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Hadron production by electron-positron colliding beams*

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Various considerations and conjectures are made on the surprisingly large total cross section for hadron production which has recently been observed in the e^+-e^- colliding-beam experiment at the Cambridge Electron Accelerator. First, they focus on the relative magnitude of cross sections for one-photon annihilation and for the two-photon process. Second, the asymptotic limit R of the ratio $\sigma(e^++e^-\to\text{hadrons})/\sigma(e^++e^-\to\mu^++\mu^-)$ is predicted to be $R\cong 12\pi^2(f_\rho^{-2}+f_\omega^{-2}+f_\phi^{-2})=5.5\pm0.7$ from the PCDC (partially conserved dilation current) anomaly and vector-meson dominance, and estimated to be less than 15 for $\Gamma(\epsilon\to\pi^++\pi^-)=400$ MeV from the first evidence found at Frascati for hadron production by the two-photon process. A comment is also made on the present and future e^+-e^- colliding-beam experiments at SLAC and DESY.

One of the most intriguing experimental discoveries recently reported is the surprisingly large total cross section ($=26\pm6$ nb) for hadron production which has been observed in the e^+-e^- colliding-beam experiment at the Cambridge Electron Accelerator (CEA).[1] The data have been analyzed in such a way that the ratio $R(q^2)$ of the observed total cross section for $e^++e^-\to$ hadrons to the theoretical cross section ($\cong 4\pi\alpha^2/3q^2$) for $e^++e^-\to\mu^++\mu^-$ at $q^2=16$ GeV^2 (where $q^2=4E^2$ is the total c.m. energy squared of colliding beams) is calculated to be 4.7 ± 1.1. A naive comparison of the CEA data with the Frascati data[2] leads to the conclusion that $R(q^2)$ continues to rise as q^2 increases from above 1 GeV^2 up to 16 GeV^2. In this paper, I shall analyze and interpret the CEA data from a theoretical point of view and comment on the colliding-beam experiments which are under way at SLAC (SPEAR). I shall also predict the asymptotic limit $R=\lim_{q^2\to\infty}R(q^2)$, if it exists, from the PCDC (partially conserved dilation current) anomaly and vector-meson dominance, and estimate its upper bound from the first evidence recently found at Frascati for hadron production by the two-photon process.[2-4] For these purposes, it may be helpful to discuss these matters by adopting a question-and-answer style. This is done in what follows.

Let us define the theoretical $R(q^2)$ by $R(q^2)=12\pi^2\Pi(q^2)$ and

$$-(g_{\mu\nu}q^2-q_\mu q_\nu)\Pi(q^2)=\sum_n (2\pi)^3\delta(q-p_n)\times\langle 0|J_\mu(0)|n\rangle\langle n|J_\nu(0)|0\rangle, \tag{1}$$

where J_μ is the hadronic electromagnetic current. To the lowest order in α, the total cross section for $e^++e^-\to$ hadrons can be expressed in terms of $R(q^2)$ as follows:

$$\sigma(e^++e^-\to\text{hadrons})=(4\pi\alpha^2/3q^2)R(q^2). \tag{2}$$

The first few of several questions and answers are the following.

Q1: Is the $R(q^2)$ really rising? A1: It depends on whether the CEA data are purely one-photon annihilation into hadrons.

Q2: Are the CEA data pure? A2: It depends on whether the possible contamination due to the two-photon process has correctly been subtracted from the data. Since the detector (BOLD) used in the CEA experiment covers the solid angle of 2π sr and particularly misses the forward and backward angles, their data before the subtraction must have been taken as events for $e^++e^-\to$ hadrons $+$anything. As is widely accepted by now, the most dangerous background to the one-photon annihilation process $e^++e^-\to\gamma^*\to$ hadrons comes from the two-photon process

$$e^++e^-\to e^++\gamma^*+e^-+\gamma^*\to e^++e^-+\text{hadrons}$$

in which electrons and positrons are scattered predominantly within a small angle $[\sim(m_e/E)^{1/2}\cong 1^\circ$ for $E=2$ GeV$]$.[4]

