[56]D. Barkai and K. J. M. Moriarty, Nucl. Phys. B50, 354 (1972).
[57]B. Sadoulet, Nucl. Phys. B53, 135 (1973).
[58]J. S. Loos and J. A. J. Matthews, Phys. Rev. D 6, 2463 (1972).
[59]B. J. Hartley and G. L. Kane, Nucl. Phys. B57, 157 (1973).
[60]M. Imachi, T. Matsuoka, K. Ninomiya, and S. Sawada, Progr. Theor. Phys. 40, 353 (1968); H. Harari, Phys. Rev. Lett. 22, 562 (1969); J. Rosner, *ibid.* 22, 689 (1969).
[61]J. Mandula, J. Weyers, and G. Zweig, Annu. Rev. Nucl. Sci. 20, 289 (1970).
[62]G. Cohen-Tannoudji, A. Morel, and Ph. Salin, Nuovo Cimento 55A, 412 (1969).
[63]M. Davier and H. Harari, Phys. Lett. 35B, 239 (1971).
[64]G. C. Fox and C. Quigg, LBL Report No. 20001, 1970 (unpublished).
[65]B. B. Brabson *et al.*, Phys. Rev. Lett. 25, 553 (1970).
[66]V. N. Bolotov *et al.*, Phys. Lett. 38B, 120 (1972).
[67]G. Höhler *et al.*, Phys. Lett. 20, 79 (1966).
[68]A. D. Martin and C. Michael, Phys. Lett. 37B, 513 (1971).

PHYSICAL REVIEW D VOLUME 9, NUMBER 5 1 MARCH 1974

Analysis of $\bar{p}p$ interactions and the quark-parton model*

A. E. Bussian†

Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48104

(Received 13 February 1973)

An analysis of $\bar{p}p$ interactions is made assuming that hadrons are composed of totally absorbing subparticles. If quarks are assumed to be the basic constituents, their mass and radius are in reasonable agreement with the Drell-Johnson model. If the subparticles are assumed to be Schwarzschild spheres, the resulting mass and radius agree with Planck's characteristic mass and length, respectively. Furthermore, the resulting mass density of partons is of the correct order of magnitude to be gravitationally bound into nucleons.

In recent years composite models of hadrons have been constructed to explain high-energy phenomena. Such entities as quarks, partons, and droplets have been proposed as the basic constituents of hadrons. For convenience we shall refer to partons as any basic subunits of hadronic matter.

In this paper $\bar{p}p$ interactions are analyzed, assuming such a composite picture in terms of the impact representation of the optical model. $\bar{p}p$ interactions are particularly amenable to such a study because of the apparent pure imaginary scattering amplitude at finite energies[1] and the absence of discrete $\bar{p}p$ resonances. Within this framework a number of authors have considered pp elastic scattering in the asymptotic energy limit where the real part of the amplitude is expected to be zero.[2] Measured differential cross sections appear to approach this limit with increasing energy.

The partial-wave expansion of the imaginary part of the scattering amplitude, neglecting spin, is given by

$$f(\theta)=\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\{\exp[-\alpha(k)g_l(k)]-1\}\times P_l(\cos\theta)\,, \qquad (1)$$

where the absorption coefficient has been written as a product of two factors. Expressions for cross sections are given by

$$\sigma_e=\frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\{1-\exp[-\alpha(k)g_l(k)]\}^2\,, \qquad (2)$$

$$\sigma_t=\frac{2\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\{1-\exp[-\alpha(k)g_l(k)]\}\,, \qquad (3)$$

$$\frac{d\sigma}{dt}=\frac{\pi}{P_{\text{c.m.}}^{\,2}}|f(\theta)|^2\,. \qquad (4)$$

The transformation to the impact-parameter representation is usually given by[3]

$$l+\tfrac{1}{2}=kb\,. \qquad (5)$$

Using (5) one can transform $g_l(k)\to g(b)$, where $g(b)$ takes into account the density distribution of two hadrons which are passing through each other at impact parameter b and is given by

$$g(b)=\int_{x_{\min}}^{x_{\max}}\int_{y_{\min}}^{y_{\max}}\int_0^{z_{\max}}\int_0^{w_{\max}}\rho_B(x,y,z)\rho_T(x,y,w)\times dw\,dz\,dy\,dx\,. \qquad (6)$$

ρ_B (ρ_T) is the beam (target) particle hadronic matter distribution normalized to one parton. The w and z coordinate axes are in the direction of the beam particle.

