PHYSICAL REVIEW D VOLUME 9, NUMBER 4 15 FEBRUARY 1974

Fixed poles and compositeness*

G. E. Hite†
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544

H. J. W. Müller-Kirsten
Fachbereich Physik, Universität Trier-Kaiserslautern, Kaiserslautern, Germany
(Received 22 August 1973; revised manuscript received 9 November 1973)

A field-theoretic model of πV scattering is constructed in order to obtain and compare the high-energy behavior of helicity amplitudes for the processes $\pi\gamma \to \pi\gamma$, $\pi\gamma \to \pi\rho$, $\pi\rho \to \pi\rho$ and hence to arrive at conclusions with regard to the occurrence of fixed J-plane poles. The model is based on the Wick-Cutkosky scalar theory, in which the dynamics is due to exchange of massless scalar mesons. Full use is made of the fact that the integral equations of the three-point and four-point functions of this theory may be solved explicitly in the ladder approximation. The solutions of the inhomogeneous equations are investigated in detail—in particular their asymptotic behavior when one of the particles is far off the mass shell. They are then inserted into our model diagrams and the asymptotic behavior of amplitudes of the processes mentioned above is derived on the assumption that the ρ meson is a composite state which lies on a Regge trajectory, whereas the photon is not. This definition of compositeness is seen to be equivalent to the condition of vanishing of the appropriate vertex renormalization constant. The occurence of significant fixed-pole contributions in Compton scattering is reaffirmed. Finally some remarks are made with regard to the vector-meson-dominance hypothesis.

I. INTRODUCTION

In a recent paper[1] we carried out a detailed investigation of the kinematics and Reggeization of the process $\pi V \to \pi V$ and its crossed reactions. In particular we considered the limiting case of Compton scattering off pions and affirmed that a finite nonzero asymptotic value of the total photoabsorption cross section as well as s-channel helicity conservation require the introduction of a singularity in the appropriate residue function such as is provided by a fixed J-plane pole at the nonsense wrong-signature point $J=1$. The question as to what is the fundamental difference between a photon and a ρ meson remained untouched in our previous investigation. This question is related to the occurrence of fixed J-plane poles at right-signature points and is the central theme of the present note. We confine our discussion to a field-theoretic model of πV scattering.

Fixed J-plane poles have for a long time been known to be related to the compositeness or noncompositeness of subnuclear particles. The central underlying idea of the vanishing of a right-signature fixed-pole residue on account of a superconvergence relation resulting from an appropriate asymptotic behavior of the amplitude may be traced back to Mandelstam's early investigations[2-4]—particularly to the first Appendix of Ref. 3. Although various aspects of this idea have been discussed from time to time—see, e.g., Refs. 5–7—there was widespread belief that Regge-pole phenomenology could ultimately be done without fixed poles or that Regge phenomenology may not really be suited for photon-induced interactions.[8] The extensive investigations of photoproduction and electroproduction processes carried out recently—both theoretically and experimentally—do not seem to confirm this belief. The existence of fixed $J=0, 1$ poles in Compton scattering off protons seems to be firmly established.[9] In fact Brandt *et al.*[10] even found arguments suggesting that there is a fixed $J=0$ pole in ρ-meson photoproduction. The compositeness of the nucleon—probed in electroproduction experiments—has again led to investigations[11,12] into the relation between fixed poles and compositeness. The important point—emphasized particularly by Mueller and Trueman[5,6]—is that fixed poles[13] can occur in weak (photonic) amplitudes of both the right signature and the wrong signature in contrast with strong-interaction amplitudes, where fixed poles occur only for the wrong signature. Mueller and Trueman also discuss simple models for $\pi\gamma$ and $\pi\rho$ scattering in order to illustrate this point (Sec. V of Ref. 5). These models are based on analyticity and unitarity. However, the fundamental difference between the two types of processes—the different asymptotic behavior of the relevant vertex functions—is not clear from these models. The purpose of this note is to clarify this point by discussing a model of πV scattering which incorporates both types of processes simultaneously. Such a model is provided by a field-theoretic Lagrangian involving spinless and vector fields. It is well known that the vanishing of the vertex renormalization constant[14-16] of the interaction of the two kinds of fields implies the removal of this interaction from the Lagrangian—the appropriate particle (in our case the vector meson) then arises as a true composite state of the fields remaining in the Lagrangian. In our model the usual mathematical difficulties associated with many-particle intermediate states compel us to resort to the Bethe-Salpeter ladder approximation. In Sec. III we will even go one step further and confine ourselves to external mesons of mass zero. With these approximations the evaluation of the $\pi V \to \pi V$ scattering amplitude in our model is made particularly simple because it reduces to the calculation of the amplitude or vertex function of the Bethe-Salpeter equation of the Wick-Cutkosky model[17,18]; it is well known that this model may be solved exactly in the ladder approximation.[19,20]

A further motivation of our investigation is the vague manner in which fixed poles are frequently associated with the Born approximation (see, for instance, Refs. 21 and 22). Here we demonstrate that the Born terms—when present—ensure that the fixed-pole residues of right-signature amplitudes do not vanish, so that the insertion of a fixed J-plane pole can indeed be effected by the use of the bare Born terms.

Finally we give a brief discussion of the relation between fixed J-plane poles and the vector-meson-dominance hypothesis.

II. THE MODEL

We consider the t-channel process $\pi\pi \to VV$ with helicity indices as shown in Fig. 1; single lines indicate pions, double lines vector particles. The t-channel amplitude

$$F^t_{\mu_2\mu_4,\mu_1\mu_3}(s,t) = (1-z)^{|\mu-\mu'|/2}(1+z)^{|\mu+\mu'|/2} \times \bar{F}_{\mu_2\mu_4,\mu_1\mu_3}(s,t), \quad z = \cos\theta_t$$

can be shown[5] to possess signatured partial-wave amplitudes $F^{J\pm}$ which have the following form near fixed J-plane poles at $J_0 = n-1, n-2, \ldots,$ where $n = \max\{|\mu|, |\mu'|\}$, $\mu = \mu_1 - \mu_3$, $\mu' = \mu_2 - \mu_4$:

$$F^{J\pm}_{\mu_2\mu_4,\mu_1\mu_3}(t) - (s,u \text{ Born contributions})$$

$$\simeq \frac{C_{J-n}(\mu,\mu',J)}{J-J_0}\ \frac{1}{\pi}\int_{z_0}^{\infty} dz[\operatorname{Im}\bar{F}_{\mu_2\mu_4,\mu_1\mu_3}(z+i\epsilon,t) \mp (-1)^n \operatorname{Im}\bar{F}_{\mu_2\mu_4,\mu_1\mu_3}(-z+i\epsilon,t)]P_{n-J-1}(z). \quad (2.1)$$

