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## VOLUME 9, NUMBER 5

## 1 MARCH 1974

## Measurement of the $K^0$ coherent regeneration amplitude in Cu from 0.6 to 1.4 GeV/c

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The magnitude of  $|f_{21}(0)|$ , the coherent  $K^0$  regeneration amplitude in Cu, has been measured for K momenta from 600 to 1400 MeV/c. Results are compared with predictions of an optical model using forward dispersion relation predictions for real parts of kaon-nucleon scattering amplitudes.

This paper reports a measurement of the momentum dependence of the  $K^{0}$  coherent regeneration amplitude in copper through the  $Y^*$  resonance region from 600 to 1400 MeV/c. The experiment was performed in a 17.5° neutral beam at the Argonne Zero-Gradient Synchrotron (ZGS) and provides the most detailed study to date of the regeneration process in the low-momentum region. The amplitude for coherent regeneration is

 $f_{21}(0) \equiv \frac{1}{2} [f(0) - \overline{f}(0)],$ 

where  $f(0)[\overline{f}(0)]$  is the S=+1[S=-1] K-mesonnucleus forward scattering amplitude. Therefore, when either f(0) or  $\overline{f}(0)$  changes rapidly with momentum, e.g., near one of the Y\* resonance peaks, one might expect rather abrupt changes in  $f_{21}(0)$ . This experiment measures  $|f_{21}(0)|$  by determining the number of coherently regenerated  $K_{S}^{0}$  mesons decaying after a 12.7-cm-thick copper regenerator set into a  $K_L^0$  beam.

The rate for producing  $\pi^+\pi^-$  pairs is

$$R_{+-} = (K_L \operatorname{flux}) \frac{\Gamma_s(\pi^+ \pi^-)}{\Gamma_s} \frac{(N \lambda \Lambda)^2}{\delta^2 + \frac{1}{4}} |f_{21}(0)|^2 f(\delta, L),$$

 $f(\delta, L) = (1 + e^{-l} - 2e^{-l/2} \cos \delta l) \exp(-N\sigma_{\tau}L),$  $\delta = |M_{K_I} - M_{K_S}| \tau_S,$ 

L is the slab length,  $\Lambda = \beta_K \gamma_K c \tau_S$ ,  $l = L/\Lambda$ ,  $\lambda = h/p_K$ ,  $\Gamma_s(\pi^+\pi^-)/\Gamma_s$  is the  $K_s$  branching ratio to  $\pi^+\pi^-$ , N is the density of scattering centers, and  $\sigma_{\tau}$  is the  $K_r$ -nucleus total cross section. To determine  $|f_{21}(0)|$ , one must measure  $R_{+-}$  as a function of  $p_{K}$ , the  $K_{L}$  flux as a function of  $p_{K}$ , and  $\sigma_{T}$ . Everything else is known.

The experimental layout is shown in Fig. 1. The  $\pi^+\pi^-$  pairs from  $K^0_s$  decay are detected in a twoarm magnetic spectrometer. The large dipion opening angles at low momentum preclude using one large magnet. Each magnet had four magnetostrictive-readout wire spark chambers mounted upstream and four mounted downstream. Each chamber recorded X and Y coordinates. They were digitized using a Science Accessories readout system on-line to a PDP-7 computer. In each arm a threefold coincidence was formed from scintillation counters defining the entrance aperture of the magnet  $(L_1, R_1)$  and covering the exit of the spark chambers  $(L_2, L_3, R_2, R_3)$ . Anticoincidence counters  $A_1, A_2, A_3$  formed a box around the regenerator to veto events having charged parti-

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where



FIG. 1. (a) Neutral beam layout at the ZGS. (b) Details of the spectrometer.

cles emanating from the material. The final trigger was

$$T = (L_1 \cdot L_2 \cdot L_3) \cdot (R_1 \cdot R_2 \cdot R_3) \cdot (\overline{A}_1 \cdot \overline{A}_2 \cdot \overline{A}_3)$$
$$= L \cdot R \cdot \overline{A} .$$

The anticoincidence signal  $\overline{A}$  was also fed to the beam-monitor coincidence, to be described later, to correct electronically for dead-time losses.

