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Understanding s-channel partial-wave amplitudes in two-body scattering*

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We study in detail the s-channel partial-wave amplitudes for vector- and tensor-exchange amplitudes by comparing the results of seven different, quantitatively successful phenomenological analyses. We consider separately the partial waves in the peripheral region, where Regge poles alone should dominate, and in the central region, where Regge cuts may also be important. We show that these regions separately show interesting regularities which are not evident when the amplitudes are plotted as functions of t. We point out difficulties with individual analyses which help explain the discrepancies between their findings. We show that the existing analyses are consistent with the assumption that the Regge poles are approximately exchange-degenerate and that exchange-degeneracy breaking occurs mainly at small impact parameter.

I. INTRODUCTION

It is clear from any recent review of two-body inelastic reactions that, while much progress has been made, we are still far from a detailed, quantitative understanding of all the amplitudes in these processes.[1-5] The only inelastic amplitudes that have been determined in a model-independent way are the ρ-exchange amplitudes in $\pi^- p \to \pi^0 n$.[6] The experimentally determined amplitudes did not agree with any existing theory. Since then, no new simple theory has been proposed which can explain the ρ-exchange amplitudes and can also fit the less restrictive data on A_2, $K_V^*(890)$, and $K_T^*(1420)$ exchange. Many models[7-15] have been proposed which can fit the data; however, they constrain only some parts of the amplitudes and freely parametrize other parts. Thus they have little predictive power. Examples of such models are the dual absorption model,[7] which does not, in general, specify the real parts of amplitudes. Other examples are the many different extensions of the absorption model[9-15] which do not specify in detail the absorption corrections. Similar in spirit to the above-mentioned models, but not formulated in the language of any specific model, are the phenomenological analyses of Chiu and Ugaz[16] and of Irving, Martin, and Barger,[17] who assume that the helicity-flip amplitudes are exchange-degenerate and then solve for the helicity-nonflip K_V^* and K_T^* amplitudes. Still other analyses have freely parametrized all the amplitudes in terms of Regge-pole-exchange amplitudes, and then tried to determine the many parameters by using both the high-energy data and, via finite-energy sum rules, the low-energy data.[18,19] While all these theoretical models and phenomenological analyses agree on the ρ-exchange amplitudes and fit the available high-energy charge and hypercharge-exchange cross section and polarization data, they give significantly different results for the individual amplitudes for A_2, K_V^*, and K_T^* exchange. A particularly dramatic illustration of the discrepancies between the different analyses occurs in the determination of the K_V^* and K_T^* amplitudes in $\pi N \to K\Lambda$ and $\bar{K}N \to \pi\Lambda$ at 4 GeV/c. The results of Refs. 5, 17, and 19 bear practically no resemblance to each other (see Fig. 17 of Ref. 1).

In this paper we attempt to gain some additional understanding of the vector- and tensor-exchange amplitudes and to resolve some of the discrepancies among the earlier analyses by looking in detail at the s-channel partial-wave amplitudes found in seven rather different, quantitatively successful, phenomenological analyses of charge- and hypercharge-exchange reactions (the seven analyses chosen are listed in the beginning of Sec. II). The utility of looking at the s-channel partial waves lies in the fact that different regions in impact-parameter space are governed by different dynamics. Division of impact-parameter (b) space into 3 regions is particularly natural: the small-b region, $b \lesssim 0.5$ fermi (F), the peripheral region, $0.5 \lesssim b \lesssim 1.4$ F, and the very high partial-wave region, $b \gtrsim 1.4$ F. Only π exchange, if allowed, can contribute significantly for $b \gtrsim 1.4$ F;[20] Regge-pole exchange should dominate the peripheral region, and both Regge poles and cuts may be significant at small b. We show that each region of impact-parameter space separately shows interesting regularities which are not evident when the amplitudes are plotted as functions of t.

The small-b region is, in principle, very complicated. This region is affected by the existence of competing channels (unitarity or absorption corrections) and by other rescattering effects (i.e., Regge-Regge cuts). No one yet has devised a reliable theory, or even recipe, for calculating these effects. Many models have been proposed which assume that, in practice, the low partial waves behave in a simple manner, i.e., that the s-wave is totally absorbed,[21] or that the low partial waves can be calculated from simple geometrical considerations,[21,22] or that only elastic rescattering corrections are important.[23-25] All these simple models have failed, and new theories for unitarity and rescattering corrections are needed.

A possible first step in constructing new theories is the determination of the systematics of the low partial-wave amplitudes that is required by the data. Extensive phenomological analysis using a variety of theoretical assumptions is one way to try to establish the systematics in the absence of a complete set of experiments. Many such analyses exist and some systematics have been proposed. There are inconsistencies, however, and we study these and try to reconcile the various results.

Knowing the systematics of the low partial waves is important, also, for judging the credibility of the amplitudes found in any particular model-dependent analysis. In the absence of a complete set of experiments, a model which assumes incorrect behavior for the low partial-wave amplitudes may still be able to fit the data by having other partial-wave amplitudes compensate for this error.

