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SU(4) × SU(4) as an approximate symmetry of hadrons*

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The $(4,4^*)+(4^*,4)$ model of broken SU(4)×SU(4) as an approximate symmetry of hadrons is investigated. Spectral-function sum rules for scalar and pseudoscalar densities are derived in this model. Sum rules based on octet-type breaking of SU(3) at $q^2=0$ and $q^2=\infty$ are also obtained. It is shown that the $q^2=0$ sum rule rules out SU(2)×SU(2) as a good symmetry of the Hamiltonian, when the vacuum is approximately SU(3)-invariant, in the present model. Thus the problem of understanding SU(2)×SU(2) and SU(3) as approximate symmetries of the Hamiltonian persists as in the case of the popular $(3,3^*)+(3^*,3)$ model of broken SU(3)×SU(3). It is shown that the $q^2=\infty$ sum rule is consistent with the idea that SU(2)×SU(2) is a good symmetry of the Hamiltonian. A mass formula for charmed pseudoscalar mesons is also derived.

I. INTRODUCTION

SU(4)×SU(4) as an approximate symmetry of hadrons has recently been proposed by several authors. The motivation for such an idea comes from the recent developments in unified gauge theories of leptons and several attempts to incorporate hadrons in such theories.[1] In unified gauge theories with hadrons, in order to restore renormalizability and eliminate sizeable strangeness-changing neutral currents, a fourth quark has been introduced carrying charm quantum number in addition to the usual triplet of quarks. It is then natural to consider SU(4) as a possible approximate symmetry[2] of hadrons. This, however, poses problems because the known spectrum of hadrons seems to fall in SU(3) multiplets. A way out of this difficulty has been proposed by Dittner and Eliezer. They suggest SU(4)×SU(4) as an approximate symmetry[3] of the Hamiltonian of hadrons where the symmetry is realized by Goldstone bosons, and in the chiral limit the vacuum is only SU(3)-invariant. In this scheme, Dittner *et al.*[3,4] have been able to obtain a solution for the symmetry-breaking parameters which shows that both SU(2)×SU(2) and SU(3) are good symmetries of the Hamiltonian. Their solution also requires that the masses of the charmed mesons be large (~5 GeV), explaining why such particles, if they exist, have not yet been detected.

The purpose of the present paper is to analyze the breaking of SU(4)×SU(4) down to the isospin group SU(2) following a method[5] recently applied to SU(3)×SU(3). The basic idea of such an approach is to obtain constraints[6,7] on the symmetry-breaking parameters by studying the spectral-function sum rules for the scalar and pseudoscalar densities. In Sec. II of this paper, spectral-function sum rules for the scalar and pseudoscalar densities in the broken SU(4)×SU(4) model are derived. In Sec. III sum rules based on the assumption of octet-type breaking of SU(3) for the two-point functions are derived. It is shown that the broken-SU(3) sum rule for the pseudoscalar density constrains the symmetry-breaking parameters in such a way that SU(2)×SU(2) cannot be a good symmetry of the Hamiltonian if the vacuum is approximately SU(3)-invariant. This is in contradiction with the result of Dittner *et al.*[3,4] who claim SU(2)×SU(2) as well as SU(3) as good symmetries of the Hamiltonian. In Sec. IV sum rules are derived assuming the validity of octet-type breaking of asymptotic SU(3) symmetry[8] for the two-point functions. It is shown that the asymptotic sum rules are consistent with a solution where SU(2)×SU(2) is a good symmetry of the Hamiltonian. In this case our sum rule also yields a formula relating the masses [in SU(3) limit] of charmed and uncharmed pseudoscalar mesons in terms of only one symmetry-breaking parameter. The mass of the charmed pseudoscalar meson can be calculated, given the value of this parameter. It is shown, as emphasized by Dicus and Mathur,[9] that the charmed pseudoscalar meson can acquire mass of the order of 5 GeV only when the value of this parameter is close to −1. Finally, in Sec. V we conclude with a summary of our result.

