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Weak neutral currents*

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Stimulated by recent indications that neutral-current effects may have been seen in high-energy neutrino reactions, we take up several issues that bear on the characterization of neutral hadron currents. Occasional guidance is sought in gauge models of the weak interactions, but the discussion is essentially phenomenological. Special emphasis is given to the question whether the neutral currents have "new" pieces, unrelated to those which appear in electromagnetism and in weak charge-changing reactions. Of particular interest here is the question: Are the neutral vector currents conserved?

I. INTRODUCTION

The possibility that neutral currents play a role in the weak interactions has long been of interest, purely as a matter of conceivable phenomenology. The issue has recently become more urgent with the development of the gauge-theory approach to weak (and electromagnetic and strong) interactions, since neutral currents make a natural appearance in certain variants of the theory. Still more recently, interest has been further stimulated by the first reports of experimental evidence suggesting (what may be) neutral-current effects: in the CERN heavy-liquid bubble chamber exposed both to neutrino and antineutrino beams, a single event interpretable[1] as $\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e$, and also at CERN[2,3] numerous events interpretable as ν_μ + nucleon $\to \nu_\mu + X$, $\bar{\nu}_\mu$ + nucleon $\to \bar{\nu}_\mu + X$. The first order of business, surely, will be to subject these indications to further experimental test, and then, if neutral-current effects persist, to embark on the task of characterizing the neutral currents in various qualitative and quantitative ways.

In strangeness-changing semileptonic reactions, wherever one has been able to look so far, neutral-current effects are known to be very tiny at best; for the present, in model building, one seeks to banish such effects altogether in lowest order (and to sufficiently suppress them as they arise indirectly in higher order). Discussion of neutral-current effects therefore tends to focus on purely leptonic and $\Delta S = 0$ semileptonic processes. The latter are our special concern here, and we restrict ourselves to processes involving only ordinary (i.e., "charmless") hadrons. The purpose of this note is to discuss a variety of issues that may form an experimental basis for characterizing these neutral currents.

To establish notation for the later discussion, let us first consider the charge-changing but strangeness- and "charm"-conserving semileptonic processes

$$\nu_l + \alpha \to l + \beta, \quad l = e \text{ or } \mu \tag{1}$$

where α and β are hadrons or systems of hadrons. The effective current-current amplitude, presumed to arise via exchange of charged, massive vector bosons (W), has the structure

$$\frac{G}{\sqrt{2}} \cos\theta_C \langle \beta | j_\mu^{1+i2} | \alpha \rangle \bar{u}_l \gamma_\mu (1 + \gamma_5) u_\nu, \tag{2}$$

where θ_C is the Cabibbo angle. The charged current j_μ^{1+i2}, so far as we presently know and as the notation suggests, is purely isovector, and it is composed of vector and axial-vector pieces in the $V - A$ combination (for which a precise meaning is given by the algebra of equal-time commutators)

$$j_\mu^{1+i2} = V_\mu^{1+i2} - A_\mu^{1+i2}. \tag{3}$$

For invariant momentum transfers small compared to the W mass, M_W, the basic constants which characterize the coupling of lepton and hadron currents to this W field are combined, along with the boson propagator factor, $\approx M_W^{-2}$, into the effective coupling constant G, $G m_p^2 \approx 10^{-5}$.

Next, consider electromagnetic processes

$$l + \alpha' \to l + \beta', \quad l = e \text{ or } \mu \tag{4}$$

as mediated solely by one-photon exchange. The amplitude is

$$\frac{e^2}{q^2} \langle \beta' | j_\mu^{em} | \alpha' \rangle \bar{u}_l \gamma_\mu u_l, \tag{5}$$

where q^2 is the invariant momentum transfer. The electromagnetic current is of course purely vectorial, and with respect to isospin, so far as we know, it is composed only of $I=1$ and $I=0$ pieces. According to the CVC (conserved vector current) hypothesis the former piece is related to V_μ^{1+i2} by an isotopic-spin rotation, hence we denote it by V_μ^3. The $I=0$ piece is written as V_μ^0; altogether, therefore,

$$j_\mu^{\text{em}} = V_\mu^3 + V_\mu^0 . \qquad (6)$$

Finally, consider the weak neutral-current process

$$\nu + \alpha' \to \nu + \beta', \qquad (7)$$

which is presumed to arise through coupling of neutral lepton and hadron currents to neutral, massive (Z) bosons. We write the effective amplitude in the form

$$\frac{G}{\sqrt{2}} \langle \beta' | j_\mu^Z | \alpha' \rangle \bar{u}_\nu \gamma_\mu (1+\gamma_5) u_\nu . \qquad (8)$$

By convention we have again displayed the factor $G/\sqrt{2}$, all other parameters of the underlying theory (e.g., the mass ratio M_Z/M_W) being absorbed into the definition of the neutral current j_μ^Z.

The first interesting questions to be faced, once neutral-current effects are at all established, will have to do with the spatial (V, A) and isotopic structure of j_μ^Z. To preserve sanity, we restrict ourselves exclusively to $I=1$ and 0 currents (in accord with present-day models, which universally build the currents in a bilinear fashion out of quarks with isospin restricted to $I=\frac{1}{2}$ and 0). In an obvious notation, then, the general structure is

$$j_\mu^Z = v_\mu^0 + a_\mu^0 + v_\mu^3 + a_\mu^3 . \qquad (9)$$

We have denoted the vector, v_μ, and axial-vector, a_μ, currents by lower-case letters to distinguish them for the general case from the currents of Eqs. (3) and (6).