The amplitudes $\bar{F}$ are as usual defined to be free of kinematic singularities in s and so are assumed to be analytic in s for fixed t. C_{J-n} are known[5] coefficients involving Clebsch-Gordan coefficients. In the process under discussion n may be as large as $2 = \sigma_2 + \sigma_4$. Now, considering $J \approx J_0$, $J_0 < n$, we have

$$P_{n-J-1}(z)\bar{F}_{\mu_2\mu_4,\mu_1\mu_3}(z,t) \sim z^{n-J_0-1-\tau} \text{ for } |z| \to \infty$$

if $\bar{F} \sim z^{-\tau}$. Thus for $|z| \to \infty$, this product falls off faster than $1/z$ provided $n - J_0 - \tau < 0$. In this case we have by Cauchy's theorem, the superconvergence relation

$$0 = \int_{-\infty}^{\infty} dz\, P_{n-J_0-1}(z)\bar{F}_{\mu_2\mu_4,\mu_1\mu_3}(z,t),$$

since the integrand is analytic (and, excepting Born terms, free of poles) in the upper half z plane. Reexpressed in terms of integrals over positive z this relation becomes

$$0 = \int_{z_0}^{\infty} dz[\operatorname{Im}\bar{F}_{\mu_2\mu_4,\mu_1\mu_3}(z+i\epsilon,t) + (-1)^{n-J_0-1}\operatorname{Im}\bar{F}_{\mu_2\mu_4,\mu_1\mu_3}(-z+i\epsilon,t)] \times P_{n-J_0-1}(z) + (s,u \text{ Born contributions}). \quad (2.2)$$

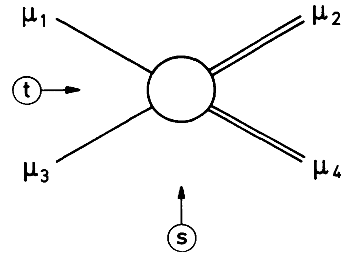

FIG. 1. The t-channel process $\pi\pi \to VV$.

We now observe that the residues of the fixed poles at $J = J_0$, i.e., the coefficient of $1/(J-J_0)$ in (2.1), may vanish as a result of the superconvergence relation (2.2), i.e., if the amplitude $\bar{F}_{\mu_2\mu_4,\mu_1\mu_3}$ has the appropriate asymptotic behavior. Thus, in order to understand when the residue of a fixed pole vanishes, it is necessary to study the asymptotic behavior of the appropriate amplitude. In our case this means that we have to examine the asymptotic behavior in s of the amplitude illustrated in Fig. 2, where the small circles denote

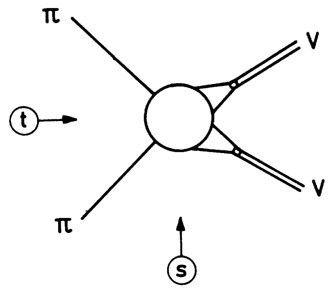

FIG. 2. Amplitude for the t-channel process $\pi\pi \rightarrow VV$.

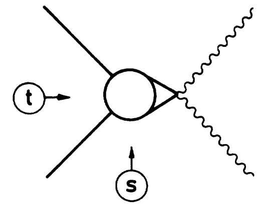

FIG. 4. The additional diagram necessary for Compton scattering.

$\pi\pi V$ vertex functions. Our method of procedure will be to consider this diagram as known if the production amplitude represented by the six-point diagram in Fig. 3 is known. Such a procedure is plausible if the vector particles are composite, that is, if they correspond to (moving) Regge poles, because compositeness implies a relation between the particle and a two-particle amplitude with the same quantum numbers, and so any result which makes use of this fact must include processes in which the particle reacts through the virtual two-particle state. If both vector particles are elementary in the sense that pole terms corresponding to them have to be introduced *ab initio*, the diagram of Fig. 2 is supplemented by the diagram of Fig. 4. In photoproduction, of course, the seagull does not arise.

The complexity of the above diagrams in general compels us to reduce them to a tractable model. This model which we shall consider is defined by the customary Bethe-Salpeter ladder diagrams or elastic unitarity approximation and may be represented by the diagram in Fig. 5. We observe that the contribution derived from P-wave poles in both ladders may be represented in terms of vertex functions as shown in Fig. 6. Since we consider the s-channel process $\pi V \rightarrow \pi V$ for high energies $\sqrt{s}$ it is clear that the asymptotic behavior of our model amplitude is determined by that of four- or three-point functions having one leg far off the mass shell. For this reason we consider now the asymptotic behavior of these functions when one of the masses becomes infinite. For motivations different from ours the calculation of the vertex or three-point function has also been considered by Furlan and Mahoux[23]; in the case of the four-point function we refer to some work of Seto.[19] But here we are using mainly the general results of Ref. 20.

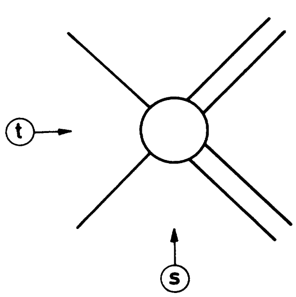

FIG. 3. The six-point amplitude.

III. THE VERTEX FUNCTION

Our interaction Lagrangian contains parts $\mathcal{L}_{\rm int}^{(0,1)}$. Here

$$\mathcal{L}_{\rm int}^{(0)} = \sum_{i=1,2} g_i \phi_i \phi_i^* \xi$$

represents the interaction of two pion fields $\phi_{1,2}$ of masses $m_{1,2}$ (which we shall occasionally assume to be equal, i.e., $m_{1,2} = m$) and a gluon scalar field ξ of mass μ which we take to be zero for calculational simplicity.[24] The other part, $\mathcal{L}_{\rm int}^{(1)}$, represents the interaction between the pion fields $\phi_{1,2}$ and a field $\psi_{\mu\nu\ldots}$ of spin L_S. In the following we are particularly interested in the case $L_S = 1$. In this case $\mathcal{L}_{\rm int}^{(1)}$ may be written

$$\mathcal{L}_{\rm int}^{(1)} = -ig[\phi_1 \partial_\mu \phi_2^* - (\partial_\mu \phi_1)\phi_2^*]\psi_\mu + g^2 \phi_1 \phi_2 \psi^\mu \psi_\mu + \text{H.c.} \tag{3.1}$$

g_i and g are the relevant coupling constants of $\mathcal{L}_{\rm int}^{(0,1)}$, respectively.

We now consider the formation of a bound state of spin L_S in the two-particle amplitude describing the scattering of the mesons $\phi_{1,2}$ by exchange of

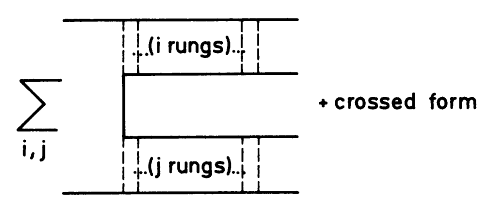

FIG. 5. The model of our six-point function.

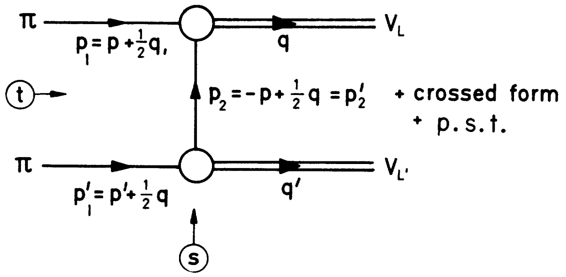

FIG. 6. The model of our four-point function.