II. EXACT SUM RULES

We begin with the Hamiltonian density of hadrons

$$H(x)=H_0(x)+\epsilon_0 u^0(x)+\epsilon_8 u^8(x)+\epsilon_{15}u^{15}(x)\,, \qquad (1)$$

where $H_0(x)$ is invariant under SU(4)×SU(4). The scalar densities $u^j(x)$ $(j=0,1,\ldots,15)$ together with the pseudoscalar densities $v^j(x)$ $(j=0,1,\ldots,15)$ transform according to the $(4,4^*)+(4^*,4)$ representation of chiral SU(4) and satisfy the following commutation relations

$$\begin{aligned}
&[Q^\alpha(t),u^j(\vec{x},t)]=if_{\alpha jk}u^k(\vec{x},t)\,,\\
&[Q^\alpha(t),v^j(\vec{x},t)]=if_{\alpha jk}v^k(\vec{x},t)\,,\\
&[Q_5^\alpha(t),u^j(\vec{x},t)]=-i\,d_{\alpha jk}v^k(\vec{x},t)\,,\\
&[Q_5^\alpha(t),v^j(\vec{x},t)]=i\,d_{\alpha jk}u^k(\vec{x},t)\,,
\end{aligned} \qquad (2)$$

where $\alpha=1,2,\ldots,15$, while j,k run from 0 to 15. The coefficients $f_{\alpha jk}$ and $d_{\alpha jk}$ can be calculated, and are tabulated in Ref. 3. The charges $Q^\alpha(t)$ and $Q_5^\alpha(t)$ are defined in terms of vector- and axial-vector-current densities

$$\begin{aligned}
Q^\alpha(t)&=-i\int d^3x\,V_4^\alpha(\vec{x},t)\,,\\
Q_5^\alpha(t)&=-i\int d^3x\,A_4^\alpha(\vec{x},t)\,.
\end{aligned} \qquad (3)$$

The vector- and axial-vector-current divergences are given by

$$\begin{aligned}
\partial_\mu V_\mu^\alpha(x)&=-i[Q^\alpha(t),H(x)]_{x_0=t}\,,\\
\partial_\mu A_\mu^\alpha(x)&=-i[Q_5^\alpha(t),H(x)]_{x_0=t}\,.
\end{aligned} \qquad (4)$$

From Eqs. (1) and (4), we get

$$\partial_\mu V_\mu^\alpha(x)=(\epsilon_8 f_{\alpha 8j}+\epsilon_{15}f_{\alpha 15j})u^j(x)\,, \qquad (5)$$

$$\partial_\mu A_\mu^\alpha(x)=-(\epsilon_0 d_{\alpha 0j}+\epsilon_8 d_{\alpha 8j}+\epsilon_{15}d_{\alpha 15j})v^j(x)\,. \qquad (6)$$

We define the two-point functions of scalar and pseudoscalar densities

$$\begin{aligned}
P_{jk}(q^2)&=i\int dx\,e^{iqx}\langle 0|T(v^j(x)v^k(0))|0\rangle\,,\\
S_{jk}(q^2)&=i\int dx\,e^{iqx}\langle 0|T(u^j(x)u^k(0))|0\rangle\,.
\end{aligned} \qquad (7)$$

These have the following Lehmann-Källén spectral representation[7]

$$\begin{aligned}
P_{jk}(q^2)&=\int dm^2\,\frac{\rho_{jk}(m^2,v)}{m^2+q^2}\,,\\
S_{jk}(q^2)&=\int dm^2\,\frac{\rho_{jk}(m^2,u)}{m^2+q^2}\,,
\end{aligned} \qquad (8)$$

where the spectral weight

$$\rho_{jk}(m^2,v)=(2\pi)^3\sum_n\langle 0|v^j(0)|n\rangle\langle n|v^k(0)|0\rangle \times\delta^4(p_n-p)\,,$$

with $p^2+m^2=0$ and $p_0>0$; similarly for the scalar density. Furthermore, both $\rho_{jk}(m^2,v)$ and $\rho_{jk}(m^2,u)$ are symmetric in j,k.