We shall make a number of comments on how to probe further into this general structure of j_μ^Z. Initially, in Sec. II, we consider the simplest possibility for the form of the neutral current: namely, the case where it is an arbitrary linear combination of the electromagnetic current and j_μ^3, the current obtained by isotopic rotation of the charged current j_μ^{1+i2} in Eq. (3). Here it becomes possible, with the added input of certain standard scaling assumptions, to obtain a useful relation connecting deep-inelastic neutrino-induced production and electroproduction processes, Eqs. (1), (4), and (7). Such a connection has already been discussed[4] for the Salam-Ward-Weinberg model,[5,6] which represents a special case in the class under discussion. For j_μ^Z an arbitrary linear combination of j_μ^{em} and j_μ^3 we shall again find an equality, contained in Eqs. (14) and (16) below, but for this more general situation the tests require separate measurements on neutron and proton targets. We cannot recommend these relations to experimentalists as being all that simple a tool. However, it seems to us important to see how far one can come with such a minimal assumption about the structure of the neutral currents.

In Sec. III we revert to the more general structure of Eq. (9) and to the possibility that "new" components appear in the neutral current. We discuss three items:

(i) comparison of the elastic scatterings $\nu + d \to \nu + d$ and $\bar{\nu} + d \to \bar{\nu} + d$, as these may bear on the presence of an isoscalar axial vector current,

(ii) weak neutral-current effects in e^+e^- annihilation reactions, and

(iii) what seems to us a fundamental issue: Are the neutral vector currents conserved? (Since this question is so broad, it may not be superfluous to state that on this one point our discussion will be independent of whether or not charm-carrying states are produced.)

The various issues raised in this paper will of course be pertinent only if experimental evidence persists for neutral currents. None of the tests that we propose bearing on one issue or another will be all that easy to carry out [with the possible exception of Eq. (11) below]. The discussion is everywhere frankly phenomenological, since until we know what is the right model (it may, after all, not even be built on the gauge strategy) many questions are open and have to be approached empirically. On the other hand, we think it useful to indicate on occasion, quite briefly, where existing gauge models fit in, and to invent (without great pride of ownership) a new gauge model for the sole purpose of illustrating that some general option for j_μ^Z is at least not excluded by the general ideas of gauge theory.

II. A SPECIAL CLASS

Among the simplest possibilities for j_μ^Z is that one or another of the currents in Eq. (9) is not really a new ingredient of the weak interactions, but instead that it is related, directly or via an isotopic-spin rotation, to a corresponding current already encountered in Eqs. (3) and (6), for example, v_μ^3 proportional to V_μ^3, or $v_\mu^0 \propto V_\mu^0$, or $a_\mu^3 \propto A_\mu^3$, where A_μ^3 is the current obtained by isotopic rotation of A_μ^{1+i2}. It is only for a_μ^0 that such a possibility does not exist, since no $I=0$ axial-vector current enters into Eqs. (3) and (6). In this sense the existence of a_μ^0 is of special qualitative significance. The discovery of effects attributable to a_μ^0, whatever their numerical size, would represent something new. For the remaining currents of Eq. (9), the simple possibilities discussed above, i.e., the question whether they represent old things, depends for its testing on quantitative comparisons among isotopically related sets of reactions, Eqs. (1), (4), and (7).

The simplest possibility of all is that j_μ^Z is exclusively composed of j_μ^{em} and $j_\mu^3 = V_\mu^3 - A_\mu^3$, i.e., that it is just a linear combination (with allowance for isotopic rotation) of the currents of Eqs. (3) and (6):

$$j_\mu^Z = \rho(V_\mu^3 + V_\mu^0) + \lambda(V_\mu^3 - A_\mu^3). \tag{10}$$

We focus on Eq. (10) in this section.

The structure implied by Eq. (10) is certainly achievable in the framework of gauge models, as witness the following two examples.

(1) $\lambda = 1$, ρ fixed by $\nu_\mu + e \to \nu_\mu + e$. This is the original SU(2)×U(1) model.[6] While the model is unsatisfactory in not making provision for *charged* strangeness-changing currents, it has nevertheless been widely utilized as a basis for discussions of neutral-current phenomenology. The parameter $\rho = -2\sin^2\theta$ $(-2 < \rho < 0)$ can be determined independently in the model from analysis of purely leptonic processes, e.g., $\nu_\mu + e \to \nu_\mu + e$. Among other things, this model has been applied[4] to analysis of the deep-inelastic cross sections for $\nu(\bar\nu)$ + nucleon $\to \nu(\bar\nu) + X$, in relation to e + nucleon $\to e + X$ and $\nu(\bar\nu)$ + nucleon $\to \mu(\bar\mu) + X$. It turns out that the neutral-current cross sections reported by the CERN group, for both ν and $\bar\nu$ beams, are in rough agreement with the theoretical predictions based on this model—for a value of the parameter ρ which also accords with the value implied by detection at CERN of a single $\bar\nu_\mu + e \to \bar\nu_\mu + e$ event (and nondetection of $\nu_\mu + e \to \nu_\mu + e$). This is a remarkable situation and suggests that additional currents in j_μ^Z, though perhaps formally present, do not contribute greatly in the deep-inelastic region at energies so far explored. In turn this leads us to consider the same deep-inelastic processes in the somewhat wider context of Eq. (10), with both ρ and λ left as free parameters. This allows for a wider range of options, in which, however, additional currents are still presumed to make only small contributions even if formally present. Before coming to this let us mention one more specific example, however.