Q3: Has contamination been correctly subtracted? A3: It depends on whether both the estimate of the asymptotic total cross section for $\gamma+\gamma\to$ hadrons, $\sigma(\gamma+\gamma\to\text{hadrons})\cong 0.3$ μb for large s (the total c.m. energy squared of colliding *photons*), and the "duality" suggested by Brodsky, Kinoshita, and the present author[5] for hadron production by two photons are correct. The reason for this is that in the analysis of the CEA data,[1] both of these assumptions have been made in order to estimate the contamination due to the two-photon process.

Q4: Is the estimate correct? A4: Probably yes. But only for $s \gg 1$ GeV^2, since it is based on the factorization[5-7]

$$\sigma(\gamma+\gamma \to \text{hadrons}) \cong \frac{[\sigma(\gamma+p \to \text{hadrons})]^2}{\sigma(p+p \to \text{hadrons})} \quad \text{for large } s\,, \qquad (3)$$

which is supported, for example, by the universal coupling of the Pomeranchukon.

Q5: Is the "duality" correct? A5: Yes and no. It is probably true that $\sigma(\gamma+\gamma \to \text{hadrons})$ with the $C=+$ resonances modulates its asymptotic value. However, one may make a big error when applying it to the following formula in the equivalent-photon approximation:

$$\sigma(e^+ + e^- \to e^+ + e^- + \text{hadrons}) = 2(\alpha/\pi)^2[\ln(E/m_e)]^2 \times \int_{s_{\rm th}}^{4E^2} \frac{ds}{s} f(\sqrt{s}/2E)\sigma(\gamma+\gamma \to \text{hadrons})\,, \qquad (4)$$

where $f(x)$ is Low's function[8] given by $f(x) = -(2+x^2)^2 \ln x - (1-x^2)(3+x^2)$. This is because the integral in (4) is weighted heavily toward small s so that it may be strongly enhanced by low-mass meson resonances with $C=+$. Unless $\sigma(\gamma+\gamma \to \text{hadrons})$ oscillates around its asymptotic value very quickly, the estimate of

$$\sigma(e^+ + e^- \to e^+ + e^- + \text{hadrons})$$

based on the factorization and the "duality" can easily be wrong by a factor of 2 or 3 or even larger. In fact, the upper bound on

$$\sigma(e^+ + e^- \to e^+ + e^- + \text{hadrons})$$

recently estimated by Gatto and Preparata,[9] based on the Cabibbo-Radicati sum rule, is roughly four times larger (~10 nb at $E=2$ GeV) than the cross section predicted in Ref. 5 (~2.6 nb), although the latter lies between the upper and lower bounds given in Ref. 9. Thus it is possible, though unlikely, that the number of two-photon events, which was claimed to be less than 2 or 3 events among the 88 events in the CEA data, is in fact as large as 20–30, accounting for 20–30% of the observed events. If this should be true, the data would turn out to be consistent with the three-triplet-colored-quark model recently advocated by Gell-Mann[10] ($R=2$) as well as the Han-Nambu model[11] ($R=4$).[12]

Q6: Can we make a better estimate of

$$\sigma(e^+ + e^- \to e^+ + e^- + \text{hadrons})?$$

A6: Yes, we can if we know the decay width $\Gamma_{M^0 \to \gamma\gamma}$ for all the $C=+$ mesons M^0 whose masses are smaller than $4E^2$. A problem is, however, that all the relevant widths we know are only those for π^0 and η, whose main decay modes are $M^0 \to \gamma\gamma$.