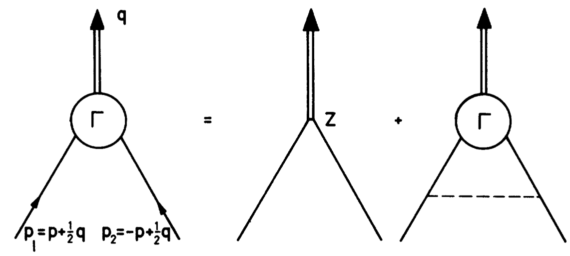

FIG. 7. Diagrammatic form of the vertex equation.

scalar mesons ξ. For convenience we set

$$p = \tfrac{1}{2}(p_1 - p_2),\quad q = p_1 + p_2,\quad \lambda = g_1 g_2/4\pi^2, \tag{3.2}$$

p_1, p_2 being the four-momenta of the two mesons of mass $m_{1,2}$, respectively. The diagrammatic form of the vertex equation in ladder approximation is shown in Fig. 7. Here the double line denotes the particle of spin L_S. The point vertex Z denotes the derivative coupling of the field $\psi_{\mu\nu\cdots}$ to the pair of massive mesons. The second diagram on the right of Fig. 7 describes the production of the particle of spin L_S as the bound state of a pair of massive mesons interacting by exchange of a massless scalar meson denoted by a dashed line. The vertex function $\Gamma_{\mu\nu\cdots}$ is therefore a tensor of rank L_S. The point vertex, denoting the bare derivative coupling, is given by the factor [25]

$$g(p_1 - p_2)_\mu \quad \text{if } L_S = 1, \tag{3.3}$$

and in general

$$g[(p_1 - p_2)_\mu (p_1 - p_2)_\nu \cdots]$$

(totally symmetrized). If $\epsilon_{\mu\nu\cdots}$ is the polarization vector of the outgoing particle of spin L_S, we write

$$\Gamma(p, q) = \epsilon^{\mu\nu\cdots}\,\Gamma_{\mu\nu\cdots}$$

and

$$Z = \epsilon^{\mu\nu\cdots} g[(p_1 - p_2)_\mu (p_1 - p_2)_\nu \cdots] \equiv \rho |p|^{L_S}. \tag{3.4}$$

The vertex equation multiplied by the polarization vector then becomes (using the metric $g_{00} = -1$, $g_{ii} = +1$ for $i = 1, 2, 3$)

$$\Gamma(p, q) = Z + \frac{\lambda}{4\pi^2 i} \times \int \frac{\Gamma(p', q)\, d^4p'}{[(p' + \frac{1}{2}q)^2 + m^2][(p' - \frac{1}{2}q)^2 + m^2](p - p')^2}. \tag{3.5}$$

We now recall that the theory defined by the Lagrangian $\mathcal{L}^{(0)}_{\text{int}}$ is superrenormalizable, whereas that defined by $\mathcal{L}^{(1)}_{\text{int}}$ is renormalizable if the field ψ_μ is massless (the renormalizability of theories of charged spin-zero bosons interacting with the electromagnetic field has been proved by Salam[26]). The renormalizability of the theory defined by $\mathcal{L}^{(1)}_{\text{int}}$ is, however, of no importance here. Thus, interpreting Γ and Z now as the physical, renormalized quantities, the condition $Z = 0$ implies the removal of the renormalized interaction $\mathcal{L}^{(1)}_{\text{int}}$ from the (renormalized) Lagrangian without implying $g = 0$. The particle of spin L_S, if it exists, must then be contained in the theory as a composite structure of the remaining fields in the Lagrangian.

We note that the Bethe-Salpeter amplitude Φ_{BS} for the binding of the two pions to a bound state of four-momentum q, i.e.,

$$\Phi_{\text{BS}} = \langle 0 | \phi_1(p_1)\phi_2(p_2) | q \rangle$$

is related to the vertex function Γ by the equation

$$\Phi_{\text{BS}} = \frac{\Gamma_{\text{BS}}(p_1, p_2)}{(p_1^2 + m_1^2)(p_2^2 + m_2^2)}. \tag{3.6}$$

Here Γ_{BS} satisfies the homogeneous part of the above vertex equation.

As usual it is convenient to make the Wick rotation which moves the integration contour of the relative energy variable to the imaginary axis and also continues the external relative energy to imaginary values (we assume the validity of this step,[27] i.e., analyticity of Γ in the lower p^2 plane and nondiverging behavior at infinity for $Z = 0$; for $Z \neq 0$ we are interested only in the asymptotic behavior of Γ, and we assume that this may be determined in either metric). We therefore work in terms of a Euclidean metric.

For $q = 0$, Eq. (3.5) is seen to possess complete four-dimensional rotational symmetry in the p space; its solutions then transform like the basis vector of an irreducible representation of the four-dimensional rotation group O(4), i.e., like the four-dimensional spherical harmonics. It is convenient and instructive to deal with this case first. It will also provide us with the zero-order terms of the perturbation method discussed in the last part of this section.

The solutions $\Gamma(p, 0)$ of (3.5) may then be written

$$\Gamma_{Llm}(p)=\Gamma_L(|p|)H_{Llm}(\psi, \theta, \varphi), \tag{3.7}$$

where

$$H_{Llm}(\psi, \theta, \varphi)=A_{Ll}(\sin\psi)^l C_{L-l}^{l+1}(\cos\psi)\, Y_{lm}(\theta, \varphi) \tag{3.8}$$

and

$$|A_{Ll}|^2=\frac{1}{\pi(L+l+1)!}\, 2^{2l+1}(L+1)(L-l)!(l!)^2, \quad |m|\leq l\leq L$$

so that

$$\int_0^\pi \sin^2\psi\, d\psi \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi |H|^2=1 .$$

Here (ψ, θ, φ) are the polar angles of p in the four-dimensional Euclidean space, and C_L^l denotes a Gegenbauer polynomial. Similarly we have

$$Z_{Llm}=|p|^L Z_L H_{Llm}, \tag{3.9}$$

where the factor $|p|^L$ [see (3.4)] has been extracted for later convenience. Using the expansions

$$\frac{1}{(p-p')^2}=\sum_{L=0}^\infty C_L^1\left(\frac{p\cdot p'}{|p||p'|}\right)R_L(|p|, |p'|),$$

$$R_L(|p|, |p'|)=\left[\frac{|p|^L}{|p'|^{L+2}}\,\theta(|p'|-|p|)+\frac{|p'|^L}{|p|^{L+2}}\,\theta(|p|-|p'|)\right], \tag{3.10}$$

$$C_L^1\left(\frac{p\cdot p'}{|p||p'|}\right)=\frac{2\pi^2}{L+1}\sum_{l=0}^L\sum_{m=-l}^l H_{Llm}(\psi, \theta, \varphi)\times H_{Llm}^*(\psi', \theta', \varphi'), \tag{3.11}$$

we see that Eq. (3.5) leads to the following integral equation for the partial-wave vertex function $\Gamma_L(|p|)$:

$$\Gamma_L(|p|)=|p|^L Z_L+\frac{\lambda}{2(L+1)}\int_0^\infty d|p'||p'|^3 R_L(|p|, |p'|)\times\frac{\Gamma_L(|p'|)}{(p'^2+m^2)^2} . \tag{3.12}$$

