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Charged vector mesons and ξ formalism*

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We discuss a finite-ξ formalism for the electromagnetic interaction of a massive charged vector meson with an arbitrary magnetic moment κ. The theory is Lorentz-covariant, renormalizable, and gauge-invariant. We consider the case $\xi=\kappa=1$ to show that the theory is unitary after introducing a fictitious particle and two new vertices in the theory.

Lee and Yang have discussed a covariant and gauge-invariant ξ-limiting formalism for the charged vector meson with a mass m and an arbitrary magnetic moment κ.[1] For finite $\xi>0$, the theory is renormalizable but the physical S matrix is not unitary. If the limit $\xi\to 0$ exists, then the limiting S matrix is unitary. Yet it is not clear whether the limit exists, because the Green's function is not renormalizable in the limit $\xi\to 0$.

We shall discuss a finite-ξ formalism for the same type of vector meson within the framework of the indefinite-metric quantum field theory. Namely, instead of adding a gauge-invariant term $-\xi[(\partial_\mu+ieA_\mu)\phi_\mu^\star][(\partial_\nu-ieA_\nu)\phi_\nu]$ to the Lagrangian and considering the limit $\xi\to 0$, we add a gauge-violating term $-\xi(\partial_\mu\phi_\mu^\star)(\partial_\nu\phi_\nu)$ and consider finite nonzero ξ. [The symbol $\star$ denotes the Hermitian conjugate in the indefinite-metric space times $(-1)^n$, n = number of "4" subscripts.] In so doing, we make the interaction vertices simpler than those in the ξ-limiting formalism.[1] The theory appears simple and natural by choosing $\xi=1$. In this case, the field $\chi\equiv\partial_\mu\phi_\mu/m$ describes a "ghost scalar boson" (i.e., the negative-metric spin-zero part of ϕ_μ) with the same mass m as the positive-metric spin-one part of $\phi_\mu(x)$. Furthermore, the propagator for ϕ_μ is greatly simplified.

The present finite-ξ formalism for ϕ_μ is equivalent to introducing a Lagrange multiplier into the usual positive-metric massive-vector-meson theory.[2,3] The Lagrange multiplier (i.e., the ghost field) removes the bad divergences in the usual theory and makes the theory renormalizable and well defined. According to our previous study of the massless Yang-Mills field and quantum electrodynamics with nonlinear gauge conditions (e.g., $\partial_\mu A_\mu+\beta A_\mu A_\mu=0$, $\beta\neq 0$), the Lagrange multiplier will upset unitarity if it does not obey the free-field equation.[3] Moreover, using the equation for the Lagrange multiplier, we can isolate the unwanted extra absorptive part in the amplitudes of all orders due to the source term. The theories are unitary after the fictitious particle is introduced to remove the extra absorptive part in the amplitudes.

Similarly, in the finite-ξ formalism the ghost field obeys a field equation with some source terms which give an extra absorptive part to the amplitudes and upset unitarity. (We note that the ghost field χ does not have a free Lagrangian and its detailed interactions cannot be seen directly from the Lagrangian.) Using the field equation for χ we obtain the effective interaction Lagrangian $\mathcal{L}_{\text{eff}}(\chi)$ of the ghost field χ. The present theory is unitary after introducing a fictitious particle which removes the extra absorptive amplitudes. The advantage of the present formalism is that the theory is both unitary and renormalizable for the same value of ξ. We may remark that the theory is really gauge-invariant because the expectation values of the gauge-violating quantities in the physical states vanish.

The Lagrangian for the vector field ϕ_μ interacting with the photon field $A_\lambda(x)$ is

$$\mathcal{L}=\mathcal{L}_\xi+\mathcal{L}_\gamma,$$
$$\mathcal{L}_\xi=-\xi(\partial_\mu\phi_\mu^\star)(\partial_\nu\phi_\nu)-\tfrac{1}{2}G_{\mu\nu}^\star G_{\mu\nu}-m^2\phi_\mu^\star\phi_\mu-ie\kappa F_{\mu\nu}\phi_\mu^\star\phi_\nu, \quad (1)$$
$$\mathcal{L}_\gamma=-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-\frac{(\partial_\mu A_\mu)^2}{2\beta},\quad \partial_\mu\equiv\frac{\partial}{\partial x^\mu}$$

where we use the notation in Ref. 1 (except the definitions of ∂_μ and of Hermitian conjugate),

$$x_\mu=x^\mu=(x_1,x_2,x_3,x_4=ix_0)\quad(x_0=t),$$
$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,$$
$$G_{\mu\nu}=(\partial_\mu-ieA_\mu)\phi_\nu-(\partial_\nu-ieA_\nu)\phi_\mu, \quad (2)$$
$$G_{\mu\nu}^\star=(\partial_\mu+ieA_\mu)\phi_\nu^\star-(\partial_\nu+ieA_\nu)\phi_\mu^\star.$$

