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Mass of the graviton*

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After emphasizing that it remains an open question whether one should try to quantize gravity theory (which would mean gravitational force is propagated by a graviton particle), we nevertheless ask whether a limit can be set on the rest mass (μ_g) of the "graviton." By recalling that gravitational force is clearly exerted over large distances in systems of galaxies and is not eliminated by a graviton-mass Yukawa cutoff, we find a limit. So, although it is not known if the graviton exists, one can still say that its rest mass is less than 2×10^{-62} g.

It is an intriguing observation that there is no experimental evidence linking the two great theories of modern physics (namely, quantum mechanics and general relativity). All the predictions of general relativity that have been subject to experiment are classical (nonquantum) predictions, be they the precession of the perihelion of Mercury or the gravitational radiation that Weber has reported.[1]

Put simply, one can validly ask the question "Should a theory of gravitation be quantized?" Note that Feynman and Schwinger, two of the men who successfully quantized classical Maxwell electromagnetic theory, have very different attitudes toward this question.

Feynman cautions, "The extreme weakness of quantum gravitational effects now poses some philosophical problems: maybe nature is trying to tell us something new here, maybe we should not try to quantize gravity.... We would like to keep an open mind. It is still possible that quantum theory does not absolutely guarantee that gravity *has* to be quantized."[2]

Schwinger, on the other hand, declares "The graviton is unknown, as yet, to experimental science. Nevertheless, we shall accept it and its conjectured properties as the proper starting point for the theory of gravitational phenomena, just as the photon with its attributes initiates the theory of (quantum) electromagnetic phenomena. The evidence for the existence of the graviton is indirect, but impressive."[3]

Schwinger's reference to the photon leads us to point out that the quantum particle of the electromagnetic field (the photon) is usually assumed to have zero rest mass (so that then indeed the limiting velocity of relativity is identical with the "velocity of light"). Actually, tests can be made of this assumption,[4] and one can say that the rest mass of the photon (μ_γ) is

$$\mu_\gamma \leq 4\times 10^{-48}\ \text{g} \equiv 10^{-10}\ \text{cm}^{-1} \equiv 3\times 10^{-15}\ \text{eV}\ . \quad (1)$$

This number is essentially a verification that the electrostatic potential between two point charges in proportional to 1/distance,

$$V = e_1 e_2 / r\ , \quad (2)$$

and is not given by the Yukawa potential for exchange of a massive quantum,

$$V = e_1 e_2 \exp(-\mu_\gamma r)/r\ . \quad (3)$$

That is, the best experimental limits on μ_γ are deduced from a failure to observe deviations from Coulomb's law, Eq. (2), and its analog in magnetostatics.

The same question can be asked about the "rest mass" of the graviton in a "quantum gravity theory." That is, even though we cannot experimentally detect a graviton (by any known technique), we can still say something about the graviton's rest mass (μ_g)—if the graviton exists. That is the main point of this note.

First it is necessary to sidetrack for a moment. It has been argued[5] that, in the *linearized* version of general relativity, a nonzero graviton rest mass, no matter how small, would change the deflection of light by the sun to $\frac{3}{4}$ its Einstein value. If this argument held also in the complete nonlinear theory, as has been suggested,[6] then the physics of massive gravitons would be discontinuous for $\mu_g \to 0$ and experiment would absolutely rule out any graviton mass. However, in Schwinger's[7] formalism one might make the linear theory continuous as $\mu_g \to 0$. Further, Vainshtein[8] has demonstrated the continuous nature of the classical nonlinear theory. (Note that quantum electrodynamics is well known[4] to be continuous as the photon mass $\mu_\gamma \to 0$.) Thus, it is indeed a valid question to ask what limit can be set on the graviton's mass. This question has meaning even if gravity should not be quantized, since it describes a modification of general relativity which would imply, for example, a dispersion in the velocity of classical gravitational waves.

To continue, from our discussion of the Coulomb electric potential it is clear that a test for a graviton mass is to ask if the Newtonian $1/r$ gravitational potential ($1/r^2$ force) shows any evidence of dying at large distances because of a Yukawa exponential cutoff, $\exp(-\mu_g r)$. One is asking, "Over what distance can we see that the force of gravitation still acts?" (The larger the distance, the better the limit on the graviton rest mass.)