This integral equation is equivalent to a differential equation supplemented by two boundary conditions. By differentiation of (3.12) these are found to be

$$\left(\frac{d^2}{d|p|^2}+\frac{3}{|p|}\,\frac{d}{d|p|}-\frac{L(L+2)}{|p|^2}+\frac{\lambda}{(|p|^2+m^2)^2}\right)\Gamma_L(|p|)=0 \tag{3.13}$$

and since $L\geq 0$,

$$\lim_{|p|\to 0} |p|^{L+1}\Gamma_L(|p|)=C_0,$$
$$\lim_{|p|\to\infty} \Gamma_L(|p|)/|p|^L\sim\begin{cases} Z_L & \text{if } Z_L\neq 0 , \\ \dfrac{C_\infty}{|p|^{2L+2}} & \text{if } Z_L=0, \end{cases} \tag{3.14}$$

C_0, C_∞ being constants. We note that (3.13) is invariant under the interchange $L\to -L-2$. Thus if a solution Γ_L is known, we also know another solution, i.e., Γ_{-L-2}. A solution of this equation satisfying the first of the boundary conditions (3.14) is

$$\Gamma_L(|p|)=A|p|^L F\left(\alpha, 1-\alpha;\, L+2;\, \frac{p^2}{p^2+m^2}\right) , \tag{3.15}$$

where

$$\alpha=\tfrac{1}{2}\left[1+\left(1+\frac{\lambda}{m^2}\right)^{1/2}\right] \tag{3.16}$$

and A is a normalization constant. Using the appropriate expansion of the hypergeometric function F, it can be shown that (3.15) satisfies also the second of the boundary conditions (3.14). [Note that for $m_1\neq m_2$ the solution cannot be expressed in the simple form of (3.15); instead a perturbation expansion such as that described in Ref. 20 would have to be derived.] From the second of the relations (3.14) and Gauss's formula we obtain

$$Z_L=AF(\alpha, 1-\alpha;\, L+2;\, 1)=A\,\frac{\Gamma(L+2)\Gamma(L+1)}{\Gamma(L+2-\alpha)\Gamma(L+1+\alpha)} . \tag{3.17}$$

The constant A may be determined by the usual condition[28] $\Gamma(p^2=-m^2)=1$. Then, introducing the (infrared) cutoff[29] ϵ defined by $\epsilon=p^2+m^2$, one finds

$$Z_L=\frac{(m^2/\epsilon)^{1-\alpha}}{(-m^2/\epsilon)^{L/2}}\,\frac{\Gamma(\alpha)\Gamma(L+1)}{\Gamma(2\alpha-1)\Gamma(L+2-\alpha)} . \tag{3.18}$$

Of course the normalization should not affect the physical content of Z_L. That this is the case may be seen by choosing $A=1$ in (3.17), i.e., the normalization $(\Gamma_L/|p|^L)_{|p|=0}=1$. The zeros of Z_L in this case, i.e.,

$$\left.\begin{aligned} L+1+\alpha&=-n \\ L+2-\alpha&=-n' \end{aligned}\right\}\; n, n'=0, 1, 2, \ldots \tag{3.19}$$

are also the zeros of (3.18), i.e.,

$$L+2-\alpha=-n' ,$$
$$2\alpha-1=-(n-n'),$$

provided $n \geq n'$. The eigenvalue conditions (3.19) may be written

$$\frac{\lambda}{4m^2} = (L+n+1)(L+n+2)\,. \tag{3.20}$$

Defining $s = -p_2^{\,2}$ and considering the particle of mass m_1 to be on the mass shell, i.e., $p_1^{\,2} = -m_1^{\,2}$, it is not possible to derive the large s (i.e., m_2 off-shell) behavior of the vertex function Γ from the relations (3.7) and (3.15). The reason is that, since

$$p^2 = -\tfrac{1}{4}q^2 - \tfrac{1}{2}(s - p_1^{\,2})$$

and (3.21)

$$p\cdot q = \tfrac{1}{2}(s + p_1^{\,2}),$$

as is easily shown, the limit $q_\mu = 0$ implies $s = -p_1^{\,2} = m_1^{\,2}$. Thus, if each component q_μ is zero, s has a fixed, finite value. It is essential therefore, in order to obtain the large-s asymptotic behavior of Γ, to consider the case $q_\mu \neq 0$. The calculations are described in Ref. 20. For $q_\mu \neq 0$ and masses m_1 and m_2 for the fields φ_1 and φ_2, we can write[20]

$$\Gamma(p,q) = K(s_1, s_2)\tilde{\Gamma}(\tilde{p})$$

and (3.22)

$$Z = K(s_1, s_2)\tilde{Z}\,,$$

where $\tilde{\Gamma}(\tilde{p})$ is the solution of the equation

$$\tilde{\Gamma}(\tilde{p}) = \tilde{Z} + \frac{\tilde{\lambda}}{4\pi^2}\int \frac{d^4\tilde{p}'\,\tilde{\Gamma}(\tilde{p}')}{(\tilde{p}-\tilde{p}')^2[(1+\tilde{\Delta})^2 + \tilde{p}'^2][(1-\tilde{\Delta})^2 + \tilde{p}'^2]}\,, \tag{3.23}$$

with

$$s_i = p_i^{\,2} + m_i^{\,2} \quad (i=1,2), \qquad \tilde{\lambda} = \lambda\Big/\left[\left(\frac{m_1+m_2}{2}\right)^2 - \eta^2\right],$$

$$q_\mu = -2i\eta_\mu\,, \qquad \xi^2 = |\xi|^2 = \tfrac{1}{2}(m_1^{\,2} + m_2^{\,2} - 2\eta^2), \tag{3.24}$$

and

$$\tilde{\Delta}^2 = \frac{(m_1-m_2)^2 - 4\eta^2}{(m_1+m_2)^2 - 4\eta^2}\,, \qquad \Delta = \tfrac{1}{4}(m_1^{\,2} - m_2^{\,2}),$$

$$M = 2\xi^2/[(m_1+m_2) + (m_1-m_2)\tilde{\Delta}],$$

$$K(s_1,s_2) = \frac{4\xi^2(\Delta^2 - \eta^2\xi^2)^{1/2}M^2\{[\tfrac{1}{2}(m_1+m_2)]^2 - \eta^2\}}{\{s_1[(\Delta^2-\eta^2\xi^2)^{1/2} - \Delta + \xi^2] + s_2[(\Delta^2-\eta^2\xi^2)^{1/2} - \Delta - \xi^2]\}}\,,$$

$$\tilde{p}^2 = \frac{\xi^2}{M^2}\left(\frac{s_1[(\Delta^2-\eta^2\xi^2)^{1/2} + \Delta - \xi^2] + s_2[(\Delta^2-\eta^2\xi^2)^{1/2} + \Delta + \xi^2]}{s_1[(\Delta^2-\eta^2\xi^2)^{1/2} - \Delta + \xi^2] + s_2[(\Delta^2-\eta^2\xi^2)^{1/2} - \Delta - \xi^2]}\right). \tag{3.25}$$

We now observe that Eq. (3.23) is O(4)-symmetric in the $\tilde{p}$ space. Its solutions may therefore be written

$$\tilde{\Gamma}(\tilde{p}) \to \tilde{\Gamma}_{Llm}(\tilde{p}) = \Gamma_L(|\tilde{p}|)H_{Llm}(\tilde{\psi}, \tilde{\theta}, \tilde{\varphi}), \tag{3.26}$$

where $\tilde{\psi}, \tilde{\theta}, \tilde{\varphi}$ are the polar angles of $\tilde{p}$ in the four-dimensional Euclidean space. Similarly we write

$$\tilde{Z} \to \tilde{Z}_{Llm} = |\tilde{p}|^L \tilde{Z}_L H_{Llm}(\tilde{\psi}, \tilde{\theta}, \tilde{\varphi})\,. \tag{3.27}$$

