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Estimate of the Pomeranchukon-ρ cut for π-nucleon scattering*

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Assuming the existence of nonsense wrong-signature zeros (NWSZ) in inclusive reactions we propose a parametrization of $g_{P\rho}{}^{\rho}(t)$ for small t and then proceed to estimate this coupling phenomenologically. Using our value of $g_{P\rho}{}^{\rho}(t)$ we estimate the contribution of the ρ-P cut to the total πN cross section and calculate the polarization in $\pi^- p$ charge-exchange scattering as a ρ-pole ρ-P cut interference. We find that it is possible to obtain a fairly good fit to the polarization for values of our parameters that are consistent with experiment, and furthermore, we find that our fit to the polarization is very sensitive to the small-t behavior of $g_{P\rho}{}^{\rho}(t)$.

I. AN ESTIMATE OF $g_{P_\rho}{}^{\rho}(t)$

In what follows, we shall use an extension of the conventional definitions of nonsense and wrong signature.[1] Let us consider the reaction $a+b\to c$ +anything, and define $s=(p_a+p_b)^2$, $t=(p_b-p_c)^2$, and $M^2=(p_a+p_b-p_c)^2$. The diagram for this reaction in the triple-Regge limit[2] is shown in Fig. 1. The signature factor in this case is given by[3]

$$\xi(t)=\frac{\tau+\exp\{-i\pi[\alpha_3(0)-\alpha_1(t)-\alpha_2(t)]\}}{\sin\pi[\alpha_3(0)-\alpha_1(t)-\alpha_2(t)]},$$

where τ is the product of the three signatures of Reggeons 1, 2, and 3, respectively. Since in all the diagrams we will deal with $\tau=1$, we see that $\xi(t)$ becomes infinite when $\alpha_3(0)-\alpha_1(t)-\alpha_2(t)=-2$, -4, -6, ... We will call this a right-signature point, whereas $\alpha_3(0)-\alpha_1(t)-\alpha_2(t)=-1$, -3, -5, . . . will be referred to as a wrong-signature point. Notice (by looking at the triple-Regge vertex) that both sets of points mentioned above are nonsense.[1]

In the dual resonance model,[4] the single-particle inclusive distribution exhibits zeros when $\alpha_0-2\alpha_t=-1$, -2, . . . , $\alpha_3(0)=\alpha_0$, and $\alpha_1(t)=\alpha_2(t)=\alpha(t)=\alpha_t$. These zeros are nonsense zeros analogous to the ones that appear in two-to-two amplitudes. Since the Gordon-Veneziano model[4] is an exchange-degenerate dual resonance model, the zeros appear at both right- and wrong-signature points. Chang, Gordon, Low, and Treiman[5] find the same result by performing a simple model calculation based on a single-type multiladder Feynman diagram.

Actually, as long as the existence of nonsense wrong-signature fixed poles is neglected, one can argue the existence of NWSZ (nonsense wrong-signature zeros) in inclusive reactions in an essentially model-independent way by performing a double O(2, 1) expansion.[6] In this case, the zeros appear only at wrong-signature points, i.e., when $\alpha_3(0)-\alpha_2(t)-\alpha_1(t)=-1$, -3, -5, . . . for $\tau=+1$.

Finally, in the particular case we are interested in, by using finite missing-mass sum rules[3] (FMMSR) one can show[6] that the NWSZ are very likely to be present in $g_{P_\rho}{}^{\rho}(t)$, in other words, if we assume $\alpha_P(0)=1$, we expect $g_{P_\rho}{}^{\rho}(t)$ to vanish linearly with t as $t\to 0$.

In view of the above, we propose the following parametrization (valid only for small t):

$$g_{P_\rho}{}^{\rho}(t)=a'te^{\gamma t}. \tag{1}$$

Let us now introduce the following notation: $\rho(b;\, c|a)$ represents $d^2\sigma/dt\,dM^2$ in the triple-Regge region for the reaction $a+b\to c$ +anything, where c is a fragment of b. If we knew the difference

$$\rho(\pi^-;\, \pi^-|p)-\rho(\pi^+;\, \pi^+|p)$$

for many values of t, we would know the t dependence of $g_{P_\rho}{}^{\rho}(t)$ whether Eq. (1) is valid or not[7]; however, the lack of data forces us to assume Eq. (1) and to try to estimate a'.

Since we do not know the value of γ, it will be considered a free parameter throughout.

Let us now try to estimate a'. We assume that a usual two-body Regge residue $\beta_i(t)$ can be parametrized as $\beta_i(t)=\beta_i(0)e^{c_i t}$, where t is in GeV^2 (this parametrization is of no fundamental importance, of course; we choose it mainly because it is a good approximation, and it will be very useful for computational purposes below). We know that the contribution of a particular triple-Regge diagram to the cross section can be written as[2]

$$\frac{d^2\sigma}{dt\,dM^2}=\frac{1}{16\pi s^2}\beta_1(t)\beta_2(t)\beta_3(0)g_{12}{}^3(t)\times\left(\frac{s}{M^2}\right)^{\alpha_1(t)+\alpha_2(t)}(M^2)^{\alpha_3(0)}, \tag{2}$$

where 1, 2, and 3 refer to the three Reggeons and we have omitted particle labels for the time being. We see from Eq. (2) that the t dependence is given by