(2) $\lambda = 0$, ρ not fixed by $\nu_\mu + e \to \nu_\mu + e$. This is the model of Bég and Zee.[7] In spite of the fact that the determination of ρ is more involved here,[8] the model has several obvious and immediate consequences. Indeed, since j_μ^Z is proportional to j_μ^{em}, the differential cross sections for $\nu + \alpha \to \nu + \beta$ and $e + \alpha \to e + \beta$ are proportional[9] up to an obvious photon propagator factor $(q^2)^{-2}$. Thus ratios such as $d\sigma(\nu p \to \nu X)/d\sigma(\nu n \to \nu Y)$, $d\sigma(\bar\nu p \to \bar\nu X)/d\sigma(\bar\nu n \to \bar\nu Y)$, and $d\sigma(ep \to eX)/d\sigma(en \to eY)$ must be equal. Moreover, since here the neutral current has no axial-vector pieces, all neutral-current processes, Eq. (7), are free of V, A interference. Hence

$$\sigma^{(\nu)} = \sigma^{(\bar\nu)}, \tag{11}$$

for any target and any channel (and, in fact, differentially as well). Needless to say, Eq. (11) holds true in *any* theory in which j_μ^Z is pure V (or pure A).

Now then to the case of general ρ and λ. Let $\sigma_p^{(\nu,\bar\nu)}$ and $\sigma_n^{(\nu,\bar\nu)}$ be the cross sections for $\nu(\bar\nu) + p \to \nu(\bar\nu) + X$ and $\nu(\bar\nu) + n \to \nu(\bar\nu) + X$. Similarly, let $\sigma_p^{(\mu,\bar\mu)}$ and $\sigma_n^{(\mu,\bar\mu)}$ refer to $\nu_\mu(\bar\nu_\mu) + p(n) \to \mu(\bar\mu) + X$. The cross sections are given by familiar expressions involving certain structure functions W_1, W_2, W_3, which depend on q^2 and ν and which have to be labeled also by the name of the target and by the nature of the process (neutral-current vs charged-current reactions). For electromagnetic processes e + nucleon $\to e + X$ there is a similar formula for differential cross sections, expressed in terms of W_1^{em} and W_2^{em}. Let us *define* σ_p^e and σ_n^e to be the total cross sections one would obtain for the neutrino reactions with the structure functions given by W_1^{em} and W_2^{em}. Thus

$$\sigma^e \equiv \frac{G^2}{2\pi}\int d\nu\, dq^2 \frac{\epsilon'}{\epsilon}\left[2W_1^{\text{em}}\sin^2(\tfrac{1}{2}\theta) + W_2^{\text{em}}\cos^2(\tfrac{1}{2}\theta)\right], \tag{12}$$

$\nu = \epsilon - \epsilon'$, $q^2 = 4\epsilon\epsilon'\sin^2(\frac{1}{2}\theta)$. Finally, let the subscript N on a cross section refer to the average over p and n, $\sigma_N \equiv \frac{1}{2}(\sigma_p + \sigma_n)$; similarly $\sigma_\Delta \equiv \frac{1}{2}(\sigma_p - \sigma_n)$. Then define

$$\begin{aligned}
\sigma_N^{(+\nu)} &= \tfrac{1}{2}(\sigma_N^{(\nu)} + \sigma_N^{(\bar\nu)}), \quad \sigma_N^{(-\nu)} = \tfrac{1}{2}(\sigma_N^{(\nu)} - \sigma_N^{(\bar\nu)}),\\
\sigma_N^{(+\mu)} &= \tfrac{1}{2}(\sigma_N^{(\mu)} + \sigma_N^{(\bar\mu)}), \quad \sigma_N^{(-\mu)} = \tfrac{1}{2}(\sigma_N^{(\mu)} - \sigma_N^{(\bar\mu)}),
\end{aligned} \tag{13}$$

and similarly for $N \to \Delta$. We now seek to relate the various cross sections on the basis of the neutral-current structure of Eq. (10). For the deep-inelastic scaling regime we invoke one additional theoretical ingredient: namely, the standard assumption for the charged-current processes that the vector and axial-vector currents make equal contributions to $\sigma^{(+\mu)}$.