If we multiply the expressions for $P_{jk}(q^2)$ and $S_{jk}(q^2)$ in Eq. (8) by $\epsilon_0 d_{\alpha 0j}+\epsilon_8 d_{\alpha 8j}+\epsilon_{15}d_{\alpha 15j}$ and $\epsilon_8 f_{\alpha 8j}+\epsilon_{15}f_{\alpha 15j}$, respectively, and use Eqs. (5), (6) and the commutation relations (2), then in the limit $q\to 0$, we get the following exact sum rules:

$$(\epsilon_0 d_{\alpha 0j}+\epsilon_8 d_{\alpha 8j}+\epsilon_{15}d_{\alpha 15j})P_{jk} =-(d_{\alpha k0}\lambda_0+d_{\alpha k8}\lambda_8+d_{\alpha k15}\lambda_{15})\,, \qquad (9a)$$

$$(\epsilon_8 f_{\alpha 8j}+\epsilon_{15}f_{\alpha 15j})S_{jk}=f_{\alpha k8}\lambda_8+f_{\alpha k15}\lambda_{15}\,, \qquad (9b)$$

where

$$\begin{aligned}
&P_{jk}\equiv P_{jk}(0)\,,\quad S_{jk}\equiv S_{jk}(0)\,,\\
&\lambda_m\equiv\langle 0|u^m(0)|0\rangle\,,\quad m=0,8,15\,.
\end{aligned} \qquad (10)$$

It is convenient to define the following parameters:

$$\begin{aligned}
&a\equiv(\tfrac{2}{3})^{1/2}\frac{\epsilon_8}{\epsilon_0}\,,\quad b\equiv(\tfrac{2}{3})^{1/2}\frac{\lambda_8}{\lambda_0}\,,\\
&e\equiv\frac{1}{\sqrt{3}}\frac{\epsilon_{15}}{\epsilon_0}\,,\quad f\equiv\frac{1}{\sqrt{3}}\frac{\lambda_{15}}{\lambda_0}\,,\\
&\eta\equiv-\frac{\lambda_0}{\epsilon_0}\,.
\end{aligned} \qquad (11)$$

In terms of the symmetry-breaking parameters defined in Eq. (11), we get from Eqs. (9a) and (9b) the following sum rules not related by SU(2) symmetry:

$$P_{33}=\eta\,\frac{1+b+f}{1+a+e}\,, \qquad (12a)$$

$$P_{44}=\eta\,\frac{1-\frac{1}{2}b+f}{1-\frac{1}{2}a+e}\,, \qquad (12b)$$

$$P_{99}=\eta\frac{1+\frac{1}{2}b-f}{1+\frac{1}{2}a-e}\,, \tag{12c}$$

$$P_{1313}=\eta\frac{1-b-f}{1-a-e}\,, \tag{12d}$$

$$\sqrt{3}\,aP_{00}+\sqrt{2}\,(1-a+e)P_{08}+aP_{015}=\sqrt{3}\,\eta b\,, \tag{12e}$$

$$\sqrt{3}\,aP_{08}+\sqrt{2}\,(1-a+e)P_{88}+aP_{815}=\sqrt{2}\,\eta(1-b+f)\,, \tag{12f}$$

$$\sqrt{3}\,aP_{015}+\sqrt{2}\,(1-a+e)P_{815}+aP_{1515}=\eta b\,, \tag{12g}$$

$$\sqrt{6}\,eP_{00}+aP_{08}+\sqrt{2}\,(1-2e)P_{015}=\sqrt{6}\,\eta f\,, \tag{12h}$$

$$\sqrt{6}\,eP_{08}+aP_{88}+\sqrt{2}\,(1-2e)P_{815}=\eta b\,, \tag{12i}$$

$$S_{44}=\eta\frac{b}{a}\,, \tag{12j}$$

$$S_{99}=\eta\frac{b+4f}{a+4e}\,, \tag{12k}$$

$$S_{1313}=\eta\frac{b-2f}{a-2e}\,. \tag{12l}$$

The positivity condition in the Hilbert space implies that the diagonal quantities P_{jj} and S_{jj} must be greater than or equal to zero. These positivity conditions are nontrivial and rule out the solutions proposed by Dittner and Eliezer[3] where

$$a=-0.053\,,\quad e=-0.943\,,\quad f=0.265$$

or (13)

$$a=-1.17\,,\quad e=0.32\,,\quad f=-0.99$$

The unacceptability of the solutions (13) for the symmetry-breaking parameters has been noted earlier by Dicus and Mathur,[9] who have studied the spectral sum rules for the currents in SU(4) × SU(4) theory.