We do not introduce the metric operator η, which is used in the old literature, because it violates the elegant and manifestly covariant property of the indefinite-metric quantum field theory.[2] The field equation for ϕ_μ is

$$(\partial_\mu-ieA_\mu)G_{\mu\nu}-m^2\phi_\nu+\xi\partial_\nu(\partial_\mu\phi_\mu)+ie\kappa\phi_\mu F_{\mu\nu}=0. \quad (3)$$

For the free field, the canonical conjugates of ϕ_μ and $\phi_\mu^\star$ are

$$\pi_k=-i(\partial_4\phi_k^\star-\partial_k\phi_4^\star),\quad \pi_4=+i\xi\partial_\mu\phi_\mu^\star\quad(k=1,2,3), \quad (4)$$
$$\pi_k^\star=-i(\partial_4\phi_k-\partial_k\phi_4),\quad \pi_4^\star=+i\xi\partial_\mu\phi_\mu,\quad \pi_4^\star\equiv\frac{\delta\mathcal{L}_\xi}{\delta(\partial_0\phi_4^\star)}.$$

The free Hamiltonian H_0 for ϕ_μ is

$$H_0=\pi_k\pi_k^\star+\xi^{-1}\pi_4\pi_4^\star+m^2\phi_\mu\phi_\mu^\star+(\vec\nabla\times\vec\phi)\cdot(\vec\nabla\times\vec\phi^\star)+i(\pi_k\nabla_k\phi_4+\pi_k^\star\nabla_k\phi_4^\star-\pi_4\nabla_k\phi_k-\pi_4^\star\nabla_k\phi_k^\star) \quad (5)$$

and the equal-time commutators are

$$[\pi_\mu(x),\phi_\nu(y)]=-i\delta_{\mu\nu}\delta^3(\vec x-\vec y)\quad(x_0=y_0),$$
$$[\pi_\mu^\star(x),\phi_\nu^\star(y)]=-i\delta_{\mu\nu}\delta^3(\vec x-\vec y)\quad(x_0=y_0), \quad (6)$$

and all others vanish. The field equation (3) with $e=0$ and its divergence lead to

$$(\Box+m^2)\phi_\mu+m(1-\xi)\partial_\mu\chi=0\qquad(-\Box\equiv\partial_\mu{}^2,\ \chi\equiv\partial_\nu\phi_\nu/m), \quad (7)$$
$$(\Box+\xi^{-1}m^2)(\Box+m^2)\phi_\mu=0\quad(\xi\neq1). \quad (8)$$

Because of Lorentz covariance, local commutativity, and (8), the commutator $[\phi_\mu(x),\phi_\nu^\star(y)]$ can only contain the terms $Q_{\mu\nu}\Delta(x-y,\rho^2)$, where $Q_{\mu\nu}=g_{\mu\nu}$ or $\partial_\mu\partial_\nu$ and $\rho^2=m^2$ or $\xi^{-1}m^2$. The coefficients of these terms are completely fixed by the equal-time commutators (6) and we have[2]

$$[\phi_\mu(x),\phi_\nu^\star(y)]=i(\delta_{\mu\nu}-m^{-2}\partial_\mu\partial_\nu)\Delta(x-y,m^2)+im^{-2}\partial_\mu\partial_\nu\Delta(x-y,\xi^{-1}m^2), \quad (9)$$
$$\Delta(x,\rho^2)\equiv-i(2\pi)^{-3}\int d^4p\,\epsilon(p_0)\delta(p^2+\rho^2)e^{ip\cdot x}.$$

One can calculate the ϕ_μ propagator from (9):

$$\Delta_{\mu\nu}^F(k)=\int d^4x\,e^{-ik\cdot x}\langle 0|T(\phi_\mu(x)\phi_\nu^\star(0))|0\rangle \quad (10)$$
$$=-i(\delta_{\mu\nu}+m^{-2}k_\mu k_\nu)(k^2+m^2)^{-1}+im^{-2}k_\mu k_\nu(k^2+\xi^{-1}m^2)^{-1}, \quad (11)$$
$$\Delta_{\mu\nu}^F(k)=-i\delta_{\mu\nu}(k^2+m^2)^{-1}\quad(\xi=1). \quad (12)$$

