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Naive quark-pair-creation model and baryon decays

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It is shown that the naive quark-pair-creation model, perhaps the simplest generalization of the additive quark model treating all the hadrons as composite, correctly describes the observed centrifugal-barrier effect in the baryon decays in which more than one partial wave contributes. Good agreement is found with the partial-wave analysis of Brody *et al.* of $\pi^- p \to \pi^+ \Delta^-$ in the 1690-MeV mass region. Comparison is made with the previous unsuccessful $SU(6)_W$ predictions and with the work of Petersen and Rosner.

I. INTRODUCTION

A serious drawback of the additive quark model of strong vertices with the static interaction

$$G_q \cdot [\vec{\sigma}(i) \cdot \vec{k}_\pi][\vec{\tau}(i) \cdot \vec{\pi}]$$

is that the π is not treated as composite, as are the other hadrons. It presents also some well-known quantitative problems [the same as $SU(6)_W$ (Ref. 1)], the most striking being related to the processes where more than one partial wave contributes. The theoretical interest of these processes has been emphasized recently.[2]

In $A_1 \to \rho\pi$ as in $B \to \omega\pi$, the model predicts[3] ρ and ω polarizations in contradiction with experiment.[4] The S and D partial waves show the same k_π^2 dependence, although one expects the lth partial wave to behave like k_π^l.[3]

The situation is similar for baryon decays $N^* \to \Delta\pi$ if $S_q = \frac{1}{2}$ for the N^*, only the helicity $\pm\frac{1}{2}$ is allowed. Then both partial waves are of the same order of magnitude, having the same k_π dependence, in contradiction with the expected centrifugal barrier $a(l+2)/a(l) \sim k_\pi^2$.

In a previous article,[5] we presented a quark-pair-creation model (QPCM), which was the simplest generalization of the additive quark model treating all hadrons as composite. This model explains the observed ρ and ω polarizations in A_1 and B decays, without any free parameter.

The purpose of this work is to present predictions of the model for baryon decays.

In Sec. II we briefly recall the principles of the model. In Sec. III our predictions are compared with the experimental analysis by Brody *et al.*[6] of the process $\pi^- p \to \pi^+ \Delta^-$. In Sec. IV we compare our model with the work of Petersen and Rosner.[2]

II. THE NAIVE QUARK-PAIR-CREATION MODEL

An intuitive picture underlying the quark-pair-creation model[7] is given by the diagram of Fig. 1. In the channel $A \to B + C$, a quark pair is created out of the "hadronic vacuum." The spectator quarks are treated as in the additive quark model: They are supposed to change neither their internal quantum numbers nor their momentum and spin states. The pair is therefore in a $^3P_0$ state ($C = +1$), SU(3)-singlet state, and $\vec{k}_q + \vec{k}_{\bar{q}} = 0$.

Then defining the $\hat{R}$ matrix operator

$$\langle BC | \hat{T} | A \rangle = 2\pi\delta(E_A - E_B - E_C) \langle BC | \hat{R} | A \rangle , \qquad (1a)$$

we write

$$\hat{R} = \sum_{i,j} \int dk_q \, dk_{\bar{q}} \left[\gamma \delta_3(\vec{k}_q + \vec{k}_{\bar{q}}) \sum_m \langle 1,1; m, -m | 0,0 \rangle \times \mathcal{Y}_1^m(\vec{k}_q - \vec{k}_{\bar{q}}) (\chi_1^{-m} \phi_0)_{ij} \right] a_i^\dagger(\vec{k}_q) b_j^\dagger(\vec{k}_{\bar{q}}), \qquad (1b)$$

where i, j are SU(6) indices, $a^\dagger$ and $b^\dagger$ are creation operators of quarks and antiquarks; Φ_0 is for SU(3) singlet, χ_1^{-m} for triplet state of spin; $\mathcal{Y}_1^m$ reflects the $L = 1$ angular momentum of the pair; γ is a dimensionless constant. We take matrix elements of $\hat{R}$ between the SU(6) harmonic-oscillator quark wave functions of $A, B,$ and C.