What emerges from the above dialogue is that the total cross sections for $e^+ + e^- \to$ hadrons and $e^+ + e^- \to e^+ + e^- +$hadrons as functions of $4E^2$ are equally difficult to predict. As an opposite extreme, in the analysis of the CEA data, one could have subtracted the one-photon annihilation events by assuming scaling, i.e., $R(q^2) = \text{constant}$, and for example, the three-triplet model ($R=2$), leaving the two-photon cross section. Of course, such an analysis may sound ridiculous. However, what I am trying to emphasize in this paper is that the relative magnitude of the one- and two-photon cross sections for hadron production is not known so precisely as one might think. This point would be more emphasized by the following ansatz: If the measured $R(q^2)$ in $e^+ + e^- \to$ hadrons is unexpectedly large (~4–6), then

$$\sigma(e^+ + e^- \to e^+ + e^- + \text{hadrons})$$

is also much larger than expected. This ansatz cannot be proved but is supported by a combination of the predicted relation for the one-photon annihilation,[13]

$$R = \sum_i Q_i^2 + \tfrac{1}{4} \sum_j Q_j^2\,, \qquad (5)$$

where Q_i and Q_j are the charges of spin-$\frac{1}{2}$ and -0 constituents of the electromagnetic current, and the rough guess for the two-photon process,

$$\sigma(\gamma+\gamma \to \text{hadrons}) \cong \left(\sum_i Q_i^4\right)\sigma(\gamma+\gamma \to \mu^+ + \mu^-) + \left(\sum_j Q_j^4\right)\sigma(\gamma+\gamma \to \pi^+ + \pi^-)\,, \qquad (6)$$

where, exclusively in (6), μ and π are symbolic of spin-$\frac{1}{2}$ and -0 *pointlike* particles, respectively. The relation (6) has been proved in the Gross-Treiman limit[14] for hadron production by highly virtual photons,[15] but is very much a conjecture for real or almost real photons.

I have so far pointed out the possibility that the CEA data may substantially be contaminated by the two-photon process. However, suppose, for a moment, that we can ignore such a possibility. Then we must admit that $R(q^2)$ is still rising, not yet reaching scaling at q^2 as large as 16 GeV^2. Then the following two extremely interesting questions are ready to be asked.

Q7: Can we estimate the asymptotic value R from presently available data? A7: Yes, we can.

Surprisingly enough, we can estimate it not directly from the CEA data for the one-photon annihilation into hadrons, but from the new Frascati data[2,3] in which the first evidence for hadron production by the two-photon process has been found. It has been reported that two of the recorded events in which both the scattered electron and the positron are tagged[16] may be interpreted as due to the two-photon process $e^+ + e^- \to e^+ + e^- + \pi^+ + \pi^-$. The upper bound on the total cross section is estimated to be[17]

$$\sigma(e^+ + e^- \to e^+ + e^- + \pi^+ + \pi^-) < 3.3\times 3 \text{ nb}$$
$$(95\% \text{ confidence level) at } 2E = 2.7 \text{ GeV}. \qquad (7)$$

On the other hand, Crewther[18] and, independently, Chanowitz and Ellis[19] have recently pointed out, on the canonical anomaly existing in PCDC,[20,21] that the coefficient of the anomalous term is completely determined by the asymptotic value R as follows:

$$\theta_\lambda^\lambda = f_\epsilon m_\epsilon^2 \epsilon + (R/32\pi^2)\tilde{F}_{\mu\nu}^i \tilde{F}_i^{\mu\nu}, \qquad (8)$$

where $\theta_{\mu\nu}$ is the stress-energy tensor, ϵ is the scalar meson field assumed to dominate the normal part of θ_λ^λ, $\tilde{F}_{\mu\nu}^i = \partial_\mu F_\nu^i - \partial_\nu F_\mu^i + h_{ijk} F_\mu^j F_\nu^k$, with the F_μ^i being external fields coupled to the SU(3) $\times$ SU(3) currents, and h_{ijk} are the structure constants of SU(3)$\times$SU(3). Furthermore, the $\epsilon\gamma\gamma$ coupling constant defined by

$$\mathcal{L}_{\epsilon\gamma\gamma}^{\text{eff}} = -(e^2 g_{\epsilon\gamma\gamma}/2)\epsilon F_{\mu\nu}F^{\mu\nu}$$

is predicted to be $g_{\epsilon\gamma\gamma} \cong R/12\pi^2 f_\epsilon$,[18,19] giving

$$\Gamma_{\epsilon\to\gamma\gamma} \cong (\alpha^2/144\pi^3)(m_\epsilon/f_\epsilon)^2 m_\epsilon R^2. \qquad (9)$$

