(1973). A detailed discussion of the definitions of the nonrelativistic c.m. variables is given in Appendix A. Two corrections should be made: (i) $\delta\vec{s}_j$ in Eq. (A3) should read $(-1)^j(\vec{q}_{NR}\times\vec{p})\times\vec{s}_j/(2mm_jc^2)$; and (ii) in *Note added in proof* of page 2051, V_E is missing in the expression for $\vec{\nabla}_x\cdot\vec{J}_2$. The correct expression should read:

$$\vec{\nabla}_x\cdot\vec{J}_2=-ieV_E(\vec{\tau}_1\times\vec{\tau}_2)_z[\delta^3(\vec{r}_1-\vec{x})-\delta^3(\vec{r}_2-\vec{x})].$$

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Quarks: Currents and constituents*

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An attempt is made to clarify the relation between current quarks and constituent quarks. Assuming that the two are related by a unitary transformation, we outline the properties of this transformation and, in the process, discover a new U(6) × U(6) × O(3) classification algebra for the hadrons. An example of this transformation is constructed in the lightlike-plane formulation of the free-quark model, where the transformation is found to be essentially unique and is just the operator solution to the problem of saturating chiral SU(3) × SU(3). Using the algebraic structure of the free-quark model phenomenologically, matrix elements of currents between different hadrons are related. This abstraction of free-quark algebraic properties works fairly well for the axial-charge and magnetic-moment operators, although it fails for bilocal operators. Nevertheless, we obtain many successful approximate relations between matrix elements of currents, not the least of which is the recovery of the ratio $\mu_T(\text{proton})/\mu_T(\text{neutron})=-\frac{3}{2}$.

I. INTRODUCTION

The subject of the relation between current quarks and constituent quarks has recently come under a great deal of discussion. It is the purpose of this paper to briefly summarize the distinction between the two types of quarks and then to explore their relationship. In this exploration we shall take the point of view that they are related by a unitary transformation, an approach which has been strongly advocated by Gell-Mann.[1,2] Furthermore, in view of the many beautiful properties possessed by lightlike charges[3] (local operators integrated over the null plane, $x^0+x^3=0$), we shall work completely within the framework of these charges, rather than within that of the more usual charges integrated over a spacelike hypersurface.

We shall find that there are rather strong constraints on the properties of the current-constituent transformation, although the transformation itself cannot yet be defined uniquely. Most importantly, we shall see that it almost certainly *cannot* be the identity. We shall also find that consideration of the properties of this transformation leads us to a larger algebra of operators than the original U(6)—we shall find a U(6) × U(6) × O(3) algebra of good operators which are invariant under boosts along $\hat{z}$, thus serving to define the complete quark-model classification of hadron states.

The transformation will be constructed in the free-quark model, where it is well defined and has no arbitrary parameters. We shall then speculate on how it might be constructed in interacting quark models.

Finally, we proceed to demonstrate how this transformation may be applied to relating matrix elements of current operators between different states in U(6) × O(3) multiplets. In these applications we shall adopt the approximate procedure of abstracting the transformation properties of operators from the free-quark model.

II. CURRENT QUARKS AND LIGHTLIKE CHARGES

Current quarks have been with us since 1964, when Gell-Mann[4] noticed that a compact and suggestive form for the generators Q_i and Q_i^5 of chiral $SU(3)\times SU(3)$ is obtained by writing them as bilinear products of local fermion fields $q(x)$, the "current quark fields":

$$Q_i \sim \int d^3x\, q^\dagger(x)\tfrac{1}{2}\lambda_i q(x)\,, \tag{1a}$$

$$Q_i^5 \sim \int d^3x\, q^\dagger(x)\gamma^5\tfrac{1}{2}\lambda_i q(x)\,, \tag{1b}$$

where λ_i is the usual 3×3 matrix representation of SU(3), $i=0,\ldots,8$. The Q_i, Q_i^5 in nature are postulated to have the same algebraic structure and Lorentz properties as the above expressions.

The form of (1a) and (1b) led Dashen and Gell-Mann[5] to consider the algebra of other operators of the form

$$\int d^3x\, q^\dagger(x)\Gamma\tfrac{1}{2}\lambda_i q(x)\,,$$

out of which only a subgroup, which we shall hereafter call $U(6)_{W,\,\mathrm{currents}}$, survives in the $p^3\to\infty$ limit. In the language of lightlike charges, only the generators of this subgroup, the F_i^α, can be meaningfully restricted to the $x^0+x^3=0$ plane. Only these "good" F_i^α can form a closed Lie algebra.

$U(6)_{W,\mathrm{currents}}$ consists of 18 generators in addition to those of chiral $SU(3)\times SU(3)$. These new generators transform like antisymmetric tensors in space, and as $\underline{8}+\underline{1}$ in SU(3). Postulating the existence of tensor currents $\mathfrak{F}_i^{\mu\nu}(x)$, the generators of the lightlike $U(6)_{W,\,\mathrm{currents}}$ can be written

$$F_i=\int d^4x\,\delta(x^+)\mathfrak{F}_i^+(x)\,, \tag{2a}$$

$$F_i^1=\tfrac{1}{2}\int d^4x\,\delta(x^+)\mathfrak{F}_i^{2+}(x)\,, \tag{2b}$$

$$F_i^2=-\tfrac{1}{2}\int d^4x\,\delta(x^+)\mathfrak{F}_i^{1+}(x)\,, \tag{2c}$$

$$F_i^3=\tfrac{1}{2}\int d^4x\,\delta(x^+)\mathfrak{F}_i^{5+}(x)\,, \tag{2d}$$

where $x^+=(1/\sqrt{2})(x^0+x^3)$, $\mathfrak{F}_i^+(x)=(1/\sqrt{2})[\mathfrak{F}_i^0(x)+\mathfrak{F}_i^3(x)]$ is a component of the vector current $\mathfrak{F}_i^\mu(x)$ and $\mathfrak{F}_i^{5+}(x)$ is the corresponding component of the axial-vector current $\mathfrak{F}_i^{5\mu}(x)$.

Before going on to the algebraic properties of the F_i^α in (2a)–(2d), we must mention some of the more striking properties of such lightlike charges. The most important of these properties is that of vacuum annihilation, $F_i^\alpha|0\rangle=0$. It is clear from the region of integration that the F_i^α commute with the transverse-momentum operator $\vec{P}_\perp$, and with $P^+=(1/\sqrt{2})(P^0+P^3)$. The vacuum has eigenvalue $p^+=0$ so that the F_i^α can only connect it with another $p^+=0$ state. However, $p^+>0$ for all states with nonzero mass and finite momentum, so that if the F_i^α decouple from infinite-momentum states ($p^3=-\infty$), they must annihilate the vacuum in the absence of zero-mass states. The pleasant thing about the F_i^α in (2a)–(2d) is that they *do* seem to decouple[6] from infinite-momentum states (disconnected pairs), and hence correspond to the "good" operators of Gell-Mann[1] (i.e., those operators with vanishing matrix elements between finite-mass and infinite-mass states in the $p^3\to\infty$ frame).

Of course, if the F_i^α act on a state at rest with mass m, they *can* lead to a state of different mass $m^*\neq m$, provided that the final state has momentum $p^3=(m^2-m^{*2})/2m$. Only if the F_i^α are conserved, $[P^-,F_i^\alpha]=0$, will they not lead to states of different mass. What the vacuum annihilation property *does* guarantee, however, is that the F_i^α cannot produce any disconnected pairs. Thus, application of various F_i^α's to a state an arbitrary number of times does not lead to an arbitrary number of pairs: There exists a possibility that we shall return to the original state after a finite number of steps, and thus obtain a finite-dimensional representation of the group. Such "good" generators are clearly the only possible candidates for the generators of a group with finite-dimensional representations.

The Lorentz properties of the F_i^α are not simple. It is clear that they are invariant under finite boosts along $\hat{z}$, $[F_i^\alpha,\Lambda_3]=0$, but they are not invariant under transverse boosts. In particular, a state with given helicity and transverse momentum $\vec{p}_\perp\neq 0$ does not have the same $U(6)_{W,\mathrm{currents}}$ classification as a state with the same helicity but different $\vec{p}_\perp$. To remedy this defect, we must introduce special transverse boost operators which leave the F_i^α invariant. Thus, defining

$$E_1=\frac{1}{P^+}(\Lambda_1+J_2)\,, \tag{3a}$$

$$E_2=\frac{1}{P^+}(\Lambda_2-J_1)\,, \tag{3b}$$

we can easily check that E_1 and E_2 commute with all F_i^α. States with transverse momenta $\vec{p}_\perp$ generated by means of the $\vec{E}_\perp$ can thus be classified in the same $U(6)_{W,\mathrm{currents}}$ representation, whatever $\vec{p}_\perp$ may be:

$$|(R,\rho)_{\mathrm{currents}},\vec{p}_\perp\rangle=e^{-i\vec{p}_\perp\cdot\vec{E}_\perp}|(R,\rho)_{\mathrm{currents}},\vec{p}_\perp=0\rangle\,, \tag{4}$$

where R signifies the representation, and ρ the member of the $U(6)_{W,\text{currents}}$ classification. The transverse boost-rotation in (4) can be decomposed into a pure Lorentz boost preceded and followed by rotations. An E-boosted state like that in (4) can be related to a certain mixture of helicity states with the same momenta.

As to other properties of the lightlike charges (2a)–(2d), we merely note that they have the same charge conjugation C as their spacelike counterparts (+ for the uncharged F_i^3, – for the uncharged F_i^1, F_i^2, and F_i), but do not have definite parity $\mathcal{P}$. Instead of parity, we can define the operation $\mathcal{R} = e^{-i\pi J_2}\mathcal{P}$ under which the F_i^α *are* eigenvectors. Thus, $\mathcal{R}^{-1}F_i\mathcal{R}^{-1} = +F_i$, $\mathcal{R}^{-1}F_i^3\mathcal{R}^{-1} = -F_i^3$, etc.

Finally, we come to the algebraic structure of the F_i^α. Just as we used the "current quark field" $q(x)$ in (1a) and (1b) as a suggestive shorthand for the algebraic and Lorentz properties of Q_i and Q_i^5, we can use a similar shorthand for the properties of the lightlike charges F_i^α. Again we use a local relativistic quark field, but now quantized on the $x^+=0$ plane, after the manner of Kogut and Soper.[7] In this scheme good operators are constructed from the independent fields $q_+(x) = \frac{1}{2}(1+\alpha^3)q(x)$, which have the anticommutation relations

$$\{q_+^\dagger(x), q_+(y)\}_{x^+=y^+} = \frac{1}{2\sqrt{2}}(1+\alpha^3)\delta(x^- - y^-) \times \delta^2(\vec{x}_\perp - \vec{y}_\perp), \tag{5}$$

$$\{q_+(x), q_+(y)\}_{x^+=y^+} = 0, \text{ etc}.$$

In terms of these $q_+(x)$ fields, we can represent the proposed $U(6)_{W,\text{currents}}$ algebra of the operators defined in (2a)–(2d) by

$$F_i \sim \sqrt{2} \int d^4x\, \delta(x^+) q_+^\dagger(x) \tfrac{1}{2}\lambda_i q_+(x), \tag{6a}$$

$$F_i^1 \sim (1/\sqrt{2}) \int d^4x\, \delta(x^+) q_+^\dagger(x) \beta\sigma^1 \tfrac{1}{2}\lambda_i q_+(x), \tag{6b}$$

$$F_i^2 \sim (1/\sqrt{2}) \int d^4x\, \delta(x^+) q_+^\dagger(x) \beta\sigma^2 \tfrac{1}{2}\lambda_i q_+(x), \tag{6c}$$

$$F_i^3 \sim (1/\sqrt{2}) \int d^4x\, \delta(x^+) q_+^\dagger(x) \sigma^3 \tfrac{1}{2}\lambda_i q_+(x), \tag{6d}$$

where the tilde is taken to mean "has the same algebra as"—it does *not* necessarily mean that we believe that the F_i^α in nature are formed out of simple bilinear combinations of some field.

