}$ is near zero. An explicit evaluation shows that $t_{\min}$ is almost zero for the whole integration region of s_1 of Eqs. (2.9) and (2.10) if $D_P \gg 1$. By taking $\alpha_R(0) = \frac{1}{2}$ and $\alpha_P(0) \approx 1$, we obtain from Eqs. (2.9) and (2.10)

$$B_1^{PPP}(s) \approx \begin{cases} 0 \quad \text{for } s < D_P^2\mu^2 \\ \dfrac{\pi^2 \epsilon_{PPP}\gamma_P^3}{4b_P} s^{2\alpha_P(0)-1}[\ln(\ln s - \ln D_P + (c_0 + c_1)/2b_P) - \ln(\ln D_P\mu^2 + (c_0+c_1)/2b_P)] \quad \text{for } s \geq D_P^2\mu^2 \end{cases} \tag{2.11}$$

and

$$B_1^{RPP}(s) \approx \begin{cases} 0 \quad \text{for } s < 4D_P\mu^2 \\ \dfrac{\pi^2 \epsilon_{RPP}\gamma_R\gamma_P^2}{4b_P} \dfrac{s^{2\alpha_P(0)-1}}{\ln s}\left(\dfrac{1}{2} - \dfrac{1}{s/D_P}\right) \quad \text{for } s \geq 4D_P\mu^2 \,. \end{cases} \tag{2.12}$$

The above results indicate that the term $B_1^{PPP}(s)$ starts to contribute at a threshold $s = D_P^2\mu^2$, and $B_1^{RPP}(s)$ starts to contribute at a threshold $s = 4D_P\mu^2$. The purely multiperipheral component in $B_0(s)$ gives a Pomeranchuk pole which starts to contribute at $s = D_P\mu^2$ in our idealized model. Therefore we may combine $B_1^{RPP}(s)$ with $B_0(s)$ as a constant background of the total cross section in the intermediate-energy region, and $B_1^{PPP}(s)$ gives a rise in the energy region $s > D_P^2\mu^2$. In order to maintain the constancy of the total cross section in the intermediate-energy region, the rising behavior of $B_1^{RPP}(s)$ must be offset by the decrease of the contribution from the ordinary meson Regge poles in the purely multiperipheral component. This can be achieved if ϵ_{RPP} is small. Then the term $B_r(s)$ of Fig. 1(c) is an order of magnitude smaller than $B_1(s)$, since the lowest-order term of $B_r(s)$ depends on the squares of the couplings ϵ_{PPP} and ϵ_{RPP}. We thus neglect the term $B_r(s)$.

We note the choice of μ. In the high-energy production experiments performed at NAL and ISR, the incoming particles are protons. The outgoing pions frequently form multipion resonances. Therefore as a rough approximation, we can choose $\mu \approx 1$ GeV. With this choice of μ, we only need $D_P \approx 20$ in order to relate the rise of $B_1^{PPP}(s)$ to the rise of the total cross section at ISR energy. Our D_P is related to the minimum rapidity gap due to the Pomeranchuk-pole exchange by the relation[9] $D_P \approx e^{\Delta}$, so Δ is about 3 for D_P about 20. Finally we consider the rising behavior of the total cross section. The term $B_1^{PPP}(s)/s$ will eventually go to zero after it reaches a maximum. But the energy to reach this maximum is

$$s_{\max} \approx D_P^2\mu^2 \exp\{\tfrac{1}{2}(\ln D_P)[1 - \alpha_P(0)]\} \,.$$

If we take $D_P \approx 20$, $1 - \alpha_P(0) \approx 0.02$, and $\mu \approx 1$ GeV, we have

$$(s_{\max})^{1/2} \approx 10^3 \text{ GeV}.$$

Before the incoming energy reaches such an extremely high energy, the total cross section in our model rises like $\ln s$ near $s = D_P^2$ and then turns to a $\ln(\ln s)$ behavior.

III. ONE-FIREBALL CONTRIBUTION TO THE SINGLE-PARTICLE INCLUSIVE CROSS SECTION

The contribution of the single-fireball events to the single-particle inclusive cross section in the pionization region of the process $a + b \to c + X$ is considered in this section. The multifireball events have a higher threshold than the single-fireball events in our model, and their contributions are neglected in this article. We must consider two terms, $I_1(s)$ and $I_2(s)$, as expressed in Figs. 2(a) and 2(b), respectively. They are discussed in Secs. III A and III B. A summary and some order-of-magnitude estimates of the effect of the single-fireball terms are presented in Sec. III C.