In order to show the various hypotheses involved, let us introduce the kernel N by

$$\langle BC | \hat{R} | A \rangle = \sum_{SU(3),\, spin} \int \prod_i d\vec{k}_i \, \psi_A N \psi_B^* \psi_C^* .$$

N is a function of the eight momenta of the quarks composing the hadrons A, B, and C and of their SU(3) and spin quantum numbers. This kernel must be invariant under unitary spin rotations, spatial rotations, and parity transformation.

Taking into account the additivity hypothesis, we first express the conservation of the SU(3) quantum numbers of the spectator quarks:

$$N = \delta_{\alpha_1 \alpha_8} \, \delta_{\alpha_2 \alpha_5} \delta_{\alpha_3 \alpha_4} N'(\alpha_6, \alpha_7).$$

$N'(\alpha_6, \alpha_7)$ has to be an SU(3) singlet in order to ensure the unitary spin invariance of the kernel. At this stage, one gets the Zweig results.[8] Our kernel N contains the momentum-conservation δ function. Applying the additivity hypothesis to the spins and momenta of spectator quarks, we get

$$N' = \delta_{s_1 s_8}\,\delta_{s_2 s_5}\,\delta_{s_3 s_4}\,\delta_3(\vec{k}_1-\vec{k}_8)\delta_3(\vec{k}_2-\vec{k}_5) \times \delta_3(\vec{k}_3-\vec{k}_4)N''(s_6,s_7;\vec{k}_6,\vec{k}_7)$$

and the conservation δ function reduces to $\delta(\vec{k}_6+\vec{k}_7)$, hence

$$N'' = \delta_3(\vec{k}_6+\vec{k}_7)N'''(s_6,s_7;\vec{k}_6-\vec{k}_7).$$

Invariance under rotation and parity implies that N''' describes a $^3P_0$ state, leaving dependence on $(\vec{k}_6-\vec{k}_7)^2$ unspecified. Note that this is not strictly the "$^3P_0$" model,[2,7,9] which is based on the enumeration of the relativistic couplings, without introducing internal quark momenta.

Last, we make the hypothesis of a minimum dependence of the kernel on $(\vec{k}_6-\vec{k}_7)^2$, the main dependence being that of the wave functions. Then one gets the model formulated above.

Note that although the diagram of Fig. 1 is common to a number of models,[2,7] the explicit form of the coupling (1b) where the quark spin of the pair is coupled to its own relative (internal) momentum, as well as the use of the harmonic-oscillator wave functions is specific to our formulation of the quark-pair-creation model.[5]

In the limit of the elementary π emission, the amplitude turns out[5] to be of the form

$$[\vec{\sigma}(i)\cdot(\vec{k}_\pi-\vec{k}(i))][\vec{\tau}(i)\cdot\vec{\pi}], \qquad (2)$$

$\vec{k}(i)$ being the internal quark momentum of the emitting quark. We thus obtain a well-defined amount of the recoil term introduced from Galilean invariance arguments by Mitra and Ross.[10]

III. PREDICTIONS AND COMPARISON WITH EXPERIMENT

Now let us consider the QPCM predictions for the $N\pi$ or $\Delta\pi$ decays of the resonances:

$$F_{15}(1690)(\underline{56},\ S_q=\tfrac{1}{2},\ L=2),$$
$$D_{15}(1690)(\underline{70},\ S_q=\tfrac{3}{2},\ L=1),$$
$$D_{13}(1520)(\underline{70},\ S_q=\tfrac{1}{2},\ L=1).$$

We have experimental information in the 1690-MeV region, where Brody *et al.* have analyzed the reaction $\pi^- p\to\pi^+\Delta^-$. The main contributions come from the F_{15} and D_{15} πN waves.