$$\frac{d^2\sigma}{dt\,dM^2} \propto e^{(\alpha_1' + \alpha_2')t \ln(s/M^2)} e^{(c_1+c_2)t} g_{12}{}^3(t)\,, \qquad (3)$$

where $\alpha_i(t) = \alpha_i(0) + \alpha_i' t$. Equation (3) clearly shows a strong exponential cutoff in t, so that if one integrates inclusive distributions over all values of t, the small-t region dominates. Therefore, whenever we integrate over all values of t, it is safe to use Eq. (1) for $g_{P_\rho}{}^\rho(t)$. Now, it is a matter of straightforward algebra to show that the product of the ρ and P signatures gives

$$\mathrm{Re}[\xi_\rho(t)\xi_P^*(t)] = \frac{\sin[\frac{1}{2}\pi(\alpha_P - \alpha_\rho)]}{\cos(\frac{1}{2}\pi\alpha_\rho)\sin(\frac{1}{2}\pi\alpha_P)}\ ,$$

which for $\alpha_P(t) = 1$ and $\alpha_\rho(t) = 0.5 + t$ becomes $\mathrm{Re}[\xi_\rho(t)\xi_P^*(t)] \approx 1$ for small t. Therefore,[8]

$$\rho(\pi^-;\,\pi^-|\,p) - \rho(\pi^+;\,\pi^+|\,p) \cong K' \equiv \left(\frac{4}{16\pi s^2}\right)\beta_{\pi\pi P}(t)\beta_{\pi\pi\rho}(t)\beta_{pp\rho}(0)\,g_{P_\rho}{}^\rho(t)\left(\frac{s}{M^2}\right)^{\alpha_P(t)+\alpha_\rho(t)}(M^2)^{\alpha_\rho(0)} \qquad (4)$$

for small t.

On the other hand, from the data on Regge residues[9] we know that $\beta_{\pi\pi P}(t)\beta_{\pi\pi\rho}(t) \propto e^{2t}$, where t is in GeV^2. So, using this fact, and Eq. (1), we can integrate Eq. (4) over all t to obtain

$$K \equiv \int_{|t_{\min}|}^{\infty} K'\,|dt| = -\left(\frac{1}{4\pi s^2}\right)\beta_{\pi\pi P}(0)\beta_{\pi\pi\rho}(0)\beta_{pp\rho}(0)s^{3/2}(M^2)^{-1}a'$$
$$\times\left\{\frac{t\exp[(\ln(s/M^2)+2+\gamma)t]}{\ln(s/M^2)+2+\gamma} - \frac{\exp[(\ln(s/M^2)+2+\gamma)t]}{[\ln(s/M^2)+2+\gamma]^2}\right\}_{t=t_{\min}}^{-\infty}\ .$$

Finally, from the inclusive kinematics[10] for these particular reactions

$$t_{\min} = \frac{M^2}{s}\left(m_\pi{}^2 - \frac{m_\pi{}^2}{(1-M^2/s)}\right)\ ,$$

so that if s/M^2 is 3 or 4, we see that $t_{\min} \approx 0$ and then

$$K = \frac{1}{4\pi s^2}\beta_{\pi\pi P}(0)\beta_{\pi\pi\rho}(0)\beta_{pp\rho}(0)\frac{s^{3/2}(M^2)^{-1}a'}{[\ln(s/M^2)+2+\gamma]^2}\ . \qquad (5)$$

Notice that for a certain value of K, $|a'|$ increases as γ increases.

A quantity usually measured in inclusive experiments is

$$F = \int dp_\perp{}^2\left(\frac{E^*}{\pi p^*_{\max}}\right)\frac{d^2\sigma}{dx\,dp_\perp{}^2}\ ,$$

where $x = (p^*/p^*_{\max})$ is the well-known Feynman variable and the asterisk represents c.m. quantities. Since from kinematics

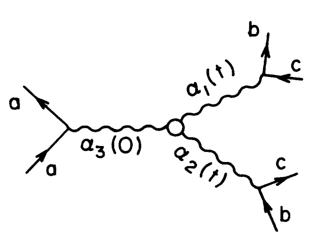

FIG. 1. Diagram for the reaction $a + b \to c + X$ in the triple-Regge region where c is a fragment of b. The open circle represents $g_{12}{}^3(t)$.

$$\frac{d\sigma}{dM^2} \equiv \int_{\text{all }t}\frac{d^2\sigma}{dt\,dM^2}\,dt = \left(\frac{\pi}{s(x)}\right)F\ ,$$

K is just the difference between two F distributions and from Eq. (5) we can estimate a'. We use the most recently available data[11] which are presented at $p_{\text{lab}} \simeq 8$ GeV/c, and what is given is the F distribution for each reaction as a function of x. Owing to the fact that the difference between the two F distributions is comparable to (and sometimes even smaller than) the experimental error in the distributions themselves, our estimate is very rough. We choose $x = 0.7$ (i.e., $M^2 = 4.54$ GeV^2, $s/M^2 = 3.35$) for our estimate. This turns out to be the best value of x if we wish to keep s/M^2 large, M^2 large, and the difference in the two distributions larger than the experimental errors.[6]

From Eq. (5) and the experimental data we obtain

$$a'/[\ln(s/M^2)+2+\gamma]^2 = 0.92 \pm 0.69\ \mathrm{GeV}^{-4}\,,$$

which gives

$$a' = -9.6 \pm 7.2\ \mathrm{GeV}^{-4} \quad \text{for } \gamma = 0\,,$$
$$a' = -16.5 \pm 12.4\ \mathrm{GeV}^{-4} \quad \text{for } \gamma = 1\,,$$
$$a' = -25.4 \pm 19.0\ \mathrm{GeV}^{-4} \quad \text{for } \gamma = 2\,,$$
$$a' = -36 \pm 27\ \mathrm{GeV}^{-4} \quad \text{for } \gamma = 3\,,$$
$$\cdots$$

etc.