In symmetric two-body decays, there is a correlation between the decay point along the beam  $(Z_{DK})$  and the  $K_L$  momentum for events that get through the spectrometer. To span the momentum range, data were taken in eight overlapping settings of regenerator position and spectrometer magnetic fields. The overlap data provided a systematic check on the Monte Carlo efficiency calculation.

The Monte Carlo program included effects of pion multiple Coulomb scattering in air from the  $K_s^0$  decay point to the first spark chamber, scattering and resolution smearing in the chambers, and scattering in scintillator L1 (R1). Simulated events that survived the geometric limits of the spectrometer apertures were processed in the same data-analysis programs as the real events to correct both for spectrometer efficiency and program efficiency. The raw data are dominated by coherently regenerated  $K_s \rightarrow \pi^+\pi^-$  events. The cleanliness of the data and the fidelity of the Monte Carlo simulation are illustrated in Fig. 2, which shows data from



FIG. 2. (a) Comparison of the dipion effective-mass distribution for all events originating within 25.4 cm of the regenerator compared with Monte Carlo predictions for coherent  $K_S^0 \rightarrow \pi^+\pi^-$ . (b) Comparison as in (a) of the  $\theta_K^2$  distribution. (c) Semilog plot of  $\theta_K^2$  distribution of data with  $1.12 < P_K < 1.20 \text{ GeV}/c$  and  $485 < M^* < 510$  MeV/ $c^2$  illustrating separation of the coherent  $K_S^0$  peak and the diffractively regenerated  $K_S^0$  background, fitted by an exponential in  $\theta_K^2$ .

one of the eight settings along with the Monte Carlo curves. The only cut applied to the data is a  $Z_{DK}$  cut to restrict the decay point to within 25.4 cm of the regenerator; no cut to enhance the coherent signal has been made. The two variables plotted are the effective mass  $M^*$  of the dipion and the quantity

$$\theta_{\kappa}^{2} = 2 \left[ 1 - \frac{(\vec{\mathbf{p}}_{1} + \vec{\mathbf{p}}_{2})}{|\vec{\mathbf{p}}_{1} + \vec{\mathbf{p}}_{2}|} \cdot \hat{Z} \right],$$

where  $\vec{P}_1$  and  $\vec{P}_2$  are the momenta of the decay pions and  $\hat{Z}$  is the beam direction. For a parallel incident beam and perfect resolution, all coherently regenerated events would have  $\theta_K^2 = 0$  and  $M^* = m_K$ . Diffraction-regenerated events show a smooth distribution with a characteristic width

$$\langle \theta^2 \rangle \sim (1/kR_{\rm nucleus})^2 \sim 6 \times 10^{-4}$$

at 1 GeV/c in Cu. Figure 2(c) shows the  $\theta_{K}^{2}$  plot for events having  $485 < M^{*} < 510 \text{ MeV/}c^{2}$ . The coherent peak has  $\theta_{K}^{2} < 10^{-4}$  and sits atop a smooth background falling like  $(kR)^{-2}$ .

The  $K_{L}^{0}$  flux measurement was perhaps the most difficult problem in the experiment. The over-all approach was to assume a spectral-shape parameterization of a form suggested by Sanford and Wang for particle production in Be and to use " $K_{\pi^{2}}$ -like" free decays, i.e., the *CP*-violating  $K_{L} - \pi^{+}\pi^{-}$  decays plus the small fraction of leptonic decays that simulate  $K_{\pi^{2}}$ , to deduce the over-all normalization. By this method the Monte Carlo program for the spectrometer acceptance is essentially the same for the flux normalization and the regeneration-efficiency calculation, so errors from the Monte Carlo calculation should be reduced.

The free-decay event rates were used to calibrate a neutral-beam-monitor telescope which served as the secondary normalization standard. The monitor M was a four-counter telescope  $M \equiv \overline{M_1} M_2 M_3 M_4$  with a 1.27-cm-thick polyethylene converter after  $M_1$  to produce charged particles. As mentioned earlier, the monitor was deadened by regenerator vetoes  $\overline{A}$ , and the secondary normalization rate is  $M\overline{A}$ .