Proceeding as before we have

$$\left(\frac{d^2}{d|\tilde{p}|^2} + \frac{3}{|\tilde{p}|}\frac{d}{d|\tilde{p}|} - \frac{L(L+2)}{|\tilde{p}|^2} + \frac{\tilde{\lambda}}{[|\tilde{p}|^2 + (1+\tilde{\Delta})^2][|\tilde{p}|^2 + (1-\tilde{\Delta})^2]}\right)\tilde{\Gamma}_L(|\tilde{p}|) = 0 \tag{3.28}$$

and

$$\lim_{|\tilde{p}|\to 0} |\tilde{p}|^{L+1}\tilde{\Gamma}_L(|\tilde{p}|) = \tilde{C}_0\,,$$

$$\lim_{|\tilde{p}|\to\infty} \tilde{\Gamma}_L(|\tilde{p}|)/|\tilde{p}|^L \sim \begin{cases} \tilde{Z}_L & \text{if } \tilde{Z}_L \neq 0\,, \\ \dfrac{\tilde{C}_\infty}{|\tilde{p}|^{2L+2}} & \text{if } \tilde{Z}_L = 0\,, \end{cases} \tag{3.29}$$

$\tilde{C}_0, \tilde{C}_\infty$ being constants. In Ref. 20 Eq. (3.28) is solved by a perturbation method.[30] We obtain the solution

$$\tilde{\Gamma}_L(|\tilde{p}|) = \tilde{A}|\tilde{p}|^L \mathfrak{F}_L\left(\tilde{\alpha}, \frac{\tilde{p}^2}{\tilde{p}^2+1}\right)\,, \tag{3.30}$$

where

$$\mathfrak{F}_L\left(\tilde{\alpha}, \frac{\tilde{p}^2}{\tilde{p}^2+1}\right) = \left[F\left(\tilde{\alpha}+\delta\tilde{\Delta}^2, 1-\tilde{\alpha}-\delta\tilde{\Delta}^2; L+2; \frac{\tilde{p}^2}{\tilde{p}^2+1}\right)\right.$$
$$\left. + \sum_{k=1}^{\infty} \tilde{\Delta}^{2k} \sum_{\substack{j=-2k \\ j\neq 0}}^{2k} P_{2k}(n,j) F\left(\tilde{\alpha}+\delta\tilde{\Delta}^2+j, 1-\tilde{\alpha}-\delta\tilde{\Delta}^2-j; L+2; \frac{\tilde{p}^2}{\tilde{p}^2+1}\right)\right] \tag{3.31}$$

and $n=0, 1, 2, 3, \ldots$. $\tilde{A}$ is a normalization constant, F is again the hypergeometric function, and P_{2k} are coefficients which are given in Ref. 20. The expansion (3.31) has a circle of convergence around $\tilde{\Delta}=0$. Jointly with (3.31) we obtain an eigenvalue expansion which may be solved for the Toller poles, $L=\alpha_n$:

$$\alpha_n = -n-2+\tilde{\alpha} + \frac{\tilde{\lambda}\tilde{\Delta}^2[(n+1)(2\tilde{\alpha}-n-1)(2\tilde{\alpha}-3)+n(2\tilde{\alpha}-n-2)(2\tilde{\alpha}+1)]}{4(2\tilde{\alpha}-3)(2\tilde{\alpha}-1)^2(2\tilde{\alpha}+1)} + O(\tilde{\Delta}^4), \tag{3.32}$$

where, according to Ref. 20,

$$\tilde{\alpha} = \tfrac{1}{2}\left[1+\left(1+\frac{4\lambda}{(m_1+m_2)^2-4\eta^2}\right)^{1/2}\right].$$

The relation between the Toller poles and the Regge poles $\alpha_{n\mu}$ is

$$\alpha_{n\mu}(\eta^2) = \alpha_n(\eta^2) - \mu; \quad n, \mu = 0, 1, 2, \ldots . \tag{3.33}$$

Knowing $\tilde{\Gamma}_L$, i.e., (3.30), we may derive $\tilde{Z}_L$ by (3.29):

$$\tilde{Z}_L = \tilde{A}\mathfrak{F}_L(\tilde{\alpha}, 1)$$
$$= \tilde{A}\frac{\Gamma(L+2)\Gamma(L+1)}{\Gamma(L+2-\tilde{\alpha}-\delta\tilde{\Delta}^2)\Gamma(L+1+\tilde{\alpha}+\delta\tilde{\Delta}^2)}\left[1+\sum_{k=1}^{\infty}\tilde{\Delta}^{2k}\sum_{\substack{j=-2k\\ j\neq 0}}^{2k} P_{2k}(n,j)\frac{\Gamma(L+2-\tilde{\alpha}-\delta\tilde{\Delta}^2)\Gamma(L+1+\tilde{\alpha}+\delta\tilde{\Delta}^2)}{\Gamma(L+2-j-\tilde{\alpha}-\delta\tilde{\Delta}^2)\Gamma(L+1-j+\tilde{\alpha}+\delta\tilde{\Delta}^2)}\right] \tag{3.34}$$

by Gauss's formula. We observe that

$$\tilde{Z}_L = 0 \text{ for } \left.\begin{matrix} L+2-\tilde{\alpha}-\delta\tilde{\Delta}^2 = -n \\ L+1+\tilde{\alpha}+\delta\tilde{\Delta}^2 = -n' \end{matrix}\right\}, \quad n, n' = 0, 1, 2, \ldots . \tag{3.35}$$

Substituting (3.30) into (3.22) and with (3.34) we have an expansion of the vertex function for $q_\mu \neq 0$. We may therefore proceed to investigate its large-s asymptotic behavior.

IV. OFF-MASS SHELL BEHAVIOR OF THE VERTEX FUNCTION

In this section we consider the behavior of the vertex function $\Gamma(p, q)$ when one of the ingoing legs is off the mass shell.

We consider first *the case* $Z=0$. In this case we have the eigensolutions

$$\tilde{\Gamma}_L(\tilde{p}) = \tilde{A}|\tilde{p}|^L \mathfrak{F}_L\left(\tilde{\alpha}, \frac{\tilde{p}^2}{\tilde{p}^2+1}\right), \tag{4.1}$$

where L and $\tilde{\alpha}$ are related to each other via the eigenvalue relation (3.32) and so are restricted to certain values characterized by an integer $n=0, 1, 2, \ldots$. Substituting

$$\tilde{\Gamma} \to \tilde{\Gamma}_{Llm}(\tilde{p})$$
$$= \tilde{\Gamma}_L(|\tilde{p}|)H_{Llm}(\tilde{\psi}, \tilde{\theta}, \tilde{\varphi})$$

into (3.22), we obtain

$$\Gamma(p, q) = K(s_1, s_2)\tilde{A}|\tilde{p}|^L \mathfrak{F}_L\left(\tilde{\alpha}, \frac{\tilde{p}^2}{\tilde{p}^2+1}\right)$$
$$\times H_{Llm}(\tilde{\psi}, \tilde{\theta}, \tilde{\varphi}) . \tag{4.2}$$