Our expressions for P_{jk} and S_{jk} in Eq. (12) simplify considerably if we replace a, b, and η by α, β, and γ, where

$$\alpha\equiv\frac{a}{1+e}\,,\quad \beta\equiv\frac{b}{1+f}\,,\quad \gamma\equiv\eta\frac{1+f}{1+e}\,. \tag{14}$$

In terms of the parameters α, β, and γ the sum rules in Eq. (12) take the form

$$P_{33}=\gamma\frac{1+\beta}{1+\alpha}, \tag{15a}$$

$$P_{44}=\gamma\frac{1-\frac{1}{2}\beta}{1-\frac{1}{2}\alpha}\,, \tag{15b}$$

$$P_{99}=\gamma\frac{\frac{1}{2}\beta+(1-f)/(1+f)}{\frac{1}{2}\alpha+(1-e)/(1+e)}\,, \tag{15c}$$

$$P_{1313}=\gamma\frac{-\beta+(1-f)/(1+f)}{-\alpha+(1-e)/(1+e)}\,, \tag{15d}$$

$$\sqrt{3}\,\alpha P_{00}+\sqrt{2}\,(1-\alpha)P_{08}+\alpha P_{015}=\sqrt{3}\,\gamma\beta\,, \tag{15e}$$

$$\sqrt{3}\,\alpha P_{08}+\sqrt{2}\,(1-\alpha)P_{88}+\alpha P_{815}=\sqrt{2}\,\gamma(1-\beta)\,, \tag{15f}$$

$$\sqrt{3}\,\alpha P_{015}+\sqrt{2}\,(1-\alpha)P_{815}+\alpha P_{1515}=\gamma\beta\,, \tag{15g}$$

$$\sqrt{6}\,eP_{00}+\alpha(1+e)P_{08}+\sqrt{2}\,(1-2e)P_{015}=\sqrt{6}\,\gamma f\,\frac{1+e}{1+f}\,, \tag{15h}$$

$$\sqrt{6}\,eP_{08}+\alpha(1+e)P_{88}+\sqrt{2}\,(1-2e)P_{815}=\gamma\beta(1+e)\,, \tag{15i}$$

$$S_{44}=\gamma\frac{\beta}{\alpha}\,, \tag{15j}$$

$$S_{99}=\gamma\frac{\beta+4f/(1+f)}{\alpha+4e/(1+e)}\,, \tag{15k}$$

$$S_{1313}=\gamma\frac{\beta-2f/(1+f)}{\alpha-2e/(1+e)}\,. \tag{15l}$$

Notice that there are only five relations, Eqs. (15e)–(15i), among the six quantities P_{00}, P_{08}, P_{015}, P_{88}, P_{815}, and P_{1515}; therefore at this stage there is no way to constrain the symmetry-breaking parameters from the positivity of P_{00}, P_{88}, and P_{1515}. In Sec. III we shall derive new sum rules based on octet-type breaking of SU(3) at $q^2=0$, and see how these sum rules, together with those in Eq. (15) enable us to solve for P_{00}, P_{08}, P_{015}, P_{88}, P_{815}, and P_{1515} in terms of the symmetry-breaking parameters α, β, γ, e, and f.

III. BROKEN SU(3) SUM RULES

In order to derive new sum rules for the two-point functions of scalar and pseudoscalar densities at $q^2=0$, we recall that P_{jk} and S_{jk} are symmetric in j and k; therefore, the most general SU(3) decomposition of these will get contributions only from the symmetric representations of SU(3). In the model under consideration, where SU(4) is broken by a term which transforms like a 15-plet and SU(3) is broken by a term which transforms like an octet, we can write the following expansion for P_{jk}:

$$\begin{aligned}P_{jk}&\equiv\int dm^2\frac{\rho_{jk}(m^2,v)}{m^2}\\&=C_1\delta_{jk}+C_2d_{8jk}+C_3d_{15jk}+C_4\delta_{j0}\delta_{k0}\\&\quad+C_5(\delta_{j0}\delta_{k8}+\delta_{j8}\delta_{k0})+C_6(\delta_{j0}\delta_{k15}+\delta_{j15}\delta_{k0})\\&\quad+C_7(\delta_{j8}\delta_{k15}+\delta_{j15}\delta_{k8})\,,\end{aligned} \tag{16}$$