The Feynman rules from the Lagrangian (1) in momentum space for the 3-vertex V [i.e., $\phi_\alpha(p)\phi_\beta(p')A_\mu$] and the 4-vertex U (i.e., $\phi_\alpha\phi_\beta A_\mu A_\nu$) are respectively

$$V=ie[\delta_{\alpha\beta}(p+p')_\mu-\delta_{\alpha\mu}(-\kappa p'+p+\kappa p)_\beta-\delta_{\beta\mu}(-\kappa p+p'+\kappa p')_\alpha], \quad (13)$$

and

$$U=-ie^2(2\delta_{\mu\nu}\delta_{\alpha\beta}-\delta_{\alpha\mu}\delta_{\beta\nu}-\delta_{\alpha\nu}\delta_{\beta\mu}), \quad (14)$$

which are the same as those in Ref. 1 with $\xi=0$. The ϕ_μ propagator is given by (12) and the photon propagator is well known. So the Feynman rules are covariant and the theory is manifestly renormalizable for $\xi>0$.

From (7) and (9), we have the following commutation relation for the field:

$$[\chi(x),\chi^\star(y)]=-i\xi^{-2}\Delta(x-y,\xi^{-1}m^2), \quad (15)$$

which implies that $\chi(x)$ is indeed a negative-metric field for $\infty>\xi\geq0$. The divergence of (3) gives the equation of motion for the interacting χ field:

$$(\Box+\xi^{-1}m^2)\chi=S(\chi), \quad (16)$$
$$S(\chi)\equiv-\frac{ie}{m\xi}\partial_\nu(A_\mu G_{\mu\nu})+\frac{ie\kappa}{m\xi}\partial_\nu(\phi_\mu F_{\mu\nu}) \quad (17)$$
$$=-ieA_\mu\partial_\mu\chi+ie\xi^{-1}mA_\mu\phi_\mu+\frac{ie\kappa}{m\xi}\phi_\mu\partial_\nu F_{\mu\nu}+(\kappa-1)\frac{(ie)^2}{m\xi}A_\mu\phi_\nu F_{\mu\nu}-(\kappa-1)\frac{ie}{m\xi}(\partial_\mu\phi_\nu)F_{\mu\nu}, \quad (18)$$

where we have used Eq. (3) and the definition of $F_{\lambda\gamma}$ in (2). The "effective" interaction Lagrangian $\mathcal{L}_{\text{eff}}(\chi)$ is

$$\mathcal{L}_{\text{eff}}(\chi)=\chi^{\star}S(\chi). \tag{19}$$

We note that the source terms in (16) contain a term with χ and some other terms without χ. This differs from that of the massless Yang-Mills field $\vec{f}_\lambda$, where the source terms in the equation for the ghost field $\vec{\chi}_y=\partial_\mu\vec{f}_\mu$ contain only the term with $\vec{\chi}_y$:

$$\Box\chi_y^a - g\epsilon^{abc}f^b_\mu\partial_\mu\chi_y^c=\Box[(\delta^{ac}-g\Box^{-1}\epsilon^{abc}f^b_\mu\partial_\mu)\chi_y^c]=0. \tag{20}$$

In this case, the source term $g\vec{f}_\mu\times\partial_\mu\vec{\chi}_y$ will contribute an extra absorptive part[3]

$$[\det(\delta^{ac}-g\Box^{-1}\epsilon^{abc}f^b_\mu\partial_\mu)]^{-1} \tag{21}$$

to the amplitude (e.g., the vacuum-to-vacuum amplitude expressed in terms of the path integral). This unwanted extra absorptive part is exactly canceled to all orders by the unitarization factor of Faddeev and Popov[4]

$$\det(\delta^{ac}-g\Box^{-1}\epsilon^{abc}f^b_\mu\partial_\mu), \tag{22}$$

which may be regarded as coming from the Feynman fictitious particle c with an effective interaction $g\vec{c}^{\star}\cdot(\vec{f}_\mu\times\partial_\mu\vec{c})$. Similarly, the source term $-ieA_\mu\partial_\mu\chi$ in (16) will contribute (see Appendix)

$$[\det(1+ie(\Box+m^2)^{-1}A_\mu\partial_\mu)]^{-1} \tag{23}$$

to the amplitude. Unfortunately, because of the presence of the source terms without χ in (16), the total extra absorptive amplitude due to χ cannot be expressed in a simple closed form as (23).

From the expressions (16) and (17), one might think that the theory will be greatly simplified by taking the limit $\xi\to\infty$. But this is not so because χ is not finite and well defined in the limit. Also, the theory is not independent of ξ, as one can see from (16) and (18). Different choices of ξ would, in general, correspond to different theories. If $\kappa=1$ the source terms $S(\chi)$ in (18) become simple because of cancellation. We may remark that even if $\kappa=0$ there are still some source terms in (18) and the theory depends on the parameter ξ.