The utility of the current quark-model expressions (6a)–(6d) lies in their suggestive power. If we take such suggestions seriously, we are led to believe that vector, axial, and tensor currents actually exist in nature and have the appropriate commutation relations. The existence of vector and axial currents in nature is well verified, since we can measure some of them directly in weak and electromagnetic interactions. Furthermore, their charges (6a) and (6d) do seem to generate the indicated chiral $SU(3)\times SU(3)$ algebra, as is attested by the success of sum rules like the Adler-Weisberger relation. The tensor currents are more speculative however. They do not seem to be coupled directly to other interactions as the vector and axial currents are. However, they do appear in more generalized schemes of current algebra, and may even have measurable matrix elements in the appropriate Bjorken scaling limits.[2] Thus, when we assume the existence of the $U(6)_{W,\text{currents}}$ algebra we are making an assumption which is in principle testable. Hereafter we shall assume that a well-defined $U(6)_{W,\text{currents}}$ algebra does exist, and that it is generated by the F_i^α defined in (2a)–(2d). The F_i^α are integrals over operators, which are in principle measurable. If the reader wishes, however, he can exclude the tensor currents F_i^1 and F_i^2 and break the group down to chiral $SU(3)\times SU(3)$ without doing violence to any of the theoretical conclusions that follow, although a great deal of phenomenological power is lost thereby.

We may now use the forms (6a)–(6d) of the F_i^α to suggest even more algebraic properties. For example, we can make guesses about the $U(6)_{W,\text{currents}}$ classification of various *local* operators like vector and axial-vector current components. We might even go so far as to classify *bad* operators (i.e., operators which do not decouple from infinite-momentum states, like scalar and pseudoscalar densities) with respect to $U(6)_{W,\text{currents}}$. Fritzsch and Gell-Mann[2] have outlined how such a scheme may work, and have established rules for determining which of the many possible algebraic relations may be plausibly abstracted from simple models (free quarks, quarks plus vector gluons).

In summary, the operators F_i^α may be assumed to form the group $U(6)_{W,\text{currents}}$, which can be used to classify various local currents (as well as integrated charges) in simple irreducible representations of the group. The picturesque term "current quark (antiquark)" can be used to describe the $\underline{6}$ ($\overline{\underline{6}}$) representation of $U(6)_{W,\text{currents}}$ out of which larger representations can be constructed. For example, the axial-vector current component $\mathcal{F}_i^{+5}(x)$ transforms as a particular member of a $\underline{35}$ of $U(6)_{W,\text{currents}}$, thus appearing to "consist" of one current quark and one current antiquark $(q\bar{q})_{\text{currents}}$.

There is only one flaw which prevents this scheme from being immediately useful: We do not know the $U(6)_{W,\text{currents}}$ classification of hadronic states.

In order to make the nature of this problem as clear as possible, we need some notational devices. Let us label a "physical state"—i.e., a hadron—by the traditional Poincaré group quantum numbers: energy-momentum $\vec{p}_\perp, p^+, p^-$, spin j, and helicity h. Furthermore, we may have to designate the channel we are interested in by other quantum numbers such as charge, isotopic spin, hypercharge, etc. We shall use the symbol Q to collectively designate these quantum numbers. Thus, we designate a "physical state" by $|\vec{p}_\perp, p^+, p^-; j, h; Q\rangle$ or, for brevity, simply by $|H\rangle$ when we do not need to specify the indices in detail.

Now if the matrix elements of the F_i^α between all such physical states were known (which is possible in principle since the F_i^α are measurable), each physical state could be decomposed into states transforming irreducibly under $U(6)_{W,\text{currents}}$ (this is always possible among the finite-mass states, since the F_i^α cannot lead out of this set). Any one of this second set of states can be designated as member ρ of representation R of $U(6)_{W,\text{currents}}$. However, such a classification is not complete, and we shall *assume* that other quantum numbers α can be found which do complete the classification. Thus, we have a complete set of states $|\vec{p}_\perp, p^+(R,\rho,\alpha)_\text{currents}\rangle$ (or just $|R_\text{currents}\rangle$ for short) whose members transform as irreducible representations of $U(6)_{W,\text{currents}}$.

Armed with these notational devices, we can expand a physical state into a sum of states which transforms irreducibly under $U(6)_{W,\text{currents}}$:

$$|\vec{p}_\perp, p^+, p^-; j, h; Q\rangle = \sum_{R,\rho,\alpha} \langle \vec{p}_\perp, p^+(R,\rho,\alpha)_\text{currents} | \vec{p}_\perp, p^+, p^-; j, h; Q\rangle |\vec{p}_\perp, p^+(R,\rho,\alpha)_\text{currents}\rangle$$

or simply

$$|H\rangle = \sum_R \langle R_\text{currents}|H\rangle |R_\text{currents}\rangle, \qquad (7)$$

the specific values of the indices being understood. In operator form this becomes

$$|H(R_\text{currents})\rangle = V|R_\text{currents}\rangle, \qquad (8)$$

where

$$V = \sum_R |H(R_\text{currents})\rangle\langle R_\text{currents}|. \qquad (9)$$

Although the matrix elements of V are defined by (7) (where $\langle R_\text{currents}|H\rangle$ can be computed from the matrix elements of the F_i^α between physical states) the *operator* V is not defined until we establish a basis, i.e., an association $(R,\rho,\alpha)_\text{currents} \leftrightarrow (p^-; j, h, Q)$. In principle, we can choose any basis we wish without changing the physical content of the transformation, but in practice we shall find a "natural" one. Note that if the $|R_\text{currents}\rangle$ states from a complete set, then V is necessarily unitary.

The formalism developed so far is, of course, just that. It does not help us to find out what the $U(6)_{W,\text{currents}}$ classification of hadron states actually is. What we really need to know are the matrix elements $\langle R_\text{currents}|H\rangle$. To throw some light on this problem, we must turn to a consideration of the regularities of the hadron spectrum.

III. CONSTITUENT QUARKS

One of the most striking regularities to emerge from our growing knowledge of the hadron spectrum is the occurrence of "supermultiplets" of hadrons with different charge and spin. It is a remarkable fact that groups of roughly degenerate hadrons can be found whose quantum numbers are predicted by a simple ansatz. Thus, the mesons can be represented as if they were composed of a spin-$\frac{1}{2}$ "constituent quark" and a "constituent antiquark," $M \sim (q\bar{q})_\text{constituent}$. Mesons of higher mass can be constructed by adding orbital angular momentum to the quark-antiquark system. Similarly, baryons behave as if they were constructed of three constituent quarks, $B \sim (qqq)_\text{constituent}$. Again, higher mass baryons are constructed by adding orbital angular momentum and insisting that the over-all wave function be completely symmetric in quark variables.

The multiplets are grouped so that states with the same orbital angular momentum seem to be roughly degenerate. The masses of the multiplets increase as the orbital angular momentum gets larger.

The most suggestive description of this situation is that proposed by Lipkin and Meshkov,[8] who postulated the existence of a set of $\underline{35}+\underline{1}$ operators W_i^α (W_i, W_i^1, W_i^2, W_i^3) whose algebra closes on a group which we shall call $U(6)_{W,\text{strong}}$. The eight W_i ($i=1,\ldots,8$) generate SU(3), while W_0 is proportional to the baryon number.

The W_i^α are the generators of an approximate $U(6)_{W,\text{strong}}$ symmetry of P_st^- (the lightlike analog of the strong-interaction Hamiltonian). Thus, we assume that $[W_i^\alpha, P_\text{st}^-] \simeq 0$, at least on single-hadron states (idealized resonances). This approximate conservation of the W_i^α explains the rough degeneracy of the multiplets, while the group structure guarantees the observed "constituent quark" structure. This latter property follows from the identification of the fundamental representation of the group, $\underline{6}$, with a constituent quark. Thus, mesons at rest belong in $\underline{6}\times\underline{\bar{6}}=\underline{35}+\underline{1}$ representations of the group (e.g., the pseudoscalar and vector octets and singlets fit into a $\underline{35}+\underline{1}$), while the baryons at rest fall into $\underline{6}\times\underline{6}\times\underline{6}=\underline{20}+\underline{56}+\underline{70}$ (e.g., the lowest-lying baryons the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decimet fill out a $\underline{56}$. The negative-parity baryons fall into a higher-lying $\underline{70}$).

The most tantalizing feature of $U(6)_{W,\text{strong}}$ is that its generators, the W_i^α, have precisely the same algebra, charge conjugation, and parity as the generators F_i^α of $U(6)_{W,\text{currents}}$. The only difference is that the F_i^α are well-defined operators, whereas most of the W_i^α are not. For some of the W_i (in particular, for $i=1,2,3$) the conserved-vector-current (CVC) hypothesis assures us that $W_i=F_i$, and it would be no great step to generalize CVC to the entire octet and set $W_i=F_i$ for *all* i in the limit that P_{st}^- is invariant under SU(3). As for the other W_i^α, we know only the empirical fact that they generate roughly degenerate multiplets of hadrons. Otherwise they are ill defined, and in the absence of recognizable multiplets would not be defined at all.

The most natural course of action under these circumstances would be to postulate that the W_i^α and the F_i^α are actually *equal*. This would immediately tell us the $U(6)_{W,\text{currents}}$ classifications of states, and many predictions could be made. The only real problem with this idea is that it does not work. Dashen and Gell-Mann[9] have attempted to classify the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decimet in a $\underline{56}$ of $U(6)_{W,\text{currents}}$ in the infinite-momentum frame (which is equivalent to the more modern usage of lightlike charges acting on states at rest). They have shown that such a classification would imply that the anomalous magnetic moments of all $\frac{1}{2}^+$ octet baryons must *vanish*, along with the octet-decimet magnetic transition amplitudes. Furthermore, they found $G_A/G_V=-\frac{5}{3}$. Since these results are far from being true in nature, one arrives at the necessity of describing the physical baryons as complex mixtures of many $U(6)_{W,\text{currents}}$ irreducible representations. This impurity of physical states under the transformations generated by $U(6)_{W,\text{currents}}$ [or its subgroup chiral $SU(3)\times SU(3)_{\text{currents}}$] is, in fact, the *raison d'etrê* of the many mixing schemes which have been proposed to obtain information about the matrix elements of currents between states of infinite momentum.[10] While these different schemes vary in detail, they all seem to agree on the need for very appreciable mixing between a variety of irreducible representations.

We can thus contrast $U(6)_{W,\text{strong}}$, whose empirical success presents the picture of nearly conserved charges and hadrons lying in simple irreducible representations, to $U(6)_{W,\text{currents}}$, whose charges are far from being conserved (the fact that many higher resonances must be included to saturate the Adler-Weisberger sum rule attests to this), and under which hadrons appear to be complicated mixtures of irreducible representations.

There is, moreover, a theoretical reason why we cannot equate the W_i^α and the F_i^α. Since the W_i^α are supposed to generate finite-dimensional multiplets of physical hadrons, they must have simple spin properties. Thus, since the smallest $U(6)_{W,\text{strong}}$ multiplets contain hadrons differing by at most one unit of angular momentum, the generators W_i^α must not be able to change spin by larger amounts—i.e., they must be parts of vectors. As we shall see in Sec. IV, the F_i^α cannot be parts of vectors unless they are *all* coupled to conserved currents. In general, the F_i^α contain unlimited angular momentum, and thus cannot be used to classify hadrons of definite spin into finite-dimensional multiplets.