The $\epsilon\pi\pi$ coupling constant defined by

$$\mathcal{L}_{\epsilon\pi\pi}^{\text{eff}} = \tfrac{1}{2} g_{\epsilon\pi\pi} \epsilon \pi_i \pi^i$$

was predicted earlier by Crewther[22] and Ellis[23] to be $g_{\epsilon\pi\pi} \cong m_\epsilon^2/f_\epsilon$, giving

$$\Gamma_{\epsilon\to\pi^+\pi^-} \cong (1/16\pi)(m_\epsilon/f_\epsilon)^2 m_\epsilon (1 - 4m_\pi^2/m_\epsilon^2)^{1/2}, \qquad (10)$$

and, therefore,[24]

$$\Gamma_{\epsilon\to\gamma\gamma}/\Gamma_{\epsilon\to\pi^+\pi^-} \cong (\alpha R/3\pi)^2 (1 - 4m_\pi^2/m_\epsilon^2)^{-1/2}. \qquad (11)$$

For ϵ production, the formula (4) can be simplified to

$$\sigma(e^+ + e^- \to e^+ + e^- + \epsilon) \cong (16\alpha^2 \Gamma_{\epsilon\to\gamma\gamma}/m_\epsilon^3)$$
$$\times [\text{[illegible]}/m_e)]^2 f(m_\epsilon/2E). \qquad (12)$$

Combining (11) and (12) and taking the branching ratio $\Gamma_{\epsilon\to\pi^+\pi^-}/\Gamma_{\epsilon\to\pi^0\pi^0} = 2$ into account, I finally estimate the upper bound on R from (7) to be

$$R < 15 \ (90\% \text{ confidence level})$$
$$\text{for } m_\epsilon \cong 700 \text{ MeV and } \Gamma_{\epsilon\to\pi^+\pi^-} \cong 400 \text{ MeV}. \qquad (13)$$

Notice that this upper bound is presumably overestimated since I have assumed that all the pion pairs in $e^+ + e^- \to e^+ + e^- + \pi^+ + \pi^-$ are produced by ϵ decay. I believe that not only a considerable improvement of this useful bound on R but also a rough estimation of R can be expected when more information on the two-photon process $e^+ + e^- \to e^+ + e^- + \pi^+ + \pi^-$ becomes available.

The asymptotic value R can also be predicted without assuming scalar-meson dominance for the stress-energy tensor, but with the usual vector-meson dominance for the electromagnetic current. To this end, let us define the photon form factor for the trace of the stress-energy tensor by

$$\langle \gamma(\epsilon_1, k_1) | \theta_\lambda^\lambda(0) | \gamma(\epsilon_2, k_2)\rangle$$
$$= -(\epsilon_1\cdot\epsilon_2 k_1\cdot k_2 - \epsilon_1\cdot k_2 \epsilon_2\cdot k_1) F((k_1-k_2)^2). \qquad (14)$$

The authors of Refs. 18 and 19 have calculated the form factor at $(k_1-k_2)^2 = 0$ $(k_1^2 = k_2^2 = 0)$ to be

$$F(0) = \tfrac{1}{12} e^2 \int dx\, dy\, x\cdot y \langle 0 | T(J_\mu(x) J^\mu(y) \theta_\lambda^\lambda(0)) | 0\rangle$$
$$= (e^2/6\pi^2) R. \qquad (15)$$