As usual, we must define the physical state $|\text{phys}\rangle$ by

$$[\partial_\mu A_\mu(x)]^{(+)}|\text{phys}\rangle=0, \tag{24}$$

where (+) denotes the positive-frequency part of the Heisenberg operator $\partial_\mu A_\mu(x)$. Because of the equation of motion[5]

$$\Box\partial_\lambda A_\lambda(x)=\beta m\xi(\phi_\nu^{\star}\partial_\nu\chi-\phi_\nu\partial_\nu\chi^{\star}) \tag{25}$$

the physical state $|\text{phys}\rangle$ must also satisfy

$$\chi(x)^{(+)}|\text{phys}\rangle=\chi^{\star}(x)^{(+)}|\text{phys}\rangle=0 \tag{26}$$

for consistency. That is, the ghost particle χ cannot be present in the physical states defined by (24).

Because of the interaction of χ, the physical S matrix defined in the subspace (spanned by the physical states) of the indefinite-metric space, is *not automatically* unitary. This is quite different from that of quantum electrodynamics with *linear* gauge condition. However, in the present formalism the problem of unitarity can be "solved" in a way similar to that in the massless Yang-Mills field theory and in quantum electrodynamics with *nonlinear* gauge conditions. Namely, we can introduce a fictitious particle F with mass m and a finite number of new vertices[6] to remove the extra absorptive amplitude due to the interaction of χ. In this way, the resultant physical S matrix is unitary.

The ghost-eliminating interactions of the fictitious particle F should be

$$F^{\star}(x)S(F(x)), \tag{27}$$

where $S(F)$ is given by (18) with χ replaced by F. When $\xi=\kappa=1$ we have

$$-ieF^{\star}A_\mu\partial_\mu F+\frac{ie}{m}F^{\star}\phi_\mu(m^2\delta_{\mu\nu}+\partial_\mu\partial_\nu-\delta_{\mu\nu}\partial_\lambda{}^2)A_\nu. \tag{28}$$

The Feynman rules which correspond to these two new vertices are respectively

$$iep_\mu \quad [\text{for } F^{\star}A_\mu(q)ip_\mu F(p)] \tag{29}$$

and

$$-\frac{e}{m}(m^2\delta_{\mu\nu}-q_\mu q_\nu+q^2\delta_{\mu\nu}) \quad [\text{for } F^{\star}\phi_\mu(k)(m^2\delta_{\mu\nu}-q_\mu q_\nu+q^2\delta_{\mu\nu})A_\nu(q)] \tag{30}$$

and a factor -1 for every closed loop.[7] If $\kappa\neq1$ we shall have two more new vertices. The fictitious particle F cannot, by definition, be present in the physical states.[8] It should be emphasized that these new vertices are introduced only for adjusting the absorptive part of the amplitude (i.e., for unitarizing the S matrix).

To demonstrate unitarity of the theory after the introduction of the fictitious particle, let us consider the absorptive part of the self-energy process of the physical vector meson ϕ [with momentum p_λ and polarization $\epsilon_\mu(p)$]

$$\phi(p)\to\phi(p-k)\gamma(k)\to\phi(p), \tag{31}$$

where γ is the photon and the external particles ϕ are physical, i.e.,

$$p_\mu \epsilon_\mu(p) = 0 . \qquad (32)$$

The absorptive part is obtained from

$$\int \frac{d^4k}{(2\pi)^4}(ie)^2\epsilon_\alpha(p)\epsilon_\beta(p)\,\frac{1}{ik^2}\,\frac{\delta_{\alpha'\beta'}}{i[(p-k)^2+m^2]}\,T_{\alpha\alpha'\beta'\beta}, \qquad (33)$$

where

$$T_{\alpha\alpha'\beta'\beta} = [\delta_{\alpha\alpha'}(2p-k)_\mu - \delta_{\alpha\mu}(p+k)_{\alpha'} - \delta_{\alpha'\mu}(p-2k)_\alpha][\delta_{\beta'\beta}(2p-k)_\mu - \delta_{\beta'\mu}(p-2k)_\beta - \delta_{\beta\mu}(p+k)_{\beta'}], \qquad (34)$$