In order to test the model independently of the constant γ, we calculate (as we did for A_1 and B decays) the ratios of partial-wave amplitudes for

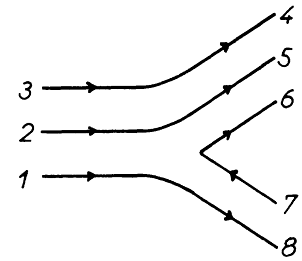

FIG. 1. Intuitive picture of quark-pair-creation process in baryon decays.

the same process. Also, in order to avoid discussing variation of γ with the energies involved in the vertex, we calculate ratios of amplitudes of decays involving particles of the same mass [e.g., $a(D_{15}\to\Delta\pi)/a(F_{15}\to\Delta\pi)$ and $a(D_{15}\to N\pi)/a(F_{15}\to N\pi)$].

The only parameters in these ratios are the baryon and meson ground-state radii which we fixed as $R_B^2=6$ GeV^{-2} and $R_M^2=8$ GeV^{-2} from Regge slopes.[11] So, once our ansatz (1b) is accepted, *our predictions do not involve free parameters.*[12]

For $F_{15}\to\Delta\pi$ and $D_{15}\to\Delta\pi$, we have, respectively, the allowed partial waves P, F and D, G. The fit by Brody *et al.* shows that experimental results are compatible with negligible F and G waves. Moreover, they can fit the experimental results with two alternative solutions:

$$\text{Solution A:}\quad FP>0 \text{ and } \left(\frac{\Gamma(F_{15}\to\Delta\pi)}{\Gamma(D_{15}\to\Delta\pi)}\right)^{1/2}=0.31,$$

$$\text{Solution B:}\quad FP<0 \text{ and } \left(\frac{\Gamma(F_{15}\to\Delta\pi)}{\Gamma(D_{15}\to\Delta\pi)}\right)^{1/2}=0.72.$$

In the QPCM we have for the decay $N^*\to N(\Delta)\pi$

$$\vec{L}+\vec{1}_{q\bar{q}}=\vec{l},$$

where $\vec{L}$, $\vec{1}_{q\bar{q}}=1$, $\vec{l}$ are, respectively, the orbital angular momentum of the quarks in the initial resonance, of the quark pair, and of the decay system πN or $\pi\Delta$.

Then, for $D_{15}\to\Delta\pi$, the G wave vanishes. Note that this is also predicted by SU(6)$_W$, the π being emitted by a quark in a P wave.

The prediction for $F_{15}\to\Delta\pi$ is model-dependent. Our model predicts

$$\left(\frac{F}{P}\right)_{F_{15}\to\Delta\pi} = -\frac{2\rho^2k_\pi^2}{5-2\rho^2k_\pi^2} = -0.204, \qquad (3a)$$

where

$$\rho^2=\frac{1}{8}\,\frac{(4R_B^2+R_M^2)(2R_B^2+R_M^2)}{(3R_B^2+R_M^2)}.$$

This is compatible with the observed negligible F wave. Note that SU(6)$_W$ gives $F/P=+1$ in contradiction with the damping of the F wave.

For $D_{13} \to \Delta\pi$ we have

$$\left(\frac{D}{S}\right)_{D_{13}\to\Delta\pi} = -\frac{2\rho^2 k_\pi{}^2}{3-2\rho^2 k_\pi{}^2} = -0.114 \tag{3b}$$

compared with the $SU(6)_W$ result $D/S = +1$.

More generally, *when two partial waves contribute to baryon decays, our model predicts a centrifugal barrier in most cases,* in agreement with experiment.[13]

Moreover for the reaction

$$\pi^- p \to F_{15} \to \pi^+ \Delta^-,$$

our model gives $PF < 0$. So we agree with solution B. Now let us compare partial decay widths. We predict

$$\left[\frac{\Gamma_{\pi N}(F_{15})}{\Gamma_{\pi N}(D_{15})}\right]^{1/2} = \frac{5}{4}\left(\frac{2R_B{}^2 + R_M{}^2}{3R_B{}^2 + R_M{}^2}\right)(R_B k_\pi) \tag{4}$$

and

$$\left\{\frac{\Gamma_{\pi\Delta}(F_{15})}{\Gamma_{\pi\Delta}(D_{15})}\right\}^{1/2} = 4\left(\frac{3}{35}\right)^{1/2}\left(\frac{5-2\rho^2 k_\pi{}^2}{4R_B{}^2 + R_M{}^2}\right)\left(\frac{R_B}{k_\pi}\right). \tag{5}$$