A. The term $I_1(s)$

The term $I_1(s)$ of Fig. 2(a) can be written as

$$I_1(s)=\sum_{n=2}\int d^4q\,\delta^+(q^2-\mu^2)\prod_{j=1}^{n}d^4k_j\,\delta^+(k_j{}^2-\mu^2)|A_M(p_a;k_1,\ldots,k_n;Q)|^2|f_P(s,s_1,s_2,t)|^2|\beta(p_b,Q,q,p_c)|^2,$$

where

$$\begin{aligned}Q&=p_b-p_c-q,\\ t&=Q^2,\\ s_1&=(k_1+k_2+\cdots+k_n)^2\\ &=(p_a+p_b-q-p_c)^2,\\ s_2&=(q+p_c)^2.\end{aligned}\tag{3.1}$$

The function β is the amplitude for the process $b+P\to q+c$. We denote

$$\tilde g(p_a{}^2,K^2,Q^2)=\sum_{n=2}\int\prod_{j=1}^{n}d^4k_j\,\delta^+(k_j{}^2-\mu^2)\times|A_M(p_a;k_1,\ldots,k_n;Q)|^2\times\delta^4\Big(K-\sum_{j=1}k_j\Big),$$

where the choice of the arguments of $\tilde g$ is based on the observation that $\tilde g$ is the forward absorptive part of the amplitude for the process $a+P\to a+P$. The four-momenta p_c and q are parameterized in the center-of-mass frame of a and b as

$$\begin{aligned}p_c&=((\mu^2+p_L{}^2)^{1/2},0,0,p_L),\\ q&=((\mu^2+q_L{}^2)^{1/2},q_L\sin\theta\cos\varphi,\ q_L\sin\theta\sin\varphi,\ q_L\cos\theta),\end{aligned}\tag{3.2}$$

where we only consider the case of zero transverse momentum for p_c. We further restrict our attention to the case $p_L=0$, which corresponds to the center of the rapidity plot, or at the point $x=0$ in terms of Feynman's variable x. The function $I_1(s)$ can then be simplified to

$$I_1(s)=\frac{\pi}{4k(\sqrt{s}-\mu)}\times\int ds_1\int_{t_{\max}}^{t_{\min}}dt\,\tilde g(\mu^2,s_1,t)\rho_P{}^2(4\mu^2s,s_1s_2)\times\left(\frac{4\mu^2s}{s_1s_2}\right)^{2\alpha_P(t)}e^{c_3t}\tilde\beta(s_2,u),\tag{3.3}$$

where we have put

$$f_P(s,s_1,s_2,t)=\rho_P(4\mu^2s,s_1s_2)\left(\frac{4\mu^2s}{s_1s_2}\right)^{\alpha_P(t)},$$

$$\beta(p_b,Q,q,p_c)=e^{c_3t}\tilde\beta(s_2,u),$$

and

$$u=(p_b-q)^2.$$

We also use the relations

$$\begin{aligned}t&=3\mu^2-(\sqrt{s}-2\mu)(\mu^2+q_L{}^2)^{1/2}-2kq_L\cos\theta-\mu\sqrt{s},\\ t_{\min}&=3\mu^2-(\sqrt{s}-2\mu)(\mu^2+q_L{}^2)^{1/2}-\mu\sqrt{s}+2kq_L,\\ t_{\max}&=3\mu^2-(\sqrt{s}-2\mu)(\mu^2+q_L{}^2)^{1/2}-\mu\sqrt{s}-2kq_L,\end{aligned}\tag{3.4}$$