The peripheral region, $1.4 \gtrsim b \gtrsim 0.5$ F, is dominated by the exchange of Regge poles, without the complicating effects of cuts. Our comparisons show that there is a strong tendency in most of the analyses considered for the peripheral partial-wave amplitudes to be exchange-degenerate for both helicity flip and helicity nonflip. The helicity-flip amplitudes have been known to be exchange-degenerate as a function of t for some time,[1-5] for ρ and A_2 at least, and so this result for the flip amplitudes is no surprise. The nonflip amplitudes, for ρ, A_2 and K_V^*, K_T^*, have been found, in all previous analyses, not to be exchange-degenerate when considered as functions of t. Our detailed comparison of the existing analyses suggests that breaking of exchange degeneracy in nonflip amplitudes occurs mainly in the low partial-wave amplitudes and that the peripheral Regge-pole amplitudes are, in fact, approximately exchange-degenerate.

Before we turn to the vector and tensor partial-wave amplitudes, which are the main subject of this paper, we wish to review briefly the status of our understanding of the s-channel partial waves of two other exchanges: π exchange and Pomeranchukon exchange.

Only π exchange can contribute significantly for $b \gtrsim 1.4$ F. The presence of this long-range π tail has been firmly established.[26,27] This contribution is real, behaves like $e^{-\mu' b}/(\mu' b)^{1/2}$, where $\mu' = (m_\pi^2 - t_{\min})^{1/2}$, and has a strength given by known coupling constants.[27] Thus, we regard the partial-wave amplitudes for $b \gtrsim 1.4$ F as well understood and do not consider this region further.

Reactions with π exchange are not, in general, useful for studying partial-wave amplitudes with $b \lesssim 1.4$ F. All that has been established so far about the low partial-wave amplitudes in π-exchange reactions is that they are much smaller than those given by π-exchange Born terms. Absorption is usually invoked to explain this effect. However, different models for the absorption cannot be distinguished at present because the experimental low partial waves give only a relatively small correction to the dominant high-partial-wave contributions.[26] Due to this difficulty we have excluded reactions with π exchange from our considerations of the $b \lesssim 1.4$ F region.

The status of Pomeranchukon-exchange partial-wave amplitudes is that their gross features are well known. The bulk of elastic cross sections can be explained by assuming that Pomeranchukon-exchange partial-wave amplitudes are predominantly imaginary in phase, Gaussian in shape, centered at $b=0$, and helicity-nonflip. Both the break[28,29] in the pp cross section at $t=-0.1$ $(\mathrm{GeV}/c)^2$ and the dip[29] at $t \approx -1.2$ $(\mathrm{GeV}/c)^2$ at CERN ISR energies can be explained by small deviations from the Gaussian form.

II. COMPARISONS OF MODELS

Of the seven analyses included in our comparisons, four are based on s-channel approaches. These are the dual absorption model (DAM) as applied by Loos and Matthews,[7] the strong central absorption prescription (SCAP) of Chiu,[16] and two of the latest versions of the conventional absorption model(AM). While numerous versions of the absorption model, or the Regge-pole-plus-cut model exist, for simplicity we include here only the analysis of Hartley and Kane[9] in which the Regge pole is "simplicity-choosing" (AM-SC) and that of Martin and Stevens[11] in which the Regge poles have exchange-degenerate form (AM-ExD). We also include in our comparisons the s-channel partial-wave amplitudes that have been found in an extensive phenomenological analysis by Desai and Stevens using the complex-pole model (CPM).[8] The CPM is based on t-channel dynamics, and in it the s-channel partial-wave amplitudes are derived rather than fundamental quantities. Finally, we also include the results of Barger and Phillips[18] (BP) on ρ and P' and the work of Irving, Martin, and Barger[17] (IMB) on K_V^* and K_T^*. These last two analyses do not assume any specific theoretical model but rather have attempted to determine the amplitudes, in the case of BP, by constraining the amplitudes by finite-energy sum rules or, in the case of IMB, by making reasonable assumptions about helicity-flip amplitudes and then solving for the nonflip amplitudes using the data directly.

The latter three analyses give as good fits to the data as any of the s-channel models without imposing any particular structure on the s-channel partial-wave amplitudes directly. These analyses therefore provide an independent check of which of the structures assumed in s-channel models or expected from geometrical arguments are really necessary to explain the data, and which are merely consistent with them.

In Fig. 1 we present the s-channel partial-wave amplitudes for ρ exchange (pole plus cut) at 6 GeV/c.[30] The ρ-exchange amplitudes as a function of t are unique in the sense that they have been determined in a model-independent way[6] at 6 GeV/c in the t range from 0 to -0.6 (GeV/c)^2. This, however, is not sufficient to determine the partial-wave amplitudes uniquely as can be seen in Fig. 1, since all the models shown agree with the model-independent amplitude analyses. Certain gross features are common to all the analyses. The imaginary parts are peripheral, i.e., peaked around $b\approx0.8$ F, while the real parts are central, more like the input Regge-pole amplitudes.