It is now an easy matter to establish the following results (for ease of writing we have set the Cabibbo factor $\cos\theta_C = 1$). First, an equation which determines the parameter ρ in terms, exclusively, of cross sections averaged over neutrons and protons:

$$\frac{\sigma_N^{(\mu)}\sigma_N^{(\bar\nu)} - \sigma_N^{(\nu)}\sigma_N^{(\bar\mu)}}{\sigma_N^{(\mu)} - \sigma_N^{(\bar\mu)}} = \rho^2\sigma_N{}^e. \tag{14}$$

Although we are regarding ρ as a free parameter, this result is not without predictive power. Whatever the value of ρ, the right-hand side of Eq. (14) must be positive. Moreover, we already know that $\sigma_N^{(\mu)} > \sigma_N^{(\bar{\mu})}$. Thus, we require that

$$\frac{\sigma_N^{(\bar{\nu})}}{\sigma_N^{(\nu)}} \geq \frac{\sigma_N^{(\bar{\mu})}}{\sigma_N^{(\mu)}} . \tag{15}$$

This constitutes a first test of Eq. (10). It just so happens, however, that it is not likely to be useful. This is because in any case, at high energies and assuming scaling, one has the bounds $\sigma_N^{(\bar{\nu})}/\sigma_N^{(\nu)} > \frac{1}{3}$, $\sigma_N^{(\bar{\mu})}/\sigma_N^{(\mu)} > \frac{1}{3}$. Since the ratio on the right-hand side of Eq. (15) has an experimental value close to the bound,[10] $\sigma_N^{(\bar{\mu})}/\sigma_N^{(\mu)} = 0.38 \pm 0.02$, there is not much room for Eq. (15) to be violated.

There is a second equation that determines the parameter ρ:

$$\left(\frac{\sigma_\Delta^{(+\nu)}}{\sigma_\Delta^e}\right)\left[1 + 2\left(\frac{\sigma_N^{(-\nu)}}{\sigma_N^{(-\mu)}}\right)\left(\frac{\sigma_\Delta^e}{\sigma_\Delta^{(+\nu)}}\right)\right]^{-1} = \rho^2 . \tag{16}$$

The requirement that Eqs. (14) and (16) yield the same value for ρ^2 constitutes the essential test of Eq. (10). Of course, again, there is also the requirement that the left-hand side of Eq. (16) be positive. Finally, the parameter λ is determined from

$$\frac{\lambda}{\rho} = 2\left(\frac{\sigma_N^{(-\nu)}}{\sigma_N^{(-\mu)}}\right)\left(\frac{\sigma_\Delta^e}{\sigma_\Delta^{(+\nu)}}\right) . \tag{17}$$

We may also note that contained in these equations are the results of Ref. 4, generalized to arbitrary λ and ρ:

$$\sigma_N^{(\nu)} - \tfrac{1}{2}[\lambda(\lambda+\rho)]\sigma_N^{(\mu)} = \sigma_N^{(\bar{\nu})} - \tfrac{1}{2}[\lambda(\lambda+\rho)]\sigma_N^{(\bar{\mu})} = \rho^2\sigma_N^e . \tag{18}$$

Recall again that the foregoing results are based on the supposition that the neutral current is built up solely out of j_μ^{em} and j_μ. We have considered this case first because of its theoretical economy. At the next level of generalization it is natural to consider the structure

$$j_\mu^Z = \rho(V_\mu^0 + V_\mu^3) + \lambda(V_\mu^3 - A_\mu^3) + \sigma(V_\mu^3 + A_\mu^3) , \tag{19}$$

which represents a completely arbitrary combination of all the currents already encountered in electromagnetic and weak charge-changing interactions. We have not discussed the deep-inelastic reactions in the context of this more general situation simply because it is no longer possible here to obtain a simple cross-section equality for arbitrary values of the parameters ρ, λ, σ, except for a special configuration that we shall note in Sec. III. However, even with its greater complexity Eq. (19) is certainly testable if one is prepared to study individual channels in sufficient detail. For example, consider the related set of processes $\nu + p(n) \to \nu + p(n)$, $e + p(n) \to e + p(n)$, $\nu + n \to \mu^- + p$. A full experimental analysis would serve to determine separately the vector and axial-vector form factors for each reaction. In this way one could test for the absence of a_μ^0 and determine the parameters ρ, λ, σ, which are to be used to now in consistency tests applied to other related sets of reactions. We do not pursue such examples here, both because the principles are obvious and because the experimental demands are excessive.

III. "NEW" CURRENTS

Let us return to the question of what the gauge models say. We have already mentioned examples which correspond to the neutral-current structure of Eq. (10), and we may also note[11] the existence of at least one model which contains a σ term. There are also situations in which the isovector currents are still as in Eq. (10), but in which "new" isoscalar currents, both vector and axial-vector, make an appearance. Such models are described by

$$j_\mu^Z = \rho V_\mu^3 + \lambda(V_\mu^3 - A_\mu^3) + v_\mu^0 + a_\mu^0 , \tag{20}$$

where a_μ^0 is necessarily a new object but also where v_μ^0 is *not* proportional to V_μ^0 (it is simply a different operator. This is the case for the model of Glashow, Iliopoulos, and Maiani,[12] for one version of the three-triplet model,[13] and for the O(4) $\times$ U(1) spinor model.[14] [As it happens, in all these cases the extra piece of v_μ^0 and the new a_μ^0 enter in the $(V-A)$ combination.] So far as the isovector currents are concerned, one can again relate their matrix elements for neutral-current processes to corresponding matrix elements for associated processes, Eqs. (1) and (4). This has been exploited by various authors[15] to set lower bounds on various neutral-current processes.