The Sanford-Wang parameterization for particle production at 17.5° and ZGS proton momentum  $p_0$  of 12.4 GeV/c reduces to

$$S(p) = C_1 p^{C_2} \left( 1 - \frac{p}{p_0} \right) \exp(-C_6 p \theta),$$

with  $\theta = 0.306$  here.<sup>1</sup> Previous experience suggests that  $K_L$  production is roughly the average of  $K^+$ and  $K^-$  production and that heavy targets viewed at wide angles shift the peak of the secondary-particle spectrum toward higher momentum than in Be.<sup>2</sup> Consequently, while the normalization constant  $C_1$  was determined from the " $K_{\pi 2}$ -like" events, the shape parameters  $C_2$  and  $C_6$  were adjusted to fit the experimental momentum distribution of the charged secondaries from a sample of  $K_{13}$  decays. These two samples were defined as follows: All events were reconstructed as  $K_{\pi 2}$  decays. Those for which the merit function

$$F = [(M^* - 497.8)/\sigma_M]^2 + (\theta_K/\sigma_\theta)^2 < 14$$

were labeled " $K_{\pi 2}$ -like." The resolution parameters  $\sigma_M$  and  $\sigma_{\theta}$  are taken from the regenerator data.  $K_{13}$  events were those having  $370 < M^* < 460 \text{ MeV}/c^2$ ,  $\theta_{\kappa} < 0.1$ , and Astier variable

$$A = \frac{(M_K^2 - M_{\pi 0}^2 - M^{*2})^2 - 4 M_{\pi 0}^2 M^{*2} - 4 M_K^2 (P_K)_T^2}{4 [(P_K)_T^2 + M^{*2}]} < 0$$

These cuts removed  $K_{\pi 2}$  and  $K_{\pi 3}$  decays and nearly all neutron-induced background.

The normalization was obtained by comparing the number of " $K_{\pi 2}$ -like" events from the data with the number of " $K_{\pi 2}$ -like" events from Monte Carlo generated  $K_L$  decays processed in the event-analysis program. These Monte Carlo  $K_L$  decays were distributed according to the tabulated  $K_L$  branching ratios. The ratio of (data)/(Monte Carlo events) calibrates the normalization constant  $C_1$  in  $K_L$  $(MeV/c)^{-1}$  (monitor count)<sup>-1</sup>. This procedure is insensitive to the exact spectral shape over a wide range of values of  $C_2$  and  $C_6$ .

In fitting the spectral-shape parameters  $C_2$  and  $C_6$ , the  $K_{13}$  data were compared with  $K_{13}$  Monte Carlo events weighted by  $S(P)e^{-D/\Lambda_L}\Delta L/\Lambda_L$ , where  $\Lambda_L$  is the  $K_L$  mean decay length at  $K_L$  momentum P, D is the 500-in. flight path from production target to decay region, and  $\Delta L$  is the 62-in. decay region. Because of the correlation of  $Z_{DK}$  and P in the spectrometer, a two-dimensional plot of  $N(Z_{DK}, P)$  showed a diagonal region that was populated by real events. Bin-by-bin agreement between the data and the weighted Monte Carlo events was excellent. The projection of these plots onto the momentum axis was subjected to a  $\chi^{\,2}$  test to select the best fit. Figure 3(a) shows the spectrum of  $K_L$  mesons at 500 in. from the production target, together with the  $\Delta \chi^2 = 1$  curves. Our bestfit parameters have  $C_1 = (0.350 \pm 0.014) \times C_1^{\text{Sanford-Wang}}$ ,  $C_2 = 1.5 \times C_2^{\text{Sanford-Wang}}$ , and  $C_6 = 0.75 \times C_6^{\text{Sanford-Wang}}$ . Converting to  $K_L$  yield, we find<sup>3</sup>

$$\frac{d^2 N}{dp \, d\Omega} = (0.01 \pm 0.002) p (1 + 0.3 p)$$
$$\times e^{-3.64 p \theta} K_L \, (\text{GeV}/c)^{-1} \text{sr}^{-1}$$
$$\times (\text{interacting proton})^{-1}.$$

The error in the flux is  $\pm 6\%$  from 600 to 1400 MeV/c. This includes contributions from the er-