and similarly for S_{jk}. Note that Eq. (16) is quite general except for the neglect of 27-plet contribution on the right-hand side, and is based on the assumption of octet-type breaking of SU(3) at $q^2=0$. From Eq. (16) the following sum rules can be obtained:

$$P_{33}+3P_{88}-4P_{44}=0\,, \tag{17a}$$

$$P_{33}-P_{44}-P_{99}+P_{1313}=0\,, \tag{17b}$$

$$2P_{44}-5P_{33}+9P_{99}-6P_{1515}=0\,. \tag{17c}$$

We wish to emphasize that the sum rules (17a)–(17c) are exact to first order in the symmetry-breaking parameter a. In view of the fact that perturbation around SU(3) works very well, we expect the sum rules (17a)–(17c) to be valid for values of a which are not necessarily small. We therefore proceed to investigate the constraints imposed by these sum rules on the symmetry-breaking parameters.

The sum rule (17a), together with those in Eqs. (15e)–(15i) are sufficient to solve for P_{00}, P_{08}, P_{015}, P_{88}, P_{815}, P_{1515}. We get

$$P_{88}=\gamma\frac{3\alpha-2\beta-\alpha\beta+2}{(2-\alpha)\,(1+\alpha)} \tag{18a}$$

$$P_{08}=\left(\frac{3}{2}\right)^{1/2}\gamma\,\frac{(\alpha-\beta)\,[2(1-e)+\alpha(1+e)]}{(2-\alpha)\,(1+\alpha)\,(1-3e)}\,, \tag{18b}$$

$$P_{815}=\gamma\,\frac{(\beta-\alpha)\,[2(1+3e)+3\alpha(1+e)]}{\sqrt{2}\,(2-\alpha)\,(1+\alpha)\,(1-3e)}\,, \tag{18c}$$

and similar more complicated expressions for P_{00}, P_{015}, and P_{1515} can be written down. The important observation to make at this point, is that the expressions for P_{33}, P_{44}, S_{44}, and P_{88} in Eqs. (15a)–(15b), (15j), and (18a) depend on α, β, and γ alone. Thus the SU(3)×SU(3) subdomain has essentially decoupled[10] itself from SU(4)×SU(4). This simplification enables us to examine directly the constraints imposed on α, β, and γ due to the positivity of P_{33}, P_{44}, P_{88}, and S_{44}. This is in direct analogy with the investigation of Ref. 5. The allowed domains for α, β, and γ are shown in Fig. 1. For a solution where the vacuum is almost SU(3)-invariant, and the Hamiltonian is close to [SU(2)×SU(2)]-invariant, the value of β must be close to zero and that of α close to -1. We see from Fig. 1 that for $\beta\simeq 0$, the *smallest* possible value of α is $\alpha\simeq -0.7$. Thus, in this scheme SU(2)×SU(2) cannot be a good symmetry of the Hamiltonian.[11] This is in contradiction to the result of Dittner, Eliezer, and Kuo,[4] who impose lepton-hadron symmetry to get a solution where both SU(2)×SU(2) and SU(3) are good symmetries of the Hamiltonian. The fact that SU(2)×SU(2) cannot be a good symmetry of the Hamiltonian in the framework discussed in this section is simply because the sum rule

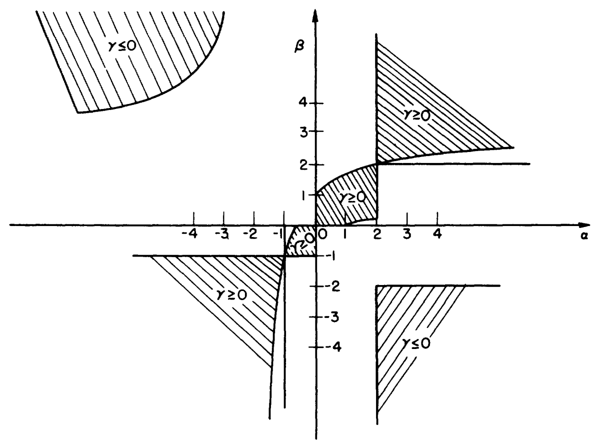

FIG. 1. The allowed domains of the parameters α and β are indicated by the shaded regions.