We conclude this section by reminding the reader that with our conventions, $g_{P_\rho}{}^{\rho}(t)$ is in GeV^{-2} and our normalization coincides with that of Rajaraman,[12] and also by mentioning that even though the errors are very large, our estimate will still enable us to study some properties of the ρ-P Regge cut in Sec. II.

II. AN ESTIMATE OF THE P-ρ CUT FOR πN SCATTERING

Regge cuts have been discussed in the past by many authors,[13] and more recently, a great deal of interest has arisen in actually trying to obtain the magnitude of the discontinuity across them and apply this result to phenomenological analyses.[14,15]

In the present section we will assume that Gribov's formula[16] holds for the P-ρ cut in πN scattering, and using this formula as well as our results of previous sections we will try to estimate the contribution of the cut to physical observables like the total cross section and the polarization in πN charge-exchange scattering. Our calculation is in a way similar to that of Muzinich *et al.*,[15] so we urge the reader to familiarize himself with this work, which is particularly useful to us because it also contains a very detailed derivation of Gribov's formula for the discontinuity across the cut.

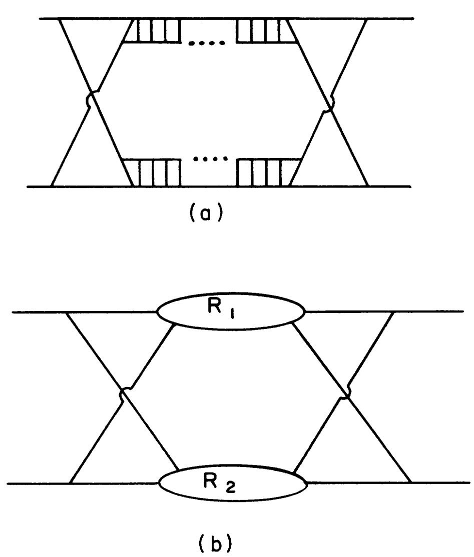

FIG. 2. (a) A diagram which when summed over all possible numbers of rungs gives rise to a Regge cut. (b) The Feynman diagram of (a) with the ladders replaced by Regge poles R_1 and R_2.

A. An estimate of the P-ρ cut contribution to the total πN cross section

Gribov's formula for the discontinuity across a two-Reggeon cut is derived by studying the asymptotic behavior of the Feynman diagram shown in Figs. 2(a) and 2(b). There is no proof that this formula holds for strong interactions in general, since it was derived only for a particular Feynman diagram. However, we will assume that Gribov's formula is valid for the ρ-P cut in πN scattering (see Fig. 3). We shall not derive the formula in the present work, but we refer the reader to the paper by Muzinich *et al.*[15] as well as the review article by Collins.[17]

A few comments are in order at this point. Some authors disagree with Gribov's sign for the discontinuity across a two-Reggeon cut. Abarbanel[18] isolates N-Reggeon irreducible amplitudes using the unitarity relation and the multiperipheral model of production and obtains a sign opposite to that of Gribov. Chew[19] argues on physical grounds that Abarbanel's sign is correct. On the other hand, White[20] seems to agree with Gribov's sign by doing an independent calculation. The basic difference between Reggeon calculus and field-theory computations, and the simple unitarity construction through pole contributions is that Reggeon calculus and field-theory calculations allow consideration of two-Reggeon contributions to the full elastic amplitude, and the total discontinuity of these amplitudes includes terms which do not arise in the unitarity construction.[21]

Let us now return to our estimate of the ρ-P cut. In what follows, we will use the usual A' and B amplitudes for πN scattering as defined by Singh.[22] In terms of A' and B, experimental quantities are given by

$$\sigma_T(s) = \frac{1}{p}\,\mathrm{Im}A'(s, t=0)\,, \tag{6}$$

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{\pi s}\left(\frac{m}{4p^*}\right)^2\left[\left(1-\frac{t}{4m^2}\right)|A'|^2 - \frac{t}{4m^2}\,\frac{4m^2p^2+st}{4m^2-t}\,|B|^2\right], \tag{7}$$

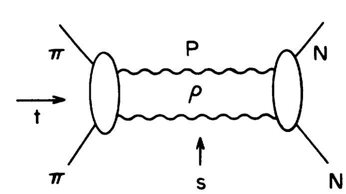

FIG. 3. ρ-P cut in πN scattering.