In general the normalization constant $\tilde{A}$ may be a function of s_1 and s_2. Thus

$$\tilde{A} = \tilde{A}(s_1, s_2) .$$

Normalizing $\Gamma(p, q)$ to 1 on the mass shell of the mesons, we may fix the value of $\tilde{A}(0, 0)$. Due to the s_2 dependence of K in (4.2) this constant depends strongly on the infrared cutoff ϵ. In order to be able to determine the s_2 dependence of A, we require additional information on the behavior of $\Gamma(p, q)$. In the second paper of Ref. 20, it is shown that the four-point function T behaves at most like $1/p^2$ when the particle of mass m_2 is off the mass shell and $|s_2| \to \infty$. In going to the pole of mass $q^2 = -m_L{}^2$ in the Lth O(4) partial wave of the four-point amplitude T as described by Fig. 8, T is related to $\tilde{\Gamma}_V$ by the relation

$$T \propto \frac{1}{p^2}\bar{T}, \quad \bar{T} \to \bar{T}_L \propto \frac{\tilde{\Gamma}_L(\tilde{p})\tilde{\Gamma}_L(\tilde{p}')C_L^1}{q^2+m_V{}^2} .$$

At the pole the entire p dependence of T is contained in Γ, which may therefore have the same asymptotic behavior, i.e., $1/p^2$. Hence, in the limit $|s_2| \to \infty$

$$\tilde{A} = \tilde{A}_\epsilon ,$$

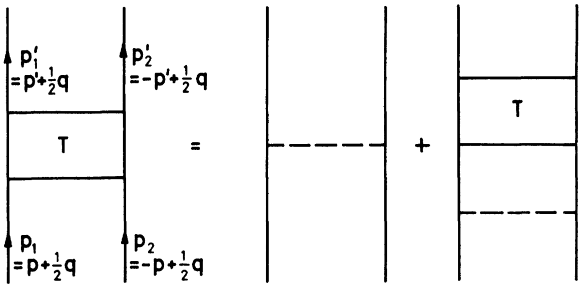

FIG. 8. The integral equation of the four-point function.

where $\tilde{A}_\epsilon$ is an infrared cutoff-dependent constant. Hence

$$\Gamma(p,q)=O(1/p^2), \tag{4.3}$$

where $p^2 \simeq -\frac{1}{2}s$ for $s\to\infty$.

Next we consider *the case* $Z\neq 0$. In this case we may again use the solution (3.30), provided n is now regarded as a parameter $n(L,\tilde{\alpha},\tilde{\Delta})$, which is found by solving the eigenvalue equation (3.32) for n.[30] Then from (3.34)

$$\tilde{A}=\tilde{Z}_L/\mathfrak{F}_L(\tilde{\alpha},1)$$

so that

$$\tilde{\Gamma}_L(|\tilde{p}|)=\tilde{Z}_L|\tilde{p}|^L\,\frac{\mathfrak{F}(\tilde{\alpha},\tilde{p}^2/(\tilde{p}^2+1))}{\mathfrak{F}_L(\tilde{\alpha},1)}\ . \tag{4.4}$$

With the use of Eqs. (3.22), (3.26), and (3.27), we find that the vertex function can be written for $Z\neq 0$ as

$$\Gamma(p,q)=Z\,\frac{\mathfrak{F}(\tilde{\alpha},\tilde{p}^2/(\tilde{p}^2+1))}{\mathfrak{F}(\tilde{\alpha},1)}\ . \tag{4.5}$$

Combining the results for the case where $Z=0$, Eq. (4.3), with those when $Z\neq 0$, we may write

$$\Gamma_{Llm}(p,q)\simeq Z_{Llm}h_L+\gamma_L/s \text{ for } s\to\infty, \tag{4.6}$$

where h_L and γ_L are independent of s.

V. FIXED POLES; DISCUSSION

We now return to our original problem and consider the contribution of Fig. 6. We denote by V_L, $V_{L'}$ the outgoing vector particles, which are characterized by the quantum numbers L, l, m and L', l', m', respectively. If we take the axis of quantization for V_L, z to be opposite to the direction of particle $V_{L'}$ as seen in the rest frame of V_L and z' to be opposite to the direction of particle V_L as seen in the rest frame of $V_{L'}$, then $m=\mu_2$ and $m'=\mu_4$. In this case the contributions of Fig. 6 to the transition amplitude for the t-channel process $\pi\pi\to V_L V_{L'}$ amount to

$$F^t_{mm',00}=\left(\Gamma_{Llm}(p,q)\frac{1}{(p_1-q)^2+m_2^2}\,\Gamma_{L'l'm'}(p_1',q')+(q\leftrightarrow q')\right)+\text{(possible seagull terms)}, \tag{5.1}$$

where $\Gamma_{Llm}(p,q)$ is the $\pi\pi V_L$ vertex function, and $s=-(p_1-q)^2$, the square of the total energy of the s channel, is assumed to be large and positive. Using (3.4) and (4.6), the helicity amplitude $F^t_{1-1,00}$ for vector fields is given by

$$F^t_{1-1,00}=\left[(\sqrt{2}Gp^t\sin\theta_t+\gamma_1/|p|^2)\,\frac{1}{m_2^2-s}\,(\sqrt{2}G'p^t\sin\theta_t+\gamma_1'/|p|^2)+(q\leftrightarrow q')\right]$$

$$=\left[(\sqrt{2}Gp^t\sin\theta_t+\gamma_1/|p|^2)(\sqrt{2}G'p^t\sin\theta_t+\gamma_1'/|p|^2)\left(\frac{1}{m_2^2-s}+\frac{1}{m_2^2-u}\right)\right]\ ,$$

where $G=h_LZ_L$, $p^{t2}+m_1^2=\frac{1}{4}t$, and θ_t is the scattering angle in the t-channel center-of-mass system. It should be noted that a gauge-invariant theory demands that both the s- and u-exchange diagrams as well as the seagull diagram shown in Fig. 9 be included and that the coupling constant for the seagull term be the square of the vertex coupling constant in the s- and u-exchange diagrams. It is clear from the invariant amplitude decomposition[1] that the seagull diagram, which is present only in one invariant amplitude, i.e., D of Ref. 1, does not contribute to either $F^t_{1-1,00}$ or $F^t_{10,00}$, which are the only helicity amplitudes that can have a fixed pole at $J=0$. Of course, the fact that gauge invariance relates the invariant amplitude containing the seagull contribution to those invariant amplitudes containing the exchange diagrams, i.e., their coupling constants are related, makes it somewhat ambiguous to say, that these t-channel helicity amplitudes are independent of the seagull diagram. This is more clearly illustrated by the fact that the s-channel helicity amplitude $F^s_{10,10}$ which is equal to $F^t_{1-1,00}$ for Compton scattering is given solely by the invariant amplitude containing the seagull diagram when t is zero (see Ref. 1). The situation here is analogous to the problem of the pion pole in $\gamma N\to\pi N$, where the pion-exchange diagram contributes due to gauge invariance only to s-channel but not t-channel helicity amplitudes.

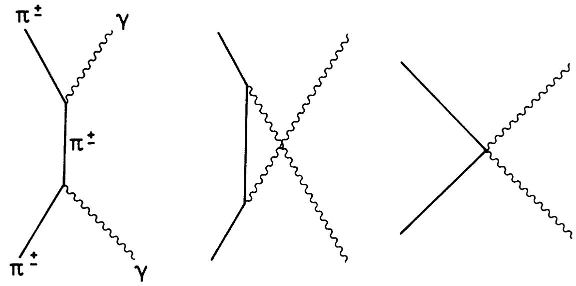

FIG. 9. Born diagrams and the "seagull" diagram.