by putting k^2 of γ and $(p-k)^2$ of ϕ on their respective mass shells. Since

$$\delta_{\alpha'\beta'}[(p-k)^2+m^2]^{-1} = \left[\left(\delta_{\alpha'\beta'} + \frac{(p-k)_{\alpha'}(p-k)_{\beta'}}{m^2}\right) - \frac{(p-k)_{\alpha'}(p-k)_{\beta'}}{m^2}\right][(p-k)^2+m^2]^{-1}, \qquad (35)$$

we see that (33) contains an extra absorptive part

$$\int \frac{d^4k}{(2\pi)^4}(ie)^2\epsilon_\alpha(p)\epsilon_\beta(p)\,\frac{1}{ik^2}\left(-\frac{(p-k)_{\alpha'}(p-k)_{\beta'}}{im^2[(p-k)^2+m^2]}\right)T_{\alpha\alpha'\beta'\beta} = \int \frac{d^4k}{(2\pi)^4}\,\frac{(ie)^2\epsilon_\alpha(p)\epsilon_\beta(p)}{m^2k^2[(p-k)^2+m^2]} \times[\delta_{\alpha\beta}(k^2+m^2)^2 - k_\alpha k_\beta(k^2+2m^2)] \qquad (36)$$

due to the interaction of the ghost χ. This will be removed from the theory by the absorptive part due to the process

$$\phi(p) \to F(p-k)\gamma(k) \to \phi(p). \qquad (37)$$

According to the Feynman rule (30), the process (37) contributes the following absorptive part[8]:

$$(-1)\int \frac{d^4k}{(2\pi)^4}\,\frac{(-e)^2\epsilon_\alpha(p)\epsilon_\beta(p)\delta_{\mu\nu}}{m^2i[(p-k)^2+m^2]ik^2}(m^2\delta_{\mu\alpha} - k_\mu k_\alpha + \delta_{\mu\alpha}k^2)(m^2\delta_{\nu\beta} - k_\nu k_\beta + \delta_{\nu\beta}k^2)$$

$$= -\int \frac{d^4k}{(2\pi)^4}\,\frac{(ie)^2\epsilon_\alpha(p)\epsilon_\beta(p)}{m^2k^2[(p-k)^2+m^2]}[(k^2+m)^2\delta_{\alpha\beta} - (k^2+2m^2)k_\alpha k_\beta], \qquad (38)$$

with k^2 of γ and $(p-k)^2$ of F on their respective mass shells. The expression (38) exactly cancels (36), and therefore the result is unitary.[9]

The expectation value of the gauge-violating ghost field χ in the physical states vanishes, i.e., $\langle\chi\rangle = 0$. Thus, the field equation (3) reduces to

$$\langle[(\partial_\mu - ieA_\mu)G_{\mu\nu} - m^2\phi_\nu + ie\kappa\phi_\mu F_{\mu\nu}]\rangle = 0, \qquad (39)$$

which is gauge-invariant. We can also verify that the gauge-noninvariant parts of the dynamical characteristics of the physical system [e.g., the energy-momentum tensor $T_{\rho\lambda}$ and the angular momentum tensor $M_{\lambda\gamma}$ derived from the Lagrangian (1)] are *not* observable because their expectation values in the physical states vanish. Thus, the observable quantities are gauge-invariant.

To conclude, the finite-ξ formalism for the electromagnetic interaction of the massive charged vector meson could be renormalizable and unitary for the same value of ξ. The equation of motion for the Lagrange multiplier (i.e., the ghost field) and the fictitious particle play an important role in the formalism. The Lagrange multiplier and the fictitious particle have been successfully applied to the Abelian gauge theory with nonlinear gauge conditions and the non-Abelian gauge theory.[3] We believe the new vertices (29) and (30) will lead to unitarity to all orders, although we have explicitly verified this only in the lowest nontrivial order. One might think that such a formalism is not so natural as that of the usual quantum electrodynamics because the field-theoretical definition of the physical S matrix is not automatically unitary. Also, although the resultant S matrix is unitary, it may not be, in general, analytic. However, the finite-ξ formalism is no worse than the quantum electrodynamics with *nonlinear* gauge conditions in the sense that in both cases the fictitious particle is necessary for unitarity. We note that the physical S-matrix elements in the quantum electrodynamics with nonlinear gauge conditions are analytic and unitary in all orders only after the fictitious particle is introduced.[3]

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APPENDIX

We shall use the equation of motion for the Lagrange multiplier χ and the Feynman path integral to express the "extra" amplitude (due to the source terms in the equation for χ) to all orders in a simple closed form. The method is independent of gauge invariance. We have also applied it to the Georgi-Glashow model, the Yang-Mills field, etc. and obtained the correct results.