Let us now see how the F_i^α and W_i^α might be related to one another in terms of the notation established in Sec. II. We have seen that we can empirically group sets of physical states $\{|H\rangle\}$ into $U(6)_{W,\text{strong}}$ multiplets, which we shall denote by $|(R,\rho,\alpha)_{\text{strong}}\rangle$ (again, we assume the existence of other quantum numbers α necessary to specify the state). Establishing the association mentioned in Sec. II is easy: We set $(R,\rho,\alpha)_{\text{currents}} \to (R,\rho,\alpha)_{\text{strong}}$. The unitary operator V which is thereby defined tells us exactly what the relation is between $U(6)_{W,\text{strong}}$ and $U(6)_{W,\text{currents}}$ representations. If $W_i^\alpha=F_i^\alpha$, then the two representations are the same, and $V=1$ in the basis established by our association. In general, $VF_i^\alpha V^{-1}$ defines a new $U(6)_W$ algebra different from $U(6)_{W,\text{currents}}$. In the above basis it makes sense to *define* the otherwise ill-defined W_i^α as[11]

$$W_i^\alpha=VF_i^\alpha V^{-1}, \tag{10}$$

that is,

$$W_i^\alpha=\sum_{R,\,\rho,\,\rho',\,\alpha}|(R,\rho',\alpha)_{\text{strong}}\rangle(F_i^\alpha)^R_{\rho'\rho}\langle(R,\rho,\alpha)_{\text{strong}}|, \tag{11}$$

where $(F_i^\alpha)^R_{\rho'\rho}$ is the matrix element of F_i^α between states ρ and ρ' in the R_{currents} representation. As such, it is just a Clebsch-Gordan coefficient.

Clearly, if the $U(6)_{W,\text{strong}}$ multiplets were exactly degenerate, and since W_i^α in (10) only takes us from one state to another in the same representation, we would have $[W_i^\alpha, P_{\text{st}}^-]=0$. In fact, the multiplets are split in mass and we have $[W_i^\alpha, P_{\text{st}}^-]\simeq 0$. In either case, the W_i^α can change spin by only one unit. Thus, this definition of W_i^α seems to fit all our requirements. If V has $C=+$ and $\mathcal{R}=+$ then the W_i^α will have the same C, $\mathcal{R}$, and algebra as the F_i^α. If $V\neq 1$ the W_i^α can be both approximately conserved and components of vectors (at least between single-hadron states) without implying similar properties for the F_i^α.[12]

The W_i^α have several properties besides C and $\mathcal{R}$ in common with the F_i^α. Since the W_i^α generate finite multiplets of hadrons, they must annihilate the vacuum $W_i^\alpha|0\rangle = 0$. By assumption, they form a closed Lie algebra [U(6)] and have meaningful restrictions to the $x^+=0$ plane. They must therefore be good operators. Azimuthal symmetry implies that the behavior of W_i^α under rotations about the $\hat{z}$ axis is the same as that of the F_i^α, requiring that $[J_3, V]=0$. Furthermore, the definition (9) of V implies that $[\Lambda_3, V]=0$, since the F_i^α have this property. Thus, the $U(6)_{W,\text{strong}}$ classification is independent of the momentum of the state along $\hat{z}$, $[\Lambda_3, W_i^\alpha]=0$. We shall see, however, that the W_i^α are probably *not* invariant under E boosts, so that the $U(6)_{W,\text{strong}}$ classification of a state *may* depend upon its transverse momentum. The classification is simple only for states with $\vec{p}_\perp = 0$.

The transformation V expresses the general idea of the phenomenological mixing schemes in a compact way. It allows one to describe, say, baryons, as simple objects, "containing" just three quarks where classification is concerned, and at the same time giving them the necessary complicated structure where current matrix elements are involved. V must be very far from unity if the results of these mixing schemes are any guide.

The usefulness and structure of such a transformation have been demonstrated phenomenologically by Buccella, Kleinert, Savoy, *et al.*[10] in the infinite-momentum frame. These authors have succeeded in fitting many coupling constants by means of this approach. The present work will be more theoretical in nature, and will concentrate more on showing how such a transformation arises physically. Our result will be seen to have transformation properties similar to that of Buccella, Kleinert, Savoy, *et al.*

The idea that a unitary transformation V is responsible for the mixing of $U(6)_{W,\text{currents}}$ representations is an old one. It appears in attempts to find representations of the current algebra at infinite momentum (Dashen and Gell-Mann,[9] Buccella, Kleinert, Savoy, *et al.*[10]) and in many other research efforts of the last seven years. The existence of such a transformation is also implicit in the phenomenological mixing schemes.[10]

IV. SPIN AND THE $U(6)\times U(6)\times O(3)_{\text{strong}}$ ALGEBRA

Before going on to the properties of the transformation V, we must digress a bit in order to discuss the important problem of the spin structure of the W_i^α. This investigation will lead us to a new and larger algebra for classifying hadron states as well as to some valuable insights into the nature and necessity of the transformation V.

Spin must be defined carefully for lightlike charges, for although they preserve p^+, lightlike charges do change p^3 when they lead to states of different mass. Since by spin we always mean the angular momentum of a state in its rest frame, the appropriate operators (for states with $\vec{p}_\perp=0$) are

$$\mathcal{J}_1 = \frac{J_1 P^0 + \Lambda_2 P^3}{M}, \tag{12a}$$

$$\mathcal{J}_2 = \frac{J_2 P^0 - \Lambda_1 P^3}{M}, \tag{12b}$$

$$\mathcal{J}_3 = J_3 . \tag{12c}$$

These $\vec{\mathcal{J}}$ form an SU(2), commute with boosts along $\hat{z}$, and $\mathcal{J}_1$ and $\mathcal{J}_2$ commute with $\vec{P}_\perp$. M is the mass operator $M^2 = 2P^+P^- - \vec{P}_\perp \cdot \vec{P}_\perp$. These are the correct spin operators for states with $\vec{p}_\perp = 0$, having matrix elements equal to those of $\vec{J}$ in the particle's rest frame.

The spin structure of the W_i^α may be derived simply by noting that in the smallest nontrivial representations (the $\underline{35}^-$ of mesons and the $\underline{56}^+$ of baryons) the particles differ in spin by, at most, *one* unit. Thus, the W_i^α must have $|\Delta \mathcal{J}| \le 1$. This condition can be maintained only if the W_i^α are parts of *vectors* under commutation with $\vec{\mathcal{J}}$. In fact, since not all the W_i^α have the same charge conjugation, the W_i^α must form *two* vectors which we shall denote:

$$\{W_i^1, W_i^2, \Omega_i^3\}C = -, \tag{13a}$$

$$\{\Omega_i^1, \Omega_i^2, W_i^3\}C = +, \tag{13b}$$

and the W_i must be scalars. The Ω_i^α operators are defined by the commutators

$$\Omega_i^1 = -i[\mathcal{J}_2, W_i^3], \tag{14a}$$

$$\Omega_i^2 = i[\mathcal{J}_1, W_i^3], \tag{14b}$$

$$\Omega_i^3 = -i[\mathcal{J}_1, W_i^2]. \tag{14c}$$

We assume that none of these commutators vanish. Given the $U(6)_{W,\text{strong}}$ algebra of the W_i^α and the commutation relations of W_i^α with $\mathcal{J}_3$,

$$[\mathcal{J}_3, W_i^3] = 0, \tag{15a}$$

$$[\mathcal{J}_3, W_i^1] = iW_i^2, \tag{15b}$$

$$[\mathcal{J}_3, W_i^2] = -iW_i^1 \tag{15c}$$

(which follow from similar relations for the F_i and and $[V, J_3]=0$), it is possible to prove that the operator sets (13a) and (13b) form vectors under the $\vec{\mathcal{J}}$ if and only if

$$[\mathcal{J}_1, W_i^1] = 0, \tag{16a}$$

$$[\mathcal{J}_1, [\mathcal{J}_1, W_i^3]] = W_i^3. \tag{16b}$$

That these relations are not so easily satisfied may become more evident when we realize that the F_i^α almost certainly *do not* satisfy (16a) and (16b). Consider the F_i^α in the rest frame of some particle. As lightlike charges, the region of integration of the F_i^α is not invariant under rotations about the $\hat{x}$ and $\hat{y}$ axes. Thus, although the Lorentz indices of the vector, axial, and tensor currents out of which the F_i^α are constructed transform correctly under rotations, the change of the region of integration will alter the value of the charge, *unless* the current is *conserved*. This is actually true for the vector currents $\mathfrak{F}_i^\mu(x)$ in the SU(3) symmetric limit, and so the F_i *are* actually scalars[13] (V may thus commute with the F_i).

It is clear, however, that for $\mathfrak{F}^{\mu 5}(x)$ and $\mathfrak{F}^{\mu\nu}(x)$ exact conservation is unlikely.[14] In this case, the F_i^α are *not* components of a vector. We shall see in Sec. V that even in the free-quark model, the F_i^α have unlimited $|\Delta \mathfrak{J}|$, and thus the states in $U(6)_{W,\,\text{currents}}$ representations *cannot have definite spin*. This is the major defect of the idea of using the F_i^α to classify hadron states. This is equally the reason why V is far from unity—V *cannot* be the identity if hadron multiplets are to contain particles with definite spin.

Conditions (16a) and (16b) thus become powerful constraints on V. Can they be considered as equations for V? Unfortunately, the answer is no. Given any V_1 such that the W_i^α satisfy both Eqs. (16), any other $\mathfrak{U}V_1$ (where $\mathfrak{U}$ is unitary and commutes with all $\vec{\mathfrak{J}}$) will also produce W_i^α satisfying (16). Nevertheless, Eqs. (16) will serve as guides for the construction of V in models.

Equations (16) have some powerful consequences. It is easy to verify (by repeated use of the Jacobi identity) that the Ω_i^1, Ω_i^2, Ω_i^3, and Ω_i (defined by $[W_i^3, \Omega_j^3] = i f_{ijk}\Omega_k$) form a $\underline{35}$ of $U(6)_{W,\,\text{strong}}$. Moreover, commutation of Ω_i^α's among themselves yields W_i^α's. The entire algebraic system closes on a $U(6)\times U(6)_{\text{strong}}$ generated by

$$G_{i\pm} = \tfrac{1}{2}(W_i \pm \Omega_i), \tag{17a}$$

$$G_{i\pm}^\alpha = \tfrac{1}{2}(W_i^\alpha \pm \Omega_i^\alpha), \quad \alpha = 1, 2 \tag{17b}$$

$$G_{i\pm}^3 = \tfrac{1}{2}(\Omega_i^3 \pm W_i^3). \tag{17c}$$

When we add the $\vec{\mathfrak{J}}$ to the algebraic system, we obtain an algebra $U(6)\times U(6)\times O(3)_{\text{strong}}$ of good operators, which are all invariant under boosts along $\hat{z}$. As such, this should be a useful classification algebra for hadron states. We also note that the algebra contains the correct generalization of the "quark spin" and "quark angular momentum" operators. Thus, the quark spin vector is defined as

$$\vec{S} = (\Omega_0^1, \Omega_0^2, W_0^3), \tag{18}$$

while the quark angular momentum is

$$\vec{L} = \vec{\mathfrak{J}} - \vec{S}, \tag{19}$$

both $\vec{S}$ and $\vec{L}$ obey an SU(2) algebra, and $[\vec{L}, \vec{S}] = 0$, as required for spin and orbital angular momentum.[15] This large $U(6)\times U(6)\times O(3)_{\text{strong}}$ algebra thus gives meaning to the scheme of classifying hadron states as if they were composed of spin-$\frac{1}{2}$ quarks with various orbital angular momenta. Unlike other $U(6)\times U(6)$ classification schemes, this one is composed entirely of good operators so that the classification still has meaning for lightlike charges (or, equivalently, in the infinite-momentum frame).