Table I shows that our model agrees *quantitatively* with solution B.[14,15]

IV. COMPARISON WITH A RELATED ANALYSIS OF PETERSEN AND ROSNER

Petersen and Rosner[2] have analyzed baryon decays using the general ideas of $SU(6)_W$ violation in the quark-pair-creation model. However, using no precise form for the spatial part of the wave functions, nor for the momentum dependence of the quark-pair-creation matrix element, they cannot make precise quantitative predictions.

As we have seen, $SU(6)_W$ relates different partial waves: $S = D$, $P = F$. As a consequence, it predicts no centrifugal-barrier effect. Petersen and Rosner introduce an $SU(6)_W$ violation "by hand" through a centrifugal-barrier factor

$$a(l) = \left(\frac{k_\pi}{p_0}\right)^l \tilde{a}(l)$$

and they fit $\tilde{S}$, $\tilde{D}$, and $\tilde{P}$, $\tilde{F}$ *in each decay* for the resonances belonging to ($\underline{70}$, 1^-) and ($\underline{56}$, 2^+), respectively. They conclude that the $\tilde{a}(l)$ do not change very much from one process to another and that, in mean value, the $\tilde{a}(l)$ verify the modified $SU(6)_W$ relations $\tilde{S} = \tilde{D}$, $\tilde{P} = \tilde{F}$ up to a sign. Note that there still remain *two parameters*.

On the contrary, in our QPC model, all the hadron decay amplitudes are related; there is only *one free parameter,* the pair creation strength γ. Instead of the two relations $\tilde{S} = \tilde{D}$, $\tilde{P} = \tilde{F}$ between waves of the same parity, we expect relations between *all the decay waves,* namely,

$$\tilde{a} = \tilde{F} = \frac{\tilde{D}}{2.22} = \frac{3\tilde{P}}{2(5-6k_\pi{}^2)} = \frac{\tilde{S}}{4.44(1-2k_\pi{}^2)}\,. \tag{6}$$

We observe that the mean values of $\tilde{F}$ and $\tilde{D}$ in the table of Petersen and Rosner satisfy $(\tilde{D}/\tilde{F}) = 2.4$ compared with our prediction $(\tilde{D}/\tilde{F}) = 2.22$. Encouraged by this success, we use the table of Petersen and Rosner to test the whole set of relations (6). With a reasonable χ^2, the result is compatible with a constant value of $\tilde{a}$ throughout Table II.

V. CONCLUSION

The naive quark-pair-creation model, based on the structure of the vertices suggested by Fig. 1, with the specific interaction (1) and with SU(6) harmonic-oscillator wave functions, describes successfully the polarizations in A_1 and B decays, the observed centrifugal-barrier effect in baryon decays, and the ratio of ($\underline{70}$, 1^-) to ($\underline{56}$, 2^+) decay amplitudes. This is a strong indication that it describes correctly spin and orbital dependence of decay amplitudes.

It is remarkable that such a simple and intuitive representation explains the features of the decay processes; it allows one to go beyond pure $SU(6)_W$ symmetry, in a very logical manner, by introducing the effect of the internal quark motion.

TABLE I. Comparison of predictions of the QPCM with the partial-wave analysis. The experimental results are taken from Brody *et al.*[6] for the $\Delta\pi$ partial width and from Particle Data Group, Phys. Lett. 39B, 1 (1972).