In general, one may introduce the Lagrange multiplier field χ in the Lagrangian $\mathfrak{L}$ for the vector-meson field ϕ_μ and derive the field equation for χ. Suppose one has an effective Lagrangian for χ (which leads to the equation for the negative-metric field χ):

$$\mathfrak{L}(\chi, \chi^\star) = \partial_\mu \chi^\star \partial_\mu \chi + \xi^{-1} m^2 \chi^\star \chi - \chi^\star S - \chi S^\star . \quad \text{(A1)}$$

We may remark that the corresponding effective Lagrangian for (1) is not renormalizable by standard power counting. Yet, the nonrenormalizable amplitudes due to χ and $\chi^\star$ cancel the nonrenormalizable amplitude due to the charged vector mesons (i.e., the physical components of ϕ_μ and $\phi^\star_\mu$), so that the theory based on the Lagrangian (1) is renormalizable.[1] However, the interaction of χ and $\chi^\star$ in the intermediate states will contribute extra *absorptive* amplitudes and upsets unitarity. Using the Feynman path integral, the amplitude due to the interaction of χ and $\chi^\star$ in the intermediate states is

$$X = \int \exp\left[i \int d^4x \, \mathfrak{L}(\chi, \chi^\star)\right] d[\chi, \chi^\star]$$
$$= \exp\left\{ -i \int d^4x \, S(x)[(\Box + \xi^{-1}m^2)^{-1} S^\star]\right\},$$
$$d[\chi, \chi^\star] = d[\chi] d[\chi^\star] \quad \text{(A2)}$$

if S does not contain χ explicitly. Thus, the unitary and renormalizable amplitude A_{ur} is

$$A_{\text{ur}} = \int X_{\text{abs}}{}^{-1} \exp\left[i \int d^4x (\mathfrak{L} + \mathfrak{L}_S)\right] d[\phi_\mu, \phi^\star_\nu, A_\rho], \quad \text{(A3)}$$

where $\mathfrak{L}_S$ = external source terms. The external particles are, by definition, physical, and the subscription abs denotes the absorptive part (which is finite after renormalization). In (A3), the extra absorptive amplitude due to χ in the Lagrangian $\mathfrak{L}$ is canceled by the factor $X_{\text{abs}}{}^{-1}$. The fictitious particle F introduced in $X_{\text{abs}}{}^{-1}$ is a scalar particle with mass $m\xi^{-1/2}$. In the momentum space, X^{-1} gives

$$(-1) S(k)[\, i(k^2 + \xi^{-1} m^2)]^{-1} S^\star(k)$$

in the lowest order. This shows a minus sign associated with the F propagator and agrees with the expression (38).

We may remove the extra absorptive amplitude in the Lagrangian (1) by considering (16) with S given by (18). In this case, the effective Lagrangian for χ and $\chi^\star$ is ($\bar{S} \equiv S + ieA_\mu \partial_\mu \chi$)

$$\bar{\mathfrak{L}}(\chi, \chi^\star) = \chi^\star(\Box + \xi^{-1} m^2 + ieA_\mu \partial_\mu)\chi - \chi^\star \bar{S} - \chi \bar{S}^\star . \quad \text{(A4)}$$

It follows that

$$\bar{X} = \int \exp\left[i \int d^4x \, \bar{\mathfrak{L}}(\chi, \chi^\star)\right] d[\chi, \chi^\star]$$
$$= [\det(1 + (\Box + \xi^{-1}m^2)^{-1} ieA_\mu \partial_\mu)]^{-1}$$
$$\times \exp\left\{ -i \int d^4x \, \bar{S}(x)[(\Box + \xi^{-1} m^2 + ieA_\mu \partial_\mu)^{-1} \bar{S}^\star]\right\} . \quad \text{(A5)}$$

Therefore, the unitary and renormalizable amplitude is

$$A_{\text{ur}} = \int \bar{X}_{\text{abs}}{}^{-1} \exp\left[i \int d^4x (\mathfrak{L} + \mathfrak{L}_S)\right] d[\phi_\mu, \phi^\star_\nu, A_\rho]. \quad \text{(A6)}$$

We note that the vertices in $\bar{X}^{-1}$ are different from those in X^{-1}. Moreover, there are two different types of the fictitious particle introduced in $\bar{X}^{-1}$. The first type, coming from the exponent part of $\bar{X}^{-1}$, is the scalar particle F with a mass $m\xi^{-1/2}$. The second type, coming from the determinant factor in $\bar{X}^{-1}$, is a "fermion" scalar particle c with a mass $m\xi^{-1/2}$. The effective coupling of $c(x)$ is $iec^\star A_\lambda \partial_\lambda c$, and each loop of c carries an additional factor (-1) because the determinant factor appears in the numerator of (A6).