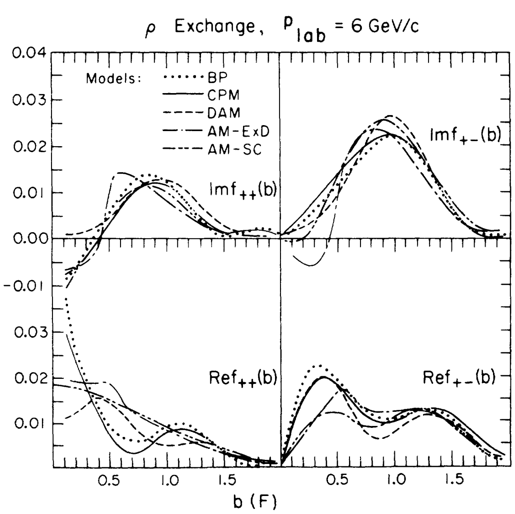

FIG. 1. The partial-wave amplitudes for ρ exchange at $p_{\rm lab} = 6$ GeV/c in five different analyses. The analyses used are described in the text.

For the imaginary parts, the discrepancies between models are readily understood. The high-b tail in $\mathrm{Im}f_{++}(b)$ in the Barger-Phillips analysis and in the complex-pole model comes, most likely, from 2π exchange.[31] Its inclusion, which is indirect in both analyses, results in slightly better fits to $\mathrm{Im}F_{++}(s, t=0)$ in these models than in the other three in which this effect is not allowed for. The small negative excursion at small b seems to be a real effect, since not one of the models specifically imposes such behavior, while the only model not showing it, the DAM, specifically excludes it.

For $\mathrm{Im}f_{+-}(b)$, the small negative excursions at small b in the AM-SC and AM-ExD analyses are a consequence of the assumption of helicity-independent rescattering and the previously shown need for a negative excursion in the nonflip amplitude. If such behavior is present it would provide some evidence that the new absorption models with the above assumptions are essentially correct. The small-b effect may not be real, however, since the BP and CPM analyses find no such behavior even though they do not exclude it.

The greatest disagreement in Fig. 1 is for $\mathrm{Re}f_{++}(b)$. While all models give nonperipheral profiles, they disagree greatly about details and so no real understanding for these partial waves can be claimed. It is interesting to note, however, that while the input Regge amplitudes for AM-SC and AM-ExD are very different, i.e., $\mathrm{Re}F_{++}^{\rm Regge}(t)$ has a single zero for AM-SC and a double zero for AM-ExD near $t=-0.6$ (GeV/c)^2, the addition of a large cut contribution produces quite similar results for the total amplitude. The $\mathrm{Re}f_{+-}(b)$ show more similarity in the various models for two reasons. First, cuts in flip amplitudes are smaller in models and also, apparently, in the data. Second, $\mathrm{Re}F_{+-}(t)$ is strongly constrained by the elastic πp polarization.[32] The AM-SC and AM-ExD partial waves imply a shallow double zero for $\mathrm{Re}F_{+-}(t)$ near $t=-0.6$ (GeV/c)^2 and explain the elastic polarization by ascribing considerable phase and t dependence to the Pomeranchukon. The other three models have $\mathrm{Re}f_{+-}(b)$ corresponding to $\mathrm{Re}F_{+-}(t)$ with a strong double zero and can explain the elastic polarization with less structure in the Pomeranchukon.

We return to the ρ-exchange results and their interpretation after discussing the A_2, K_V^*, and K_T^* exchanges.

In Fig. 2 we present the results for A_2 exchange (pole plus cut) at 6 GeV/c in four different analyses. The data included in the analyses were mainly differential cross-section data for $\pi^- p\to\eta n$ for $p_{\rm lab}$ from 4 to 18 GeV/c and for $|t|\lesssim2.0$ (GeV/c)^2. Very few polarization data are available and do not con-

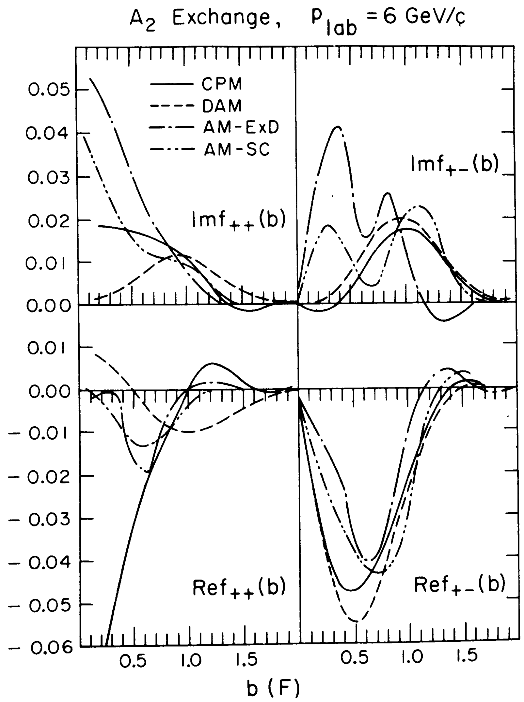

FIG. 2. The partial-wave amplitudes for A_2 exchange at p_{lab} = 6 GeV/c in four different analyses. See text for description of the analyses and for normalization used.

strain the models very much. Since the differential cross section for $\pi^- p \to \eta n$ is dominated by $\mathrm{Re}F_{+-}(t)$ for $t \lesssim 1.5$ $(\mathrm{GeV}/c)^2$, it is not surprising that the models agree reasonably well on $\mathrm{Re}f_{+-}(b)$. The data have a dip for $t \simeq -1.4$ $(\mathrm{GeV}/c)^2$ which is best fit with an unabsorbed Regge-pole amplitude of slope 1, intercept 0.5, and exchange-degenerate form. The AM-SC and AM-ExD analyses, with more parameters than a simple pole model, do not fit the data as well.[11] In the absence of a complete set of experiments, it is also not surprising that there is considerable disagreement about the other amplitudes: $\mathrm{Im}f_{++}(b)$, $\mathrm{Im}f_{+-}(b)$, and $\mathrm{Re}f_{++}(b)$.