(18a) is inconsistent with the Gell-Mann, Oakes, and Renner[12] type of solution for the symmetry-breaking parameters. This can be easily checked, as shown in Ref. 5, by neglecting η-X mixing and using pole dominance for the spectral weights in (18a). We also note in passing that the sum rule (18a) is consistent with the type of solution advocated by Brandt and Preparata.[13] Thus we conclude that in SU(4)×SU(4) theory, as in the case of SU(3)×SU(3) theory, the problem of understanding SU(2)×SU(2) and SU(3) as approximate symmetries of the Hamiltonian with the vacuum almost SU(3)-invariant remains.

IV. BROKEN ASYMPTOTIC SU(3) SUM RULES

We now turn to an examination of the breaking of SU(4)×SU(4) on the basis of sum rules that follow from the octet-type breaking of asymptotic SU(3). The assumption of octet-broken SU(3) at $q^2=\infty$ for the two-point functions gives, instead of Eq. (16), the following:

$$\begin{aligned}\lim_{q^2\to\infty} q^2P_{jk}(q^2)\equiv R_{jk} &= C_1'\delta_{jk}+C_2'd_{8jk}+C_3'd_{15jk}+C_4'\delta_{j0}\delta_{k0}\\ &\quad+C_5'(\delta_{j0}\delta_{k8}+\delta_{j8}\delta_{k0})\\ &\quad+C_6'(\delta_{j0}\delta_{k15}+\delta_{j15}\delta_{k0})\\ &\quad+C_7'(\delta_{j8}\delta_{k15}+\delta_{j15}\delta_{k8})\,,\end{aligned} \tag{19}$$

where $R_{jk}=\int dm^2\rho_{jk}(m^2,v)$. Equation (19) gives the following sum rules:

$$R_{33}+3R_{88}-4R_{44}=0\,, \tag{20a}$$

$$R_{33}-R_{44}-R_{99}+R_{1313}=0\,, \tag{20b}$$

$$2R_{44}-5R_{33}+9R_{99}-6R_{1515}=0\,. \tag{20c}$$

Since R_{jk} is not directly related to α, β, γ, e, and f, there is no straightforward way to constrain these parameters from the positivity of R_{jj}. However, if we dominate the spectral weights by single particle states, then we can constrain the parameters in terms of masses and decay constants. If we neglect η-X mixing, we can easily verify[5] that the sum rule (20a) on using pole dominance is consistent with the Gell-Mann, Oakes, and Renner[12] type of solution:

$$m_\pi^2 = (1+\alpha) m_8^2 , \tag{21a}$$

$$m_K^2 = (1-\tfrac{1}{2}\alpha) m_8^2 , \tag{21b}$$

$$m_\eta^2 = (1-\alpha) m_8^2 , \tag{21c}$$

and $\beta \simeq 0$, $F_\pi \simeq F_K \simeq F_\eta$, where m_8 is the mass of the uncharmed pseudoscalar-meson octet in SU(3) limit; $F_\pi = 94$ MeV, F_K, and F_η are pion, kaon, and η decay constants. Thus within the framework of octet-type broken SU(3) at $q^2=\infty$, there exists a solution for the symmetry-breaking parameters where SU(2)×SU(2) is a good symmetry of the Hamiltonian.

We next investigate the consequence of the sum rule (20b). For this purpose we define the following matrix elements:

$$\begin{aligned}
\langle 0 | V^3(0) | \pi^3(p)\rangle &= \frac{Z_\pi}{(2\omega_\pi)^{1/2}} ,\\
\langle 0 | V^4(0) | K^4(p)\rangle &= \frac{Z_K}{(2\omega_K)^{1/2}} ,\\
\langle 0 | V^9(0) | P^9(p)\rangle &= \frac{Z_9}{(2\omega_9)^{1/2}} ,\\
\langle 0 | V^{13}(0) | P^{13}(p)\rangle &= \frac{Z_{13}}{(2\omega_{13})^{1/2}} ,
\end{aligned} \tag{22}$$

where the pseudoscalar mesons P^9 and P^{13} belong to an SU(3) triplet of charmed particles consisting of isospin doublet (P^9, P^{10}) and the isosinglet P^{13}. In analogy with the mass formulas (21a)–(21c), we may relate the masses of charmed pseudoscalar mesons in terms of the symmetry-breaking parameters α and e. We get