To substantiate the above approach, let us consider the Georgi-Glashow model in the absence of fermions[10]:

$$\mathfrak{L}_G = -\tfrac{1}{4}\vec{\mathrm{B}}_{\mu\nu} \cdot \vec{\mathrm{B}}^{\mu\nu} + \tfrac{1}{2}[(\partial_\mu + g\vec{\mathrm{B}}_\mu \times)\vec{\phi}]^2 - V(\vec{\phi})$$
$$- \tfrac{1}{2}\xi'\left(\partial^\mu \vec{\mathrm{B}}^t_\mu - \frac{1}{\xi'} g \vec{v} \times \vec{\phi}^t\right)^2 - \frac{1}{2\alpha}(\partial_\mu B^\mu_3)^2,$$
$$\vec{\mathrm{B}}_{\mu\nu} = \partial_\mu \vec{\mathrm{B}}_\nu - \partial_\nu \vec{\mathrm{B}}_\mu + g\vec{\mathrm{B}}_\mu \times \vec{\mathrm{B}}_\nu , \quad \text{(A7)}$$
$$\vec{\phi} = \vec{\phi}^t + \hat{\eta}\phi_3 , \quad \vec{\phi}^t \cdot \hat{\eta} = 0, \quad \hat{\eta} \parallel z \text{ axis},$$

where $x^\lambda = (t, x, y, z)$, $g_{00} = -g_{kk} = 1$ ($k = 1, 2, 3$), $\hat{\eta}$ is a unit vector, and $V(\vec{\phi})$ is an isospin-invariant quartic polynomial of $\vec{\phi}$. The addition of fermions as source only makes additional algebraic complications, and no new physical problem. Introducing the Lagrangian multiplier $\vec{\chi}$, we modify (A7) to read

$$\mathfrak{L}_\chi = -\tfrac{1}{4}\vec{\mathrm{B}}_{\mu\nu}{}^2 + \tfrac{1}{2}[(\partial_\mu + g\vec{\mathrm{B}}_\mu \times)\vec{\phi}]^2 - V(\vec{\phi}) + \chi_3 \partial^\mu B^3_\mu$$
$$+ \tfrac{1}{2}\alpha(\chi_3)^2 + \vec{\chi}^t \cdot (\partial^\mu \vec{\mathrm{B}}^t_\mu - \frac{1}{\xi'} g\vec{v} \times \vec{\phi}^t) + \frac{1}{2\xi'}(\vec{\chi}^t)^2 , \quad \text{(A8)}$$

which leads to the following equations:

$$(\partial^\mu \vec{B}^t_\mu - \frac{1}{\xi'} g \vec{v} \times \vec{\phi}^t) + \frac{1}{\xi'} \vec{\chi}^t = 0, \quad \text{(A9)}$$

$$\partial_\mu B_3^\mu + \alpha \chi_3 = 0, \quad \text{(A10)}$$

$$-\partial_\mu \vec{B}^{\mu\nu} + \partial^\nu \vec{\chi} + g \vec{B}^{\mu\nu} \times \vec{B}_\mu + (\partial^\nu \vec{\phi} + g \vec{B}^\nu \times \vec{\phi}) \times \vec{\phi} = 0, \quad \text{(A11)}$$

$$\partial^\mu (\partial_\mu \vec{\phi} + g \vec{B}_\mu \times \vec{\phi}) + \frac{\partial V(\vec{\phi})}{\partial \vec{\phi}} + \frac{g^2}{\xi'} \vec{\chi}^t \times \vec{v} - g(\partial_\mu \vec{\phi} + g \vec{B}_\mu \times \vec{\phi}) \times \vec{B}^\mu = 0. \quad \text{(A12)}$$

It follows from (A11) and (A12) that

$$\partial^2 \vec{\chi} - \frac{g^2}{\xi'} (\vec{\chi}^t \times \vec{v}) \times \vec{\phi} + g \vec{B}_\mu \times \partial^\mu \vec{\chi} = 0, \quad \partial^2 \equiv \partial_\mu \partial^\mu \quad \text{(A13)}$$

which completely determines the interactions of the Lagrange multiplier field $\vec{\chi}$. The fact that $\vec{\chi}$ does not obey the free equation implies that the gauge condition $\vec{\chi} = 0$ [cf. Eqs. (A9) and (A10)] does not persist for all times. Consequently, the source terms in (A13) will upset gauge invariance and unitarity of the theory. As usual, one may regard the source term $g \vec{B}_\mu \times \partial^\mu \vec{\chi}$ as coming from an effective interaction $g \vec{\chi}^\star \cdot (\vec{B}_\mu \times \partial^\mu \vec{\chi})$. The two degrees of freedom $\vec{\chi}$ and $\vec{\chi}^\star$ "correspond" to the two unphysical components in the 4-vector photon field A_λ (i.e., the combinations of the longitudinal and the timelike photons). Equation (A13) could be derived from