It is instructive to consider the special case of a single pointlike parton incident on a target particle of radius R. Assuming a uniform distribution of static partons in the target particle gives

$$\alpha(k)g(b)=(R^2-b^2)^{1/2}/\lambda(k)$$
$$=n\sigma(k)\frac{(R^2-b^2)^{1/2}}{4\pi R^3/3}\ ,$$

where

$\lambda(k)$= inelastic interaction mean free path for the incident parton traversing the target particle,

$\sigma(k)$= inelastic parton-parton cross section,

n= number of partons in target particle.

Thus we can write

$$\alpha(k)=n\sigma(k)\ ,$$
$$g(b)=\frac{(R^2-b^2)^{1/2}}{4\pi R^3/3}\ .$$

Generalizing to two identical hadrons of arbitrary parton distribution we obtain (6) for $g(b)$ and $\alpha(k)$ $=n^2\sigma(k)$.

In (6) a static parton approximation is made so that $g(b)$ is energy-independent. This assumption is justified on the grounds that the electromagnetic proton form factors

$$G_p(t)\ \text{\textasciitilde}G_E(t)\text{\textasciitilde}[u^2/(u^2-t)]^2\ ,$$

with

$$u^2=0.71\ (\text{GeV}/c)^2$$

give a good fit to existing data.[4] Thus, to the extent that the distribution of partons is similar to the distribution of charge, we can take $\rho_B=\rho_T$ $\propto\exp(-\sqrt{12}\,r/r_m)$, where

r_m (root-mean-square radius)=0.8 F.

$g(b)$ can be solved numerically for a given density distribution. The results of a Monte Carlo digital computer calculation for a number of density distributions are shown in Fig. 1. Knowing $g(b)$ and using (5) one can determine $g_t(k)$. One can then solve for $\alpha(k)$ in both (2) and (3) for measured values of σ_e and σ_t using Newton's method.[5] The results of a computer calculation for $\bar{p}p$ data

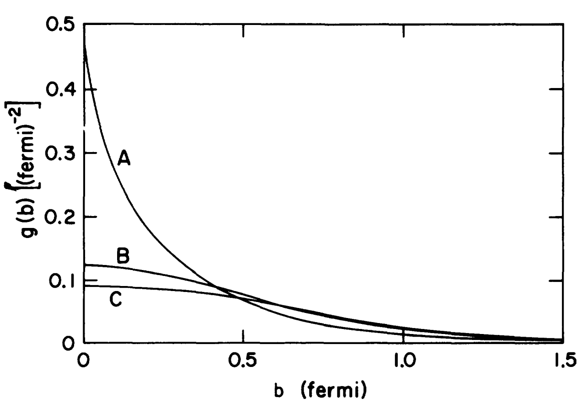

FIG. 1. Results of a numerical calculation of $g(b)$, assuming density distributions of $r^{-2}e^{-r/r_0}$ (curve A), e^{-r/r_0} (curve B), and e^{-r^2/r_0^2} (curve C), all with a root-mean-square radius of 0.8 F.