The final aspect of $U(6)\times U(6)_{\text{strong}}$ which we shall discuss is its relation to a $U(6)\times U(6)_{\text{currents}}$. Just as $W_i^\alpha = VF_i^\alpha V^{-1}$, we should expect that operators Φ_i^α are defined by $\Omega_i^\alpha = V\Phi_i^\alpha V^{-1}$, and that the F_i^α and Φ_i^α form a $U(6)\times U(6)_{\text{currents}}$ algebra. These Φ_i^α are explicitly worked out for the free-quark model in Appendix A. It is easily seen from the form of the free-quark model results that the Φ_i^α have a model-independent form, given the lightlike plane commutation relations of Gell-Mann and Fritzsch.[1] Thus,

$$\Phi_i = \int d^4x\, \delta(x^+)\, \frac{1}{\pi}\, \mathrm{Re}\left(\frac{1}{y^- - x^- + i\epsilon}\right) \times \mathfrak{F}_i^+(\vec{x}_\perp, x^-; \vec{x}_\perp, y^-), \tag{20a}$$

$$\Phi_i^1 = \tfrac{1}{2}\int d^4x\, \delta(x^+)\, \frac{1}{\pi}\, \mathrm{Re}\left(\frac{1}{y^- - x^- + i\epsilon}\right) \times \mathfrak{F}_i^{2+}(\vec{x}_\perp, x^-; \vec{x}_\perp, y^-), \tag{20b}$$

$$\Phi_i^2 = -\tfrac{1}{2}\int d^4x\, \delta(x^+)\, \frac{1}{\pi}\, \mathrm{Re}\left(\frac{1}{y^- - x^- + i\epsilon}\right) \times \mathfrak{F}_i^{1+}(\vec{x}_\perp, x^-; \vec{x}_\perp, y^-), \tag{20c}$$

$$\Phi_i^3 = \tfrac{1}{2}\int d^4x\, \delta(x^+)\, \frac{1}{\pi}\, \mathrm{Re}\left(\frac{1}{y^- - x^- + i\epsilon}\right) \times \mathfrak{F}_i^{5+}(\vec{x}_\perp, x^-; \vec{x}_\perp, y^-), \tag{20d}$$

where the $\mathfrak{F}_i^\alpha(x, y)$ are the usual bilocal currents.[16] Note that we have implicitly assumed factors which damp the $1/(x^- - y^-)$ at very large values of $(x^- - y^-)$.

It can be explicitly verified that these Φ_i^α and the F_i^α form a $U(6)\times U(6)_{\text{currents}}$ algebra. It should also be clear that the Φ_i^α are invariant under finite boosts along $\hat{z}$, and that they are almost certainly[17] good operators just as the F_i^α are. At the present time, however, these Φ_i^α do not seem to be particularly useful, so we shall not consider them further.

V. PROPERTIES OF THE TRANSFORMATION

We have now amassed a great deal of information about the properties of V from the study and comparison of the properties of the W_i^α and the F_i^α. We do not, unfortunately, have a definition of V which will allow it to be unambiguously computed at the present time. What we have discovered will, however, be found to be very helpful in constructing such a transformation in models.

In order to summarize our deductions about the structure of V, we present here a list of the properties V can be expected to possess:

(a) V transforms states lying in irreducible representations of $U(6)_{W,\text{currents}}$ into states with definite energy and spin:

$$|H(R_{\text{currents}})\rangle = V|R_{\text{currents}}\rangle .$$

(b) V is unitary.

(c) V transforms the F_i^α in such a way that when acting on single-hadron states the $VF_i^\alpha V^{-1}$ are conserved in some sensible limit not "too" far removed from reality. In other words, hadrons fall into roughly degenerate $U(6)_{W,\text{strong}}$ multiplets.

(d) V transforms the F_i^α in such a way that the $VF_i^\alpha V^{-1}$ are parts of vectors with respect to the particle spin operators $\vec{\mathcal{J}}$. This is guaranteed if and only if

$$[\mathcal{J}_1, VF_i^1V^{-1}] = 0 ,$$

$$[\mathcal{J}_1, [\mathcal{J}_1, VF_i^3V^{-1}]] = VF_i^3V^{-1} .$$

[This ensures that $U(6)_{W,\text{strong}}$ multiplets can contain particles with definite spin.]

(e) V contains only good operators or, at least, takes good operators only into good operators. This ensures that finite-mass states decouple from infinite-mass states.

(f) V is an SU(3) singlet, $[F_i, V]=0$, in the limit where all physical processes are SU(3)-invariant.

(g) V has $C=+$, $\mathcal{R}=+$ and is invariant under O(2), $[J_3, V]=0$, and boosts along $\hat{z}$, $[\Lambda_3, V]=0$.

In order to get further insight into the structure of V, it seems necessary to resort to explicitly constructing it in a simple model.

VI. EXPLICIT CONSTRUCTION OF V IN THE FREE-QUARK MODEL

In the lightlike version of the free-quark model,[7] the fundamental operator of the theory is $q_+(x)$, a local relativistic field obeying equal-x^+ commutation relations given by (5). Good local current densities can be constructed from bilinear products of these fields, being of the form $q_+^\dagger(x)\Gamma\frac{1}{2}\lambda_i q_+(x)$, where Γ is any 4×4 Dirac matrix which commutes with α^3. The generators F_i^α of $U(6)_{W,\text{currents}}$ can be defined in this model by means of such bilinear products. Thus, for the free-quark model we write

$$F_i^\alpha = (1/\sqrt{2})\int d^4x\, \delta(x^+)q_+^\dagger(x)\Gamma^\alpha\tfrac{1}{2}\lambda_i q_+(x), \tag{21}$$

where the $\Gamma^\alpha=(2, \beta\sigma^1, \beta\sigma^2, \sigma^3)$ sequentially. The $U(6)_{W,\text{strong}}$ generators are defined as

$$W_{i,\text{free}}^\alpha = V_{\text{free}}F_i^\alpha V_{\text{free}}^{-1} . \tag{22}$$

Finally, the analog of the Hamiltonian in this theory is

$$P_{\text{free}}^- = \frac{1}{2\sqrt{2}}\int d^4x\, \delta(x^+)q_-^\dagger(x)i\partial_-q_-(x)$$
$$= \frac{i}{2\sqrt{2}}\int d^4x\, d\xi\, \delta(x^+)\epsilon(x^- - \xi)q_+^\dagger(x) \times(\vec{\partial}_\perp{}^2 - m^2)q_+(\vec{x}_\perp, \xi) . \tag{23}$$

From (23) we easily see that $[F_i^\alpha, P_{\text{free}}^-]=0$, so that the motivation for condition (c) on V vanishes. Although several authors[18] have taken this to mean that there is no way to determine V in the free-quark model, we shall see that condition (d) now comes into full play, and actually determines V_{free} uniquely.

Since we are going to discuss spin, it will be advantageous to work in the rest frame of the particle states, where spin becomes simple. Only—what do we mean by "particle states" in the free-quark model? Certainly the "rest frame" is not the rest frame of an individual quark.

What we mean by a "particle" in this model is a wave packet, one containing either a quark and antiquark (to represent mesons) or three quarks (to represent baryons). The quantum numbers (charge, parity, spin, momentum) of the wave packet are arranged to match those of some physical hadron. Thus, the state has the same $U(6)\times U(6)\times O(3)_{\text{strong, free}}$ classification as does the corresponding hadron under $U(6)\times U(6)\times O(3)_{\text{strong}}$. In this way we make the algebraic aspects of the free-quark model as similar to those of the real world as possible. Hence, when we talk of the rest frame of a particle, we mean that frame in which the sum of the individual quark momenta is zero. This frame does *not* coincide with the rest frame of individual quarks except for states with only one quark—which are of no phenomenological interest.

For the time being, let us assume that we are in such a rest system. Then $\vec{\mathcal{J}}=\vec{J}$ and we must impose the requirement that the $W_{i,\text{free}}^\alpha$ have $|\Delta J|\le 1$. It is easily checked that the F_i^α do not have this property. Thus,

$$[J_1, F_i^1] = -\frac{1}{2\sqrt{2}}\int d^4x\, d\xi\, \delta(x^+)\epsilon(x^- - y^-) \times q_+^\dagger(x)\gamma_1\partial_1 q_+(\vec{x}_\perp, y^-) , \tag{24}$$

which is certainly not zero. The structure of the F_i^α is even more clearly revealed in Fock space:

$$F_i^\alpha = \frac{1}{2}\sum_{r,s}\int \frac{d^3p}{E/m}[a_p^{\dagger(r)}a_p^{(s)}X^{\dagger(r)}S^{-1}\Gamma_i^\alpha S X^{(s)} - b_p^{\dagger(s)}b_p^{(r)}X^{\dagger(r)}S^{-1}\Gamma_i'^\alpha S X^{(s)}], \tag{25}$$

where we have used the usual spin basis for the $a_p^{(r)}$ and $b_p^{(r)}$'s, the $X^{(r)}$'s are 2-component Pauli spinors, and the notation is otherwise standard: $\Gamma_i^\alpha = (\lambda_i, \sigma^1\frac{1}{2}\lambda_i, \sigma^2\frac{1}{2}\lambda_i, \sigma^3\frac{1}{2}\lambda_i)$ and $\Gamma_i'^\alpha = (\lambda_i, -\sigma^1\frac{1}{2}\lambda_i, -\sigma^2\frac{1}{2}\lambda_i, \sigma^3\frac{1}{2}\lambda_i)$. The unitary matrix S is the important part:

$$S = \frac{(\sqrt{2}\,p^+ + m) + i(\vec{p}\times\vec{\sigma})_3}{[2\sqrt{2}\,p^+(E+m)]^{1/2}}. \tag{26}$$

As a result of S, the F_i^α are clearly *not* simple under rotations. For example,

$$S^{-1}\Gamma_i^3 S = \left[\frac{m}{E+p^3}\sigma^3 + \left(1+\frac{p^3}{E+m}\right)\frac{\vec{\sigma}\cdot\vec{p}}{E+p^3}\right]\frac{1}{2}\lambda_i. \tag{27}$$

Terms like $1/(E+p^3) = (1/E)\sum_{n=0}^\infty(-p^3/E)^n$ carry unlimited orbital angular momentum. It is worth noting that, e.g., (27) has appeared before in physics—it is nothing less than the solution of the angular condition which Dashen and Gell-Mann[19,20] obtained for the free-quark model. It would seem that we are on the right track with these lightlike charges. The V_{free} which we shall obtain will thus turn out to be the free-quark solution to the angular condition, but in operator form.

It is clear that the Fock space form we require for the $W_{i,\text{free}}^\alpha$ is simply

$$W_{i,\text{free}}^\alpha = \frac{1}{2}\sum_{r,s}\int\frac{d^3p}{E/m}[a_p^{\dagger(r)}a_p^{(s)}X^{\dagger(r)}\Gamma_i^\alpha X^{(s)} - b_p^{\dagger(s)}b_p^{(r)}X^{\dagger(r)}\Gamma_i'^\alpha X^{(s)}]. \tag{28}$$

These expressions clearly have $|\Delta J| \leq 1$ and, in fact, are exactly what one expects the $U(6)_{W,\text{strong}}$ generators to be like in naive quark models. It is now simple to construct the transformation V_{free} which takes us from (25) to (28). Writing

$$V_{\text{free}} = e^{iY_{\text{free}}} \tag{29}$$

then in Fock space

$$Y_{\text{free}} = \sum_{r,s}\int\frac{d^3p}{E/m}\left[a_p^{\dagger(r)}a_p^{(s)}X^{\dagger(r)}\tan^{-1}\left(\frac{(\vec{p}\times\vec{\sigma})_3}{\sqrt{2}\,p^+ + m}\right)X^{(s)} - b_p^{\dagger(s)}b_p^{(r)}X^{\dagger(r)}\tan^{-1}\left(\frac{(\vec{p}\times\vec{\sigma})_3}{\sqrt{2}\,p^+ + m}\right)X^{(s)}\right], \tag{30}$$

which becomes

$$Y_{\text{free}} = \sqrt{2}\int d^4x\,\delta(x^+)q_+^\dagger(x)\tan^{-1}\left(\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{\kappa}\right)q_+(x) \tag{31}$$

in configuration space. In this expression $\kappa = \sqrt{2}\,|\partial_-| + m$, where $|\partial_-|$ stands for

$$m\left[1-\left(1+\frac{\partial_-^2}{m^2}\right)\right]^{1/2},$$

whose power-series expansion in $(1+\partial_-^2/m^2)$ converges rapidly when the quark's momentum is much smaller than its mass. All functions of derivatives are to be understood in terms of power-series expansions. The configuration space forms of the $W_{i,\text{free}}^\alpha$ and $\Omega_{i,\text{free}}^\alpha$ may be found in Appendix A.