It happens that all published gauge models (among those which feature neutral currents at all) stop short of the full generality of Eq. (9), in that none of them contain "new" $I=1$ currents. The quark content of these models embraces either one strong isodoublet $(\mathcal{P}, \mathcal{N})$, or three of them—as in the (Han-Nambu or color) models of the SU(3)$\times$SU(3)$'$ variety. It is easily seen, whenever the quark content is in the above categories, that no new $I=1$ currents can arise, independent of what choice is made for the weak-electromagnetic gauge group. In spite of this the reader can convince himself, by examples, that gauge models *can* be constructed (of course outside the above categories) which *do* contain new isovector currents and which do meet the usual requirements, including the constraint imposed by the $\pi^0 \to 2\gamma$ rate.

As already emphasized, one reason for contemplating the simple current of Eq. (10) is the possibility that (for appropriate ρ and λ) any additional "new" pieces may happen in typical cases to give small contributions to neutral-current cross sections. After all, just because such added pieces (as they might arise explicitly in some model) are new, we have no way to assess their effects from estimates based on reactions (1) or (4). For this very reason, however, such currents, if they exist, would be objects of great interest, so one wants to consider special situations which may serve qualitatively (perhaps at the cost of low counting rates) to signal their existence. We wish to make several points in this connection.

(1) Consider first the question of an $I=0$ axial-vector current. In principle this can be looked for without reference to any but neutral-current processes. For example, consider again the processes $\nu+p\to\nu+p$ and $\nu+n\to\nu+n$. A complete experimental analysis would clearly serve to isolate the matrix element $\langle p|a_\mu^0|p\rangle=\langle n|a_\mu^0|n\rangle$, but the practical difficulties are of course enormous. A more attractive possibility, though still difficult enough, is to study elastic scattering of neutrinos and antineutrinos on an $I=0$ target with nonvanishing spin, e.g., deuterium:

$$\begin{aligned}&\nu+d\to\nu+d\,,\\&\bar\nu+d\to\bar\nu+d\,.\end{aligned}\tag{21}$$

The important point here is that only the $I=0$ vector and axial-vector currents can contribute. Therefore discovery of any V,A interference would in itself signal the existence of the new current a_μ^0, independent of whether the vector current v_μ^0 is old or new. The differential cross section has the familiar form

$$\frac{\partial\sigma^{(\nu,\bar\nu)}}{\partial q^2}=\frac{G^2}{2\pi}\left[W_2\left(1-\frac{q^2}{2M\epsilon}\right)+(2W_1-W_2)\frac{q}{4\epsilon^2}\pm W_3\left(1-\frac{q^2}{4M\epsilon}\right)\frac{q^2}{2M\epsilon}\right],\tag{22}$$

where the + (−) sign in the coefficient of W_3 refers to the ν ($\bar\nu$) process. Here M is the deuteron mass, ϵ the (laboratory) energy of the incident neutrino, and q^2 the invariant momentum transfer. The form factors W_i depend only on q^2, and it is W_3, in particular, that measures V,A interference. In principle the form factor W_3 can be disentangled from W_1 and W_2 at each value of q^2 by holding q^2 fixed and varying ϵ. Alternatively, one can integrate over q^2 to obtain the total elastic cross sections $\sigma^{(\nu)}(\epsilon)$ and $\sigma^{(\bar\nu)}(\epsilon)$ at any ϵ. Any nonvanishing difference between these signals the existence of V,A interference. For $q^2=0$ the various form factors might well be comparable to those describing, say, $\nu+p$ elastic scattering. However, since the deuteron is a loosely bound system its form factors will begin to drop off rapidly at an early stage (in comparison with the situation for $\nu+p\to\nu+p$), namely when $q^2R_d{}^2\gg1$, where R_d is the the deuteron radius. This already leads one to expect that the total elastic deuteron cross section will be much smaller than that for elastic $\nu+p$ scattering (something which itself has not yet been seen) if $q^2{}_{\max}R_d{}^2=2M\epsilon R_d{}^2\gg1$. What is worse, W_3 in Eq. (22) is multiplied by a factor which vanishes as $q^2\to0$, so information on W_3 comes from large q^2, where the event rate is bound to be especially low. On the other hand, at very low energies where $q^2{}_{\max}$ is sufficiently small ($\epsilon<$ several tens of MeV) there is a chance that the deuteron cross sections would be comparable to that of the proton (all would of course be small) and that the difference $\sigma^{(\nu d)}-\sigma^{(\bar\nu d)}$ could be a substantial fraction of the sum if an $I=0$ axial-vector currents exists. At any rate, insofar as deuterium-filled bubble chambers are going to be exposed to ν and $\bar\nu$ beams, one ought to be on the lookout for the elastic reactions of Eq. (21).

(2) Electron-positron annihilations represent another potential source of information on weak neutral currents. To get at both the hadron and lepton neutral currents one wants to consider the processes $e^++e^-\to$ hadrons. At foreseeable energies such reactions will usually be dominated by pure electromagnetic (one-photon exchange) contributions, but one may hope to see small interference effects arising from the weak interactions. The weak amplitude would have the structure of Eq. (8), with the neutrino current replaced by an electron current (this will in general have both vector and axial-vector parts, in a combination that depends on the model). The weak and electromagnetic amplitudes are expected to stand roughly in the ratio $Gs/4\pi\alpha$; interference effects at the highest presently accessible energies might therefore be of order of 1%.