Table I	$[\Gamma(F_{15}\to N\pi)/\Gamma(D_{15}\to N\pi)]^{1/2}$	$[\Gamma(F_{15}\to \Delta\pi)/\Gamma(D_{15}\to \Delta\pi)]^{1/2}$	Sign of FP of F_{15}
QPCM	1.32	1.03	−
Experiment	$1.10^{+0.36}_{-0.45}$	B: $0.72^{+0.30}_{-0.36}$	−
		A: $0.37^{+0.18}_{-0.12}$	+

TABLE II. Calculated values of reduced amplitude $\tilde{a}(l)$, using Eq. (6) (the centrifugal barrier factor being removed). The average $\tilde{a}$ appears to be almost equal for the four types of partial waves. The general average is $\tilde{a}_0 = 2.7$:

$$\chi^2 = \sum_i \frac{(\tilde{a}_i - \tilde{a}_0)^2}{(\Delta\tilde{a}_i)^2} = 33.5 \text{ for 32 points (31 degrees of freedom).}$$

F wave			P wave dominant			D wave			S wave dominant		
Process	Γ	$\tilde{a}$	Process	Γ	$\tilde{a}$	Process	Γ	$\tilde{a}$	Process	Γ	$\tilde{a}$
$\frac{7}{2}^+$			$\frac{5}{2}^+$			$\frac{5}{2}^-$			$\frac{3}{2}^-$		
$\Delta(1950) \to \pi N$	80^{+64}_{-25}	$2.3^{+0.8}_{-0.4}$	$N(1690) \to \Delta\pi$	51^{+23}_{-20}	$2.4^{+0.2}_{-0.8}$	$N(1670) \to N\pi$	56^{+35}_{-17}	$3.5^{+1.0}_{-0.6}$	$N(1520) \to \Delta\pi$	$64^{+4.0}_{-15}$	$1.5^{+0.2}_{-0.6}$
ΣK	4	~ 2.7	$\Lambda(1860) \to Y_1^*\pi$	$14^{+18}_{-\sim}$	$1.7^{+0.9}_{\sim}$	$\Delta\pi$	100^{+74}_{-38}	$1.3^{+0.25}_{-0.15}$	$\frac{1}{2}^-$		
$\Delta\pi$	115^{+35}_{-65}	$5.0^{+0.5}_{-1.7}$	$\frac{3}{2}^+$			$\Sigma(1765) \to N\bar{K}$	53^{+15}_{-31}	$3.0^{+0.4}_{-1.1}$			
$\Sigma(2030) \to N\bar{K}$	25^{+23}_{-13}	$2.8^{+11}_{-0.9}$	$N(1860) \to N\pi$	75^{+13}_{-29}	$2.3^{+0.2}_{-0.4}$	$\Lambda\pi$	18^{+25}_{-3}	$2.4^{+1.3}_{-0.2}$	$\left.\begin{matrix} N(1535) \\ N(1700) \end{matrix}\right\} \to N\pi$	37	$4.3^{\cdot\sim}_{-2.1}$
$\Lambda\pi$	35	$2.7^{+0.2}_{-0.8}$	ΛK	$18^{+50}_{\sim}$	$2.0^{+2.0}_{\sim}$	$Y_1^*\pi$	16^{+26}_{-10}	$5.2^{+3.2}_{-2.2}$		160^{+80}_{-110}	
$\Sigma\pi$	$5^{+15}_{-?}$	$1.5^{+1.5}_{-?}$				$N(1830) \to N\bar{K}$	11		$\left.\begin{matrix} N(1535) \\ N(1700) \end{matrix}\right\} \to N\eta$	58^{+90}_{-40}	$2.0^{+0.9}_{-1.0}$
$\frac{5}{2}^+$			$\frac{1}{2}^+$			$\Sigma\pi$	24^{+56}_{-0}	$2.0^{+1.3}_{-0}$		~ 0	
$N(1690) \to N\pi$	91^{+30}_{-30}	$3.3^{+0.5}_{-0.6}$	$\Delta(1910) \to N\pi$	66^{+64}_{-50}	$2.8^{+1.2}_{-1.6}$	$\frac{3}{2}^-$			$\Delta(1630) \to N\pi$	46^{+14}_{-22}	$5.7^{+2.5}_{-1.8}$
$\Lambda(1815) \to \Sigma\pi$	$9^{+5}_{-1.5}$	$2.4^{+1.6}_{-0.2}$				$\Delta(1670) \to N\pi$	36^{+25}_{-6}	$3.0^{+0.9}_{-0.3}$	$\Lambda(1405) \to \Sigma\pi$	40^{+22}_{-11}	$2.0^{+0.5}_{-0.3}$
$N\bar{K}$	51^{+26}_{-16}	$3.9^{+0.7}_{-0.7}$				$N(1520) \to N\pi$	64^{+30}_{-14}	$2.2^{+0.5}_{-0.3}$	$\Lambda(1670) \to \Lambda\eta$	~ 9	~ 1.5
$\Sigma(1915) \to N\bar{K}$	$8^{+22}_{-1.5}$	$4.7^{+4.5}_{-0.4}$				$\left.\begin{matrix} \Lambda(1520) \\ \Lambda(1690) \end{matrix}\right\} \to N\bar{K}$	7.4	$2.7^{+0.8}_{-1.2}$			
$\Delta(1990) \to N\pi$	45^{+25}_{-33}	$2.4^{+0.2}_{-0.8}$					17^{+5}_{-12}				
						$\left.\begin{matrix} \Lambda(1520) \\ \Lambda(1690) \end{matrix}\right\} \to \Sigma\pi$	6.6	$2.7^{+0.3}_{-1.3}$			
							22^{+5}_{-15}				
						$N(1520) \to N\eta$	~ 0.7	$\sim 7.4^{+\sim}_{-3.1}$			
Average $\tilde{a} = 3.1$			Average $\tilde{a} = 2.3$			Average $\tilde{a} = 3.3$			Average $\tilde{a} = 2.8$		