We see that Y_{free}, hence V_{free}, is a complicated nonlocal operator. It is, however, good. Furthermore, $[Y_{\text{free}}, P_{\text{free}}^-] = 0$, hence $Y_{\text{free}}|0\rangle = 0$. Y_{free} is simple algebraically: It belongs to a $\underline{35}$ of $U(6)_{W,\text{currents},\text{free}}$, or to a $(3,\bar{3})\oplus(\bar{3},3)$, $L_3(F) = \pm 1$ of the subgroup $U(3)\times U(3)_{\text{currents},\text{free}}$ [we here use $L_3(F) = J_3 - F_0^3$ to define current quark orbital angular momentum]. Y_{free} cancels the $|\Delta J| > 1$ terms in F_i^α only at the expense of itself containing all angular momenta. Note that Y_{free} (thus the W_i^α) does not commute with transverse boost-rotations $\vec{E}_\perp$. The $U(6)_{W,\text{strong},\text{free}}$ classification of states will thus depend upon their transverse momenta, being simple only for $\vec{p}_\perp = 0$.

In one respect, however, V_{free} is not a good model of the transformation V used in nature. Young[21] has demonstrated that exotic representations of $U(6)_{W,\text{currents}}$ (e.g., representations which cannot be written as $q\bar{q}$ for mesons) must be present in the solution of the angular condition when a potential is acting between the quarks. V_{free} does not generate any such representations—it makes no pairs. As has been clearly pointed out by Eichten *et al.*,[18] V_{free} is nothing more than a change of spin basis in the free-quark model. The reason for this defect is clearly that the quarks, being free, cannot exchange momentum and create pairs. We thus expect that V will produce exotic representations as soon as some sort of interaction is turned on. For this reason, one must be somewhat skeptical of abstracting properties from V_{free} and applying them to V. Although this procedure does seem to work in some cases, it also fails badly in relating the deep-inelastic structure function of the proton to that of the neutron. It is in precisely such circumstances that pairs would be expected to play an important role.

The final property of Y_{free} we must note is that it is *not* invariant under boosts along $\hat{z}$, contrary to our condition (g). This fact should come as no surprise—after all, Y_{free} was constructed in the rest frame. Furthermore, since Y_{free} was constructed to satisfy a spin requirement, and since the $\vec{J}$ are not invariant under boosts unless specially supplemented to form $\vec{\mathfrak{J}}$'s, it is practically certain that the Y_{free} will have to be similarly supplemented. This is easy to do. We can simply *define* the value of a matrix element of W_i^α between states with $p^3 \neq 0$ to be precisely the same as the value of the matrix element between $p^3 = 0$ states. This, of course, automatically ensures invariance under boosts along $\hat{z}$. This condition may also be put into operator language exactly as was done for the boost-invariant $\vec{\mathfrak{J}}$'s. Thus we expand (31) as a power series in $|\partial_-|$. Each term with a given power of $|\partial_-|$ is multiplied by a similar power of $M_{\text{free}}/P^+_{\text{free}}$, where $M^2_{\text{free}} = 2P^+P^-_{\text{free}} - \vec{P}_\perp \cdot \vec{P}_\perp$. In the rest frame $M_{\text{free}}/P^+_{\text{free}} = 1$, whereas the ratio $|\partial_-|/P^+_{\text{free}}$ is invariant under boosts along $\hat{z}$. Since $M_{\text{free}}/P^+_{\text{free}}$ commutes with Y_{free} there is no ambiguity in the order of factors.

In order to avoid long, ugly expressions, we write the invariant Y_{free} *formally* as

$$Y_{\text{free}} = \sqrt{2} \int d^4x\, \delta(x^+) q_+^\dagger(x) \tan^{-1}\left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{\kappa_{\text{inv}}}\right) q_+(x), \tag{32}$$

where

$$\kappa_{\text{inv}} = (\sqrt{2}\, M_{\text{free}} |\partial_-| / P^+_{\text{free}}) + m,$$

and we always mean that the $M_{\text{free}}/P^+_{\text{free}}$ will appear *outside* the integral when the power-series expansion is performed. Thus, we see that Y_{free} (hence the W_i^α) can be made explicitly invariant under boosts along $\hat{z}$, although the result is somewhat cumbersome. Fortunately, we can always eliminate the $M_{\text{free}}/P^+_{\text{free}}$ factors by going to the rest frame when we take matrix elements.

In summary, we have seen that a transformation V_{free} may be defined in the free-quark model. V_{free} explicitly satisfies all constraints (a)–(g) of Sec. V. The operators $W^\alpha_{i,\text{free}}$ form a group $\text{U}(6)_{W,\text{strong},\text{free}}$ which classifies particles having definite spin and $\vec{p}_\perp = 0$ into small irreducible representations, whereas the irreducible representations of the F_i^α contain large spin mixtures. V_{free} is a nonlocal operator, as are the $W^\alpha_{i,\text{free}}$ and $\Omega^\alpha_{i,\text{free}}$. In spite of the defect of not producing exotic states, V_{free} may thus serve as a useful model for the transformation V used by nature.

Note that this is not the first time that a nonlocal operator has been proposed as the generator of a U(6) hadron classification group. Several authors[22] have suggested that the Foldy-Wouthuysen transformation might be useful for creating conserved charges. The trouble with the Foldy-Wouthuysen transformation is that it is not a good operator, and cannot be meaningfully restricted to the $x^+ = 0$ plane. It also leads to infinite-mass states. This undesirable property has caused us to reject it as a candidate for V [furthermore, V_{FW} does not satisfy condition (d) for lightlike charges F_i^α].

VII. CONSTRUCTION OF V IN INTERACTING QUARK MODELS

Having constructed V in the free-quark model, the next logical step would be to construct it in interacting models. This step, however, proves to be a formidable one. The main difficulty seems to be our complete ignorance of any interacting field theoretical model which exhibits degenerate (or nearly degenerate) multiplets of particles of different spin. The models thus lack the very feature which motivated us to study $\text{U}(6)_{W,\text{strong}}$ in nature.

In the absence of such models, the only approach left open to us is a perturbative one. We start with the free-quark model, where V_{free} is known, and then turn on some sort of interaction. The effect of the interaction on the $\text{U}(6)_{W,\text{currents}}$ classification of states can be examined in various orders of the coupling constant for individual matrix elements, and we may hope to draw some conclusions about the way the classifications change and thus learn something about the structure of V. As was speculated upon in the author's dissertation,[23] V may take the form

$$V = \mathfrak{U}\, V_{\text{free}}\,, \tag{33}$$

where V_{free} solves the spin criterion (d), serving only to rotate the spin basis of the quarks in an underlying Fock space, while $\mathfrak{U}$ is a scalar which commutes with $\vec{\mathfrak{J}}$, and which contains all the pair states and exotic representations resulting from interactions.

Unfortunately, this very form allows $\mathfrak{U}$ to escape our most powerful constraints on V, with the result that we can say very little about $\mathfrak{U}$. Can one find additional constraints, constraints powerful enough to determine $\mathfrak{U}$ as well? We do not know at present.

The effects of interactions have been very little explored at the present time, although the apparent (and unexplained) phenomenological success of V_{free} in roughly predicting the $\text{U}(6)_{W,\text{currents}}$ classification of states would make such a study very interesting. It seems unlikely that simple theories like the vector-gluon model can account for the preservation of the free-quark $U(6)_{W,\text{currents}}$ structure when interactions become strong. Is there any theory which can? This question remains to be answered.

VIII. APPLICATION TO CURRENT MATRIX ELEMENTS

We have now come to the end of our theoretical discussion of the transformation V. As we have seen, we can *deduce* very little about the structure of V in nature. We do, however, have an *example* of V in the free-quark model, and we have seen that V_{free} satisfies many of the requirements which the V used in nature must satisfy. For phenomenological purposes, then, it may make sense to abstract the *algebraic* properties of V_{free} from the free-quark model, and apply them to the algebraic structure of matrix elements in nature. The *only* justification for such a procedure can be in the success of its predictions.

The type of matrix element we shall attempt to evaluate is that of a current and its various moments [classified by $U(6)_{W,\text{currents}}$] sandwiched between two hadron states [classified by $U(6)_{W,\text{strong}}$]. Allowing the hadrons to have arbitrary transverse momenta, the matrix element will be of the form

$$\left\langle A, \vec{p}_\perp' \,\middle|\, \int d^4x\, \delta(x^+) \mathcal{F}_i^\alpha(x) e^{i(\vec{p}_\perp' - \vec{p}_\perp)\cdot \vec{x}_\perp} \,\middle|\, B, \vec{p}_\perp \right\rangle, \tag{34}$$

where the states are normalized as $\langle A, \vec{p}\,' | B, \vec{p}\rangle = \delta_{AB}\delta^3\vec{p}\,'\vec{p}$. If the transverse momenta of $|A, \vec{p}_\perp'\rangle$ and $|B, \vec{p}_\perp\rangle$ are generated by E boosts, then the matrix element (34) will depend only upon the *difference* $(\vec{p}_\perp' - \vec{p}_\perp)$ of the transverse momenta (there is no dependence, of course, upon the longitudinal momentum—the difference $p'^3 - p^3$ is fixed by the requirement that p^+ is the same for both states). This property is easily demonstrated: If the transverse momenta are generated by E boosts, we can write (34) as

$$\left\langle A, \vec{p}_\perp' = 0 \,\middle|\, \int d^4x\, \delta(x^+) \mathcal{F}_i^\alpha(x) e^{i(\vec{p}_\perp' - \vec{p}_\perp)\cdot(\vec{x}_\perp + \vec{E}_\perp)} \,\middle|\, B, \vec{p}_\perp = 0 \right\rangle = \langle A, \vec{p}_\perp' = 0 | F_i^\alpha(\vec{p}_\perp' - \vec{p}_\perp) | B, \vec{p}_\perp = 0\rangle, \tag{35}$$

where we have defined

$$F_i^\alpha(\vec{p}_\perp) = \int d^4x\, \delta(x^+) \mathcal{F}_i^\alpha(x) e^{i\vec{p}_\perp\cdot(\vec{x}_\perp + \vec{E}_\perp)}. \tag{36}$$

The matrix element (35) obviously depends only upon $\vec{p}_\perp' - \vec{p}_\perp$, as required. The operators $F_i^\alpha(\vec{p}_\perp)$ have many nice properties—for example, they commute with the transverse momentum operator $\vec{P}_\perp$, so that moments of $\mathcal{F}_i^\alpha(x)$ can be expressed in terms of forward matrix elements. This is a useful property for $U(6)_{W,\text{strong}}$ calculations, since we only know how to classify states with $\vec{p}_\perp = 0$.

We can now use V to make the algebraic structure of these matrix elements evident. We remember that V is defined so that $|A, \vec{p}_\perp = 0\rangle = V|R_{\text{currents}}(A)\rangle$, where $R_{\text{currents}}(A)$ is defined by the association $(R, \rho, \alpha)_{\text{currents}} \leftrightarrow (R, \rho, \alpha)_{\text{strong}}$. Thus, the $U(6)_{W,\text{currents}}$ classification of the state $|R_{\text{currents}}(A)\rangle$ will be precisely the same as the $U(6)_{W,\text{strong}}$ classification of $|A, \vec{p}_\perp = 0\rangle$.

Using this trick, we can throw all of the complexity of mixing onto the operators. Thus, the matrix element

$$\langle A, \vec{p}_\perp = 0 | F_i^\alpha(\vec{p}_\perp) | B, \vec{p}_\perp = 0\rangle = \langle R_{\text{currents}}(A) | V^{-1} F_i^\alpha(\vec{p}_\perp) V | R_{\text{currents}}(B)\rangle. \tag{37}$$

The problem now reduces to the $U(6)_{W,\text{currents}}$ classification of $V^{-1}F_i^\alpha(\vec{p}_\perp)V$. We shall abstract this structure (but not the numerical values of matrix elements, of course) from the free-quark model.