For AM-SC and AM-ExD, the mechanism that produces the negative excursion in $\mathrm{Im}f_{++}(b)$ and $\mathrm{Im}f_{+-}(b)$ for ρ exchange produces the central $\mathrm{Im}f_{++}(b)$ for A_2 exchange and also the double-peaked structure in $\mathrm{Im}f_{+-}(b)$. The double-peaked behavior of $\mathrm{Im}f_{+-}(b)$ seems to be unique to these absorption-model treatments and not forced by the tensor-exchange data, as can be seen from the other model results. The positive peak at small b for A_2 exchange is larger than the negative excursion for ρ exchange because the real part of the A_2–Regge-pole amplitude, by reason of its signature factor, is more central than the real part of the corresponding ρ amplitudes. The central $\mathrm{Im}f_{++}(b)$ is confirmed by the CPM analysis and seems to hold for other tensor exchanges also (see below). The apparent failure of DAM to describe $\mathrm{Im}f_{++}(b)$ for tensor exchange is significant and will be discussed further below. The very different behavior of $\mathrm{Re}f_{++}(b)$ in the various models is easy to understand but sheds no light on the correct description. Because DAM assumes $\mathrm{Im}f_{++}(b)$ is peripheral, $\mathrm{Re}f_{++}(b)$ must also be peripheral if the polarization in $\pi^- p \to \eta n$ is to be small. Since all other analyses give $\mathrm{Im}f_{++}(b)$ nonperipheral, this determination of $\mathrm{Re}f_{++}(b)$ is very unreliable. In both AM-SC and AM-ExD, $\mathrm{Re}f_{++}(b)$ is suppressed at $b \approx 0$ if the rescattering used for tensor exchange is the same as for vector exchange. Finally, $\mathrm{Re}f_{++}(b)$ in the CPM has the behavior shown because in this analysis the nonflip A_2 amplitude was assumed to have Regge phase to reproduce the small $\pi^- p \to \eta n$ polarization. In summary, the CPM and DAM make no prediction for $\mathrm{Re}f_{++}(b)$ while the AM-SC and AM-ExD models predict $\mathrm{Re}f_{++}(b) \approx 0$ for $b \approx 0$. At present there are insufficient data to reach any conclusions about $\mathrm{Re}f_{++}(b)$ for A_2 exchange.

In Figs. 3 and 4, we present the s-channel partial-wave amplitudes for $K_V^*(890)$ and $K_T^*(1420)$ exchange (pole plus cut) at 4 GeV/c. The magnitudes and relative signs of the amplitudes shown correspond to the reaction $\pi^+ p \to K^+ \Sigma^+$. The analyses include also the line-reversed reaction $K^- p \to \pi^- \Sigma^+$ and other isospin related reactions. The CPM and DAM analyses include data from 4–18 GeV/c, while the IMB and the SCAP analyses use data mainly at 4 GeV/c. Absorption models such as those of Hartley and Kane (AM-SC) and Martin and Stevens (AM-ExD) give K_V^* and K_T^* amplitudes

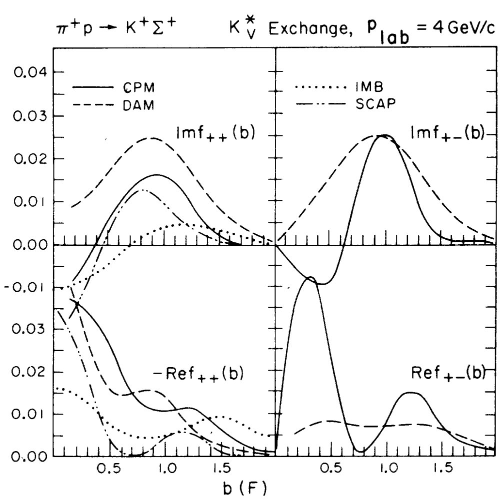

FIG. 3. The partial-wave amplitudes for K_V^* exchange in the reaction $\pi^+ p \to K^+ \Sigma^+$ at 4 GeV/c as found in four different analyses (see text).

similar in shape to those of ρ and A_2, and we have not included these in Figs. 3 and 4.

In Fig. 3, we see that the gross features of the K_V^*-exchange amplitudes are similar to those of ρ exchange; namely, the imaginary parts are peaked at $b \approx 0.8$ F, while the real parts are central and seem to have a double-peaked structure characteristic of amplitudes with a double zero as a function of t, i.e., Regge-pole amplitudes.