One limit on the graviton rest mass can be obtained by referring to the odd tails and bridges connecting pairs of galaxies in Arp's *Atlas of Peculiar Galaxies*.[9] Toomre and Toomre[10] have demonstrated that these bridges and tails are due to the gravitational tidal forces exerted during the near encounter of these galaxies. Taking a galactic diameter to be about 30 kpc, this would give from these encounters a distance on the order of 100–300 kpc over which one can be sure gravity works.

The best graviton mass limit comes from recalling that there are clusters of galaxies that are bound.[11] From data on groups of Shapley-Ames galaxies Holmberg concluded, "With a Hubble parameter $H = 80$ km/(sec Mpc), the... mean separation corresponds to 118 kpc and the maximum separation to 400 kpc."[12] Using Sandage's new value for the Hubble constant,[13] $H = 55 \pm 7$ km/(sec Mpc), one can conservatively take 580 ± 70 kpc for the distance over which gravity is not drastically weakened. (As there are many bound clusters of galaxies with scale sizes of order 1–10 Mpc,[14] one could even argue for a longer distance.) In any event, using the distance $r = 580$ kpc to bound $e^{-1} \leq \exp(-\mu_g r)$ yields a limit for μ_g of

$$\mu_g \leq 2\times 10^{-62}\ \mathrm{g} \equiv 5.6\times 10^{-25}\ \mathrm{cm}^{-1} \equiv 1.1\times 10^{-29}\ \mathrm{eV}. \qquad (4)$$

Note that a graviton rest mass given exactly by Eq. (4) would correspond to a graviton Compton wavelength of

$$\lambda_g = 2\pi/\mu_g = 3.7\ \mathrm{Mpc} = 6.7\times 10^{-4} R\,, \qquad (5)$$

where $R = c/H$ is the "Hubble radius of the universe."

To conclude, whether or not gravity should be quantized, one can say that the graviton's rest mass is less than 2×10^{-62} g. Therefore, somewhat paradoxically, one has experimental evidence on the rest mass of a particle which may not exist.[15–17]

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[1]J. L. Logan [Phys. Today 26, No. 3, 44 (1973)] gives a review of gravitational wave experiments. Weber's original experiments are described in J. Weber, M. Lee, D. J. Gretz, G. Rydbeck, V. L. Trimble, and S. Steppel, Phys. Rev. Lett. 31, 779 (1973) and references therein.

[2]R. P. Feynman, "Lectures on Gravitation, 1962-1963," Caltech report (unpublished), lecture 1, pp. 12–13.

[3]J. Schwinger, *Particles, Sources, and Fields* (Addison-Wesley, Reading, Mass., 1970), p. 81.

[4]A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. 43, 277 (1971).

[5]H. van Dam and M. Veltman, Nucl. Phys. B22, 397 (1970); V. I. Zakharov, Zh. Eksp. Theor. Fiz. Pis'ma Red. 12, 447 (1970) [JETP Lett. 12, 312 (1970)].

[6]D. G. Boulware and S. Deser, Phys. Lett. 40B, 227 (1972). This paper contains a number of assertions which are either not relevant to the question of continuity of the metric tensor as the mass of the graviton goes to zero, or are contradicted by the paper of Vainshtein, Ref. 8. However, there is one assertion which, as far as we know, has not been contradicted in the literature, and that is the statement that the Fierz-Pauli form of massive gravitation is not consistent in the sense that there exist solutions which have arbitrarily large negative energy. This objection might be fatal to the entire idea of massive gravitation theory, and since it is so critical, we believe that it deserves further theoretical attention. Nevertheless, it clearly is possible that, perhaps by a change in the perspective from which this statement is viewed, one will be able to show that this objection does not invalidate finite-mass gravitation. Therefore, it remains a worthy goal to consider, as in the present paper, the experimental status of limits on the mass of the graviton.

[7]J. Schwinger, Ref. 3, pp. 78–85. A μ_g-dependent term, added to the stress-energy tensor, would be required.

[8]A. I. Vainshtein, Phys. Lett. 39B, 393 (1972). There remains an open question in Vainshtein's work. He solves differential equations in a limited range of radii about the mass center. It is conceivable that if his solution for the metric is continued to a larger radius, always imposing the condition that the differential equations are obeyed locally, then the metric will not obey reasonable boundary conditions at infinity. This matter should be investigated, but we believe no difficulty will be found.

[9]H. Arp, Astrophys. J. Suppl. 14, 1 (1966).

[10]A. Toomre and J. Toomre, Astrophys. J. 178, 623 (1972).