Assuming vector-meson dominance, the left-hand side of Eq. (14) can be approximated by

$$(e^2/f_\rho^2)\langle \rho(\epsilon_1, k_1) | \theta_\lambda^\lambda(0) | \rho(\epsilon_2, k_2)\rangle$$
$$+ (\omega \text{ and } \phi \text{ terms}), \qquad (16)$$

where f_ρ is the γ-ρ coupling constant ($f_\rho^2/4\pi \cong 2.2 \pm 0.3$).[25] Furthermore, the matrix element in (16) can be evaluated by the definition of the stress-energy tensor when $k_1 = k_2$, namely,

$$\langle \rho(\epsilon_1, k) | \theta_{00}(0) | \rho(\epsilon_2, k)\rangle$$
$$= -2k_0 k_0 (\epsilon_1\cdot\epsilon_2 - \epsilon_1\cdot k \epsilon_2\cdot k/k^2). \qquad (17)$$

Thus we obtain the following approximate result:

$$F(0) \cong 2e^2/f_\rho^2 + (\omega \text{ and } \rho \text{ terms}). \qquad (18)$$

The isoscalar (ω and ϕ) contribution [$f_\omega^2/4\pi = 19 \pm 4$ and $f_\phi^2/4\pi \cong 14 \pm 1$ (Ref. 25)] can be calculated in a similar way so that

$$F(0) \cong 2e^2(f_\rho^{-2} + f_\omega^{-2} + f_\phi^{-2}). \qquad (19)$$

Comparing these two results (15) and (19), I finally predict

$$R \cong 12\pi^2(f_\rho^{-2} + f_\omega^{-2} + f_\phi^{-2}) = 5.5 \pm 0.7 . \qquad (20)$$

It is striking that this predicted value agrees perfectly with the preliminary data for $q^2 = 25$ GeV^2 from CEA[2] as well as the rough estimate from the Frascati two-photon data.[26] Notice also that this prediction, though it looks similar, is independent of and different from those made by Bramón, Etim, and Greco[27] and by Sakurai,[28] based on an infinite series of vector-meson resonances or on the "new duality," since I have assumed the most conventional form of vector-meson dominance for real photons.

The relation (20) can also be interpreted in the following way[29]: Suppose one considers the probability of finding hadronic components in a real photon; it is roughly estimated to be $e^2(f_\rho^{-2} + f_\omega^{-2} + f_\phi^{-2})$ in the vector-dominance model. If one takes the vector mesons as bound states of quark-antiquark pairs, the same probability is roughly proportional to $\sum_i (eQ_i)^2$. Combining this consideration and the relation (5), one would end up with the conclusion that a sum of the fundamental constants $f_\rho^{-2} + f_\omega^{-2} + f_\phi^{-2}$ may be simply related to the constant R by Eq. (20). I think this is a very important result. Notice also that the relation (20) needs two corrections: One is due to the extrapolation of the matrix element from $k^2 = m_V^2$ to $k^2 = 0$ and the other is due to vector states with negative charge conjugation other than ρ, ω, and ϕ, such as ρ'. These corrections are supposed to be less than 10%. In any case, future e^+-e^- colliding-beam experiments at SLAC (SPEAR) and DESY (Doris) will decide whether our prediction based on the PCDC anomaly agrees with the data.

The other question related to the rising $R(q^2)$ is the following.