$$P(s,t) = -\left(\frac{\sin\theta}{16\pi s^{1/2}}\right)\frac{\mathrm{Im}(A'B^*)}{d\sigma/dt}\ , \qquad (8)$$

where s is the invariant square of the total energy, p is the pion lab momentum, p^* is the c.m. momentum, θ is the c.m. angle, m is the nucleon mass, and $P(s,t)$ is the polarization parameter defined relative to the normal $\vec{p}_i \times \vec{p}_f$, where p_i and p_f are the initial and final pion momenta. The contribution of the forward cut to $\mathrm{Im}A'$ is given by[15-17]

$$\mathrm{Im}A'_{\rm cut}(s,0) = \frac{1}{32\pi s}\int_{-\infty}^{0} dt\,\phi_{P_\rho}(t) N_{P\pi\to\rho\pi}(t) N_{PN\to\rho N}(t) \times\left(\frac{s}{s_0}\right)^{\alpha_P(t)+\alpha_\rho(t)}, \qquad (9)$$

where[3,15,23]

$$N_{P\pi\to\rho\pi}(t) = \int_0^{\overline{M}_0^2} A'_{P\pi\to\rho\pi}(\overline{M}^2,t)\,d\overline{M}^2 - \frac{\beta_{\rho\pi\pi}(0)\,g_{P\rho}{}^{\rho}(t)}{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)}\,(\overline{M}_0{}^2)^{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)}\ , \qquad (10)$$

$$N_{PN\to\rho N}(t) = \int_0^{\overline{M}_0^2} A'_{PN\to\rho N}(\overline{M}^2,t)\,d\overline{M}^2 - \frac{\beta_{\rho N N}(0)\,g_{P\rho}{}^{\rho}(t)}{1+\alpha_\rho(0)-\alpha_{\cdot}(t)-\alpha_\rho(t)}\,(\overline{M}_0{}^2)^{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)}\ . \qquad (11)$$

Here $\overline{M}^2 = M^2 - t - m_\pi^2$ (M is the missing mass), $s_0 = 1$ GeV^2, $\beta_{\rho\pi\pi}$ and $\beta_{\rho N\overline{N}}$ are the usual nonflip two-body residues, and

$$\phi_{P_\rho} = 2\,\mathrm{Re}(\xi_P\xi_\rho)\,,$$

where ξ_i is the usual signature factor. The absorptive parts of the $P\pi\to\rho\pi$ and $PN\to\rho N$ Reggeon-particle amplitudes are represented by $A'_{P\pi\to\rho\pi}(\overline{M}^2,t)$ and $A'_{PN\to\rho N}(\overline{M}^2,t)$, respectively; N can be recognized as the residue of the nonsense wrong-signature fixed pole in the appropriate Reggeon-particle scattering amplitude at $J=\alpha_P(t)+\alpha_\rho(t)-1$ (for forward cuts both Reggeons are at the same mass). Equations (10) and (11) are obtained from the FMMSR.[3,23]

Since we are only interested in a rough estimate of the cut, we will make the following approximations: We assume $\alpha_\rho = 0.5+t$, $\alpha_P = 1$.[24] Then,

$$\xi_P\xi_\rho = -i\,\frac{\cos[\frac{1}{2}\pi(\alpha_P+\alpha_\rho)] - i\,\sin[\frac{1}{2}\pi(\alpha_P+\alpha_\rho)]}{\sin(\frac{1}{2}\pi\alpha_P)\cos(\frac{1}{2}\pi\alpha_\rho)}$$

so that

$$\mathrm{Re}(\xi_P\xi_\rho) = -\frac{\sin[\frac{1}{2}\pi(\alpha_P+\alpha_\rho)]}{\sin(\frac{1}{2}\pi\alpha_P)\cos(\frac{1}{2}\pi\alpha_\rho)} \approx -1$$

for small t; on the other hand, we will assume that $A_{P\pi\to\rho\pi}$ is dominated by the π pole, and $A_{PN\to\rho N}$ is dominated by the N pole. With this in mind, we can write

$$\int_0^{\overline{M}_0^2} A_{P\pi}(\overline{M}^2,t)\,d\overline{M}^2 = \beta_{P\pi\pi}(t)\beta_{\rho\pi\pi}(t)\,, \qquad (12a)$$

$$\int_0^{\overline{M}_0^2} A_{PN\to\rho N}(\overline{M}^2,t) = \beta_{PNN}(t)\beta_{\rho NN}(t)\,, \qquad (12b)$$

and choose $\overline{M}_0{}^2 = 1$ in the rest of the calculation. Equations (12a) and (12b) might seem a little bit surprising; for example, one might ask about higher contributions like A_1 and A_2 in Eq. (12a) or N^* contributions to Eq. (12b). Let us therefore forget about the cut for one moment, and show that Eqs. (12a) and (12b) are indeed a reasonable approximation. First, let us calculate the contributions of the A_1 and A_2 to the integral in Eq. (12a). From the data of Antipov *et al.*[25] we know that for the reaction $\pi^-p\to A_2^-p$ the cross section can be parametrized as

$$\frac{d\sigma}{dt} = -Ate^{Bt}\,,$$

with $B = 8$ GeV^{-2} and $A = 900$ μb/GeV^4. Analogously, for the reaction $\pi^-p\to A_1^-p$ one has

$$\frac{d\sigma}{dt} = ae^{bt}\,,$$

with $b = 6.7$ GeV^{-2} and $a = 218$ μb/GeV^2. We immediately see that A_2 production is dominated by the flip amplitude, and for small t the A_2 contribution is expected to be negligible. In order to estimate the A_1 contribution, we shall use a logical (although phenomenologically unfounded) extension of Carlitz, Green, and Zee's P-P' universality.[26] Using an obvious notation for Regge couplings at $t=0$ we will assume that

$$\frac{(\pi^-A_1^+P)}{(\pi^-A_1^+P')} = \frac{(ppP)}{(ppP')} \approx 1\,. \qquad (13)$$