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[9]Our model possesses, of course, the coplanar symmetry studied by J. L. Rosner [Phys. Rev. D 6, 1781 (1972)] as is displayed by writing our transition operator in the form

$$\begin{aligned}\sum_m \mathcal{Y}_1^m \chi_1^{-m}\langle 1,1;m,-m\,|\,0,0\rangle &\simeq |L_z=0,\,S_z=0\rangle\\ &\quad+|L_x=0,\,S_x=0\rangle\\ &\quad+|L_y=0,\,S_y=0\rangle\\ &=|L_z=0,\,W=0,\,W_z=0\rangle\\ &\quad+i\,|L_x=0,\,W=1,\,W_y=0\rangle\\ &\quad-i\,|L_y=0,\,W=1,\,W_x=0\rangle.\end{aligned}$$

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[12]In fact, the radii can be estimated from a lot of different processes. All these estimations agree within 25%; the results in this paper are not very sensitive to the precise value of R^2. For instance, with $R_B^{\,2}=10\ \mathrm{GeV}^{-2}$, as estimated in a recent paper [LPTHE Report No. 73/15 (unpublished)] centrifugal barrier effects are conserved, and in Table I we get 1.62 instead of 1.32 and 0.65 instead of 1.03, both results compatible with experiment. For F_{35}, increasing ρ^2 would make things better.

[13]In a recent experiment, U. Methani *et al.* [Phys. Rev. Lett. 29, 1634 (1973)] found in $F_{35}\to\Delta\pi$ an effect contrary to the centrifugal barrier: $F/D>2$. D. Faiman, J. L. Rosner, and J. Weyers [Nucl. Phys. B57, 45 (1973)] interpret this as requiring mixing between ($\underline{56}$, 2^+) and ($\underline{70}$, 2^+). In our model, formula (3a) suggests a possible cancellation in the P wave; but ρ^2 is actually twice too small to give such a cancellation in $F_{35}\to\Delta\pi$.

[14]A wide partial-wave analysis of $\pi N\to\pi\pi N$ has been made, since the paper of Brody *et al.*, by D. J. Herndon *et al.* [LBL Report No. 1065 (unpublished), SLAC Report No. SLAC-PUB-1108, 1972 (unpublished)], which favors solution A of Brody *et al.*

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