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Sum rule for deep-inelastic electroproduction from polarized protons*

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A sum rule is derived for the asymmetry in deep-inelastic scattering of polarized electrons from polarized protons: $\int_0^1 d\xi\, g^{ep}(\xi) \approx 0.15 g_A$. The result follows from the quark light-cone algebra and the assumption that strange quarks do not contribute to the asymmetry. The latter is justified by conventional parton-model arguments.

Measurements of the asymmetry in the inelastic scattering of polarized electrons from polarized protons will begin soon.[1] Several years ago Bjorken[2] derived a sum rule for the difference in polarization asymmetry in deep inelastic scattering from protons and neutrons. The sum rule has been rederived recently using the parton model[3] and the quark light-cone algebra.[4,5] Unfortunately, testing the Bjorken sum rule would require data on scattering from polarized deuterons which will be unavailable for some time. Here we derive a sum rule for the asymmetry in scattering from polarized protons alone. We use the standard quark light-cone algebra, the usual parton-model assumptions that the only isosinglet (λ-type) quarks in the proton are in the "sea" (Pomeranchukon) and that the spins of partons in the "sea" are paired. These are discussed further later.

The structure functions for scattering polarized electrons from polarized protons are defined as follows:

$$\tfrac{1}{2}(W_{\mu\nu} - W_{\nu\mu}) = i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda n^\sigma}{M^2}G_1(\nu, q^2) + i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda}{M^4}(\nu n^\sigma - n\cdot q p^\sigma)G_2(\nu, q^2),$$

where $\nu \equiv p\cdot q$, n^σ is the proton's polarization vector ($n^2 = -M^2$, $p\cdot n = 0$) and

$$W_{\mu\nu}(q,p) = \frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p, n|[J_\mu(x), J_\nu(0)]|p, n\rangle .$$

It is expected that νG_1 and $\nu^2 G_2$ should scale in the Bjorken limit ($\nu, q^2 \to \infty$, $\xi \equiv -q^2/2\nu$ fixed):

$$\lim_{\text{Bj}} \frac{\nu}{M^2}G_1(q^2,\nu) \equiv g_1(\xi),$$

$$\lim_{\text{Bj}} \frac{\nu^2}{M^4}G_2(q^2,\nu) \equiv g_2(\xi).$$

Conventional quark light-cone algebra[6] yields the following expression for $g_1(\xi)$:

$$g_1^{ep}(\xi) = \frac{1}{4\pi}\int_{-\infty}^{\infty} d(x\cdot p)e^{i\xi x\cdot p} \times\left[\tfrac{1}{6}S_3^5(x\cdot p) + \frac{1}{6\sqrt{3}}S_8^5(x\cdot p) + (\tfrac{2}{27})^{1/2}S_0^5(x\cdot p)\right],$$

$$g_1^{en}(\xi) = \frac{1}{4\pi}\int_{-\infty}^{\infty} d(x\cdot p)e^{i\xi x\cdot p} \times\left[-\tfrac{1}{6}S_3^5(x\cdot p) + \frac{1}{6\sqrt{3}}S_8^5(x\cdot p) + (\tfrac{2}{27})^{1/2}S_0^5(x\cdot p)\right], \tag{1}$$

where the $S_a^5(x\cdot p)$ are defined from the bilocal operators:

$$\langle p, n|S_{a\sigma}^5(x|0)|p, n\rangle|_{x^2=0} = n_\sigma S_a^5(x\cdot p) + \cdots,$$

where $S_{a\sigma}^5(x|0)$ is the axial-vector bilocal which is symmetric under interchange of x and 0, and has the following form in free-field theory:

$$S_{a\sigma}^5(x|0) = \bar\psi(x)\frac{\lambda_a}{2}\gamma_\sigma\gamma_5\psi(0) + \bar\psi(0)\frac{\lambda_a}{2}\gamma_\sigma\gamma_5\psi(x).$$

Bjorken's sum rule follows from inverting the Fourier transform in Eq. (1), setting $x\cdot p$ to zero, taking the ep, en difference and noting that

$$\langle p, n|S_{3\sigma}^5(0|0)|p, n\rangle = 2g_A n_\sigma$$

since the local limit of the bilocal is just the neutral isovector axial-vector current.