Before going on to the matrix elements of the electromagnetic and axial currents, we must add an element to our classification of states. Since J_3 commutes with both the W_i^α and F_i^α, the helicity of a $\vec{p}_\perp = 0$ state is compatible with both classifications. We shall, however, define a quark "orbital angular momentum" component, $L_3(W) = J_3 - W_0^3$ for a $U(6)_W \times O(2)_{\text{strong}}$ classification, and $L_3(F) = J_3 - F_0^3$ for a $U(6)_W \times O(2)_{\text{currents}}$ classification. Thus, for example, a proton would be classified as a member of a $\underline{56}$, $L_3(W) = 0$, while the low-lying negative-parity baryons would be in a $\underline{70}$, $L_3(W) = -1, 0, 1$ under $U(6)_W \times O(2)_{\text{strong}}$. In the corresponding $U(6)_W \times O(2)_{\text{currents}}$ classification, the axial charge would belong to a $\underline{35}$, $L_3(F) = 0$, while Y_{free} (say) would be in a $\underline{35}$, $L_3(F) = \pm 1$. This classification will make it easy to see at a glance which matrix elements vanish and which do not. Furthermore, since the $U(6)_{W,\text{currents}}$ classification $\underline{35}$ is not particularly enlightening (*all* our transformed currents will be members of $\underline{35}$'s), we shall instead use the subgroup $U(3) \times \overline{U}(3) \times O(2)_{\text{currents}}$ classification to note the structure of the operator in more detail. Thus, a $(3, \overline{3}) \oplus (\overline{3}, 3)_{\text{currents}}$ operator can flip "current quark spin" F_0^3, while $(1, 8) \oplus (8, 1)_{\text{currents}}$ cannot. Nevertheless, we shall use the full $U(6)_W \times O(2)_{\text{currents}}$ group to evaluate Clebsch-Gordan coefficients. The subgroup notation is only employed to make the character of the operators involved more explicit.

The truly remarkable thing about the transformation properties derived from the free quark is their simplicity: Since Y_{free} is bilinear in quark fields (in the rest frame), bilinear operators are transformed only into bilinear operators. Thus, the resulting transformed current can contain the irreducible representations $(1,8)\oplus(8,1)$ and $(3,\bar{3})\oplus(\bar{3},3)$ and *nothing else*. This property of rapid termination is unique to the free-quark model, and unless we have some special reason to think that the free-quark algebra should be preserved, we would not expect it to show up in an interacting model. In general, interactions should introduce products of these simple irreducible representations, invalidating the free-quark results. We must thus expect some deviations of the following predictions from the experimental values, since the validity of the V_{free} structure can only be approximate.

A. The axial-vector current

The axial-vector charge $F_i^3 = F_i^3(\vec{p}_\perp = 0)$ yields the first interesting results. Referring to Appendix B, where the form of $V^{-1}F_i^3 V$ is explicitly written out for the free-quark model, we see that the structure of the transformed operator is

$$V^{-1}F_i^3 V \sim (1,8)\oplus(8,1),\ L_3(F)=0 \\ +(3,\bar{3})\oplus(\bar{3},3),\ L_3(F)=\pm 1\,. \qquad (38)$$

The first term transforms like $(\sigma^3 \frac{1}{2}\lambda_i)$, and the second term like $[(\vec{\gamma}\times\vec{\partial})_3 \frac{1}{2}\lambda_i]$. The first term transforms like the original charge F_i^3 (although there is no reason whatsoever to think that it *is* F_i^3; it is not equal to F_i^3 even in the free-quark model). The second term is new and can lead from $L_3(W)=0$ representations [like the baryon $\underline{56}$, $L_3(W)=0$] to higher ones [like $\underline{70}$, $L_3(W)=\pm 1$, or $\underline{56}$, $L_3(W)=\pm 1$]. This is the kind of behavior we expect of the physical F_i^3 —such behavior is actually seen in, for example, the Adler-Weisberger sum rule, where we find many resonances contributing to the empirical sum over states.

More detailed results are obtained by sandwiching the transformed charge (38) between well-known states, like the baryons, classified as $\underline{56}$, $L_3(W)=0$ under $U(6)_W\times O(2)_{\text{strong}}$. We see that only the first $L_3(F)=0$ term can contribute to the matrix element. [Remember that $V^{-1}F_i^3 V$ is taken between $L_3(F)=0$ states, which correspond to the $L_3(W)=0$ classification of the physical states. This jumping back and forth between $L_3(F)$ and $L_3(W)$ may seem confusing at first, but, by enabling us to put V on the operator, it actually results in a great simplification.] Since the first term of (38) has precisely the same $U(6)_W\times O(2)_{\text{currents}}$ structure as F_i^3 itself, we see that we simply get back all the old $U(6)_W$ results for these matrix elements—with one important proviso. The difference is that the first term of (38) is *not* a generator of $U(6)_W\times O(2)_{\text{currents}}$. The values of its matrix elements are not determined by the algebra; there is always some reduced matrix element η, which is in general different from 1 (we have defined η so that $\eta=1$ if there is no transformation). Thus, we find the traditional $U(6)_W$ results modified by factors

$$\frac{G_A}{G_V} = -\eta\,\tfrac{5}{3}\,,\quad G^* = -\eta\,\tfrac{4}{3}\,,\quad (D/F)_{\text{axial}} = \tfrac{3}{2}\,. \qquad (39)$$

The D/F ratio is the same as before, since η cancels out. Whether or not $\eta = 1/\sqrt{2}$ is a question of dynamics, which the present work cannot decide.

The structure (38) which we have proposed for the transformed axial-vector charge has been applied to the decays of meson and baryon resonances by several groups,[24] who use partial conservation of axial-vector current (PCAC) to relate matrix elements of the axial-vector current F_i^3 to pion emission amplitudes. The predictions seem generally satisfactory (although there are some unresolved difficulties with the signs of the amplitudes) and tend to indicate the dominance of the $(3,\bar{3})\oplus(\bar{3},3)$, $L_3(F)=\pm 1$ term for the baryonic decays of $\underline{70}$, $L=1$ to the $\underline{56}$, $L=0$ and for the mesonic decays $\underline{35}$, $L=1$ to $\underline{35}$, $L=0$.

The dominance of this term raises some interesting problems for the original suggestion that $U(6)_{W,\text{strong}}$ might be an approximate vertex *symmetry*. Such a symmetry would require the $(1,8)\oplus(8,1)$, $L_3(F)=0$ term to dominate the pion decay amplitudes, so that the apparent dominance of $(3,\bar{3})\oplus(\bar{3},3)$, $L_3(F)=\pm 1$ in the decay amplitudes mentioned above would seem to rule out any vestige of $U(6)_{W,\text{strong}}$ symmetry *even for vertices*. $U(6)_{W,\text{strong}}$ can thus probably be looked upon as a classification group for single particles and resonances *only*, and some new prescription must be adopted for the discussion of strong-interaction vertices.[25]

There are also some deep (and unresolved) theoretical questions about the consistency of using PCAC and the $U(6)_{W,\text{currents}}$ algebra of lightlike charges. As it is normally understood, PCAC involves the idea that matrix elements behave smoothly as the pion mass is taken to zero. However, if there is a massless pion, then we have no guarantee that $W_i^3|0\rangle = 0$, so that the utility of the W_i^α for classifying states becomes questionable. Moreover, the limit in which $[P_{\bar{s}t}, W_i^\alpha]=0$ should be very singular (since it would involve the degeneracy of the vector mesons with the pseudoscalar mesons). The problem of discovering how PCAC and $U(6)_{W,\text{strong}}$ mesh together is thus a very important one.[26]

Theoretical difficulties aside, however, the algebraic structure of the transformed F_i^3 seems to give adequate results for forward matrix elements. The area where the old $U(6)_W$ schemes first broke down, however, was in the first moments of the electromagnetic current. We must therefore turn to these to see that the proposed algebraic structure really does make the corrections we intended it to make.

B. Moments of the electric current

The matrix elements of the electromagnetic charge $F_{em} = Q$ are, of course, trivial since $[V, F_{em}] = 0$. The moments are more interesting, however. In particular, the first moment, $(\partial/\partial k_x) \times F_{em}(k_x)|_{k_x=0}$ is nothing less than the anomalous-magnetic-moment operator. For spin-$\frac{1}{2}$ particles, the anomalous magnetic moment μ_A is given by

$$\frac{\mu_A}{2M} = \frac{\partial}{\partial k_x} \langle A; \text{rest}; -\tfrac{1}{2} | F_{em}(k_x) | A; \text{rest}; +\tfrac{1}{2} \rangle \Big|_{k_x=0}. \quad (40)$$

This identification can be readily checked by expanding out the matrix element in terms of the traditional invariants, being careful to remember the spin rotations induced by the E boosts. One then sees that only the Pauli form factor $F_2(0)$ is projected out.

We can also differentiate $F_{em}(k_x)$ directly, finding

$$\frac{\mu_A}{2M} = i \Big\langle A; \text{rest}; -\tfrac{1}{2} \Big| \Big[\int d^4x\, \delta(x^+)\, x\, \mathcal{F}^+_{em}(x) + QE_1 \Big] \Big| A; \text{rest} + \tfrac{1}{2} \Big\rangle \quad (41)$$

The second term is just kinematic: Q is the net charge of the state $|A\rangle$, while $E_1 = (J_2 + \Lambda_1)/P^+$. But $\langle \text{rest} | E_1 | \text{rest} \rangle = (1/M) \langle \text{rest} | J_2 | \text{rest} \rangle$. Λ_1 has negative parity, and therefore has no diagonal matrix elements between states at rest. Since $\langle \text{rest}, -\frac{1}{2} | J_2 | \text{rest}, +\frac{1}{2} \rangle = \frac{1}{2} i$, we find

$$\frac{\mu_A}{2M} = i \Big\langle A; \text{rest}; -\tfrac{1}{2} \Big| \int d^4x\, \delta(x^+)\, x\, \mathcal{F}^+_{em}(x) \times \Big| A; \text{rest}; +\tfrac{1}{2} \Big\rangle - \frac{Q}{2M}. \quad (42)$$

The second term is just the Dirac moment. Note that this identification is a bit more subtle than it seems, since E_1 and $\int d^4x\, \delta(x^+)\, x\, \mathcal{F}^+_{em}(x)$ individually can change $\vec{p}_\perp$, and their forward matrix elements may not be well defined. However, if the evaluations are done in terms of symmetric wave packets, rather than plane-wave states, no ambiguities arise and the matrix elements are perfectly well defined. The upshot of this argument is that we can write

$$\begin{aligned} \frac{\mu_T}{2M} &= i \Big\langle A; \text{rest}; -\tfrac{1}{2} \Big| \int d^4x\, \delta(x^+)\, x\, \mathcal{F}^+_{em}(x) \Big| A; \text{rest}; +\tfrac{1}{2} \Big\rangle \\ &= i \Big\langle R_{\text{currents}}(A); \text{rest}; -\tfrac{1}{2} \Big| V^{-1} \int d^4x\, \delta(x^+)\, x\, \mathcal{F}^+_{em}(x) V \Big| R_{\text{currents}}(A); \text{rest}; +\tfrac{1}{2} \Big\rangle, \end{aligned} \quad (43)$$

where μ_T is the total magnetic moment of the particle. The algebraic structure of this operator is readily determined (see Appendix B):

$$V^{-1} \int d^4x\, \delta(x^+)\, x\, \mathcal{F}^+_{em}(x) V \sim (1, 8) \oplus (8, 1),\ L_3(F) = \pm 1 + (3, \bar{3}) \oplus (\bar{3}, 3),\ L_3(F) = 0, \pm 2\ , \quad (44)$$

where the $(3, \bar{3}) \oplus (\bar{3}, 3)$, $L_3(F) = 0$ term transforms like $(\gamma^1 \frac{1}{2} \lambda_i)$. With the assignment of the nucleon, spin-up to $(6, 3) L_3(W) = 0$ and the nucleon, spin-down to $(3, 6) L_3(W) = 0$, we find that the $(1, 8) \oplus (8, 1)$ parts give no contribution, the $(3, \bar{3}) \oplus (\bar{3}, 3)$, $L_3(F) = 0$ term alone connecting the two states. It is easy to verify that this yields

$$\frac{\mu_T(\text{proton})}{\mu_T(\text{neutron})} = -\tfrac{3}{2}. \quad (45)$$

We have thus recovered this famous ratio. The fact that one obtains the result $\mu_A = 0$ when the transformation $V \to 1$ is, alone, a striking proof of how badly such a transformation is needed, since a $U(6)_{W,\text{currents}}$ hadron classification would predict that $\langle \text{rest}; -\frac{1}{2} | E_1 | \text{rest}; +\frac{1}{2} \rangle = 0$—in clear contradiction to the Lorentz algebra. It is again clear that states belonging to irreducible representations of $U(6)_{W,\text{currents}}$ cannot have definite spin.