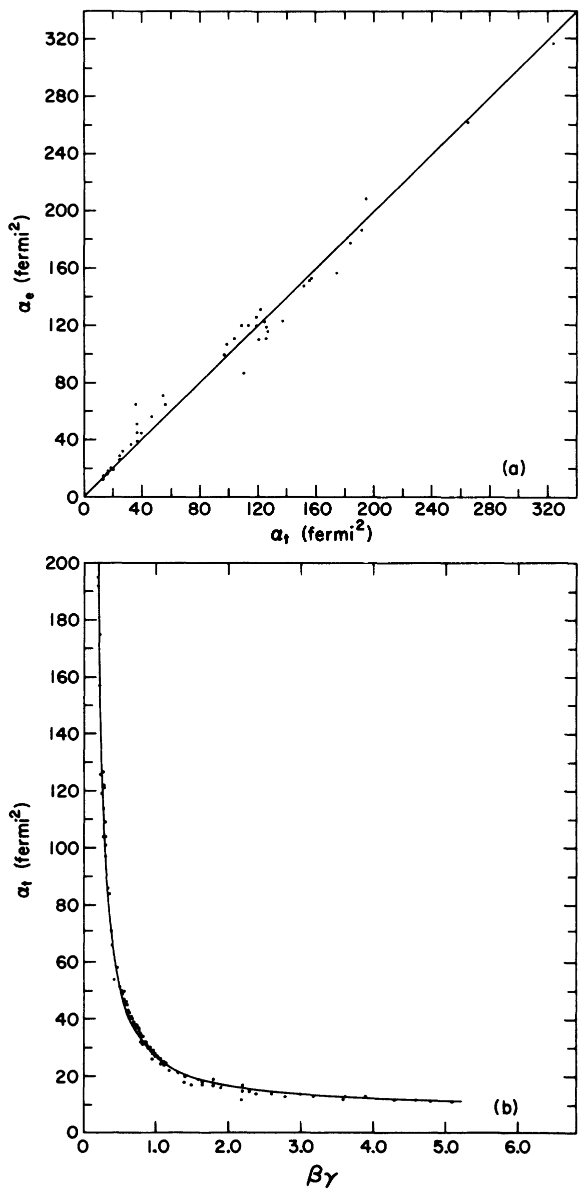

FIG. 2. (a) A plot of α_e [calculated from (2)] vs α_t [calculated from (3)] using measured values of σ_e and σ_t at the same momenta (see Ref. 7). (b) Calculated values of α_t vs $\beta\gamma$ using measured values of σ_t (see Ref. 7). The curve is a best fit of (7) combined with (8).

assuming an exponential density distribution with $r_m = 0.8$ F are shown in Fig. 2(a).[6] (Data in Fig. 2 are from Ref. 7.) The result is in good agreement with the required condition that $\alpha_t = \alpha_e$.

In this formulation with the density distribution normalized to one parton we have

$$\alpha(k) = n^2\sigma(k), \quad (7)$$

where

n = number of partons which comprise the proton (antiproton),

$\sigma(k)$ = inelastic parton-parton cross section.

In the picture of two hadrons passing through each other, an interaction occurs when a beam parton interacts with a target parton. If we assume that partons are perfectly absorbing or black spheres of radius a, then