and

$$q_L=\frac{[(s-s_1)(s-s_1+4\mu^2-4\mu s)]^{1/2}}{2(\sqrt{s}-\mu)}.$$

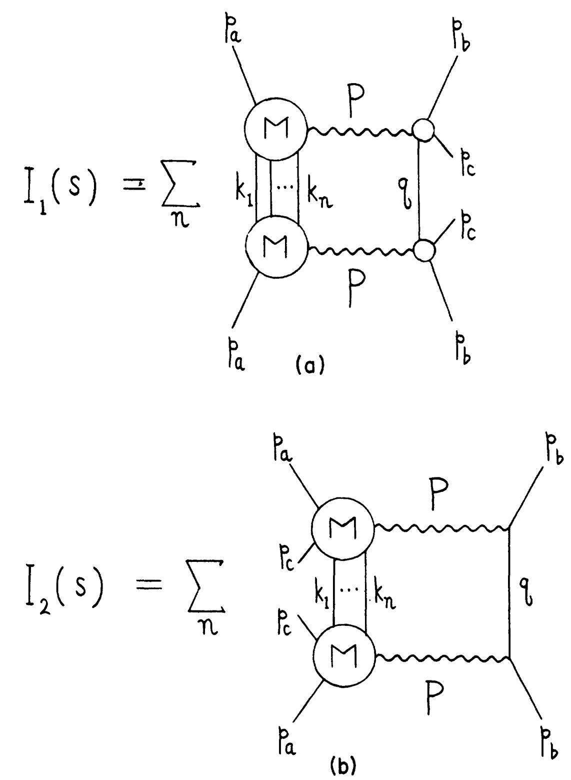

FIG. 2. (a) The first kind of single-fireball event in a single-particle inclusive cross section. (b) The second kind of single-fireball event in a single-particle inclusive cross section.

The important contribution of the t integration of $I_1(s)$ comes from the region $t\approx t_{\min}$. The main contribution of the s_1 integration of $I_1(s)$ in Eq. (3.3) comes from the region where $t_{\min}$ is near zero. The variable s_2 can be calculated from Eqs. (3.1), (3.2), and (3.4) as a function of s_1. The function

$$Y(s_1)=s_1 s_2$$

is an increasing function of s_1 in the region $s_1<\frac{1}{2}s$. The region $s_1\geq\frac{1}{2}s$ can be neglected due to the large value of $|t_{\min}|$ in this region. The existence of the switch-on factor ρ_P implies that the s_1 integration of Eq. (3.3) is further restricted by the inequality

$$s>\frac{D_P}{4\mu^2}\,Y(s_1)\geq\frac{D_P}{4\mu^2}\,Y(s_1=4\mu^2)\text{ for }s_1<\tfrac{1}{2}s\ .$$

Since $Y(s_1=4\mu^2)\approx 4\mu^3\sqrt{s}$ for $\sqrt{s}\gg\mu$, we obtain

$$s>\mu^2 D_P^{\,2}\ ,$$

otherwise $I_1(s)$ will vanish. This property of $I_1(s)$ is derived before assuming the detailed properties of $\tilde{g}$. This should be contrasted to the case of the total cross section, where only a piece of $B_1(s)$, i.e., B_1^{PPP}, vanishes if $s<D_P^{\,2}\mu^2$, and this property is derived from an assumption about the detailed properties of $\tilde{g}$ in Eq. (2.6).

The more detailed behavior of $I_1(s)$ can be studied by assuming a form of $\tilde{g}$ as in Eq. (2.6); then we obtain

$$I_1(s)=I_1^{PPP}(s)+I_1^{RPP}(s)\,,$$

with

$$I_1^{PPP}(s)\approx\frac{\pi^2\epsilon_{PPP}\gamma_P}{4k(\sqrt{s}-\mu)}\int_{D_P\mu^2}^{s_c}ds_1 s_1^{\,\alpha_P(0)}e^{(c_1+c_3)t_{\min}}\left[\frac{4\mu^2 s}{Y(s_1)}\right]^{2\alpha_P(t_{\min})}\frac{\tilde{\beta}(s_2,u_{\min})}{c_1+c_3+2b_P\ln(4\mu^2 s/Y(s_1))}$$

and

$$I_1^{RPP}(s)\approx\frac{\pi^2\epsilon_{RPP}\gamma_R}{4k(\sqrt{s}-\mu)}\int_{D_P\mu^2}^{s_c}ds_1 s_1^{\,\alpha_R(0)}e^{(c_2+c_3)t_{\min}}\left[\frac{4\mu^2 s}{Y(s_1)}\right]^{2\alpha_P(t_{\min})}\frac{\tilde{\beta}(s_2,u_{\min})}{c_2+c_3+2b_P\ln(4\mu^2 s/Y(s_1))}\ ,$$

where the t integration has been performed by keeping only the region of t near $t_{\min}$, and the parameter s_c is the upper bound of the s_1 integration obtained from the inequality $s>D_P Y(s_1)/4\mu^2$, and is