Q8: Why has the scaling in the one-photon annihilation into hadrons not started yet at such high mass of the virtual photon as $q^2 = 16$ GeV^2, as it has in the SLAC-MIT deep-inelastic electroproduction experiments at $-q^2 = 1-2$ GeV^2? A8: There are two differences between these processes: (1) the timelike photon vs the spacelike photon and (2) the vacuum expectation value for the product of currents vs the matrix element between the hadron states. A small negative value for q^2 is sufficient to keep the virtual photon away from the resonance region, while a timelike photon with such high mass may still suffer from possible vector-meson resonances with $C = -$. If the difference (2) is the more important one, then an approximate Bjorken scaling of the structure functions in the inclusive process, for example $e^+ + e^- \to \gamma^* \to \pi^+ +$anything, will be seen at presently available energies, although the scaling of the total cross section has not yet been observed. This possibility, considered by Pestieau and the present author,[30] is taken seriously in a new formalism of the parton model by Sanda,[31] who has shown an interesting relation between the rising $R(q^2)$ and the more rapidly rising multiplicity $n(q^2)$.

The last question is addressed to experimentalists.

Q9: Is there any reliable method to distinguish between one-photon annihilation and the two-photon process? A9: Yes, there are at least two such methods. One is the electron tagging already established at Frascati[16]; the other is the performance of both e^+e^- and e^-e^- colliding-beam experiments, which is possible exclusively at DESY (Doris). There are a few other methods which are less reliable: (1) Measure the total energy of produced particles and find whether it agrees with the total energy of colliding beams. (2) Find the exact collinearity of two tracks for two-particle productions by one-photon annihilation or find the approximately coplanar but noncollinear two tracks for two-particle productions by the two-photon process. Notice, however, that this is only applicable to two-particle production and that the coplanarity in the two-photon process has a large error of an order $(m_e/E)^{1/4}$ (~7° for $E = 2$ GeV).[5] (3) Identify particles produced with large transverse momentum with those from one-photon annihilation. The criterion (3) is the least reliable since the ratio

$$\frac{d\sigma(e^+ + e^- \to e^+ + e^- + \pi + \text{anything})}{d\sigma(e^+ + e^- \to \gamma^* \to \pi + \text{anything})}$$

is as model-dependent as the ratio

$$\frac{\sigma(e^+ + e^- \to e^+ + e^- + \text{hadrons})}{\sigma(e^+ + e^- \to \text{hadrons})} .$$

Gatto and Preparata[9] have calculated the former ratio to be less than 1% for $p_\pi^{tr} > 1$ GeV at $2E = 5$ GeV, assuming Bjorken scaling for one-photon annihilation as well as Feynman scaling plus the exponential falloff with transverse momentum for the two-photon process, while Berman, Bjorken, and Kogut[32] estimated the same ratio to be about 20% from the parton model applied to both processes. I shall give another argument which suggests even larger values for the ratio. Pais and Treiman[33] have proposed the inclusive soft-pion production by one-photon annihilation, $e^+ + e^- \to \gamma^* \to \pi(\text{soft}) +$anything, as an experiment for measuring the spectral function of the axial-vector current propagator defined by

$$-(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_A^{(3)}(q^2) + q_\mu q_\nu \Pi_P^{(3)}(q^2) = \sum_n (2\pi)^3\delta(q-p_n)\langle 0|A_\mu^{(3)}(0)|n\rangle\langle n|A_\nu^{(3)}(0)|n\rangle . \quad (21)$$

Assuming the PCAC hypothesis and current algebra, they have shown ($f_\pi \cong 95$ MeV)

$$E_\pi \frac{d\sigma(e^+ + e^- \to \gamma^* \to \pi + \text{anything})}{d^3p_\pi} \cong \frac{(16\pi^3\alpha^2/q^2)\Pi_A^{(3)}(q^2)}{2(2\pi)^3 f_\pi^2} \text{ for small } p_\pi . \quad (22)$$

If one further assumes the SU(3) symmetry $\Pi_A^{(3)} \cong \frac{3}{4}\Pi_A^{(\mathrm{em})}$ and the asymptotic chiral symmetry $\Pi_A \cong \Pi_V$ for large q^2, one can obtain

$$E_\pi \frac{d\sigma(e^+ + e^- \to \gamma^* \to \pi + \text{anything})}{d^3p_\pi} \cong \frac{\frac{3}{4}\sigma(e^+ + e^- \to \text{hadrons})}{2(2\pi)^3 f_\pi^2} \text{ for small } p_\pi . \quad (23)$$