The K_T^*-exchange amplitudes shown in Fig. 4 also have the same gross features as the A_2-exchange amplitudes discussed earlier; namely, the $\mathrm{Im}f_{++}(b)$ are central, except for DAM, the $\mathrm{Im}f_{+-}(b)$ are peripheral, the $\mathrm{Re}f_{+-}(b)$ are polelike, while the $\mathrm{Re}f_{++}(b)$ vary considerably from model to model.

III. THE SMALL-b REGION

As clearly shown in Figs. 1–4, the small-b partial-wave amplitudes are not systematically suppressed, as would be expected in naive geometrical pictures or in the original absorption models. Unitarity corrections and other rescattering effects are clearly not simple.

The new absorption models which use an effective rescattering amplitude with significant real part instead of the predominantly imaginary elastic rescattering alone (such as AM-SC and AM-ExD) have had some success in explaining the complicated systematics of the small-b region. For vector exchange, those models provide a ready explanation of the sign change in the low partial waves of $\mathrm{Im}f_{++}(b)$ and the more polelike behavior of $\mathrm{Re}f_{++}(b)$. In these models, the same assumptions for tensor exchange lead to nonperipheral $\mathrm{Im}f_{++}(b)$. Tying together these three rather well-established features of the small-b region represents a significant improvement over earlier absorption models. The treatment of the small-b region in these models is still not satisfactory, however, for several reasons. First, these models predict various complicated structures for other partial-wave amplitudes, such as other sign changes at small b and the double-peaked structures discussed above. While there is no strong evidence against these structures, the other independent analyses do not show them.[33] Second, these models are quite arbitrary in the sense that the effective rescattering is not well defined, so much so that models with extremely different input Regge-pole amplitudes can give rather similar output amplitudes. Third, the amplitudes that result bear no resemblance to the amplitudes expected in the geometrical picture which inspired these pole-plus-cut approaches in the first place. Fourth, the effective rescattering is found in all the analyses to vary strongly with energy. This, Worden[34] has shown, results in amplitudes which do not satisfy finite-energy sum rules.

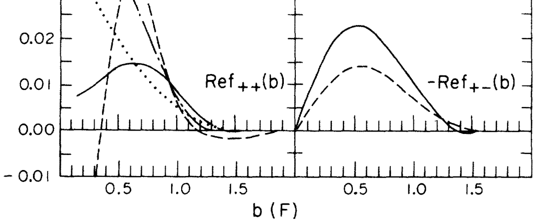

FIG. 4. The partial-wave amplitudes for K_T^* exchange in the reaction $\pi^+p \to K^+\Sigma^+$ at 4 GeV/c.

As far as describing the small-b region is concerned, the dual absorption model as formulated by Harari[35] and as implemented by Loos and Matthews fails completely. The most important failure is its prediction that $\mathrm{Im}f_{++}(b)$ for tensor exchange is suppressed at small b. This prediction is contradicted by all other analyses. Less significant is its inability to reproduce the negative excursion at small b in $\mathrm{Im}f_{++}(b)$ for vector exchange. Finally, the inability of DAM to specify $\mathrm{Re}f_{++}(b)$ for either vector or tensor exchange severely restricts its usefulness.

The strong central absorption prescription (SCAP) of Chiu[3,16] is a theoretical conjecture for the systematic behavior of the small-impact-parameter partial-wave amplitudes. The SCAP assumption is that for central collisions the two colliding hadrons momentarily form a "droplet," which can be in any of a large number of states described by some statistical distribution. In this view the probability for the droplet to correspond to some simple two-body state is small, resulting in the suppression of the low-partial-wave amplitudes in two-body channels. There are two versions of SCAP, i.e., "strong SCAP" in which s-waves are assumed completely absorbed in all two-body inelastic reactions and "weak SCAP" in which total absorption of the s waves is expected only if the droplet corresponds to the usual "nonexotic quark state." Since helicity-flip amplitudes are constrained to vanish at $b=0$, these hypotheses are relevant primarily to the behavior of nonflip amplitudes.

Not one of the seven analyses we have displayed in Figs. 1–4 shows the systematic suppression of the low-partial-wave amplitudes for all exchanges that is predicted by "strong SCAP."

Chiu and Ugaz[16] have performed a model-dependent determination of the nonflip amplitudes in $\pi^+ p \to K^+\Sigma^+$ and $K^- p \to \pi^-\Sigma^+$ at 4 GeV/c. Their results, labeled SCAP in Figs. 3 and 4, agree well with the weak-SCAP hypothesis, i.e., $\mathrm{Im} f_{++}(b=0) \simeq 0$ and $\mathrm{Re} f_{++}(b=0) \simeq 0$ for $(K_V^* + K_T^*)$. The analysis of Irving, Martin, and Barger[17] involves the same data and very similar assumptions, yet a check of the appropriate curves in Figs. 3 and 4 for the IMB analysis shows that the weak-SCAP predictions are not satisfied by $\mathrm{Re} f_{++}(b=0)$ and only approximately satisfied by $\mathrm{Im} f_{++}(b)$; i.e., $\mathrm{Im} f_{++}(b=0)$ is not very small but does seem to be suppressed relative to its value in the peripheral region.