In trying to derive a sum rule for scattering off polarized protons it is necessary to know $\langle p, n|S_{8\sigma}^5(0|0)|p, n\rangle$ and $\langle p, n|S_{0\sigma}^5(0|0)|p, n\rangle$. The first of these can be estimated using SU(3) for the baryon matrix elements of the axial charges. Hyperon decays are known to agree well with SU(3) and the Cabibbo theory, and the F/D ratio for axial charges is well determined.[7] However, nothing is known *a priori* about $\langle p, n|S^5_{0\sigma}(0|0)|p, n\rangle$. To evaluate it we argue on the basis of the parton model that the matrix elements of the combination

$$\frac{1}{\sqrt{2}}S^5_{0\sigma}(x|0) - S^5_{8\sigma}(x|0) \tag{2}$$

between nucleons are expected to be small.

The combination (2) gets contributions only from λ- and $\bar{\lambda}$-type quarks. Data from neutrino reactions[8] already suggest that the nucleons' wave functions have small $(\lambda, \bar{\lambda})$ components, and that it is reasonable to assume they only occur in the "sea" associated with the Pomeranchukon. The Pomeranchukon does not contribute to the spin-dependent structure functions G_1 and G_2,[9] so one might suspect that the $(\lambda, \bar{\lambda})$ contribution to them should be small. In fact

$$\left\langle p, n\left|\left[\frac{1}{\sqrt{2}}S^5_{0\sigma}(0|0) - S^5_{8\sigma}(0|0)\right]\right|p, n\right\rangle \propto \int_0^1 d\xi[\lambda_+(\xi) + \bar{\lambda}_+(\xi) - \lambda_-(\xi) - \bar{\lambda}_-(\xi)],$$

where $\lambda_\pm(\xi)$ are the distributions of λ quarks in the nucleon with helicities parallel (antiparallel) to the helicity of the nucleon itself,[3] and $\bar{\lambda}_\pm(\xi)$ are defined similarly. The usual spin and charge conjugation properties of the Pomeranchukon imply

$$\lambda_+(0) = \bar{\lambda}_+(0) = \lambda_-(0) = \bar{\lambda}_-(0).$$

Our assumption is that this remains at least approximately valid for $\xi \neq 0$ for the "sea" of $q\bar{q}$ pairs:

$$\lambda_+(\xi) \approx \bar{\lambda}_+(\xi) \approx \lambda_-(\xi) \approx \bar{\lambda}_-(\xi). \tag{3}$$

Assuming that the "sea" has the property (3) it follows that[10]

$$\langle p, n|S^5_{0\sigma}(x|0)|p, n\rangle \simeq \sqrt{2}\,\langle p, n|S^5_{8\sigma}(x|0)|p, n\rangle$$

and separate sum rules for the proton and neutron asymmetries follow immediately:

$$\int_0^1 g_1^{ep}(\xi)d\xi = \frac{g_A}{12}\left(1 + \frac{5}{3}\,\frac{3(F/D) - 1}{(F/D) + 1}\right), \tag{4}$$

$$\int_0^1 g_1^{en}(\xi)d\xi = \frac{g_A}{12}\left(-1 + \frac{5}{3}\,\frac{3(F/D) - 1}{(F/D) + 1}\right). \tag{5}$$

The difference of Eqs. (4) and (5) is just Bjorken's sum rule.[2] In writing (4) and (5) we have used SU(3) to reexpress the ratio

$$\frac{\langle p, n|S^5_{3\sigma}(0|0)|p, n\rangle}{\langle p, n|S^5_{8\sigma}(0|0)|p, n\rangle}$$

in terms of the F/D ratio of the axial charges. Since the axial charges seem to form an excellent octet[7] we regard this as a minor assumption. Using the currently accepted value[7]

$$\frac{F}{D} = 0.581 \pm 0.029,$$

we obtain[11]

$$\int_0^1 d\xi\, g_1^{ep}(\xi) = \frac{g_A}{12}(1.78), \tag{6}$$

$$\int_0^1 d\xi\, g_1^{en}(\xi) = \frac{g_A}{12}(-0.22), \tag{7}$$

where $g_A = 1.248 \pm .010$.