Furthermore, we know[6] that in the exchange-degenerate case

$$\kappa \equiv \frac{(ppP')(\pi^-A_1^+P')}{(pp\rho)(\pi^-A_1^+\rho)} = -5.5\,. \qquad (14)$$

Since the data of Antipov *et al.* are measured at $s = 40$ GeV^2, we will assume that P exchange dominates differential cross sections, so that at $t=0$

$$\frac{d\sigma}{dt}(\pi^-p\to A_1^-p) = 0.56\ \mathrm{GeV}^{-4} = \left(\frac{1}{16\pi}\right)|(\pi^-A_1^+P)(ppP)|^2\,.$$

From this and the known value of (ppP) we can calculate $(\pi^- A_1^+ P)$ (up to a sign) and then using Eqs. (13) and (14) we can estimate $(\rho\pi^- A_1^+)$. In our normalization we find

$$(P\pi\pi)(\rho\pi\pi) \approx 34\,,$$

while

$$|(P\pi^- A_1^+)(\rho\pi^- A_1^+)| \approx 0.4\,.$$

By an entirely analogous reasoning, the A_2 contribution to Eq. (12a) is also shown to be negligible:

$$|\rho\pi^- A_2^+|^2 = 0.61\,.$$

Now, regarding the N^* contribution to Eq. (12b), from the data of Foley *et al.*[27] for the Roper $N^*(1400)$ production the cross section is given as

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^- N^*(1400)) = 6.4\times e^{12.1t}\ \mathrm{GeV}^{-4},$$

$$\frac{d\sigma}{dt}(\pi^+ p \to \pi^+ N^*(1400)) = 6.2\times e^{15.6t}\ \mathrm{GeV}^{-4},$$

$s \approx 31\ \mathrm{GeV}^2$. Therefore, at $t=0$

$$\frac{d\sigma}{dt}(\pi^- p \to \pi^- N^*) = \left(\frac{1}{16\pi}\right)|f_P + f_\rho(1+k)s^{-1/2}|^2,$$

$$\frac{d\sigma}{dt}(\pi^+ p \to \pi^+ N^*) = \left(\frac{1}{16\pi}\right)|f_P + f_\rho(-1+k)s^{-1/2}|^2,$$

where we have assumed $\alpha_{P'}(0) = \alpha_\rho(0) = \frac{1}{2}$, have written $f_R \equiv (\pi\pi R)(NN^*R)$ for any Reggeon R, and have defined

$$k \equiv \frac{f_{P'}}{f_\rho}\,.$$

If we now use universality again (i.e., $f_P \simeq f_{P'}$), we have

$$16\pi\left[\frac{d\sigma}{dt}(\pi^- p \to \pi^- N^*) - \frac{d\sigma}{dt}(\pi^+ p \to \pi^+ N^*)\right] \approx 4 f_P[f_\rho(s^{-1/2} + s^{-1}]\,.$$

Since f_P is known, we can calculate f_ρ and we find

$$(NNP)(NN\rho) \simeq 35\,, \quad |(NN^*P)(NN^*\rho)| \simeq 0.36\,,$$

so that the N^* contribution to Eq. (12b) is of the order of 1%. The contributions of other N^* resonances turn out to be negligible too, and actually even smaller than the contribution of the $N^*(1400)$. For example, by using the same reasoning as above we find for $N^*(1600)$,

$$|(NN^*P)(NN^*\rho)| \approx 0.26\,,$$

and so on. Actually, for $N^*(1400)$ one can estimate $(NN^*\rho)$ independently from the N^*-production charge-exchange data,[28] and the result is identical to the one we just mentioned. Therefore, we feel that Eq. (12) is a reasonable approximation, and we can now get back to the estimate of

$$\sigma_c \equiv p\ \mathrm{Im}\,A'_{\mathrm{cut}}(s,0).$$

Once Eq. (12) is established, we see that knowledge of $g_{P_\rho}{}^{\rho}(t)$ is all we need to calculate σ_c, and this is precisely the triple-Regge residue we estimated in Sec. I. Since the experimental errors in the determination of a' are fairly large, and we do not know the value of γ, we present our results as follows: We choose $a' = 9.6\ \mathrm{GeV}^{-4}$, which is the central value for $\gamma=0$ (see our discussion at the end of Sec. I), and then we plot the ratio $(\sigma_c/\sigma_{\mathrm{pole}})$ as a function of s, for different values of γ.