$$\mathcal{L}' = \chi^{\star a} \left[\partial^2 \delta^{ac} - \frac{g^2 v}{\xi'} \phi^3 (\delta_{ac} - \eta_a \eta_c) - \frac{g^2 v}{\xi'} \phi^t_c \eta_a + g \epsilon^{abc} B^b_\mu \partial^\mu \right] \chi^c, \quad \text{(A14)}$$

which is renormalizable. The amplitude due to the interaction of $\vec{\chi}$ in the intermediate states is

$$Y = \int e^{i \int d^4 x \mathcal{L}'} d[\vec{\chi}, \vec{\chi}^\star] = \left\{ \det \left[\delta^{ac} - \partial^{-2} \frac{g^2 v}{\xi'} \phi^3 (\delta_{ac} - \eta_a \eta_c) - \partial^{-2} \frac{g^2 v}{\xi'} \phi^t_c \eta_a + \partial^{-2} g \epsilon^{abc} B^b_\mu \partial^\mu \right] \right\}^{-1}, \quad \text{(A15)}$$

which must be removed from the amplitude. The unitary and gauge-invariant amplitude A_{ug} is

$$A_{ug} = \int Y^{-1} \exp \left[i \int d^4 x (\mathcal{L}_G + \mathcal{L}_S) \right] d[\vec{B}_\mu, \vec{\phi}]. \quad \text{(A16)}$$

This result agrees with that obtained from another approach.[10] We may remark that if Y^{-1} in (A16) is replaced by $Y_{abs}{}^{-1}$, the theory will not be gauge-invariant.

If the gauge symmetry is spontaneously broken, we have[10]

$$\vec{\phi} = \vec{\phi}_t + \hat{\eta}(v + \phi).$$

It just gives rise to a mass term $-(g^2 v^2/\xi')\vec{\chi}^t$ in (A13). The amplitude corresponding to (A16) can easily be obtained. The result is the same as that in Ref. 10. It is important to note that although B_1^λ and B_2^λ become massive there are still two unphysical components in $\vec{B}_\lambda$. Thus, there are two degrees of freedom $\vec{\chi}$ and $\vec{\chi}^\star$ in (A14). The massive vector particles with three physical components are made up of the two transverse components associated with B_a^λ ($a = 1, 2$) and the would-be Goldstone bosons as the longitudinal components.

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[1]T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962); T. D. Lee, *ibid.* 128, 899 (1962); G. Feldman and P. T. Matthews, *ibid.* 130, 1633 (1963).

[2]See, for example, N. Nakanishi, Prog. Theor. Phys. Suppl. 51, 1 (1972); Phys. Rev. D 5, 1324 (1972).

[3]J. P. Hsu, Phys. Rev. D 8, 2609 (1973).

[4]L. D. Faddeev and V. N. Popov, Phys. Lett. 25B, 29 (1967).

[5]This equation is derived from the Lagrangian (1). Note that the symbol ★ is defined in the second paragraph of the paper and χ is a *negative*-metric particle.

[6]In fact, it does not matter if there is an infinite number of vertices in a theory. The gravitational theory is an example. The author would like to thank Professor R. Feynman for the comment.

[7]G. 't Hooft and M. Veltman, Nucl. Phys. B50, 318 (1972). Based on the gauge-transformation property of the gauge-violating term in the Lagrangian, a prescription for the interactions of the fictitious particle is given in this paper. However, this prescription does not give the correct results when it is applied to the Lagrangian (1).

[8]The fictitious particle F can only appear as the internal line in the Feynman diagrams. Its propagator is $-i(p_\gamma{}^2 + m^2/\xi)^{-1}$, where p_λ is the momentum carried by F, whose mass is assumed to be the same as that of χ.

[9]In this case, the cancellation occurs even if $\xi \neq 1$. Because of this and the close similarity between the gauge-violating terms $-(\partial_\mu A_\mu)/2\beta$ in $\mathcal{L}_\gamma$ and $-\xi(\partial_\mu \varphi^\star_\mu)(\partial_\nu \varphi_\nu)$ in $\mathcal{L}_\xi$ [cf. Eqs. (1) and (11)], one might speculate that the unitary S matrix may be independent of ξ *after* renormalization. Yet so far there is no proof for this.

[10]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972).