We do not expect the sum rules, Eqs. (6) and (7), to be exact because of SU(3) breaking, the uncertainty in the F/D ratio, and the parton-model assumption about the $(\lambda, \bar{\lambda})$ distributions. However, as long as the momentum of strange quarks extracted from neutrino experiments (see footnote 8) remains small, we expect Eqs. (6) and (7) to be quite accurate estimates of polarization asymmetries in scattering off protons and neutrons separately. The proton sum rule (6) is particularly interesting because experiments using polarized proton targets will soon be performed, and because (following Bjorken[12]) it predicts that a large mean asymmetry of the order of 35% will be observed.

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studied by Zee from a somewhat different approach [A. Zee, Phys. Rev. D 6, 3011 (1972)].

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[7]H. Ebenhoh *et al.*, Z. Physik 241, 473 (1971).

[8]In the quark model

$$\int_0^1 d\xi\, \xi[\lambda(\xi)+\bar\lambda(\xi)] = 9\int_0^1 d\xi\, F_2^{\gamma N}(\xi) - \tfrac{5}{2}\int_0^1 d\xi\, F_2^{\nu N}(\xi)\Big|_{\Delta S=0},$$

where $\lambda(\xi)$ and $\bar\lambda(\xi)$ are the distributions of λ and $\bar\lambda$ quarks in the nucleon, and $F_2^{\gamma N}(\xi)$ and $F_2^{\nu N}(\xi)$ are structure functions averaged over neutrons and protons. D. Perkins [*Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 189] has estimated

$$\int_0^1 d\xi\, F_2^{\gamma N}(\xi) = 0.14\pm 0.02,\quad \int_0^1 d\xi\, F_2^{\nu N}(\xi)\Big|_{\Delta S=0} = 0.49\pm 0.07$$

implying that

$$\int_0^1 d\xi\, \xi[\lambda(\xi)+\bar\lambda(\xi)] = 0.02\pm 0.25.$$

[9]See for example, Kuti and Weisskopf (Ref. 3) and references therein.

[10]M. Kugler informs us that this relation can also be obtained using the Melosh transformation from current to constituent quarks.

[11]These sum rules are expected to converge in conventional Regge theory. Only singularities at $\alpha\geq 1$ would cause a divergence, and as noted earlier, the Pomeranchukon at $\alpha=1$ decouples from $g_1(\xi)$.

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Universal slope and the nonlinear Regge trajectories

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We have investigated constraints imposed by duality on the slopes of nonlinear trajectories. In Argand-loop analysis for square-root-type trajectories we show analytically that the leading and daughter trajectories in the *s* channel have the same form, except for an additive constant, as the input trajectory in the *t* channel. This result is also true for all leading nonlinear trajectories in a model proposal by Cohen-Tannoudji, Henyey, Kane, and Zakrzewski.

I. INTRODUCTION

In a model world of zero-width resonances, duality requires a universal slope for all *linear* trajectories. Thus, in Argand-loop analysis the slope of the output trajectory has been shown equal to the slope of the input trajectory.[1] The dual resonance amplitudes (Veneziano or Virosoro representation) also require a universal slope for all linear trajectories.[2,3] One important consequence of a universality of slope is that a unique scale parameter emerges for high-energy processes. However, in the real world, resonances have finite widths and consequently trajectories are nonlinear. This prompts the question: Does duality also require a unique scale parameter for nonlinear trajectories?

The purpose of the present paper is to examine the constraints imposed by duality on the slopes of nonlinear trajectories. Firstly, we investigate this problem without using a definite dual resonance representation. We show analytically that the partial-wave projection of a crossed-channel nonlinear Regge-pole contribution can result in partial-wave amplitudes which produce spiral traces in the Argand diagram. The resulting Regge trajectory in the direct channel is identical to the input trajectory apart from a constant, i.e., the input and output trajectories are parallel. Evaluation of the partial-wave projections of Regge amplitudes requires a definite form for the Regge trajectory. We have taken square-root type of trajectories and our proof can be extended to nth-root type of trajectories.

We then study a model recently proposed by CHKZ.[4] Crossing symmetry is exactly satisfied by this model and Regge asymptotic behavior is ensured by the requirement[5]

$$|\alpha(t)| \lesssim |C|t|^{1/2}\ln(t)|, \text{ as } |t|\to\infty . \quad (1)$$

We derive a condition on the asymptotic behavior of nonlinear Regge trajectories, employing a mod-