FIG. 3. (a) Relative  $K_L$  flux at the regenerator position, normalized to the average flux of 20  $K_L$  (GeV/c)<sup>-1</sup> (monitor count)<sup>-1</sup> from 0.6 to 1.4 GeV/c. Also shown are the  $\Delta\chi^2 = 1$  curves from the spectral-shape studies. For the best fit,  $\chi^2 = 18$  for 28 degrees of freedom. (b) Comparison of the experimental results for  $|f_{21}(0)|$  as a function of momentum with optical-model predictions, using real parts taken from dispersion-relation fits by Martin and Poole (Ref. 6). Also shown is the optical-model result for purely imaginary amplitudes (dashed line).

ror in  $C_1$ , the variations due to spectral shape, and an error of 5% assigned to the Monte Carlo efficiency. The Monte Carlo error is an estimate based on the good agreement of overlapping data sets in the regenerator data, of low-field and highfield settings in the free-decay normalization, and of the two-dimensional  $Z_{\rm DK}$  vs P plots in the spectral analysis. An efficiency error of more than 5% would have led to detectable inconsistencies.

To make the attenuation correction for the  $e^{-N\sigma_T L}$  term,  $\sigma_T$  was taken from optical-model calculations. This factor is insensitive to the real-part ratios used and also is not very momentum-dependent. Therefore, using  $\sigma_T$  from the optical model to test the optical-model calculations of  $f_{21}(0)$  is valid. Moreover, the optical model used predicts total cross sections of 972 mb at 2.5 GeV/c and 957 mb at 2.7 GeV/c, where they have been measured to be 940 ± 20 mb and 957 ± 10 mb.<sup>4</sup> The values of  $\sigma_T$  used at each momentum are listed in Table I, along with the number of coherent  $2\pi$  events, the experimental  $|f_{21}(0)|$ , and the quantity  $|f(0) - \overline{f}(0)|/k$ . Corrections were made for  $\pi$  de-

cay and absorption losses at each setting, using the mean pion momentum. These corrections were typically  $1.25 \pm 0.03$ , and the error encompasses the uncertainty due to spread in pion momentum and kaon momentum at a single setting.

In the optical model, the meson-nucleus scattering amplitude f(0) [f(0)] is computed from the nuclear density  $\rho(r)$  and the meson-nucleon amplitudes  $f_p(0)$  and  $f_n(0)$  by an impact-parameter integral:

$$f(\theta) = ik \int_0^\infty bdb J_0(kb\sin\theta) [1 - e^{i\chi(b)}]$$

for phase shift

$$\chi(b) = \frac{2\pi}{k} \left[ \frac{Z}{A} f_p(0) + \frac{A-Z}{A} f_n(0) \right] \int_{-\infty}^{\infty} dz \,\rho(r) \,,$$

where  $r^2 = b^2 + z^2$  and, at  $\theta = 0$ ,  $J_0(kb\sin\theta) \rightarrow 1$ . This has been evaluated using a Saxon-Woods function

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - r_0)/d]},$$

with falloff parameter d = 0.57 F and mean radius

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P <sub>K</sub> (GeV/c)	(N <sub>+</sub> _) coh	$\overline{\sigma}_T$ (b)	$ f_{21}(0) ^2$ (fermi) <sup>2</sup>	f <sub>21</sub> (0)  (fermi)	$ f - \overline{f} /k$ (mb)
0.60-0.68	$107 \pm 11$	0.966	$103.5 \pm 12.4$	$10.17 \pm 0.61$	$62.7 \pm 3.8$
0.68-0.76	$225 \pm 15.5$	1.00	$117.3 \pm 9.8$	$10.83 \pm 0.45$	$59.4 \pm 2.5$
0.76-0.84	$207 \pm 15.1$	1.04	$145.6 \pm 12.4$	$12.07 \pm 0.52$	$59.5 \pm 2.6$
0.84-0.92	$117 \pm 5.7$	1.07	$153.4 \pm 16.3$	$12.39 \pm 0.66$	$55.6 \pm 3.0$
0.92 - 1.00	$77 \pm 9.3$	1.11	$158.2 \pm 20.1$	$12.58 \pm 0.80$	$51.7 \pm 3.3$
1.00 - 1.12	$152 \pm 6.6$	1.11	$153.2 \pm 14.8$	$12.38 \pm 0.60$	$46.1 \pm 2.2$
1.12 - 1.20	$116 \pm 11.2$	1.07	$143.0 \pm 16.4$	$11.96 \pm 0.69$	$40.7 \pm 2.3$
1.20-1.28	$64 \pm 8.5$	1.05	$85.0 \pm 12.5$	$9.22 \pm 0.68$	$29.3 \pm 2.2$
1.28-1.40	60±8	1.03	$143.7 \pm 21.1$	$11.99 \pm 0.88$	$34.8 \pm 2.6$