However, for certain special final states the electromagnetic contribution cannot arise in lowest (one-photon exchange) order. This is the case for states with charge conjugation $C=+1$. For such channels the electromagnetic contribution requires exchange of two photons, whereas in appropriate cases the weak axial-vector currents ($C=+1$) can still contribute in lowest order. In this situation the cross sections will certainly be small, but the weak contributions have a chance, relatively, to play a big role. An example is the system $K^0\bar K^0+\pi^0$'s, where the $K^0\bar K^0$ subsystem is in a $C=+1$ state, which we select by looking for K_SK_S decays, i.e., by observing the final state

$K_SK_S+\pi^0$'s. Another example, of course, is the state $3\pi^0$.

However, let us return to those final-state channels which *can* be reached via one-photon exchange. This will then provide the dominant mechanism for the reaction, but what is of interest here are the effects produced by possible weak neutral-current contributions. One effect which would clearly signal a weak contribution is the occurrence of a parity-violating term in the differential cross section. There is more, however. The structure of such parity-violating effects, in their dependence on the various kinematic variables of the reaction, can serve to reveal something of the nature of the neutral lepton and hadron currents (vector versus axial-vector).

It is also the case that the parity-conserving terms in the spectrum can provide information on the presence and nature of weak neutral-current contributions. Namely, in the absence of such contributions the spectrum arising from one-photon exchange has a restricted form with respect to dependence on certain variables of the reaction.[16] On the other hand, the weak neutral currents (Z exchange) can produce new kinds of parity-conserving terms in the spectrum. Unfortunately, however, the new terms can also arise at the level of two-photon exchange, in interference with the dominant one-photon exchange, i.e., at a level of order α relative to the dominant terms. At this level, therefore, the observation of such terms would not be decisive as a signal of weak contributions. These qualifications of course do not apply to the parity-violating terms, which necessarily require the intervention of Z exchange. Here it is enough to restrict consideration to interference between Z exchange and one-photon exchange.

These issues are discussed and illustrated in more detail in the Appendix. There we note that the presence of special correlations can yield information about the vector and axial-vector content of *both* the leptonic and the hadronic parts of the weak current.

(3) Our last topic has to do with the question how to discover whether the weak neutral currents v^0_μ, v^3_μ, a^3_μ have "new" pieces (a^0_μ, as repeatedly emphasized, is necessarily new if it is there at all). We earlier discussed how this could be tested in principle, namely, by assuming the contrary, Eq. (19), and studying related sets of neutral-current, charged-current, and electromagnetic processes. However, here we want to ask how one might come to a decision on the basis of more qualitative considerations, restricted solely to neutral-current processes.

That this can be done may seem hard to imagine, in the absence of characterizations that go beyond the spatial (V,A) and internal symmetry (e.g., isospin) properties of the various currents. But the electromagnetic currents V^0_μ and V^3_μ in fact *do* have another distinctive property, reflecting, respectively, hypercharge and isospin conservation in the strong interactions; they are conserved. If the weak vector currents v^0_μ and v^3_μ have new pieces, not simply proportional to V^0_μ and V^3_μ, are these new pieces conserved?

For the last time in this paper we shall ask: What do gauge theories have to say on the issue? Neglecting electromagnetic and weak effects, one clearly cannot construct a nonconserved vector $\Delta S=0$ current from $\mathcal{P}, \mathfrak{N}, \lambda$ only—but then, as far as is known, one cannot make a sensible gauge model only out of $\mathcal{P}, \mathfrak{N}, \lambda$ either. In turn, this has led to the introduction of additional SU(3)-singlet quarks in many models. This, of course, opens the possibility for the existence of new vector conservation laws (charm), but that is not our concern here. However, it is not difficult to show, once quarks of the latter variety are admitted, that also nonconservation of the vector part of j^Z_μ becomes a logical (if perhaps not attractive) possibility.[17]

To test the conservation question, we can take over ideas first proposed by Adler[18] in connection with the issues of CVC and PCAC (partial conservation of axial-vector current) for charge-changing weak reactions. Namely, consider the process

$$\nu+\alpha\to\nu+\beta_1+\beta_2+\cdots,$$

where $\{\beta_1,\beta_2,\ldots\}$ is a system of hadrons, and specialize to the configuration where the outgoing neutrino is moving in the same direction as the incident neutrino. Let q_μ be the momentum transfer between the neutrinos. In the "parallel" configuration q^2 is strictly zero and the differential cross section is proportional to

$$|\langle\beta_1\beta_2\cdots|q_\mu j^Z_\mu|\alpha\rangle|^2=\left|\left\langle\beta_1\beta_2\cdots\left|\frac{\partial j^Z_\mu}{\partial x_\mu}\right|\alpha\right\rangle\right|^2. \quad (23)$$

Thus the differential cross section is determined here by the divergence of the neutral current j^Z_μ. The divergence of the axial-vector part of j^Z_μ is certainly nonvanishing; the conservation question has to do rather with the vector currents. If they are conserved there can be no V,A interference effects and therefore no parity-violating terms in the spectrum. The test consists in looking for parity-violating terms, of the sort

$$\vec{k}_1\cdot\vec{k}_2\times\vec{k}_3, \quad (24)$$

where the $\vec{k}_i$ are any three momenta that remain independent as $q^2\to 0$. For nonforward neutrino configurations such terms are always, in general, to be expected. If they are nevertheless invisible, even for the general neutrino configuration, that in itself may be interesting but the conserved-current test becomes empty. If such terms *are* detected, the question is then whether they persist as $q^2 \to 0$. If so, the vector current is not conserved. In the same way, if some of the final particles have spin, one looks in the general configuration for parity-violating terms of the sort $\vec{\sigma}\cdot\vec{k}$, where k is some momentum in the problem. If such a term is detected, one then asks whether it persists as $q^2 \to 0$. These tests are of course equally applicable to exclusive or inclusive studies.