$$s_c\approx 4\mu\sqrt{s}\,/D_P\,.$$

The variable $u_{\min}$ is the value of $u=(p_b-q)^2$ at $\cos\theta=-1$. From the above expressions we see that $I_1^{PPP}(s)$ vanishes unless $s>\mu^2 D_P^{\,4}/16$, and $I_1^{RPP}(s)$ vanishes unless $s>\mu^2 D_P^{\,2}D_R^{\,2}/16$. Furthermore we have $Y(s_1)\approx\mu s_1\sqrt{s}$ for $s\gg\mu^2$, and may put

$$\tilde{\beta}^2(s_2,u_{\min})=\gamma_R^{\,2}\phi^2 e^{c_8 u_{\min}}(s_2/\mu^2)^{2\alpha_R(u_{\min})}\,,$$

where ϕ is the Reggeon-particle-Pomeron coupling. By approximating $t_{\min}\approx u_{\min}\approx 0$, and $\alpha_R(0)=\frac{1}{2}$, $\alpha_P(0)\approx 1$, we have

$$I_1^{PPP}(s)\approx\frac{32\pi^2\gamma_P\gamma_R^{\,2}\phi^2\epsilon_{PPP}\mu^2}{b_P D_P}\,\frac{s}{\ln s}\left(1-\frac{D_P^{\,2}\mu}{4\sqrt{s}}\right)$$

and (3.5)

$$I_1^{RPP}(s)\approx\frac{16\pi^2\mu^{3/2}\gamma_R^{\,3}\phi^2\epsilon_{RPP}}{b_P\sqrt{D_P}}\,\frac{s^{3/4}}{\ln s}\left\{1-\left(\frac{D_P D_R}{4s}\right)^{1/2}\right\}.$$

B. The term $I_2(s)$

The term $I_2(s)$ of Fig. 2(b) can be expressed as

$$I_2(s)=\int d^4q\ \delta^+(q^2-\mu^2)\int d^4K\,\delta^4(p_c+q+K-p_a-p_b)|f_P(s,s_2,t)|^2|\beta_P(t)|^2 G(\mu^2,s_1,s_2,t,t_1)\,,$$

where

$$G(\mu^2,s_1,s_2,t,t_1)=\sum_{n=2}\int\prod_{j=1}^{n}d^4k_j\,\delta^+(k_j^{\,2}-\mu^2)\delta^4\Bigl(K-\sum_{j=1}^{n}k_j\Bigr)\,|A_M(p_a;k_1,\ldots,k_n;Q)|^2\,,$$

and it depends on μ^2, s_1, s_2, t, and $t_1=(p_a-p_c)^2$ since it is the single-particle inclusive cross section of the process $a+P\to c+X$. The invariant variables are defined as

$$Q=p_b-q\,,\quad t=Q^2\,,$$
$$t_1=(p_a-p_c)^2\,,$$
$$s_1=(k_1+\cdots+k_n)^2=K^2\,,$$
$$s_2=(k_1+\cdots+k_n+p_c)^2=(K+p_c)^2\,.$$

The four-momenta p_c and q are parameterized as in Eq. (3.2) with the choice $p_L=0$. Choosing the Pomeranchuk-pole propagator f_P and the Pomeron-particle coupling β_P as in Sec. II, i.e.,

$$f_P(s,s_2,t)=\rho_P(s,s_2)\left(\frac{s}{s_2}\right)^{\alpha_P(t)}\,,$$
$$\beta_P{}^2(t)=\gamma_P{}^2e^{c_0t}\,,$$

we have

$$I_2(s)=\frac{\pi\gamma_P{}^2}{4k(\sqrt{s}-\mu)}\times\int ds_1\rho_P{}^2(s,s_2)\int_{t_{\max}}^{t_{\min}}dt\;e^{c_0t}\left(\frac{s}{s_2}\right)^{2\alpha_P(t)}\times G(\mu^2,s_1,s_2,t,t_1)\,,$$

where

$$t=2\mu^2-\sqrt{s}\,(\mu^2+q_L{}^2)^{1/2}-2kq_L\cos\theta\,,$$
$$t_{\min}=2\mu^2-\sqrt{s}\,(\mu^2+q_L{}^2)^{1/2}+2kq_L\,,$$
$$t_{\max}=2\mu^2-\sqrt{s}\,(\mu^2+q_L{}^2)^{1/2}-2kq_L\,,$$
$$q_L=\frac{[(s-s_1)(s-s_1+4\mu^2-4\mu\sqrt{s}\,)]^{1/2}}{2(\sqrt{s}-\mu)}\,,\tag{3.6}$$

and

$$t_1=(p_a-p_c)^2=2\mu^2-\mu\sqrt{s}\,.$$

The subenergy s_2 is, from its definition,

$$s_2=(K+p_c)^2=(p_a+p_b-q)^2=\mu\sqrt{s}+\frac{s_1\sqrt{s}-\mu^3}{\sqrt{s}-\mu}\,.\tag{3.7}$$