To get a more complete picture of the evidence for or against the weak-SCAP hypothesis, we list in Table I how well all the analyses we have considered satisfy the hypothesis. To allow for the uncertainty in the determination of the low partial waves because the models fit only over limited t regions, we consider the weak-SCAP hypotheses successful if $\mathrm{Im} f_{++}(b \approx 0)/\mathrm{Im} f_{++}(b \approx 0.8) \leq 0.5$, with a similar condition for the real parts.

From the Table we see that only the Chiu-Ugaz analysis shows weak-SCAP behavior for both real and imaginary parts. Several other analyses show suppression near $b=0$ of $\mathrm{Im} f_{++}(b)$, i.e., CPM, DAM, and IMB. No analysis except SCAP shows such suppression for $\mathrm{Re} f_{++}(b)$.

To see how conclusive the various entries in the Table are, we consider each analysis in turn.

In the CPM analysis the most uncertain quantity was $\mathrm{Re} f_{++}(b)$ for tensor exchange. In that paper the arbitrary assumption was made that the tensor nonflip amplitude has approximate Regge phase. The assumption was made for simplicity, mainly, and affected the fits very little, since the data which constrain $\mathrm{Re} F_{++}(t)$ for tensor exchange, for example the polarization data in $\pi^- p \to \eta n$, are rather meager. Thus weak SCAP is not excluded by the CPM analysis nor is it inconsistent with the theory of complex poles in general.

TABLE I. Comparison of the weak-SCAP hypothesis at $b=0$ with the results of various phenomenological analyses. See the text for criteria used to define satisfactory agreement.

	$\rho + A_2$		$K_V^* + K_T^*$	
Model	$\mathrm{Im} f_{++}$	$\mathrm{Re} f_{++}$	$\mathrm{Im} f_{++}$	$\mathrm{Re} f_{++}$
CPM	Yes	No	Yes	No
DAM	Yes	No	Yes	No
AM-ExD	No	No	···	···
AM-SC	No	No	···	···
IMB	···	···	Yes	No
SCAP	···	···	Yes	Yes

Neither of the new absorption models, AM-SC or AM-ExD, seem to satisfy the weak-SCAP hypothesis. Since the ρ-exchange amplitudes are quite well determined, we look for the discrepancy in the treatment of the A_2-exchange amplitudes. We have already noted that the use of the same effective rescattering for tensor exchange as for vector exchange gives poorer fits to the $\pi^- p \to \eta n$ data than a pure pole model or than a model in which the rescattering is given by elastic rescattering alone. If the conventional absorption correction were applied to A_2 exchange (but not to ρ exchange) the resultant amplitudes would fit the data better and would differ in the following ways from the A_2-exchange amplitudes shown in Fig. 2: $\mathrm{Im} f_{++}(b \approx 0)$ would be smaller, while $\mathrm{Re} f_{++}(b \approx 0)$ would be larger. Such amplitudes would come close to satisfying weak SCAP. Of course, use of different rescattering for vector and tensor exchange is against the spirit of the original absorption models and the extensions we have discussed.[36]

Finally, as we remarked earlier, the low-partial-wave amplitudes in DAM disagree with all other treatments so that the DAM analysis is not useful for testing any hypotheses of low-partial-wave systematics.

In summary, evidence for weak SCAP is weak and evidence against it is inconclusive.

IV. THE PERIPHERAL-b REGION

The peripheral-b region, $1.4 \gtrsim b \gtrsim 0.5$ F, is dominated by the pure Regge-pole amplitude. The most interesting question to ask here is: Is there any evidence for the exchange degeneracy of the Regge poles? The imaginary parts of the ρ- and A_2-exchange amplitudes, helicity-nonflip and -flip, are replotted in Figs. 5 and 6, model by model, to answer this question.

In Fig. 5 we see clearly the tendency for exchange degeneracy in $\mathrm{Im} f_{++}(b)$ in the peripheral region. In the CPM analysis exchange degeneracy of the ρ and A_2 was imposed at $t=0$ in the fitting so that the weighted areas of the ρ and A_2 curves were constrained to be equal. In the other three analyses no such constraint was imposed.

Fig. 6 shows the corresponding comparison for the helicity-flip amplitudes. For the CPM and the DAM the agreement with exchange degeneracy is again excellent. For the flip amplitude exchange degeneracy was not imposed in CPM. The two absorption models agree with exchange degeneracy only in some average sense, since the b depen-

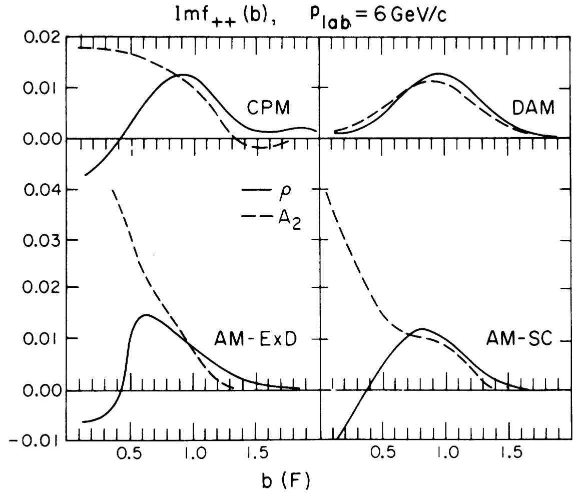

FIG. 5. Comparison of $\mathrm{Im}f_{++}(b)$ for ρ and A_2 exchange in four different analyses.