Suppose that the vector current turns out to be conserved, for whatever reasons [either it contains only the old pieces of Eq. (19) or else any new pieces happen themselves to be conserved]. What else can then be learned from the forward (inelastic) lepton scattering? Here the answer is: We can learn whether the axial-vector current is exclusively "old" [as in Eq. (19)] or whether it has new pieces. To bring this out in its simplest form, suppose that the target particle has zero isospin (for illustration we take it to be a deuteron) and consider the inclusive reactions

$$\nu + d \to \nu + X, \tag{25}$$

$$\nu + d \to \mu^- + X'. \tag{26}$$

In both cases we restrict ourselves to the parallel configuration of the outgoing lepton (forward scattering). Insofar as the muon mass can be neglected, forward scattering for both processes corresponds to $q^2=0$; we are therefore dealing with the differential cross section

$$\left.\frac{\partial\sigma}{\partial q^2 \partial M^2}\right|_{q^2=0} \equiv f(\epsilon, M) \tag{27}$$

at $q^2=0$, as a function of incident neutrino energy ϵ and "missing mass" M. If Eq. (19) is correct, along with Eq. (3), it is evident from the reasoning that has gone before that

$$f^{(\nu\to\nu)}(\epsilon, M) = \tfrac{1}{2}(\lambda-\sigma)^2 f^{(\nu\to\mu^-)}(\epsilon, M), \tag{28}$$

i.e., the forward differential cross sections are proportional for all ϵ and M. If the idea of PCAC is correct, then, as Adler has shown, $f^{\nu\to\mu}(\epsilon, M)$ would in turn be related to the total cross section $\sigma_\pi(M)$ for $\pi^+ + d \to X$; but Eq. (28) does not rest on PCAC.

Returning to the important issue of conservation of the neutral vector currents, we regret that it must rest on such difficult experimentation, but we have not been able to think up more practical tests. If one were someday to discover the heavy neutral leptons (call them L^0) that are suggested by certain gauge models, and if these were to decay via Z exchange according to

$$L^0 \to \nu + \text{hadrons},$$

a whole new range of interesting tests would become available. For example, consider the decay process

$$L^0 \to \nu + \pi^+ + \pi^-,$$

where one encounters the matrix element $\langle k^+k^-|v_\mu^3|0\rangle$. In greatest generality this has the structure

$$f_1(k^+ - k^-)_\mu + f_2(k^+ + k^-)_\mu,$$

where f_1 and f_2 depend on the invariant dipion mass. If the current v_μ^3 is conserved, f_2 must vanish. One can readily think up other similar tests, but this whole subject is clearly for the remote future, at least as it now seems.

APPENDIX

Consider the annihilation reaction

$$e^+ + e^- \to \beta_1(k_1) + \beta_2(k_2) + \beta_3(k_3) + \cdots,$$

where the β_i are the names of the hadrons and the k_i are the corresponding momenta. For simplicity suppose that the leptons are unpolarized and that one integrates the cross section over all final-state variables other than the momenta k_1 and k_2 of particles β_1 and β_2 (hadron spins, if any, are also summed over). The remaining variables of the problem are the total center-of-mass energy $\sqrt{s}$ and, in addition, five final-state quantities. Expressed in terms of center-of-mass parameters, these latter are taken to be: E_1 and E_2, the energies of β_1 and β_2; the angle θ_1 between $\vec{k}_1$ and the positron momentum $\vec{p}$; the angle θ_2 between $\vec{k}_2$ and $\vec{p}$; and the angle θ_{12} between $\vec{k}_1$ and $\vec{k}_2$. Let $Z_1 = \cos\theta_1$, $Z_2 = \cos\theta_2$, $Z_{12} = \cos\theta_{12}$. We write the spectrum in final-state variables in the form

$$d^5 w = W \Delta^{-1/2} dE_1 dE_2 dZ_1 dZ_2 dZ_{12},$$

where

$$\Delta = 1 - Z_1^2 - Z_2^2 + 2Z_1 Z_2 Z_{12} - Z_{12}^2$$

and

$$W = W(s, E_1, E_2, Z_1, Z_2, Z_{12}).$$

Ignoring exchange of more than a single photon, but allowing for contributions from weak neutral currents, one finds for W the following structure, which is simple and explicit in its dependence on Z_1 and Z_2:

$$W = A_1 + A_2 Z_1^2 + A_3 Z_2^2 + A_4 Z_1 Z_2 + B_1 Z_1 + B_2 Z_2 + (C + D_1 Z_1 + D_2 Z_2)\vec{p}\cdot\vec{k}_1\times\vec{k}_2,$$

where the A_i, B_i, D_i, and C depend on s, E_1, E_2, and Z_{12} but are *independent* of Z_1 and Z_2. These coefficients have been labeled in groups according to the following scheme.