As a function of s_1, s_2 increases monotonically as s_1 increases. The switch-on factor $\rho_P(s,s_2)$ requires $s>s_2D_P$, and this means that $I_2(s)$ will vanish unless

$$s>D_P{}^2\mu^2\,.$$

This property is derived again without assuming the detailed structure of G, as in the case of $I_1(s)$.

The detailed structure of G is more complicated than that of $\tilde{g}$ of Eq. (2.6). To see its structure we first consider the upper limit of the s_1 integration of Eq. (3.5) from the inequality $s>D_Ps_2$ indicated by $\rho_P(s,s_2)$. From Eq. (3.7) we obtain s_c, the upper limit of the s_1 integration,

$$s_c\approx\frac{s}{D_P}-\mu\sqrt{s}\quad\text{for } s\gg\mu^2,\; D_P\gg1\,.\tag{3.8}$$

From Eq. (3.6) we see that $t_{\min}$ is near zero in the region $s_1\leq s_c$. The function $X(s_1)$ is defined as

$$X(s_1)=\frac{s_2}{s_1}\,.$$

From Eq. (3.7) we see that it decreases as s_1 increases. At $s_1=4\mu^2$, $X\approx\sqrt{s}/4\mu\gg1$, and at $s_1=s_c$, $X\approx1+\mu D_P/\sqrt{s}$. This implies that the purely multiperipheral amplitude of the process $a+P\to c+K$, $A_M(p_a+Q\to p_c+K)$, is in its Regge asymptotic region for small s_1. We extrapolate this Regge representation for A_M to the region $s_1\approx s_c$ by duality. The exchange Reggeon of A_M, of course, can only be an ordinary meson Regge pole. We consider an invariant variable u_1,

$$u_1=(p_c-Q)^2$$
$$=(p_c-p_b+q)^2$$
$$=\mu^2-\mu\sqrt{s}+2\mu(\mu^2+q_L{}^2)^{1/2}+t$$

as well as $t_1=(p_c-p_a)^2$. The invariant variable t_1 is of the order $-\mu\sqrt{s}$ and is independent of s_1. We denote

$$u_1^{\min}=\mu^2-\mu\sqrt{s}+2\mu(q_L{}^2+\mu^2)^{1/2}+t_{\min}\,,$$

which is a monotonically decreasing function of s_1. At $s_1=4\mu^2$, $u_1^{\min}\approx-\mu^3/\sqrt{s}$, and at $s_1=s_c$, $u_1^{\min}\approx-\mu\sqrt{s}/D_P$. This implies that $A_M(p_a+Q\to p_c+K)$ should be represented as a u_1-channel Reggeon exchange amplitude. Thus $I_2(s)$ can be expressed as in Fig. 3.

The term $I_2(s)$ in Fig. 3 can then be written as

$$I_2(s)=\frac{\pi\gamma_P{}^2}{4k(\sqrt{s}-\mu)}\int_{4\mu^2}^{s_c}ds_1\int_{t_{\max}}^{t_{\min}}dt\;e^{c_0t}\left(\frac{s}{s_2}\right)^{2\alpha_P(t)}\left(\frac{s_2}{s_1}\right)^{2\alpha_R(u_1)}\phi^2(t,u_1)\tilde{g}(\mu^2,s_1,u_1)\,,\tag{3.9}$$

where $\tilde{g}$ is the forward absorptive part of the purely multiperipheral amplitude of the process $a+R\to a+R$, and $\phi(t,u_1)$ is the Reggeon-particle-Pomeron coupling. We assume that $\tilde{g}$ has the form

$$\tilde{g}(\mu^2, s_1, u_1) = \pi\gamma_P\epsilon_{PRR}e^{c_5u_1}\left(\frac{s_1}{\mu^2}\right)^{\alpha_P(0)}\rho_P(s_1, \mu^2) + \pi\gamma_R\epsilon_{RRR}e^{c_6u_1}\left(\frac{s_1}{\mu^2}\right)^{\alpha_R(0)}\rho_R(s_1, \mu^2).$$