TABLE I. Summary of results, listing the number of coherent  $K_{S}^{0} \rightarrow \pi^{+}\pi^{-}$  events observed in each momentum bin; the  $K_{L}$  total cross section in Cu at the mean momentum, taken from the optical model; the result for  $|f_{21}(0)|$ ; and the related quantity  $|f(0) - \overline{f}(0)| / k$ .

 $r_0 = 1.15 \ A^{1/3}$  F, as has been used to study  $\pi$ -nucleus scattering.<sup>5</sup>

The meson-nucleon amplitudes were obtained from total-cross-section data for charged kaons, using charge independence and the optical theorem, combined with kaon forward dispersion relations, which predict the ratio of real to imaginary parts of these amplitudes.<sup>6</sup> Figure 3(b) shows a comparison of the optical-model predictions and the data.

The general behavior of the model predictions agrees well with the data when nonzero real parts are included. If the real parts are set to zero,  $|f_{21}(0)|$  is underestimated, as seen in Fig. 3(b). If  $K^+N$  dispersive real parts are used but  $K^-N$  real parts are set to zero, the agreement is improved, but  $|f_{21}(0)|$  is still too small at  $p_K < 900 \text{ MeV}/c$ and in general agrees with the data more poorly than the full model. In this sense these data confirm the general correctness of K-nucleon forward dispersion relations in the Y\* resonance **region**.

There is an interesting discrepancy between the model and the data for  $1000 \le p_{\kappa} \le 1200 \text{ MeV}/c$ . The  $K^-N$  cross sections drop markedly after the  $\Sigma(1770)$  peak near 950 MeV/c and the  $\Lambda(1815)$  peak near 1050 MeV/c. Independent of real-part behavior, which must be small near resonance peaks in any case, the optical potential must change markedly in this vicinity, and the model predicts that  $|f_{21}(0)|$  will fall by 33% from 1000 to 1100 MeV/c. Experimentally, the measured  $|f_{21}(0)|$ does not drop until a substantially higher momentum, shifted by ~100 MeV/c from the model. Furthermore, the measured structure, even allowing for the smoothing effects of histogramming, is not as pronounced as that in the model. Because this shift of the falloff of  $|f_{21}(0)|$  is ~ 10% of the kaon momentum at which it occurs, there is no chance that it is instrumental. Moreover, we have a calibration of the spectrometer momentum determination from the value of the regenerated  $K_s^0$ 

mass. The other available data in Cu in this region from Vishnevskii *et al.* also show no marked fall in  $|f_{21}(0)|$  at 1 GeV/c.<sup>7</sup>

An attempt was made to understand the shift on the basis of the Fermi motion of the nucleons. Because for coherent regeneration the kaon interacts with the entire block of material, the role of the Fermi motion of the nucleons is to smear the optical potential seen by an incident kaon of momentum P. This smearing has been approximated by taking the convolution of the optical model  $|f_{21}(0)|$  with a Gaussian smearing function. The smeared  $|f_{21}(0)|$  has a slightly lower peak near 950 MeV/c, and the deep valley at 1150 MeV/c is filled in somewhat, but these effects are small and do not shift the abrupt decrease in  $|f_{21}(0)|$  away from 1100 MeV/c.

Corrections to the optical potential have been investigated. Nuclear-physics effects are rather highly constrained by the coherence of the process and by the smallness of the  $K_L$ - $K_S$  mass difference. The extreme sharpness of the coherent forward peak tends to suppress any interactions involving transverse momentum transfer, and the near identity of the  $K_L$  and  $K_S$  masses allows few intermediate-state processes. We have evaluated an intermediate-state term suggested by Kisslinger<sup>8</sup> of the form  $(\partial V/\partial E)\Delta E$  arising from phonon-type excitations of the nuclear states. Again the effects are quite small—of order 0.1 F—and in fact tend to sharpen the slope.