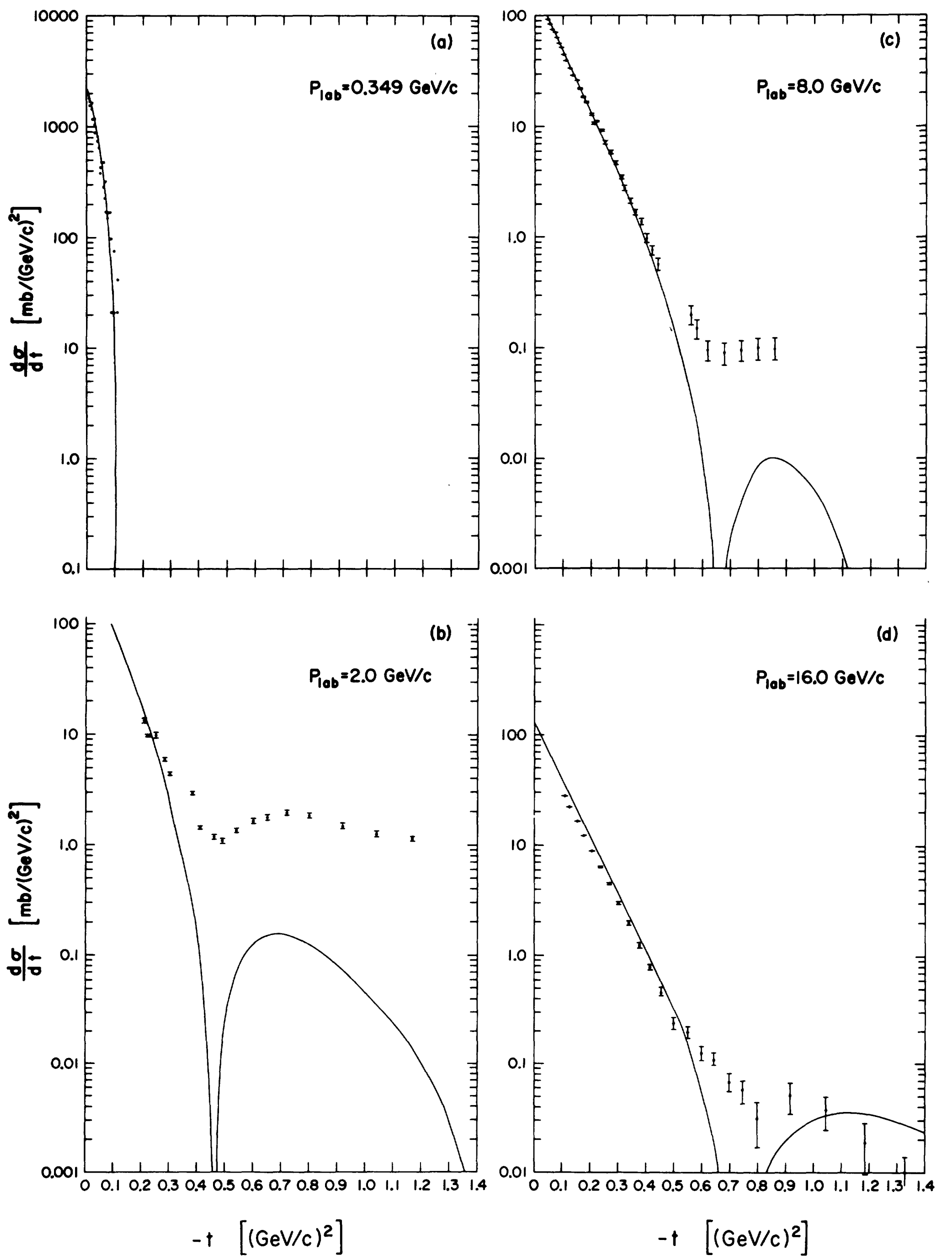

FIG. 3. Comparison of differential cross section data with (1) and (4) as explained in the text (see Ref. 8). (a) Data from Ref. 9; (b) data from Ref. 10; (c) and (d) data from Ref. 11.

$$\sigma(k)=\pi\left(2a+\frac{1}{2k}\right)^2 .$$

To the extent that partons can be treated as free particles one can write

$$\sigma(k)=4\pi a^2\left(1+\frac{1}{4ma\beta\gamma}\right)^2 , \quad (8)$$

where m equals the parton mass and β and γ refer to the parton velocity, which equals the proton (antiproton) velocity in the c.m. system.

In Fig. 2(b) the calculated values of α_t from (3) are plotted vs $\beta\gamma$. The curve is a best fit to (7), combined with (8), which gives

$$na=0.85\times10^{-13}\text{ cm}, \quad (9)$$

$$m/n=0.14\times10^{-24}\text{ g}. \quad (10)$$

The asymptotic value of $\alpha_t(k)$ is 9.05 F^2. Figure 2(b) illustrates the approach to this value as a function of energy. If the assumptions leading to (8) are valid, then one would expect $\bar{p}p$ cross sections to approach a constant at increasingly higher energies.

Using (7) and (8) with the best-fit parameter from total cross section measurements one can also calculate differential cross sections with *no additional parameters*. Using (1) and (4), digital computer calculations were made for a number of energies; these are compared with experiment in Fig. 3.[8] (Data in Fig. 3 are from Refs. 9–11.) Note that the slope expands with increasing energy[12] and that the dips and maxima are essentially at the correct $-t$ values. The discrepancy in the magnitude of $d\sigma/dt$ at large $-t$ values could be due to spin effects or a small real part of the scattering amplitude.