Then $I_2(s)$ can be written as

$$I_2(s) = I_2^P(s) + I_2^R(s),$$

where

$$I_2^P(s) = \frac{\pi^2\gamma_P^3\epsilon_{PRR}}{4k(\sqrt{s}-\mu)}\int_{D_P\mu^2}^{s_c} ds_1 \int_{t_{\max}}^{t_{\min}} dt\, e^{c_0t+c_5u_1}\left(\frac{s}{s_2}\right)^{2\alpha_P(t)}\left(\frac{s_2}{s_1}\right)^{2\alpha_R(u_1)}\left(\frac{s_1}{\mu^2}\right)^{\alpha_P(0)}\phi^2(t, u_1)$$

and

$$I_2^R(s) = \frac{\pi^2\gamma_R\gamma_P^2\epsilon_{RRR}}{4k(\sqrt{s}-\mu)}\int_{D_R\mu^2}^{s_c} ds_1 \int_{t_{\max}}^{t_{\min}} dt\, e^{c_0t+c_6u_1}\left(\frac{s}{s_2}\right)^{2\alpha_P(t)}\left(\frac{s_2}{s_1}\right)^{2\alpha_R(u_1)}\left(\frac{s_1}{\mu^2}\right)^{\alpha_R(0)}\phi^2(t, u_1).$$

These expressions impose a stronger constraint on the nonvanishing regions of $I_2^P(s)$ and $I_2^R(s)$. The term $I_2^P(s)$ vanishes unless $s > \frac{1}{2}\mu^2 D_P^2(3+\sqrt{5})$ from the condition $s_c > D_P\mu^2$, and the term $I_2^R(s)$ will vanish unless $s > (D_P^2 + 2D_PD_R)\mu^2 \approx D_P^2\mu^2$.

The expressions of $I_2^P(s)$ and $I_2^R(s)$ can further be simplified by assuming

$$\phi(t, u_1) = e^{c_7u_1}\phi,$$

since $t_{\min}$ is near zero and t must be close to $t_{\min}$ in order to make considerable contribution to $I_2(s)$. We have

$$I_2^R(s) \approx \frac{\pi^2\gamma_R\gamma_P^2\epsilon_{RRR}\phi^2}{2b_P\mu^2}\,\frac{s^{2\alpha_P(0)-1}}{\ln s}\left(\frac{\sqrt{s}}{D_P} - D_P\right)^{1/2}, \tag{3.10}$$

for s not very far from $\mu^2D_P^2$. For s large, we have

$$I_2^R(s) \approx \frac{\pi^3\gamma_R\gamma_P^2\epsilon_{RRR}\phi^2}{8b_P}\,\frac{s^{2\alpha_P(0)-1-1/4}}{\ln s}. \tag{3.11}$$

For the term $I_2^P(s)$ we obtain

$$I_2^P(s) \approx \frac{\pi^2\gamma_P^3\epsilon_{PRR}\phi^2}{4\mu^2b_P}\,\frac{s^{2\alpha_P(0)-1}}{\ln s}\ln 2$$

for (3.12)

$$s \gg \tfrac{1}{2}\mu^2D_P^2(3+\sqrt{5}).$$

C. Conclusion

The results of Secs. III A and III B imply that the contribution to the inclusive cross section in the pionization region from the term $I_2^P(s)$ is as important as that from the term $I_1^{PPP}(s)$ at very high energy. The terms $I_2^R(s)$ and $I_1^{RPP}(s)$ are important for s not far from $D_P^2\mu^2$, where the thresholds of $I_2^P(s)$ and $I_1^{PPP}(s)$ have not yet been reached. At higher energy both $I_2^R(s)$ and $I_1^{RPP}(s)$ damp out like $s^{-1/4}$ compared to $I_2^P(s)$ and $I_1^{PPP}(s)$. The threshold of $I_1^{PPP}(s)$ is at $\mu^2D_P^4/16$, and is much higher than the threshold of $I_2^P(s)$ and that of $B_1^{PPP}(s)$ of the total cross section. Therefore we can expect that the rise of the single-particle inclusive cross section in the pionization region will be more marked over a larger energy region than the rise of the total cross section.