dence of the two exchanges is rather different. Again, the double-peaked structure for A_2 and the small negative excursion for ρ are consequences of using the same effective rescattering for the flip amplitude as for the nonflip (helicity-independent rescattering) and using the same rescattering for ρ and A_2. As noted earlier there is no evidence for these structures. In fact what little evidence there is, i.e., the elastic πp polarization, the energy dependence of the $\pi^- p \to \pi^0 n$ differential cross section, and the $t = -1.4$ $(\mathrm{GeV}/c)^2$ dip in $\pi^- p \to \eta n$, would suggest that the cut in flip amplitudes is smaller than in these models. In summary, the analyses shown are consistent with exchange degeneracy for the flip amplitudes of ρ and A_2 exchange.

Fig. 7 compares $\mathrm{Im}f_{++}(b)$ for K_V^* and K_T^* in $\pi^+ p \to K^+\Sigma^+$ at 4 GeV/c. Here there is evidence for exchange degeneracy only in CPM and SCAP, and not in DAM or IMB. Several points should be made. First, in the CPM analysis exchange degeneracy at $t=0$ was imposed during the fitting. The fits are excellent and no improvement was found when this constraint was removed in the final stage of the fitting. Second, the CPM results are consistent with simple breaking of SU_3 as found in other analyses.[37] SCAP approximately satisfies similar simple SU_3 breaking, while the DAM and IMB violate SU_3 by large factors. Furthermore, these latter two analyses disagree as to whether vector or tensor exchange dominates. Third, we note that the SCAP and IMB analyses are very similar and based on the same data. In view of these points and the results for (ρ, A_2)-exchange discussed above, we tend to favor the CPM and SCAP results and conclude that there is some evidence, not very conclusive, that the K_V^* and K_T^* nonflip Regge-pole amplitudes are exchange-degenerate in the peripheral region, and that the breaking of exchange degeneracy occurs mainly at small impact parameter for these exchanges also.

In Fig. 8 we compare $\mathrm{Im}f_{+-}(b)$ for K_V^* and K_T^* exchange for the CPM and DAM analyses. The SCAP and IMB curves are not shown because they were assumed exchange-degenerate. Again we discount the DAM results because they violate so strongly SU_3. The CPM results pose an interesting problem. This model was formulated in terms of t-channel amplitudes. The s-channel amplitudes vary from reaction to reaction depending on the

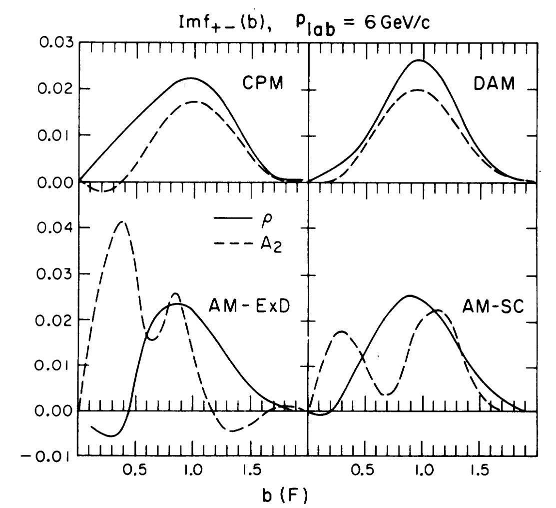

FIG. 6. Comparison of $\mathrm{Im}f_{+-}(b)$ for ρ and A_2 exchange in four different analyses.

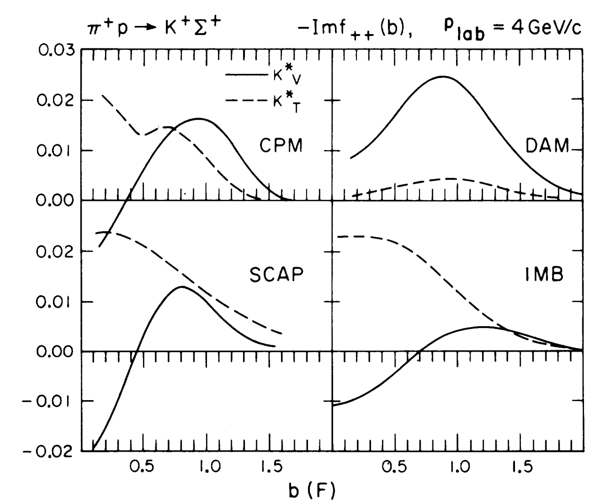

FIG. 7. Comparison of $-\mathrm{Im}f_{++}(b)$ for K_V^* and K_T^* exchange in $\pi^+ p \to K^+\Sigma^+$ in four different analyses.