Equations (9) and (10) impose two conditions on the mass, size, and number of partons. A third condition must be assumed in order to determine m, n, and a uniquely.

In the quark model the third condition is that $n=3$, which gives $a=0.28$ F and $m=240$ MeV. This is in reasonable agreement with the dynamical quark model of Drell and Johnson,[13] which has a quark mass of approximately 300 MeV and a repulsive interaction between quarks mediated by neutral vector gluons with a range of approximately 0.2 F.[14]

The binding energy of a proton made up of n partons of mass m is given by

$$\text{B. E.}=nm-m_{\text{proton}} . \quad (11)$$

In this analysis the quark model predicts

$$\text{B. E.}=-220\text{ MeV},$$

which means that the proton would decay into three quarks unless some mechanism were to prevent it. Drell and Johnson introduce an effective single-particle Hartree potential with infinitely rising walls to accomplish this purpose.

Speculations on a collapsed-matter parton model. Partons are usually considered to be pointlike or much smaller than the dimensions of the proton. If partons are assumed to have mass, then, from the general theory of relativity, there is a minimum radius that can be associated with them, i.e., the Schwarzschild radius given by[15]

$$a=\frac{2Gm}{c^2}=1.48\times10^{-28}m\text{ cm}\quad(\text{with } m \text{ in g}), \quad (12)$$

where G is Newton's constant and c is the velocity of light. If we speculate that partons have this minimum radius, then (9), (10), and (12) give[16]

$$a=1.3\times10^{-33}\text{ cm},$$

$$m=0.9\times10^{-5}\text{ g},$$

$$n=0.6\times10^{20}.$$

Note that a and m are approximately Planck's characteristic length $[(\hbar G/c^3)^{1/2}=1.6\times10^{-33}\text{ cm}]$ and mass $[(\hbar c/G)^{1/2}=2.2\times10^{-5}\text{ g}]$, respectively.[17] In this case the binding energy of the proton using (11) is approximately 3×10^{38} GeV.

What force could give rise to such a tremendous binding energy? The clue may lie in the heavens, as it did in the formulation of the Bohr atom. Our picture of $\sim10^{20}$ partons distributed as e^{-r/r_0} resembles a globular cluster of 10^4 to 10^6 stars displaying strong central condensation. Is it possible that gravitational force which binds globular clusters is also responsible for the tremendous binding energy required for a proton composed of many massive partons?

The gravitational binding energy of a spherically symmetric body of mass density ρ is given by

$$\text{B. E.}=\int_0^\infty \frac{GM_r}{r}\rho\, 4\pi r^2\, dr ,$$

where

$$M_r=\int_0^r \rho\, 4\pi r^2\, dr .$$

Thus Eq. (11) gives

$$m_{\text{proton}}=nm-\frac{5n^2m^2G}{32r_0c^2} \quad (13)$$

for

$$\rho=\frac{nm}{8\pi r_0^3}e^{-r/r_0}.$$

For many massive partons the observed mass m_{proton} is orders of magnitude smaller than both the constituent mass (nm) and the mass defect ($5n^2m^2G/32r_0c^2$). Hence, setting $m_{\text{proton}}=0$ in (13) gives

$$nm=\frac{32r_0c^2}{5G}=2.0\times10^{15}\ \text{g},$$

where $r_0=(0.8/\sqrt{12})\times10^{-13}$ cm. Thus, the value of nm, obtained from $\bar{p}p$ data assuming collapsed-matter partons, is of the correct order of magnitude for gravitational binding. From Eq. (13) it is evident that only for extreme mass density is the mass defect important relative to the constituent mass. For example, the constituent mass of the earth is approximately 10^9 times greater than its mass defect.

It has previously been suggested that gravitation is the underlying foundation of the theory of elementary particles, and, in fact, that particles corresponding to Planck's characteristic mass might actually exist.[18] This analysis gives impetus to the idea by suggesting that partons are collapsed bits of matter corresponding to Planck's characteristic mass and gravitationally bound together to form hadrons.