The over-all rise of the inclusive cross section in the pionization region is determined eventually by $I_2^P(s)$ and $I_1^{PPP}(s)$. Due to the higher threshold of $I_1^{PPP}(s)$, the rise of the inclusive cross section at ISR energy may be expected to come from the term $I_2^P(s)$. The contribution of $I_2^P(s)$ depends on the Pomeron-Reggeon-Reggeon coupling ϵ_{PRR}, the Reggeon-particle-Pomeron coupling ϕ, and the Pomeron-particle-particle coupling γ_P. Since the momentum transfer of the Regge poles involved is negligible, and a Regge pole with a negligible momentum transfer can be roughly approximated by a small-mass particle in an order-of-magnitude estimate, we may put

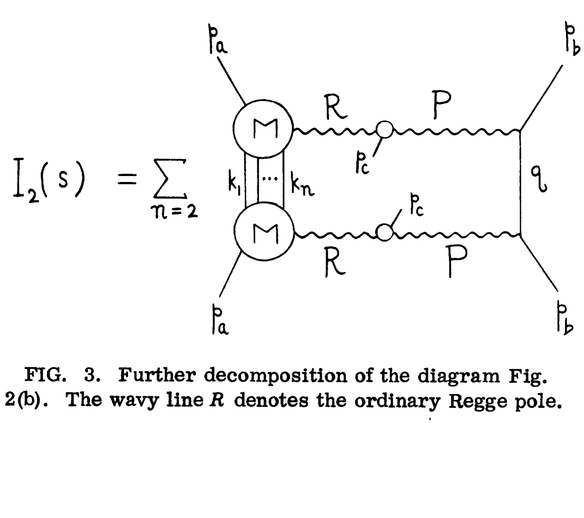

FIG. 3. Further decomposition of the diagram Fig. 2(b). The wavy line R denotes the ordinary Regge pole.

$$\epsilon_{PRR} \approx \phi \approx \gamma_P .$$

The order of magnitude of γ_P can be estimated from the pion-pion-Pomeron coupling in the intermediate-energy region, where the single-fireball effect can be neglected. We use the factorizability of the total cross sections in that energy region. We use the relations

$$\mathrm{Im}\, A_{\pi\pi}(s, t=0) = \pi\gamma_P{}^2 s^{\alpha_P(0)} ,$$

$$\sigma^T_{\pi\pi}(s) = \frac{16\pi \mathrm{Im}\, A_{\pi\pi}(s, t=0)}{s} \approx 15 \text{ mb},$$

and obtain $\gamma_P{}^2 \approx 0.3$. The over-all contribution of $I_2^P(s)$ to the rise of the inclusive cross section is thus

$$\Delta\sigma^{\text{in}}(I_2^P) \approx 2\frac{16\pi^3\gamma_P{}^3\epsilon_{PRR}\phi^2 \ln^2}{4b_P}$$

$$\approx \frac{8\pi^3\gamma_P{}^6 \ln 2}{b_P},$$

where the factor 2 comes from the existence of a diagram with a and b interchanged in Fig. 2(b). Taking $b_P = \frac{1}{3}$ we have

$$\Delta\sigma^{\text{in}}(I_2^P) \approx 16 \text{ mb}.$$

The contribution of $I_1^{PPP}(s)$ comes in at much higher energy and may overlap with the contribution from the neglected multifireball events. But the estimate of its contribution will give a possible lower limit of the rise in the inclusive cross section at higher energy. The over-all rise from the term $I_1^{PPP}(s)$ depends on ϕ, γ_P, γ_R, and ϵ_{PPP}, the triple-Pomeron coupling. If we assume that the over-all rise of the total cross section at ISR energy is about 4 mb, the triple-Pomeron coupling ϵ_{PPP} can be estimated. The coupling γ_R is estimated from a $\pi\pi$ Veneziano model.[17] The over-all rise due to $I_1^{PPP}(s)$ is, by taking $\gamma_R{}^2 \approx 0.5$, $\gamma_P{}^3\epsilon_{PPP} \approx 0.025$, and $D_P \approx 20$,

$$\Delta\sigma^{\text{in}}(I_1^{PPP}) \approx 7 \text{ mb}.$$

Our model thus indicates that the single-particle inclusive cross section in the pionization region can exhibit a rise of the order of 16 mb at ISR energy, and it further rises at least about 7 mb at higher energy.

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