Another example of a sizable shift of a resonance peak in complex nuclei, although a shift to lower energy, is seen in  $\pi$ -C<sup>12</sup> total cross sections near the  $\Delta(1236)$  resonance.<sup>9</sup> This has been analyzed and the shift accommodated in terms of a nonlocal *P*-wave correction to the optical potential.<sup>10</sup> The effect in  $\pi$ -nucleus scattering is seemingly correlated with a dominant strong resonance, analogous to the  $\Lambda(1815)$ , that leads to nuclear mean free paths which are small compared with nuclear dimensions at the resonance.<sup>11</sup> These conditions certainly obtain here also, but the shift is to higher momentum, not lower, near an *F*-wave resonance, in a quantity not so simply related to the resonance amplitude as is  $\sigma_T$ . The nature of the effect in these data is still not clear.

In summary, these data provide the first precise  $K^{\circ}$  coherent regeneration amplitude measurements in the low-momentum region. They test the conventional optical model with rapidly varying input amplitudes. Under most conditions the general agreement of the model and the data is good, but near a dominant resonance peak in one of the four available strangeness and isotopic spin channels,

there is a discrepancy which thus far has not been explicable.

We would like to thank the ZGS staff, especially the operations shift supervisors, for their cheerful assistance, and Dr. R. Lamb, Dr. F. Peterson, and Dr. D. Chesire for their help in data taking. One of us (J. R.) thanks Professor L. Kisslinger for a useful discussion. We give special thanks to J. Smith and F. Johns for many hours of work on this experiment. We also acknowledge the fine work of the C.-M. U. machine shop and electronics shop in preparing the apparatus.

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- <sup>3</sup>The conversion from particles/(monitor count) to particles  $(\text{GeV}/c)^{-1}\text{sr}^{-1}$ (interacting proton)<sup>-1</sup> has an additional 20% systematic uncertainty from the determination of the incident proton flux and the fraction which interact. This additional uncertainty does not affect the  $K_L$  flux used in computing the regeneration intensity.
- ${}^{4}\!K_{L}$ -Cu total cross section data come from S. Bennett, D. Nygren, H. Saal, J. Sunderland, and J. Steinberger [Phys. Lett. <u>27B</u>, 239 (1968)] at 2.5 GeV/c, and R. Carnegie, R. Cester, V. Fitch, M. Strovink, and L. Sulak [Phys. Rev. D 6, 2335 (1972)].
- ${}^{5}r_{0}$  is enlarged over the value 1.08 ordinarily used in electron-scattering analyses of nuclear charge radii to account for the finite interaction radius of the incident particle. Modification of  $r_{0}$  of this type have been used in analysis of  $\pi$ -nucleus scattering; see J. W. Cronin, R. L. Cool, and A. Abashian, Phys. Rev. 107, 1121

(1957). Also, the electron-scattering result tends to underestimate  $\sigma_T$  slightly, as compared with the  $1.15A^{1/3}$  result. In any case, the shifts are small, of the order of 30 mb in  $\sigma_T$  and 0.2 F in  $|f_{21}(0)|$ , and these data are not sensitive to such effects.

- <sup>6</sup>Cross sections used have been taken from R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontić, K. K. Li, A. Lundby, J. Teiger, and C. Wilkin [Phys. Rev. D <u>1</u>, 1887 (1970)] and recent unpublished results below 1.1 GeV/c by the Kycia group. We thank Dr. D. Michael for giving us these data prior to publication. Real-part ratios used were taken from K-nucleon forward dispersion-relation calculations. Similar results were obtained from the calculations of M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Nuovo Cimento <u>45A</u>, 792 (1967); <u>49A</u>, 705 (1968); A. D. Martin and F. Poole, Nucl. Phys. <u>B4</u>, 467 (1968); A. D. Martin and F. Perrin, *ibid.* <u>B20</u>, 287 (1970); A. Carter, Phys. Rev. Lett. <u>18</u>, 801 (1967); Univ. of Cambridge Report No. HEP 68-10 (unpublished).
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