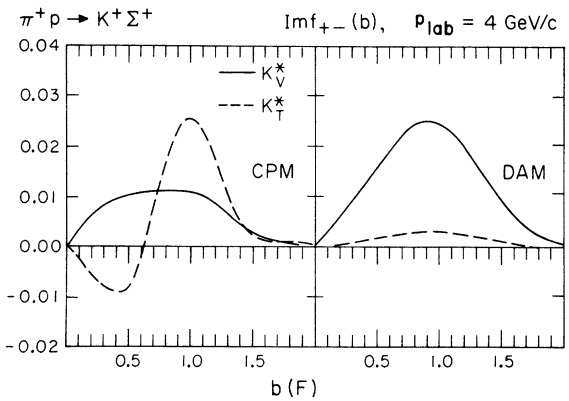

FIG. 8. Comparison of Imf_{+-}(b) for K_V^* and K_T^* exchange in $\pi^+p \to K^+\Sigma^+$ in the CPM and DAM analyses [the Imf_{+-}(b) for the IMB and SCAP analyses were assumed exchange-degenerate].

flip, nonflip ratio of the t-channel amplitudes. Since $\pi^+p \to K^+\Sigma^+$ has a larger nonflip contribution than $K^-p \to \bar{K}^0n$, the K_V^* and K_T^* s-channel flip amplitudes have a somewhat different profile than the corresponding ρ- and A_2-exchange flip amplitudes. R and A parameter measurements are needed to determine whether the various exchange amplitudes have a common profile in the s channel or in the t channel. For our purposes here, the CPM results are roughly consistent with exchange degeneracy of the helicity-flip amplitudes.

V. RESULTS FOR RELATED EXCHANGES

Other vector-exchange partial-wave amplitudes that have been determined from the data are Im$f_{++}(b)$ for ω exchange in KN and NN elastic scattering.[38] These results are very similar to those quoted here, with the minor quantitative difference that the ω-exchange low partial waves seem to have a larger negative excursion than the ρ-exchange amplitudes.

The isolation of the P' amplitude has been the subject of considerable controversy.[1,39] The structure of Im$f_{++}(b)$, i.e., whether it is peripheral or has a central component, depends greatly on what is assumed for the slope of the Pomeranchukon. P' is found to have a central component if the slope of the Pomeranchukon is less than 0.7.[1] In Fig. 9 we compare Im$f_{++}(b)$ for P' as found by Barger and Phillips[18] with the corresponding quantity for A_2 in the various analyses we considered. The P' seems quite similar to the A_2.

VI. CONCLUSIONS

We have compared the s-channel partial-wave amplitudes for vector and tensor exchange in a number of different, quantitatively successful phenomenological analyses. While the amplitudes often differ significantly from analysis to analysis, we have found some interesting regularities by considering the peripheral region, which is Regge-pole dominated, separately from the small-b region, $b \leq 0.5$ F, which is strongly affected by Regge cuts.

In the small-b region three features seem well established: Im$f_{++}(b)$ for vector exchange changes sign at small b, Re$f_{++}(b)$ for vector exchange has a central component much like that of a Regge pole, and Im$f_{++}(b)$ for tensor exchange has a central component of the same sign as its peripheral component. This behavior is inconsistent with exchange degeneracy of the vector- and tensor-exchange amplitudes in this region.

The DAM cannot explain any of the small-b features. The new absorption models explain them all by the same mechanism: the presence of a little understood real part in the rescattering amplitude. The weak-SCAP hypothesis for the behavior of the small-b partial-wave amplitudes is not confirmed by any analysis other than that of Chiu and Ugaz, but neither is it ruled out because Re$f_{++}(b)$ for tensor exchange is, for all practical purposes, not restricted by the data.

Most analyses find the ρ- and A_2-exchange partial-wave amplitudes to be exchange-degenerate

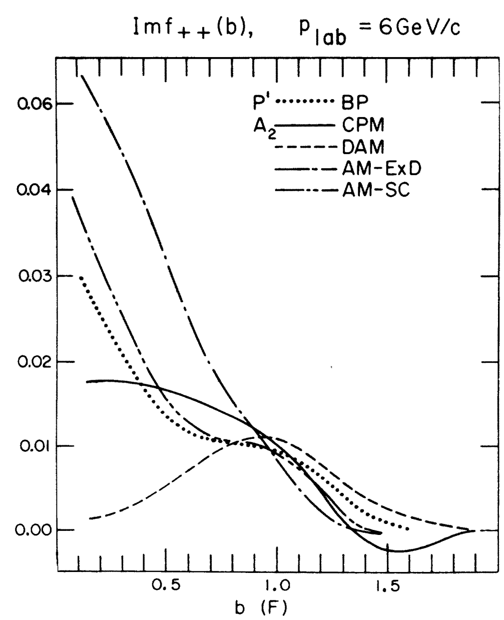

FIG. 9. Comparison of Imf_{++}(b) for A_2 exchange in four different models with Imf_{++}(b) for P' exchange as found by Barger and Phillips.

in the peripheral region, both for the flip and the nonflip amplitudes. The K_V^* and K_T^* amplitudes are not as well determined, but the more reliable analyses show a similar exchange degeneracy in the peripheral region for these amplitudes also.

Our study of existing analyses therefore suggests that Regge poles are approximately exchange-degenerate and that exchange-degeneracy breaking occurs mainly at small impact parameter.

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