Conclusion. The picture that hadrons are composed of smaller, perfectly absorbing constituents is in good agreement with $\bar{p}p$ data. This analysis suggests that if quarks are the fundamental subparticles of hadrons they are relatively light and have a non-negligible radius relative to the radius of hadrons. The speculation that hadrons are gravitationally bound "black holes" is an intriguing possibility which merits further investigation.

I wish to acknowledge helpful discussions with B. Durney, F. Henyey, A. Krisch, D. Lyon, M. Ross, and Y. Yao. I am particularly grateful to T. Hildebrandt and the Computing Facility of the National Center for Atmospheric Research[19] for the use of the CDC 6600/7600. I am indebted to P. Swarztrauber for his many helpful suggestions regarding mathematical analysis and computer programming. Finally, I would like to thank L. W. Jones for his continued help and support.

---

*Work supported by the National Science Foundation.
†Present address: Community College of Denver, Red Rocks Campus, Denver, Colorado 80215.

[1]There is no evidence for the existence of a real part of the scattering amplitude. See for example R. Armenteros and B. French, in *High Energy Physics*, edited by E. H. S. Burhop (Academic, New York, 1969), Vol. IV.

[2]Loyal Durand III and Richard Lipes, Phys. Rev. Lett. 20, 637 (1968); T. T. Chou and C. N. Yang, *ibid.* 20, 1213 (1968); Phys. Rev. 170, 1591 (1968).

[3]The analysis was also carried out assuming $l=kb$. This gave somewhat different quantitative results, but the qualitative conclusions are unchanged.

[4]B. Richter, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, Stanford, California, 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D.C., 1968).

[5]In calculating summations over l the summations were terminated when additional terms gave negligible contributions.

[6]A number of other density distributions were also tried. However, because of a number of uncertainties no conclusive results were obtained with regard to a best fit. Therefore, all calculations in this paper use an exponential density distribution with $r_m=0.8$ F.

[7]Data were taken from James E. Enstron *et al.*, LBL Report No. LBL-58, 1972 (unpublished). In Fig. 2(a) all compiled data were used where both σ_t and σ_e were measured at the same momenta. In Fig. 2(b) all measured values of σ_t in this compilation were used. The momenta range is approximately 0.3 to 16.0 GeV/c.

[8]In evaluating Eq. (1) the exact expressions for $P_l(\cos\theta)$ were used for $l\leq10$. For $l>10$ the approximation $P_l(\cos\theta)\simeq J_0(l\theta)$ was used.

[9]B. Conforto *et al.*, Nuovo Cimento 54A, 441 (1968).

[10]B. C. Barish *et al.*, Phys. Rev. Lett. 17, 720 (1966); 19, 142(E) (1967).

[11]D. Birnbaum *et al.*, Phys. Rev. Lett. 23, 663 (1969).

[12]A changing slope is often interpreted in terms of a simple disk model as a change in the radius of interaction. In the present formulation the proton-antiproton radius of interaction is energy-independent. The variation of the slope and the cross sections as a function of energy is due to the energy dependence of the opacity (parton-parton cross section).

[13]S. D. Drell and K. Johnson, Phys. Rev. D 6, 3248 (1972).

[14]In the Drell-Johnson model the short-range interaction is one of strong repulsion between pairs of quarks and one of strong attraction between quark-antiquark pairs. It is the latter case which is applicable to proton-antiproton interactions.

[15]See for example R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1965).

[16]That many extremely massive partons are treated as free particles may be a matter of concern. However, I know of no convincing arguments which would invalidate this assumption.

[17]Max Planck, *The Theory of Heat Radiation* (Dover, New York, 1959).

[18]See M. A. Markov, Zh. Eksp. Teor. Fiz. 51, 878 (1966) [Sov. Phys.—JETP 24, 584 (1967)]; J. A. Wheeler, Rev. Mod. Phys. 34, 873 (1962).

[19]The National Center for Atmospheric Research is supported by the National Science Foundation.