We can also compute μ^*, the $M1$ transition moment for $p \to \Delta^+$, assigning the Δ^+, spin $+\frac{1}{2}$ to (6, 3), $L_3(W) = 0$. As before, we can verify that

$$\mu^* = 2\frac{\partial}{\partial k_x}\left\langle \Delta^+, \tfrac{1}{2}, \text{rest} \right| F_{\text{em}}(k_x) \times \left| p, -\tfrac{1}{2}, p^3 = \frac{m_\Delta{}^2 - m_p{}^2}{2m_\Delta} \right\rangle \Bigg|_{k_x=0}. \quad (46)$$

Again, only the $(3, \bar{3}) \oplus (\bar{3}, 3)$, $L_3(F) = 0$ term contributes, yielding the traditional

$$\mu^* = \frac{2\sqrt{2}}{3}\mu_T(\text{proton}), \quad (47)$$

which is within about 30% of the measured value. Finally, we can work out the $E2$ transition moment for $p \to \Delta^+$:

$$E2 = \frac{1}{2}\frac{\partial}{\partial k_x}\left[\frac{-1}{\sqrt{2}}\left\langle \Delta^+, \tfrac{1}{2}, \text{rest} \right| F_{\text{em}}(k_x) \left| p, \tfrac{1}{2}, p^3 = \frac{m_\Delta{}^2 - m_p{}^2}{2m_\Delta}\right\rangle + \tfrac{1}{2}(\tfrac{2}{3})^{1/2}\left\langle \Delta^+, \tfrac{3}{2}, \text{rest} \right| F_{\text{em}}(k_x) \left| p, \tfrac{1}{2}, p^3 = \frac{m_\Delta{}^2 - m_p{}^2}{2m_\Delta}\right\rangle\right]_{k_x=0}. \quad (48)$$

With the assignment of Δ^+, spin $+\frac{3}{2}$, to (10, 1), $L_3(W) = 0$, this yields $E2 = 0$ in good agreement with experiment. This moment has a very special importance for our work, since neither $(1, 8) \oplus (8, 1)$ terms nor $(3, \bar{3}) \oplus (\bar{3}, 3)$ can give any contributions to $E2$. Products of these representations, however, *can* contribute, so that the vanishing of $E2$ provides a test for the absence of such terms. Experimentally,[27] $E2/M1 = 0.02 \pm 0.02$, which seems to indicate that any terms transforming like products of currents (i.e., terms not bilinear in quark fields) are absent, or at least contribute very little to $\Delta L_3(F) = 0$ transitions.

The free-quark structure (44) of the first moment of the electromagnetic current has been applied[28] to electromagnetic decays of higher resonances and to photoproduction, with generally satisfactory results. There thus seems to be a fair amount of evidence that the free-quark algebraic structure (44) is roughly like that of the current in nature.

This is actually something of a surprise, even in the free-quark model, for since $[P^-_{\text{free}}, F_{\text{em}}(k_x)] \neq 0$, we would expect the $M_{\text{free}}/P^+_{\text{free}}$ factors in V_{free} to produce terms in (44) transforming like products of the simple $(3, \bar{3}) \oplus (\bar{3}, 3)$ and $(1, 8) \oplus (8, 1)$ representations. As long as matrix elements of $F_{\text{em}}(k_x)$ are taken between states that would be degenerate in the free-quark model (like the baryon $\underline{56}$), the $M_{\text{free}}/P^+_{\text{free}}$ can consistently be set equal to one. However, when we consider matrix elements like

$$\langle R_{\text{currents}}(A) | V_{\text{free}}{}^{-1} F_{\text{em}}(k_x) V_{\text{free}} | R_{\text{currents}}(B) \rangle,$$

where A and B have different values of $M_{\text{free}}/P^+_{\text{free}}$, we find the presence of multilinear products of quark fields with coefficients proportional to $M_{\text{free}}(A)/P^+_{\text{free}} - M_{\text{free}}(B)/P^+_{\text{free}}$. This problem does not arise for F^3_i since $[P^-_{\text{free}}, F^3_i] = 0$, although it does make the abstraction of the transformation properties (38) from the free-quark model seem a bit unrealistic where transitions between states with very different masses are involved.

C. Bilocal currents

Finally, we come to a matrix element in which the free-quark model algebraic structure does not seem to be like that of the algebraic structure used by nature. Deep-inelastic structure functions can be related[16] to forward matrix elements of the vector bilocal operator $\mathfrak{F}^+_i(x, y)$. Thus, in the Bjorken scaling limit, we are interested in matrix elements like

$$\left\langle A, \text{rest} \right| \int d^4x\, \delta(x^+) e^{i\xi(x^- - y^-)} \times \mathfrak{F}^+_i(\vec{x}_\perp, x^-; \vec{y}_\perp, y^-) \left| A, \text{rest} \right\rangle. \quad (49)$$

Now, from Appendix B we find

$$V^{-1}\int d^4x\, \delta(x^+) e^{i\xi(x^- - y^-)} \mathfrak{F}^+_i(\vec{x}_\perp, x^-; \vec{x}_\perp, y^-) V \sim (1, 8) \oplus (8, 1),\ L_3(F) = 0 + (3, \bar{3}) \oplus (\bar{3}, 3),\ L_3(F) = \pm 1, \quad (50)$$

where the $\Delta L_3(F) = 0$ piece transforms like the charge (λ_i). Thus, the free-quark model prediction is that the octet portion of the bilocal currents is purely F-coupled within the $\frac{1}{2}^+$ octet. This implies that the ratio of the neutron to proton structure function $F^n_2(\xi)/F^p_2(\xi)$ should be $\geq \frac{2}{3}$, independent of ξ. Inasmuch as this is in violent disagreement with the data, we must regard this case as a failure of the abstraction of the bilocal operator algebraic structure from the free-quark model. The bilocal operators thus seem to be quite sensitive to the absence of pairs in V_{free}—something we might have expected on the basis of the parton model.

IX. CONCLUSION

We have seen that a clear distinction must be made between current quarks and constituent quarks; that is, between the group which classifies currents into simple irreducible representations and the group which similarly classifies hadrons. Assuming that the two groups are related by a unitary transformation V, we have outlined the properties we expect V to possess. In order to show how such a set of requirements might be satisfied, we have constructed a simple *example* of V in the free-quark model. Although this example does not possess all the properties that V probably has in nature, it may nevertheless be useful as the only model for V we know of at present.

There remain many important problems for future study. Perhaps the most straightforward problem is that of SU(3) breaking. This problem can be studied in the free-quark model, where we can explicitly see how V_{free} must be modified in order to ensure that the SU(3) generators, the W_i, do not change particle spin. Then there is the problem of models with interactions—can these be simply treated? An interesting approach to interactions has been taken by Eichten *et al.*[18] who study the Bardakci-Halpern scheme[29] (although in a slightly different context from the one we used to construct V_{free} in this paper). Although pairs are not produced in the Bardakci-Halpern scheme, it would be instructive to see how a potential modifies V_{free}. Another very important problem is the relation of the approach taken in this paper to the $\text{U}(6)_{W,\,\text{currents}}$ saturation schemes via the angular condition. We have seen that V_{free} is the operator solution to the angular condition for free quarks. The angular condition can be formulated without restrictions on the masses of states—can the angular condition thus be used to provide an unambiguous equation for V? All these questions, and many others, have yet to be answered.

The most difficult question is, as always, why does the free-quark structure abstracted from V_{free} work so well for matrix elements of F_i^3 and $F_{\text{em}}(\vec{k}_\perp)$? There seems to be no doubt that known models with interactions will add extra terms to the classifications of $V_{\text{free}}^{-1}F_i^3\,V_{\text{free}}$ and $V_{\text{free}}^{-1}F_{\text{em}}(\vec{k}_\perp)V_{\text{free}}$. Only, to a good approximation, such terms do not seem to be there phenomenologically. What suppresses these extra terms? We do know that this mechanism is not universal, since the bilocal current structure in nature *is* quite different from that found in the free-quark model.

We have thus raised more questions than one can pretend to answer. Nevertheless, we believe that we *have* shed some light on the relation between the two kinds of quark, and shown how the paradoxes arising from confounding the two may be resolved. We have shown how to recover the successes of the "naive quark model" schemes and how to temper the failures.

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APPENDIX A: $\text{U}(6)\times\text{U}(6)_{\text{strong}}$ GENERATORS IN THE FREE-QUARK MODEL

Note that $\kappa_{\text{inv}} = (\sqrt{2}\,M_{\text{free}}\,|\partial_-|\,/P^+_{\text{free}}) + m$ (see Sec. VI on the free-quark model for discussion). The $M_{\text{free}}/P^+_{\text{free}}$ is to be interpreted as always appearing *outside* the integral in the power-series expansion of an integrated operator containing κ_{inv}:

$$W_{i,\text{free}} = \sqrt{2}\int d^4x\,\delta(x^+)q_+^\dagger(x)\tfrac{1}{2}\lambda_i\,q_+(x)\,,$$

$$W^1_{i,\text{free}} = \frac{1}{\sqrt{2}}\int d^4x\,\delta(x^+)q_+^\dagger(x)\left\{\beta\sigma^1 - \frac{2i\partial_1[\kappa_{\text{inv}}\sigma^3 + i\beta\vec{\sigma}_\perp\cdot\vec{\partial}_\perp]}{\kappa_{\text{inv}}{}^2 - \partial_\perp{}^2}\right\}\tfrac{1}{2}\lambda_i\,q_+(x)\,,$$

$$W^2_{i,\text{free}} = \frac{1}{\sqrt{2}}\int d^4x\,\delta(x^+)q_+^\dagger(x)\left\{\beta\sigma^2 - \frac{2i\partial_2[\kappa_{\text{inv}}\sigma^3 + i\beta\vec{\sigma}_\perp\cdot\vec{\partial}_\perp]}{\kappa_{\text{inv}}{}^2 - \partial_\perp{}^2}\right\}\tfrac{1}{2}\lambda_i\,q_+(x)\,,$$

$$W^3_{i,\text{free}} = \frac{1}{\sqrt{2}}\int d^4x\,\delta(x^+)q_+^\dagger(x)\left\{\sigma^3 + \frac{2[\partial_\perp{}^2\sigma^3 + i\kappa_{\text{inv}}\beta\vec{\sigma}_\perp\cdot\vec{\partial}_\perp]}{\kappa_{\text{inv}}{}^2 - \partial_\perp{}^2}\right\}\tfrac{1}{2}\lambda_i\,q_+(x)\,,$$

$$\Omega_{i,\text{free}} = -i\sqrt{2}\int d^4x\,\delta(x^+)q_+^\dagger(x)\frac{\partial_-}{|\partial_-|}\tfrac{1}{2}\lambda_i\,q_+(x)\,,$$

$$\Omega^1_{i,\text{free}}=\frac{-i}{\sqrt{2}}\int d^4x\,\delta(x^+)q_+^\dagger(x)\left\{\beta\sigma^1-\frac{2i\partial_1[\kappa_{\text{inv}}\sigma^3+i\beta\vec{\sigma}_\perp\cdot\vec{\partial}_\perp]}{\kappa_{\text{inv}}{}^2-\partial_\perp{}^2}\right\}\frac{\partial_-}{|\partial_-|}\tfrac{1}{2}\lambda_i\,q_+(x),$$

$$\Omega^2_{i,\text{free}}=\frac{-i}{\sqrt{2}}\int d^4x\,\delta(x^+)q_+^\dagger(x)\left\{\beta\sigma^2-\frac{2i\partial_2[\kappa_{\text{inv}}\sigma^3+i\beta\vec{\sigma}_\perp\cdot\vec{\partial}_\perp]}{\kappa_{\text{inv}}{}^2-\partial_\perp{}^2}\right\}\frac{\partial_-}{|\partial_-|}\tfrac{1}{2}\lambda_i\,q_+(x),$$

$$\Omega^3_{i,\text{free}}=\frac{-i}{\sqrt{2}}\int d^4x\,\delta(x^+)q_+^\dagger(x)\left\{\sigma^3+\frac{2[\partial_\perp{}^2\sigma^3+i\kappa_{\text{inv}}\beta\vec{\sigma}_\perp\cdot\vec{\partial}_\perp]}{\kappa_{\text{inv}}{}^2-\partial_\perp{}^2}\right\}\frac{\partial_-}{|\partial_-|}\tfrac{1}{2}\lambda_i\,q_+(x).$$