(i) The terms with coefficients A_i are the only ones that would survive in the absence of neutral-current contributions. They are therefore expected to be the dominant terms. However, the A_i can also receive contributions from the weak neutral currents, in interference with one-photon exchange—if, and only if, the weak lepton and hadron currents *both* have *vector* parts.

(ii) The parity-conserving terms with coefficients B_i arise from interference between neutral-current contributions and one-photon exchange if, and only if, the weak lepton and hadron currents *both* have *axial-vector* parts.

(iii) Similarly, the parity-violating term with coefficient C arises if, and only if, the weak lepton current has an *axial-vector* part and the hadron current has a *vector* part.

(iv) The parity-violating terms with coefficients D_i arise if, and only if, the weak lepton current has a *vector* part and the hadron current has an *axial-vector* part.

Several points are worth making here. Two-photon exchange, which has been ignored in the above analysis, leads to a more complicated structure for the parity-conserving part of the spectrum in its dependence on Z_1 and Z_2 and, in particular, can produce contributions to the B_i. Thus, detection of the B_i at a level of order α relative to the dominant A_i terms would not conclusively signal the presence of weak neutral-current effects. On the other hand, observation of any of the parity-violating terms would surely reveal the presence of weak-current effects, and distinction between the C and D_i terms would provide the kind of information about these currents that has been noted above.

We may also remark that since the variables E_1 and E_2 played no interesting role in the above analysis, nothing is lost if one integrates the spectrum over these variables. Finally, we may comment on the special interest that attaches to annihilation reactions leading to channels of even G parity, e.g., 4π, 6π, etc. Here the only weak neutral hadron currents that can play a role are v_μ^3 and a_μ^0, so, in particular, detection of the terms with coefficients D_i would establish the existence of the "new" isoscalar axial-vector current a_μ^0.

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necessitate that $m(\mathcal{P}'_1) - m(\mathcal{P}'_2)$ be small on a quark-mass scale, however.]

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Singularities in three-body final-state amplitudes*

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Three-body amplitudes have two-body threshold singularities over and above those coming from the usual treatments of final-state interactions. These are analyzed both formally and numerically. In the numerical model studies, they account for an important variation of the amplitude. The singularity comes from the "next-to-last" rescattering, and hence, may be represented correctly by any approximate amplitude that has that rescattering, even if the approximation scheme diverges. This may account for the correct shape (not magnitude) of multibody spectra determined with a divergent multiple-scattering series.

I. INTRODUCTION

How to extract two-body information from a multibody final state is an old problem. Empirically, we have learned much from such situations in nearly every branch of physics, but when there are more than two strongly interacting particles in the final state, there exists little in the way of a firm theoretical basis for the analysis. In this paper we investigate the nature and consequences of an important singularity in the final-state amplitude that has been ignored in most previous analyses. This is a threshold singularity in the pair subenergy of each final-state pair. It is therefore on the boundary of the physical region, and produces considerable variation of the amplitudes over that region. Its neglect can lead to incorrect final-state parameters. We carry out our analysis in the context of the three-body problem, but the existence and nature of the singularity is by no means restricted to that case.

The existence of this singularity was already implicit in 1967[1,2] in a different guise, but we were not then aware of its importance for phenomenology. Much more recently we have demonstrated its presence in a general way through unitarity.[3] In this paper we explore further its origin, nature, and numerical importance.

It is customary to decompose a three-body final-state amplitude into a sum of terms, depending on which pair interacts last,

$$M = \sum_{i=1}^{3} f_i \, \tau_i(\sigma_i) \, , \qquad \text{(a)}$$

where τ_i is the j-kth pair's two-body t matrix ($i \neq j \neq k$) and σ_i is that pair's center-of-mass energy. f_i is the coefficient of τ_i in the decomposition. This form is closely related to the Faddeev or multiple-scattering expansion of nuclear physics and to the isobar expansion of particle physics.[4] Most empirical analyses proceed by assuming that f_i is slowly varying and that f_i and f_j ($i \neq j$) are totally independent. But what we have already shown is that for small σ_i (Ref. 1–3)

$$f_j = A_j + i(\sigma_j)^{1/2} B_j$$

and

$$B_j = \sum_{k \neq j} f_k \, \tau_k \, . \qquad \text{(b)}$$

This means that f_i has a square-root singularity in the pair subenergy of the j-kth pair and the coefficient of the singularity is the "non-i" term in the decomposition. Near $\sigma_i = 0$ we can write

$$M \cong (A + iBq)\, e^{i\delta} \frac{\sin\delta}{q} + B \, ,$$

where $q = (\sigma_i)^{1/2}$, $\tau_i = (e^{i\delta} \sin\delta)/q$ and we have dropped the i label to keep the expression simple. Elementary algebra then gives

$$M = A\, e^{i\delta} \frac{\sin\delta}{q} + B\, e^{i\delta} \cos\delta \, .$$

This result on the coherence of the amplitude shows that in some sense it is the entire amplitude that carries the phase δ and not just a part. This is a kind of Watson theorem which has been known for some time in some circles,[1,2,5] but its