$$m_9^2 = \left(1 + \frac{\alpha}{2}\frac{1+e}{1-e}\right) m_3^2 , \tag{23a}$$

$$m_{13}^2 = \left(1 - \alpha \frac{1+e}{1-e}\right) m_3^2 , \tag{23b}$$

where m_3 is the mass of the charmed pseudoscalar-meson triplet in the SU(3) limit. If we dominate the spectral weights in Eqs. (15a)–(15d) and (20b) by single particle states and use the mass formulas (21a), (21b), (23a), and (23b), we get

$$m_3^2 = \frac{1-e}{1+e} m_8^2 . \tag{24}$$

In the model of Gell-Mann, Oakes, and Renner[12], one has, further, $m_8^2 = \frac{1}{3}(m_\pi^2 + 2m_K^2)$; we therefore get

$$m_3^2 = \frac{(1-e)}{(1+e)} \left(\frac{m_\pi^2 + 2m_K^2}{3}\right) . \tag{25}$$

It is clear from Eq. (25) that m_3 can be large (~ 5 GeV) only when e is very close to -1. If we take $m_3 = 5$ GeV, we get $e = -0.987$. This value of e is similar to the value obtained by Dittner, Eliezer, and Kuo.[4] On the other hand, Dicus and Mathur, who have studied[9] the η-X mixing problem within the framework of the SU(4)×SU(4) model, claim that the allowed value of e closest to -1 is -0.58. Using $e=-0.58$, we get $m_3 \approx 800$ MeV, which is rather small. Thus the viability of the SU(4)×SU(4) model with m_3 as large as 5 GeV crucially depends on the value of the parameter e. In order to determine this parameter independently we shall have to study the η-X or η-X-E mixing problem, which we have avoided in this paper.

V. SUMMARY AND CONCLUSION

The basic aim of this investigation has been to analyze the SU(4)×SU(4) model of strongly interacting particles. The approach towards this analysis begins with a study of the spectral-function sum rules for the scalar and pseudoscalar densities. These sum rules impose useful constraints on the symmetry-breaking parameters in the theory via the positivity requirement of the spectral weights. We find that the assumption of octet-type breaking of SU(3) at $q^2=0$ provides additional sum rules whose consequences are far reaching. It is shown that these sum rules constrain the symmetry-breaking parameters in such a way that SU(2)×SU(2) cannot be a good symmetry of the Hamiltonian if the vacuum is approximately SU(3)-invariant. This contradicts the result of Dittner *et al.*,[3,4] who claim that SU(2) ×SU(2) as well as SU(3) are good symmetries of the Hamiltonian in the SU(4)×SU(4) model. Their conclusion is unacceptable within the framework of broken SU(3) at $q^2=0$ because if SU(3) is a good symmetry, we expect the sum rule in Eq. (17a) to be valid to an excellent approximation. However, the sum rule (17a) together with the sum rules in Eq. (15) rule out a solution where SU(2) ×SU(2) is a good symmetry of the Hamiltonian.

When we turn to octet-broken SU(3) at $q^2=\infty$, we find that the sum rules obtained under this assumption are consistent with a solution for the symmetry-breaking parameters where SU(2) ×SU(2) is a good symmetry of the Hamiltonian. In particular, we find that the solution of Gell-

Mann, Oakes, and Renner[12] is consistent with the asymptotic sum rules. In this case we also obtain a mass formula for charmed pseudoscalar mesons in terms of the parameter e alone. We find that the mass of charmed pseudoscalar meson can be large (~ 5 GeV) only when $e \simeq -0.99$. This result is consistent with the work of Dittner, Eliezer, and Kuo.[4] Dicus and Mathur, who assume exact SU(3) for certain matrix elements, show that the smallest value of e close to -1 which is acceptable is $e \simeq -0.6$. This value of e yields $m_3 \simeq 800$ MeV, implying that charmed pseudoscalar mesons are not heavy. Thus an independent evaluation of e is needed to verify whether the value of this parameter can indeed be close to -1. For this purpose, the η-X or η-X-E mixing problem has to be studied. In this paper we do not attempt such an investigation.

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