[11]The conclusion that clusters of galaxies are bound is reached by observing that clusters are found much more commonly than could come about by statistical fluctuations. Further, the observed velocities of the component galaxies would have greatly dispersed them over the age of the universe, if there were no binding. However, a long-standing problem has been that the virial masses are greater than the observed masses for many clusters. [One does a virial theorem on the observed parameters of a particular system and hopes to find $V_{tot} = -2$(total kinetic energy).] It is generally agreed that there are two possible solutions to this problem: (1) Either at various times in the past, up to quite recently, the clusters lost some of their existing masses, for instance, by quasar explosions, or (2) there are intergalactic "missing masses," probably in the form of ionized hydrogen, which account for the discrepancies. (There is preliminary evidence for this solution.) But the important point to observe is that even under solution (1) we are not prevented from establishing our conservative mass limit because the dispersion that may have occurred in the possibly recently disrupted clusters would have to be relatively small. Finally, we note that another solution would be to postulate some unknown long-distance force other than gravity. If one assumes this undefined *ad hoc* hypothesis then, of course, nothing at all can be said about large-scale dynamics. We should mention that Y. Yamaguchi (private communication) has suggested that a magnetically contained plasma between galaxies might conceivably yield such a force. Although this in an interesting speculation, it is doubtful that such a mechanism could mimic accurately the effects of gravitation. See H. J. Rood, V. C. A. Rothman, and B. E. Turnrose, Astrophys. J. 162, 411 (1970); G. B. Field and W. C. Saslaw, *ibid.* 170, 199 (1971); D. S. De Young, *ibid.* 173, L7 (1972), and references therein.

[12]E. Holmberg, Ark. Astron. 5, 305 (1969). See p. 309.

[13]A. Sandage, Astrophys. J. 178, 1 (1972). See p. 22.

[14]G. de Vaucouleurs, Publ. Astron. Soc. Pac. 83, 113 (1971).

[15]Somewhat weaker limits on the graviton mass, using a variety of less sensitive methods, have been obtained by M. G. Hare, Can. J. Phys. 51, 431 (1973).

[16]It is interesting to note that as long ago as 1957 Zwicky suggested that clustering up to certain sizes might imply a gravitational cutoff at distances of around 3×10^6 pc. Since the Hubble scale has changed by roughly a factor of 10 in the meantime, this implies that Zwicky was speculating on a cutoff limit roughly 50 times our own more conservative estimate. Among the possibilities suggested for the origin of this speculated cutoff was the existence of a cosmological term in Einstein's equations. See F. Zwicky, Publ. Astron. Soc. Pac. 69, 518 (1957).

[17]Earlier K. Hiida and Y. Yamaguchi [Prog. Theor. Phys. Suppl., extra number, 262 (1965)] suggested that the analysis of the dynamics of clusters of galaxies could yield a limit on the graviton mass as low as 5×10^{-62} g. (See p. 264 of the above reference.)

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Addendum to Wilson's theory of critical phenomena and Callan-Symanzik equations in $4-\epsilon$ dimensions

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In a previous work, it was shown how to derive the scaling laws near a critical point using renormalized perturbation theory. The calculations of the Callan-Symanzik functions β and γ which lead to the critical exponents are extended to next order in ϵ. The existence of a solution to the eigenvalue conditions $\beta(g)=0$ in four dimensions, at fourth order in the coupling constant, is shown to be renormalization-dependent.

In the framework of Wilson's theory of critical phenomena,[1] we have discussed in a recent article[2] how scaling laws for the correlation functions near the critical point may be derived from the Callan-Symanzik equations in $4-\epsilon$ dimensions applied to a $g(\vec{\varphi}^2)^2$ interaction, where $\vec{\varphi}(x)$ is an n-component order parameter. Higher-order corrections in ϵ have now been computed.[3] The purpose of this addendum is to give various quantities which are useful in these calculations,[2,4] like the expansions of the renormalization constants, of the Callan-Symanzik β and γ functions, and of the solution of the eigenvalue condition. The notations are identical to those of Ref. 2.

For simplicity we have done the calculations in the massless theory[4] with the following convenient renormalization conditions for the vertex functions:

$$\Gamma^{(2)}(p,-p;u)|_{p^2=0}=0\,, \tag{1a}$$

$$\frac{\partial}{\partial p^2}\Gamma^{(2)}(p,-p;u)\Big|_{p^2=e^l}=1\,, \tag{1b}$$