It should also be noted that our model as shown in Fig. 2 (apart from possible seagull contributions) contains the exchange diagrams of Fig. 9 and thus should not be compared directly with Fig. 2 of Landshoff and Polkinghorne of Ref. 9. In particular, when the vector particles are elementary, i.e., Compton scattering, our model (apart from possible seagull contributions) is just the renormalized Born diagrams responsible for the so-called current-algebra fixed pole.

We now distinguish three cases:

(a) $F^t_{1-1,00} \sim$ constant if both $Z_1, Z_1 \neq 0$, i.e., in $\pi\pi \to \gamma\gamma$

(b) $F^t_{1-1,00} \sim 1/s^2$ if only one $Z_1 \neq 0$, i.e., in $\pi\pi \to \gamma\rho$,

and

(c) $F^t_{1-1,00} \sim 1/s^4$ if both $Z_1 = 0$, i.e., in $\pi\pi \to \rho\rho$.

Case (a) implies that the vector particle γ is elementary in the sense that it does not lie on a Regge trajectory and that its field is contained in the Lagrangian of the theory, like the photon field in quantum electrodynamics. Case (b), however, implies that the vector particle ρ, i.e., the ρ meson, is composite in the sense that it lies on a Regge trajectory—determined by (3.32) and (3.33)—and that its field is not contained in the (renormalized) Lagrangian. In case (c) neither vector particle is elementary—the Lagrangian reduces to that of the Wick-Cutkosky model.

We now return to our earlier discussion of fixed poles and consider the amplitude $F^t_{1-1,00}$. We have

$$\tilde{F}_{1-1,00}(s,t) = F^t_{1-1,00}(s,t)/(1-z_t^2)$$

$$\sim F^t_{1-1,00}/s^2 .$$

As noted before, the superconvergence relation (2.2) holds provided

$$\tilde{F} \sim s^{-\tau}$$

and

$$\tau > n - J_0, \quad J_0 = n-1, n-2, \ldots .$$

Thus, if $n=2$ (as in the case under discussion) we may have fixed poles at $J_0 = 1, 0, \ldots$ and $n - J_0$ is $1, 2, \ldots$. Because the initial state consists of two identical spinless mesons, the amplitudes of negative spin-parity are zero.

If the vector particles are elementary, i.e., photons, so that $Z_1 \neq 0$, irrespective of our model, the amplitudes of interest, i.e., $F^t_{1-1,00}$ and $F^t_{10,00}$, possess contributions coming from the Born exchange diagrams shown in Fig. 9. Thus when $Z_1 \neq 0$ fixed poles are possible at $J = 1, 0, \ldots$ in amplitudes of the right signature as well as in those of the wrong signature.

The situation is different, however, when $Z_1 = 0$. In this case the Born diagrams of Fig. 9 do not contribute or, expressed more precisely, these diagrams are now sums of ladder diagrams because only these imply a relation between the outgoing vector particles and two-particle amplitudes with the same quantum numbers, i.e., only these diagrams imply that the vector particles are composite and lie on Regge trajectories. In this case the asymptotic behavior of the amplitude discussed above is such as to ensure the vanishing of the integral in (2.1) for amplitudes of the right signature (i.e., even signature for the fixed pole at $J = 0$, odd signature for the fixed pole at $J = 1$).

For simplicity we assumed the meson fields in our model to be elementary. Since the pion is known to possess structure and to be a strongly interacting particle, it is essential to remove this assumption in a more realistic theory. This can in fact be done, e.g., in the framework of the quark model or a (nonrenormalizable) nonlinear spinor theory.[27,31] In this case the diagrams of Fig. 6 would be replaced by those of Fig. 10 (Q meaning quark). Here each of the vertices represents a vertex function which can be calculated in ladder approximation so that the asymptotic behavior can (in principle) be derived explicitly. Such a model would be related to models discussed recently by Drell and T. D. Lee,[11] S. Y. Lee,[11] and Brodsky *et al.*[12]

We conclude with some remarks on the vector-meson-dominance (VMD) hypothesis. The question is: Are fixed poles in conflict with the VMD model? We recall that the VMD hypothesis connects the hadronic electromagnetic current with

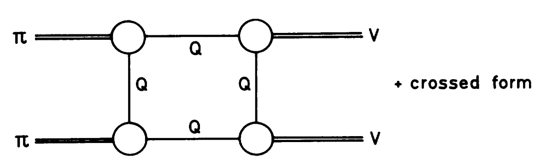

FIG. 10. Our model amplitude in a spinor theory.

the fields of the vector mesons ρ (isospin $I=1$) and ω, ϕ ($I=0$) which have the same quantum numbers as this current and the photon. Photonic processes—such as photoproduction and Compton scattering—are therefore related to vector-meson-induced strong interactions. Thus the VMD model takes into account only purely hadronic parts of the photon's interaction with a hadron; it leaves unaccounted the interaction between a bare photon and the hadron. Since all solely strongly interacting particles are believed to be composite and so to lie on (moving) Regge trajectories, those contributions to a scattering amplitude which result from fixed J-plane poles must be interpreted as due to interactions with noncomposite, i.e., elementary particles. One may picture the interaction in the manner of suri and Yennie.[32] According to this picture, the incoming photon in Compton scattering is composed of hadrons for a fraction of the time. At sufficiently high energies the points at which the photon changes from a bare elementary particle to a system of hadrons will lie within the target hadron, so that in this case the hadron-hadron interaction will be supplemented by an interaction between the target hadron and the bare photon. Clearly this latter interaction is not taken into account by the VMD hypothesis and so there are non-VMD contributions in Compton scattering, as has also been conjectured by Brodsky *et al.*[12] and Ezawa.[33] These bare photon contributions are, of course, directly related to the Born terms or fixed poles. Their contributions may again be subdivided into fixed-pole (pure Born term) and Regge-pole contributions.

*This work performed under the auspices of the U. S. Atomic Energy Commission Contract No. W-7405-Eng. 36.

†Permanent address: Fachbereich Physik, Universität Trier-Kaiserslautern, 675 Kaiserslautern, Germany.

[1] G. E. Hite and H. J. W. Müller, Phys. Rev. D 7, 3409 (1973).

[2] S. Mandelstam, Nuovo Cimento 30, 1113 (1963).

[3] S. Mandelstam, Nuovo Cimento 30, 1127 (1963).

[4] S. Mandelstam, Nuovo Cimento 30, 1148 (1963).

[5] A. H. Mueller and T. L. Trueman, Phys. Rev. 160, 1296 (1967).

[6] A. H. Mueller and T. L. Trueman, Phys. Rev. 160, 1306 (1967).

[7] H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. 167, 1441 (1967).

[8] H. D. I. Abarbanel, in Proceedings of the Regge Pole Conference, Irvine, California, 1969 (unpublished).