A similar relation holds for the two-photon process, based on the same assumptions plus the approximate chiral symmetry even for small q^2, which means that the cross section for vector-photon–axial-vector-photon scattering is approximately equal to that for photon-photon scattering. From these two relations I conjecture

$$\frac{d\sigma(e^+ + e^- \to e^+ + e^- + \pi + \text{anything})}{d\sigma(e^+ + e^- \to \gamma^* \to \pi + \text{anything})} \cong \frac{\sigma(e^+ + e^- \to e^+ + e^- + \text{hadrons})}{\sigma(e^+ + e^- \to \text{hadrons})} \text{ for small } p_\pi , \quad (24)$$

which shows that the relevant ratio can be of order unity. This indicates that the inclusive experiments $e^+ + e^- \to \pi + \text{anything}$ may seriously be contaminated by the two-photon process unless scattered leptons are tagged or unless observed pion momenta are much larger than, say 1 GeV.

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Note added in proof. It cannot be stressed too strongly that *our application of the vector-meson dominance to the vacuum expectation value of the "softer" operator product $T(J_\mu(x)J_\nu(y)\theta_\lambda^\lambda(0))$ for real photons is perfectly consistent with the PCDC anomaly and the nonvanishing constant R.* We have never applied vector-meson dominance directly to the vacuum expectation value of the "hard" operator product $T(J_\mu(x)J_\nu(0))$. If one were to do this, one would reach the contradictory result $R=0$ unless one adopts the "new duality".[27,28] In this connection, it is worth mentioning here that the old application of vector-meson dominance to the decay of π^0 into 2γ by Gell-Mann, Sharp, and Wagner is still consistent with the present data, independent of the PCAC anomaly.

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[24]Notice that in (11) the errors due to the extrapolation needed from $m_\epsilon^2 = 0$ to $m_\epsilon^2 = (700\text{ MeV})^2$ would best be compensated by each other.
[25]To determine $f_V^2/4\pi$, we have used the formula

$$\Gamma_{V\to l^+l^-} = (e^4/12\pi f_V^2)m_V(1+2m_l^2/m_V^2)(1-4m_l^2/m_V^2)^{1/2}$$

for the decay width of the vector meson V into a lepton pair l^+l^-, and the present particle data,

$$\Gamma_{\rho\to e^+e^-} = (146\pm10)(0.0043\pm0.0005)\times10^{-2}\text{ MeV},$$

$$\Gamma_{\omega\to e^+e^-} = (9.8\pm0.5)(0.0076\pm0.0017)\times10^{-2}\text{ MeV},$$

and

$$\Gamma_{\phi\to e^+e^-} = (4.2\pm0.2)(0.032\pm0.003)\times10^{-2}\text{ MeV}.$$

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PHYSICAL REVIEW D VOLUME 9, NUMBER 5 1 MARCH 1974

Analysis for hypercharge-exchange reactions in terms of dispersion relations

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An analysis of $\pi N\to K\Lambda$ and $\bar{K}N\to\pi\Lambda$ is presented in which the imaginary parts of the amplitudes are in approximate agreement with finite-energy sum rules and the real parts are calculated in terms of the imaginary parts via fixed-t dispersion relations. The resulting differential cross sections and polarizations are in fair agreement with experiment. Certain features and implications of the amplitude structure are discussed.

I. INTRODUCTION

Line-reversed hypercharge-exchange reactions present a number of intriguing features which cannot be understood in terms of simple Regge phenomenology. For example, it has been known for quite some time[1] that a model with a pair of exchange-degenerate Regge poles K^*-K^{**} is in clear contradiction with basic experimental facts. One may hope that amplitude analysis, which has thrown much light on the structure of πN amplitudes, will provide some insight into the problems of hypercharge exchange as well. One may also hope that through such an analysis a common pic-