Here σ_{pole} is the ρ-pole contribution to the $\pi^- p$

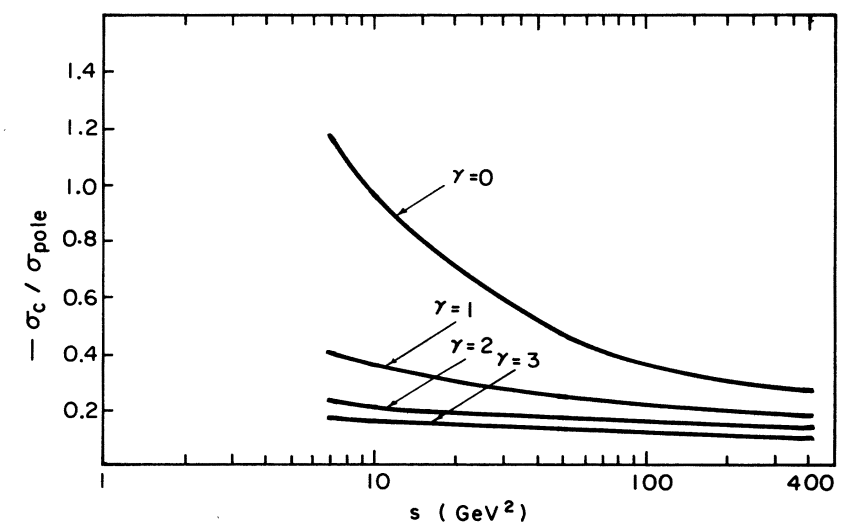

FIG. 4. Ratio of ρ pole to ρ-P cut contributions to $\sigma_T(\pi^- p)$ as a function of s and γ.

total cross section. We should mention that in our calculation we have parametrized the two-body Regge residues as simple exponentials:

$$\beta_{PNN}(t)=\beta_{PNN}(0)e^{2t},$$

$$\beta_{\rho\pi\pi}(t)=\beta_{\rho\pi\pi}(0)e^{t},$$

$$\beta_{\rho NN}(t)=\beta_{\rho NN}(0)e^{t}\quad\text{(nonflip residue)},$$

$$\beta_{P\pi\pi}(t)=\beta_{P\pi\pi}(0)e^{t}.$$

This parametrization is consistent with the Regge fits we are using[9] and the fact that we have simple exponentials will become extremely useful for computational purposes below.

Our results are shown in Fig. 4. We have not said much about the a' dependence of the cut yet. We will talk about it in detail after we calculate the polarization for πN charge-exchange scattering. In the meantime, let us see what happens when one wants to calculate the contribution of the cut to the B amplitude, what should be done about nonforward cuts, etc.

B. Calculation of nonforward cuts

When we wish to calculate other observables like, for example, the differential cross section or the polarization, we must know the B amplitude also. Calculating the forward contribution of the ρ-P cut to the B amplitude does not pose a big problem. We can write equations entirely analogous to Eqs. (9), (10), and (11), with the only basic difference being that we take the $\beta_{\rho NN}$ to be the flip residue instead of the nonflip residue. For example, if we define $\beta_{\rho NN}^{\Delta h=1}=\beta'_{\rho NN}$, where Δh is the change in helicity at the vertex in question, we have

$$\nu\,\mathrm{Im}\,B_{\mathrm{cut}}(s,0)=\frac{1}{32\pi s}\int_{-\infty}^{0}dt\,N'_{P\pi\to\rho\pi}(t)N'_{PN\to\rho N}(t)\left(\frac{s}{s_0}\right)^{\alpha_P(t)+\alpha_\rho(t)},$$

where $\nu\equiv(s-u)/4m$ and

$$N'_{P\pi\to\rho\pi}(t)=N_{P\pi\to\rho\pi}(t)\approx\beta_{P\pi\pi}(t)\beta_{\rho\pi\pi}(t)-\frac{\beta_{\rho\pi\pi}(0)g_{P\rho}{}^{\rho}(t)}{(1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t))}(\overline{M}_0{}^2)^{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)}$$

and

$$N_{PN\to\rho N}(t)\approx\beta_{PNN}(t)\beta'_{\rho NN}(t)-\frac{\beta'_{\rho NN}(0)g_{P\rho}{}^{\rho}(t)}{(1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t))}(\overline{M}_0{}^2)^{1+\alpha_\rho(0)-\alpha_P(t)-\alpha_\rho(t)}.$$

Again, $\beta'_{\rho NN}$ can be approximated by a simple function. It turns out that

$$\beta'_{\rho NN}(t)\approx\text{constant}$$

is a good approximation for small t.[29] This approximation will of course break down for $|t|$ near 0.5 GeV^2 since we have not included the usual nonsense wrong-signature zero when α_ρ vanishes; however, the approximation is fairly good for, say, $|t|\leq 0.3$ GeV^2.

So far, we have been able to calculate forward cuts in terms of fairly simple functions, and integrals that can be done analytically. On the other hand, we know from kinematics that the polarization vanishes at $t=0$, therefore we would like to calculate the contributions of nonforward cuts to the A' and B amplitudes. This is a slightly more complicated problem; however, we shall soon see that as long as all our Regge residues are expressed in terms of exponential functions, we still get an analytic expression for the cut.

When $t\neq 0$, instead of having a single integral over t, we have a double integral.[15-17,30] Let us forget about factors multiplying the integral, and just say that we have an integral of the form

$$H(t)=\frac{1}{\pi}\int dt_1\int dt_2\,\theta(-\Delta)[\Delta(t_1,t_2,t)]^{-1/2}\times g_1(t_1)g_2(t_2),\tag{15}$$

where

$$\Delta(t_1,t_2,t)\equiv t_1{}^2+t_2{}^2+t^2-2t_1t-2t_2t-2t_1t_2.$$

The functions g_1 and g_2 are essentially the N functions of Eqs. (10) and (11). Therefore, let us assume that

$$g_i(t_i)=e^{\gamma_i t_i},\quad i=1,2.\tag{16}$$

Equation (16) only reflects the fact that we can approximate all our Regge residues by exponentials, so that if we can obtain a closed expression for the integral in Eq. (15) when Eq. (16) holds, we will be able to calculate nonforward cuts analytically.