[9] M. Damashek and F. J. Gilman, Phys. Rev. D 1, 1319 (1970); C. A. Dominguez, J. F. Gunion, and R. Suaya, Phys. Lett. 31B, 365 (1970) and Phys. Rev. D 6, 1404 (1972); M. J. Creutz, S. D. Drell, and E. A. Paschos, Phys. Rev. 178, 2300 (1969); Y. Matsumoto, T. Muta, H. Nakajima, A. Niegawa, Y. Okumura, and T. Uematsu, Phys. Lett. 39B, 258 (1972); J. M. Cornwall, D. Corrigan, and R. E. Norton, Phys. Rev. D 3, 536 (1971); R. A. Brandt, P. Vinciarelli, and M. Breidenbach, Phys. Lett. 40B, 495 (1972); T. P. Cheng and Wu-Ki Tung, Phys. Rev. Lett. 24, 851 (1970); M. Elitzur, Phys. Rev. D 3, 2166 (1971); H. F. Jones, Nuovo Cimento Lett. 4, 545 (1970); P. V. Landshoff and J. C. Polkinghorne, Phys. Rev. D 5, 2056 (1972).

[10] R. A. Brandt, W. C. Ng, P. Vinciarelli, and G. Preparata, Nuovo Cimento Lett. 2, 937 (1971).

[11] S. D. Drell and T. D. Lee, Phys. Rev. D 5, 1738 (1972); S. Y. Lee, Nucl. Phys. B45, 449 (1972).

[12] S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D 5, 1384 (1972); 6, 177 (1972).

[13] A basic discussion of fixed poles and their properties may be found in P. D. B. Collins and E. J. Squires, *Regge Poles in Particle Physics* (Springer, New York, 1968).

[14] The original idea is due to A. Salam, Nuovo Cimento 25, 224 (1962) and Phys. Rev. 130, 1287 (1963).

[15] Many of the earlier references on this topic are reviewed by H. Osborn, Ann. Phys. (N.Y.) 47, 308 (1968).

[16] For some more recent investigations see T. Pradhan and J. N. Passi, Phys. Rev. 160, 1336 (1967); J. N. Passi, Nucl. Phys. B46, 497 (1972).

[17] G. C. Wick, Phys. Rev. 96, 1124 (1954); R. E. Cutkosky, *ibid.* 96, 1135 (1954).

[18] Reviews of the Bethe-Salpeter equation, involving the Wick-Cutkosky model, have been given by A. Pagnamenta, Colorado lectures, 1968 (unpublished), and N. Nakanishi, Prog. Theor. Phys. Suppl. 43, 1 (1969).

[19] K. Seto, Prog. Theor. Phys. 40, 605 (1968).

[20] G. E. Hite, H. J. W. Müller-Kirsten, and N. Vahedi, Nuovo Cimento 17A, 652 (1973); G. E. Hite and H. J. W. Müller-Kirsten, University of Kaiserslautern, report, 1973 (unpublished).

[21] Fayyazuddin, Riazuddin, and M. Ahmad, Nuovo Cimento Lett. 4, 633 (1970).

[22] J. Layssac and F. M. Renard, Nuovo Cimento 10A, 407 (1972).

[23] G. Furlan and G. Mahoux, Nuovo Cimento 36, 215 (1965).

[24] This restriction is of little significance in the context of the present investigation, although, of course, it necessitates the introduction of an infrared cutoff. In principle, the case $\mu \neq 0$ may be dealt with perturbation-wise by using the O(4)-symmetric solutions obtained for $\mu = 0$ as unperturbed solutions of the Bethe-Salpeter equations. For a discussion along these lines we refer to J. H. Connell, Phys. Rev. D 7, 1221 (1973).

[25] S. Schweber, *Introduction to Relativistic Quantum Field Theory* (Harper and Row, New York, 1961), p. 483.

[26] A. Salam, Phys. Rev. 96, 1124 (1954).

[27]Wick rotation of the solutions to Bethe-Salpeter equations has been stated to be justified in renormalizable theories by G. Domokos, P. Suranyi, and A. Vancura, Nucl. Phys. 60, 1 (1964).

[28]Note that

$$p^2 = \vec{p}^2 - p_0^2 = \vec{p}^2 + p_4^2 = |p|^2 > 0$$

in the four-dimensional Euclidean space. Thus in going to the point $p^2 = -m^2$, one has to make an analytic continuation to negative values of p^2; this can, of course, be done.

[29]ϵ is an infrared cutoff which comes in because the exchanged particle is assumed to have zero mass.

[30]For a similar procedure applied to the Yukawa potential see, for instance, H. J. W. Müller and K. Schilcher, J. Math. Phys. 9, 255 (1968).

[31]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1960) and 124, 246 (1961); H. J. W. Müller, Nucl. Phys. B21, 493 (1970).

[32]A. suri and D. R. Yennie, Ann. Phys. (N.Y.) 72, 243 (1972).

[33]Z. F. Ezawa, Phys. Rev. Lett. 27, 1092 (1971).

PHYSICAL REVIEW D VOLUME 9, NUMBER 4 15 FEBRUARY 1974

Theory of the electromagnetic structure functions of the proton

A. Baiquni* and A. O. Barut†
International Centre for Theoretical Physics, Trieste, Italy
(Received 31 August 1973)

The knowledge of the wave function of a relativistic composite system constitutes a complete description of its intrinsic properties such as mass spectrum, elastic and inelastic form factors, and structure functions. The wave functions of the proton obtained from the O(4,2) infinite-multiplet model, which were used previously to calculate mass spectra and form factors, are applied to reevaluate in a more complete manner the structure functions in closed form. The resultant scaling functions obey the Drell-Yan relation $F_2(\xi) \sim (1-\xi)^3$ and, under certain conditions, the Callan-Cross relation $F_2(\xi) \approx 2\xi F_1(\xi)$.

I. INTRODUCTION

A good model of the nucleon as a relativistic composite object must account for all its properties which are usually associated with the internal structure of the nucleon. It must give a good description of processes which are determined by the nucleon structure alone. Only then can we say that we have a good over-all picture of the nucleon. These properties are the elastic form factors, the spectrum of the excited states, the inelastic transition form factors, the decay rates of the excited states, and the structure functions. In ordinary quantum theory these properties are all determined by the wave function of the system. Hence the exact knowledge of the wave function constitutes a complete description of the system. Experimentally the intrinsic properties of the system are measured by probes which are themselves structureless. For the nucleons, the above-mentioned properties have been and are being analyzed from γN, eN, and νN scattering processes, and considerable information has been obtained, in particular through the inelastic electron-nucleon scattering.[1]

The purpose of this paper is to apply the explicit wave function of the proton and its excited states obtained from an infinite-component wave equation to evaluate the structure functions in inelastic electron-proton scattering. The wave function has previously been used to predict the elastic form factors,[2] the mass spectrum,[2,3] the inelastic form factors,[4] and the partial decay rates.[5] Some aspects of the structure functions have also been reported.[6-9] We present here the details of a more complete calculation, briefly reported earlier,[10] in particular the explicit form of the so-called scaling functions $F_1(\xi)$ and $F_2(\xi)$.

There is an underlying physical picture of the description of the proton by a wave equation. It corresponds to an atomic-type composite system.[11] The relativistic H atom itself, conversely, can be described completely by an infinite-component wave equation.[12] This picture and all the calculations indicated taken together lead to the conclusion that, as far as electromagnetic probes are concerned, the proton, in a very wide range of energy and momentum transfer, behaves like an "atom," the inelastic process proceeding via the excitation of the "atom" (including continuum) and its subsequent decay. The limitations of the picture will come when particle production without the excitation of the proton will be a dominant process.