One can check directly that the above forms for $W^\alpha_{i,\text{free}}$ satisfy the equations

$$[\mathcal{J}_1,W^1_{i,\text{free}}]=0,\qquad [\mathcal{J}_1,[\mathcal{J}_1,W^3_{i,\text{free}}]]=W^3_{i,\text{free}},$$

where we have used the rest frame commutator

$$\left[J_1,\int d^4x\,\delta(x^+)q_+^\dagger(x)\Theta(\vec{\partial}_\perp,\partial_-)q_+(x)\right]$$
$$=\frac{1}{4\sqrt{2}}\int d^4x\,d\xi\,\delta(x^+)\epsilon(x^--\xi)q_+^\dagger(x)\left\{i(2\partial_-^2-\partial_\perp{}^2+m^2)\frac{\delta\Theta}{\delta\partial_2}-2i\partial_2\partial_-\frac{\delta\Theta}{\delta\partial_-}-[\Theta,\beta\sigma^1(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp+im)]\right\}q_+(\vec{x}_\perp,\xi).$$

All derivatives act to the right. The expressions

$$\frac{\delta\Theta(\vec{\partial}_\perp,\partial_-)}{\delta\partial_2}\quad\text{and}\quad\frac{\delta\Theta(\vec{\partial}_\perp,\partial_-)}{\delta\partial_-}$$

are formal derivatives of $\Theta(\vec{\partial}_\perp,\partial_-)$, with respect to its arguments (treated as variables). These terms arise from partial integrations on the x^2 and x^- moments in J_1.

The operators $\Phi^\alpha_{i,\text{free}}=V_{\text{free}}{}^{-1}\Omega^\alpha_{i,\text{free}}V_{\text{free}}$ are given by

$$\Phi_{i,\text{free}}=\frac{-i\sqrt{2}}{\pi}\int d^4x\,d\xi\,\delta(x^+)\,\text{Re}\left(\frac{1}{\xi-x^-+i\epsilon}\right)q_+^\dagger(x)\tfrac{1}{2}\lambda_i\times q_+(\vec{x}_\perp,\xi),$$

$$\Phi^1_{i,\text{free}}=\frac{-i}{\sqrt{2}\,\pi}\int d^4x\,d\xi\,\delta(x^+)\,\text{Re}\left(\frac{1}{\xi-x^-+i\epsilon}\right)q_+^\dagger(x)\beta\sigma^1\tfrac{1}{2}\lambda_i\times q_+(\vec{x}_\perp,\xi),$$

$$\Phi^2_{i,\text{free}}=\frac{-i}{\sqrt{2}\,\pi}\int d^4x\,d\xi\,\delta(x^+)\,\text{Re}\left(\frac{1}{\xi-x^-+i\epsilon}\right)q_+^\dagger(x)\beta\sigma^2\tfrac{1}{2}\lambda_i\times q_+(\vec{x}_\perp,\xi),$$

$$\Phi^3_{i,\text{free}}=\frac{-i}{\sqrt{2}\,\pi}\int d^4x\,d\xi\,\delta(x^+)\,\text{Re}\left(\frac{1}{\xi-x^-+i\epsilon}\right)q_+^\dagger(x)\sigma^3\tfrac{1}{2}\lambda_i\times q_+(\vec{x}_\perp,\xi),$$

where we have used

$$\int d^4x\,\delta(x^+)q_+^\dagger(x)\frac{\partial_-}{|\partial_-|}\Gamma q_+(x)=\frac{1}{\pi}\int d^4x\,d\xi\,\delta(x^+)\,\text{Re}\left(\frac{1}{\xi-x^-+i\epsilon}\right)q_+^\dagger(x)\Gamma q_+(\vec{x}_\perp,\xi).$$

The presence of suitable damping factors as $\xi-x^-\to\infty$ is assumed.

APPENDIX B: TRANSFORMATION PROPERTIES OF LIGHTLIKE CHARGES AND MOMENTS IN THE FREE-QUARK MODEL

Note that $\kappa_{\text{inv}}=(\sqrt{2}\,M_{\text{free}}|\partial_-|/P^+_{\text{free}})+m$ and $\kappa=\sqrt{2}\,|\partial_-|+m$.

1. The axial charge

$$V_{\text{free}}{}^{-1}F^3_i\,V_{\text{free}}=\frac{1}{\sqrt{2}}\int d^4x\,\delta(x^+)q_+^\dagger(x)\left(\frac{\kappa_{\text{inv}}{}^2+\partial_\perp{}^2-2i\kappa_{\text{inv}}\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{\kappa_{\text{inv}}{}^2-\partial_\perp{}^2}\right)\sigma^3\tfrac{1}{2}\lambda_i\,q_+(x).$$

2. The electromagnetic current-magnetic moment

$$V_{\text{free}}{}^{-1}\int d^4x\,\delta(x^+)\,x\,\mathfrak{F}^+_{\text{em}}(x)V_{\text{free}}=\int d^4x\,\delta(x^+)\,x\,\mathfrak{F}^+_{\text{em}}(x)-\sqrt{2}\int d^4x\,\delta(x^+)q_+^\dagger(x)\frac{1}{\kappa^2-\partial_\perp{}^2}\times\left[\kappa\beta\sigma^2+i\partial_2\sigma^3+\frac{2\kappa\partial_1(\kappa+i\vec{\gamma}_\perp\cdot\vec{\partial}_\perp)}{\kappa^2-\partial_\perp{}^2}\right]\tfrac{1}{2}\lambda_{\text{em}}\,q_+(x).$$

This expression is to be evaluated in the rest frame of the external states (which must be of equal mass in the free-quark model).

3. The bilocal vector current

$$V_{\text{free}}^{-1}\sqrt{2}\int d^2x_\perp q_+^\dagger(\vec{x}_\perp, x^-)\tfrac{1}{2}\lambda_i\, q_+(\vec{x}_\perp, y^-)V_{\text{free}} = \sqrt{2}\int d^2x_\perp q_+^\dagger(\vec{x}_\perp, x^-)\left\{\frac{\overleftarrow{\kappa}\,\overrightarrow{\kappa} - \partial_\perp{}^2 - i(\overrightarrow{\kappa} - \overleftarrow{\kappa})\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{[(\overleftarrow{\kappa}^2 - \partial_\perp{}^2)(\overrightarrow{\kappa}^2 - \partial_\perp{}^2)]^{1/2}}\right\}\tfrac{1}{2}\lambda_i\, q_+(\vec{x}_\perp, y^-),$$

where the expression is to be evaluated in the rest frame of the external states, as for 2. The derivatives in $\overrightarrow{\kappa}$ act to the right, those in $\overleftarrow{\kappa}$ act to the left. All other derivatives act to the right.

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[11]Note that we could use Eq. (10) as our starting point, first distinguishing the two $U(6)_W$'s (implicitly assuming that the W_i^α are well defined in some way) and then *assuming* that they are related by a unitary transformation V. We have avoided this approach (perhaps at the risk of causing considerable confusion) in order to bring out the main issues involved: viz, the poorly defined nature of the W_i^α and the fact that the irreducible representations of $U(6)_{W,\text{currents}}$ (plus a set of commuting operators) must define a complete set of states in order that V be unitary.

[12]In spite of these concordances, however, let the reader beware. V is not really as well defined as one would like. The problem is, as before, a choice of basis. *If* it is possible to group hadron states into $U(6)_{W,\text{strong}}$ multiplets in some sort of narrow-resonance approximation, then a basis can be defined as above. However, as masses become higher and resonances are harder to find, this empirical process may break down. We are then back to picking some sort of arbitrary association in order to define V (hence W_i^α). For the purposes of this paper we shall ignore the problem, although it must be faced if one wishes to make a rigorous theory out of $U(6)_{W,\text{strong}}$.

[13]If SU(3) is not an exact symmetry, then the lightlike charges F_i may no longer be spin singlets. Since the generators of the SU(3) hadron classification group certainly do not change spin, SU(3) breaking may require a transformation between the F_i and the W_i, which makes the W_i spin singlets. This point is presently under investigation.

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[25]M. Gell-Mann has made a suggestion on how to resolve this problem. PCAC and vector dominance allow us to evaluate the consequences of V_{free} for certain strong-interaction vertices. Strong-interaction vertices in general can be expanded in terms of invariants, each of which has definite $U(6)_{W,\text{strong}}$ properties, after the manner of Sakita and Wali [B. Sakita and K. C. Wali, Phys. Rev. 139, B1355 (1965)]. In this scheme, mesons and baryons are described as objects with appropriate sets of U(12) indices. Vertices are described by means of invariant terms consisting of the various possible contractions of these indices with each other and with the external momenta. We can see what the structure of V_{free} implies for these invariants in processes where V_{free} can be applied. The next step of this generalization would be to *assume* this structure applies for all strong-interaction processes, whether or not V_{free} can be applied directly. Hey, Rosner, and Weyers (see Ref. 24) have shown that this procedure is equivalent to the $^3P_0$ prescription for decays of mesons or baryons where a pion is emitted.

[26]Discussions with R. Carlitz and W.-K. Tung have been very enlightening with regard to these difficulties. They are also discussed by S. P. de Alwis (unpublished).

Note added in proof: The PCAC limit brings to light the vast difference between the chiral $U(3)\times U(3)$ subgroups of $U(6)_{W,\text{currents}}$ and $U(6)_{W,\text{strong}}$. In the limit of zero pion mass $F_i^3|0\rangle \neq 0$, since F_i^3 can produce massless pion states with $p^+=0$. In this limit F_i^3 is a scalar under $\vec{\mathcal{J}}$, so that F_i and F_i^3 form a chiral $U(3)\times U(3)_{\text{currents}}$ group whose generators *do* have simple spin properties. This group can qualify by itself as a hadron classification group, since its multiplets may have definite spin (all particles in a multiplet have the same spin). When the divergence of the axial-vector current is nonzero, F_i^3 is no longer a scalar, and it may be useful to define a new operator $XF_i^3X^{-1}$ which *is* a scalar. The new transformation X has nothing to do with V, as is obvious from the fact that W_i^3 is a vector component, not a scalar. As such, $W_i^3|0\rangle=0$, even when the pion is massless, since W_i^3 cannot couple the vacuum to a zero-spin state. Thus, the chiral $U(3)\times U(3)_{\text{currents}}$ of F_i and F_i^3 is a very different group from the chiral $U(3)\times U(3)_{\text{strong}}$ of F_i and W_i^3. In discussing PCAC one must be careful to avoid confusing the two groups.

[27]R. Walker has kindly supplied the following data for the $\gamma p \to n\pi^+$ transition (resonance parameters are taken to be $M=1233$ MeV, $\Gamma=120$ MeV): $M_{1+}=2.49 \pm 0.04$, $E_{1+}=0.038\pm 0.03$ (errors are approximate).

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