In order to evaluate the integral in Eq. (15), let us make the following change of variables:

$$\tfrac{1}{2}(t_1+t_2)=u,\quad t_1=u+\tfrac{1}{4}t+(ut)^{1/2}z,$$

$$\frac{t_1-t_2}{2(ut)^{1/2}}=z,\quad t_2=u+\tfrac{1}{4}t-(ut)^{1/2}z,$$

which implies that

$$(1-z^2)^{1/2}=\frac{[-\Delta(t_1,t_2,t)]^{1/2}}{2(ut)^{1/2}} .$$

Remembering that the phase-space boundary is given by $\Delta=0$, i.e., $z^2-1=0$, we have

$$H(t)=\frac{1}{\pi}\int_{-\infty}^{0}du\int_{-1}^{+1}dz\,g_1(t_1)g_2(t_2)(1-z^2)^{1/2},$$

which upon substitution of Eq. (16) becomes,

$$H(t)=\frac{1}{\pi}\int_{-\infty}^{0}du\int_{-1}^{+1}\frac{dz}{(1-z^2)^{1/2}}\exp[(\gamma_1+\gamma_2)(u+\tfrac{1}{4}t)+(ut)^{1/2}(\gamma_1-\gamma_2)z]$$

$$=\int_{-\infty}^{0}du\,e^{(\gamma_1+\gamma_2)(u+t/4)}\frac{1}{\pi}\int_0^{\pi}d\phi\,e^{(ut)^{1/2}(\gamma_1-\gamma_2)z},$$

where $z=\cos\phi$. Now put $u=-v$, then all the integrals are known[31] and we have

$$H(t)=\int_0^{\infty}dv\,e^{-(\gamma_1+\gamma_2)(v-t/4)}I_0[(\gamma_1-\gamma_2)(-vt)^{1/2}]$$

$$=\frac{e^{(\gamma_1+\gamma_2)t/4}}{\gamma_1+\gamma_2}\,e^{-(\gamma_1-\gamma_2)^2t/4(\gamma_1+\gamma_2)},$$

so that finally,

$$H(t)=\frac{e^{\gamma_1\gamma_2t/(\gamma_1+\gamma_2)}}{\gamma_1+\gamma_2} . \tag{17}$$

Equation (17) is precisely what we wanted. It tells us that as long as we use exponential functions, the only difference between forward and nonforward cuts is the extra "nonforward correction factor" $e^{\gamma_1\gamma_2t/(\gamma_1+\gamma_2)}$. We are not quite done, however. In Eqs. (10) and (11) we were able to use the triple-Regge residue of inclusive reactions, because the calculation of a forward cut guarantees that the mass of one of the three legs in a triple-Regge coupling is zero. When we go to nonforward cuts, however, this is no longer true, and the use of the triple-Regge residue of inclusive reactions is no longer justified. Since we are interested in small values of t, ($|t|\leq 0.3$ GeV^2), and we have to make some assumption about the value of this triple-Regge coupling, we will assume that the coupling is the same even when all three Reggeon masses are nonvanishing, in other words, we will keep on using $g_{P\rho}{}^{\rho}$ as the coupling, even though as we said before this is not justified.

C. Estimate of the polarization for π^-p charge-exchange scattering

We have seen in Sec. II B that we are able to estimate the contribution of the ρ-P cut for both the A' and the B amplitudes as long as t is small. In this section we will assume that the polarization in π^-p charge-exchange scattering is produced by the combined effect of the ρ pole and the ρ-P cut,[32] so that once we know the cut contribution to the amplitudes we can immediately calculate the polarization from Eq. (8). Our results will obviously depend on the value of γ we choose, and furthermore, they are extremely sensitive to the value of a', as can be seen by recalling from Eqs. (9), (10), and (11) that the cut contribution to the amplitude has linear as well as quadratic terms in a'. In fact, these terms have opposite signs, so changing a' will greatly affect the cut contribution, and perhaps even the sign of the cut contribution. This effect is further enhanced by the fact that in Eq. (8) either products or squares of amplitudes appear, so that one has terms of order as large as $(a')^4$, and several terms have opposite relative signs. We also notice that as $|a'|$ becomes larger and larger, the quadratic terms in a' must dominate the amplitudes, and then the amplitudes become so large in magnitude that the cross section as calculated from Eq. (7) becomes much larger than the experimental value. In particular, if the cut contribution to the B amplitude is too large, we see from Eq. (7) that as t goes away from zero, $d\sigma/dt$ will become too large, and this might bring down the value of the polarization considerably, as soon as we move away from $t=0$. This phenomenon can actually be seen in the graphs that we will present shortly. We also remind the reader that if we decide to calculate the polarization for $-t\approx 0.5$ GeV^2, our approximation for $\beta'_{\rho NN}$ breaks down, and again $d\sigma/dt$ from Eq. (7) becomes too large. Therefore, the predicted polarization beyond, say, $-t=0.3$ GeV^2 cannot be taken seriously.

As for the s dependence, our model predicts a decrease proportional to $(\ln s)^{-1}$ as s increases.

Because of our unknown parameters, as well as all our assumptions, we do not feel that it is useful to present very detailed numerical predictions. Instead, we present the results at $p_{\text{lab}}=5$ GeV/c for several typical values of a', γ, and t in Fig. 5. We see that it is possible to obtain a fairly good fit to the data. We have also compared the cross sections to those of Giacomelli *et al.*[33] For the highest values of $|a'|$ presented the predicted cross sections are too large. However, when we get a good fit to the polarization data, the cross sections are also reasonable (say, within 30% of the experimental data). Since in terms like

$$\mathrm{Im}\xi_\rho(\xi_{\text{cut}})^*=-\frac{\sin[\frac{1}{2}\pi(\alpha_\rho-\alpha_c)]}{\cos(\frac{1}{2}\pi\alpha_\rho)\cos(\frac{1}{2}\pi\alpha_c)} ,$$

the slope of the Pomeranchukon trajectory is important,[34] we have taken $\alpha_P=1+0.3t$. Notice that this affects very little our former estimate of the cut amplitudes.[24]

The experimental data shown in Fig. 5 are those

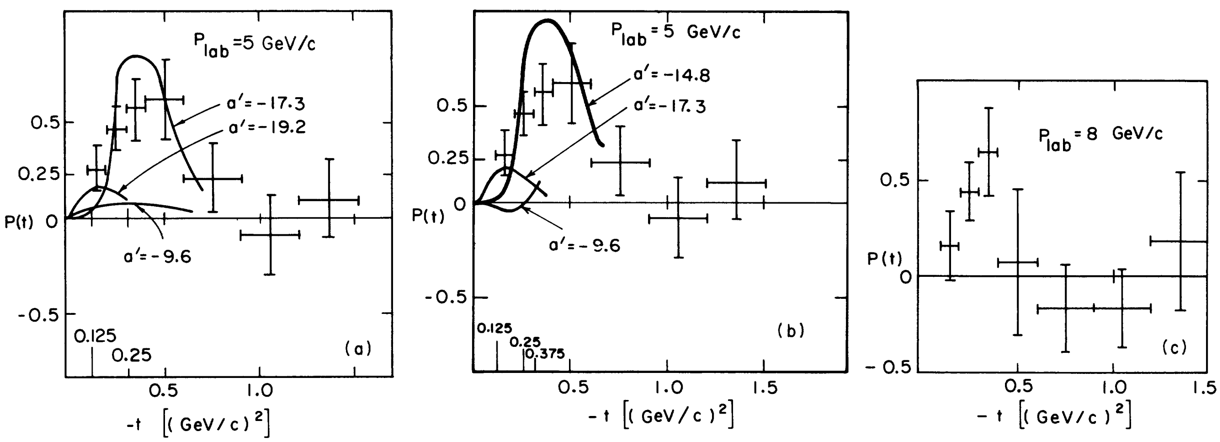

FIG. 5. $\pi^- p$ charge-exchange polarization data and predictions of our model (a) for $\gamma = 2$ and (b) for $\gamma = 1$; (c) experimental data at 8 GeV/c.

of Yokosawa,[35] and we see that the s dependence of our model is also consistent with the data [see Fig. 5(c)].

III. CONCLUSION

We have mentioned the possibility of the existence of NWSZ in inclusive reactions both in specific models as well as in a model-independent way. In particular, we mentioned that it is very likely that $g_{P_\rho}{}^{\rho}(t)$ has a NWSZ at $t=0$. With this basis, we estimated $g_{P_\rho}{}^{\rho}(t)$ for small t in terms of an unknown parameter γ and a parameter a' which is determined up to order of magnitude only, due to experimental errors. Then, in Sec. II we showed how to calculate the ρ-P Regge cut contribution to πN scattering in terms of $g_{P_\rho}{}^{\rho}(t)$ and under certain approximations we estimated the polarization for $\pi^- p$ charge-exchange scattering. We found that it is possible to get a fit consistent with experiment for certain values of γ and a' which are also consistent with experiment. Even though our calculation is very rough (and we caution the reader not to take our numbers too seriously), we feel that the results are encouraging; much work needs to be done, both theoretically and experimentally, to determine whether Gribov's formula is valid in general, and whether the $\pi^- p \to \pi^0 n$ polarization is indeed produced by the type of pole-cut interference we have worked with.

More important than the precise numerical results, however, is the fact that we have shown the possibility that triple-Regge residues might be very closely connected with physical observables (like the polarization) in two-body reactions. This we feel is our most interesting result.

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[8]In Eq. (4) diagrams involving lower trajectories have been neglected. This can always be done as long as the values of s and M^2 are large enough. See M. Dub-

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$$\begin{aligned}\alpha_c &= \alpha_P(0) + \alpha_\rho(0) - 1 + \frac{\alpha'_P\,\alpha'_\rho}{\alpha'_P + \alpha'_\rho}\,t\\ &\approx 0.5 + 0.23t\end{aligned}$$

if $\alpha_P = 1 + 0.3t$. If the Pomeranchukon were flat, then $\alpha_c \simeq 0.5$, and therefore we see that $\alpha_\rho - \alpha_c$ is considerably affected even by a small Pomeranchukon slope.

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