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Broken SU(4)⊗SU(4) symmetry*

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Starting with chiral SU(4)⊗SU(4) invariance realized in the Nambu-Goldstone manner, with the vacuum invariant under SU(3), we discuss in this paper the various implications of explicit symmetry-breaking terms that transform as the $(4,4^*)\oplus(4^*,4)$ representation. The motivation for this work comes from recent investigations in connection with the unified gauge theories, which suggest the relevance of SU(4)⊗SU(4) symmetry of strong interactions with $(4,4^*)\oplus(4^*,4)$ breaking. The domain structure for the allowed values of the symmetry-breaking parameters is discussed in analogy with a similar investigation for the SU(3)⊗SU(3) theory, and the ratio of charmed- to uncharmed-particle masses is derived in terms of these parameters. Solutions are sought for these parameters through an analysis of the η-χ and η-χ-E mixing problems, as well as $\eta, \chi\to 2\gamma$ decays. Within the framework of our assumptions, we are unable to obtain a solution that agrees with the recent results discussed by Dittner *et al.* on the basis of a hadron-lepton analogy.

I. INTRODUCTION

Recently there has been a great deal of interest in the problem of how to incorporate hadrons into a unified gauge theory. Although no credible model has yet emerged, many general features of such a theory have been isolated and emphasized. Within the framework of a quark model, it is well known that the usual triplet of quarks runs into difficulties. In the Weinberg-type theories,[1] for instance, the three-quark model leads to strangeness-changing neutral currents in contradiction with the experimental analysis. A way out of this difficulty was suggested by Glashow *et al.*[2] who introduced a fourth quark carrying a quantum number referred to as charm, and constructed the charged currents in analogy with the corresponding leptonic currents.

In terms of the four quarks, it is natural to consider the SU(4) symmetry of strong interactions. Extended to chiral symmetry, this suggests consideration of the SU(4) ⊗ SU(4) symmetry. The fact that the particle spectra seem to follow recognizable patterns on the basis of SU(3) classification, rather than SU(4) or SU(4) ⊗ SU(4), strongly suggests that the SU(4) ⊗ SU(4) symmetry of the Hamiltonian should be realized as a Nambu-Goldstone symmetry with the vacuum invariant under the SU(3) group. Thus not only do the pseudoscalar mesons π, K, η, and X (or η') appear as Goldstone particles, there are several "charmed" mesons that also appear as zero-mass particles in the theory. Additional explicit symmetry-breaking terms must also be considered so that the eventual symmetries of the Hamiltonian and the vacuum state are appropriately reduced to the usual isospin group SU(2), and the Goldstone particles acquire suitable masses.

Dittner *et al.*[3,4] have recently considered such a scheme and have invoked[4] a lepton-hadron analogy to obtain some interesting results. Using this analogy, they have obtained a solution for the symmetry-breaking parameters which shows that the Hamiltonian is, to an excellent approximation, almost SU(3) ⊗ SU(3)-invariant [so that both chiral SU(2) ⊗ SU(2) and SU(3) are very good symmetries], and the charmed (would be) Goldstone mesons acquire rather large masses compared with the masses generated for the usual pseudoscalar mesons. The large masses of the charmed particles (~5 GeV) could explain why these objects may have escaped detection. Furthermore the pair production of such particles may well account for the upturn in the total p-p cross section at high energies.

The purpose of this paper is to study chiral SU(4) theory in some detail but using more conventional ideas, rather than the somewhat illusive lepton-hadron symmetry. The main thrust of this investigation is to see if the conventional methods can lead to a solution having features claimed by Dittner *et al.*[4] Throughout this work we follow the SU(3) ⊗ SU(3) analysis of Gell-Mann, Oakes, and Renner[5] (GMOR) and of Okubo and Mathur[6,7] with appropriate generalization.

The paper is planned as follows. In Sec. II the allowed domains for the symmetry-breaking parameters are discussed. These allowed values follow from the requirements of positivity of the two-point spectral functions. It is shown that the SU(3) ⊗ SU(3) subdomain structure essentially decouples from the theory and leads to the conventional solution which is reproduced in Sec. III. Section IV is devoted to obtaining the ratio of charmed- to uncharmed-particle masses. Since we do not assume approximate SU(4) symmetry of the vacuum, a generalization of the method of GMOR to obtain this mass ratio is inadequate. Instead, we use techniques based on asymptotic symmetries. It is clear that in the limit the Hamiltonian is SU(3) ⊗ SU(3)-invariant with the vacuum SU(3)-invariant, the conventional uncharmed mesons π, K, and η would be massless, whereas the charmed particles would generally acquire finite masses, so that the ratio of charmed- to uncharmed-particle masses would go to infinity. The departure of the solution from exact SU(3) ⊗ SU(3) invariance is studied in Secs. V and VI. To this end, we study in Sec. V the η-X mixing problem. By enlarging the original symmetry group to U(4) ⊗ U(4), we also investigate the three-particle mixing problem (η-X-E mixing). In Sec. VI we use the triangle anomaly to compute the rates for $\pi, \eta, X \to 2\gamma$ decays, and use some of this information to study the symmetry-breaking solution. Finally, in Sec. VII we summarize our results and conclusions.

II. SW(4) SPECTRAL CONDITIONS

We take the strong-interaction Hamiltonian density to be of the form

$$H = H_0 + \epsilon_0 u^0 + \epsilon_8 u^8 + \epsilon_{15} u^{15}\,, \tag{2.1}$$

where we assume H_0 is invariant under the chiral group $SW(4) \equiv SU^{(+)}(4) \otimes SU^{(-)}(4)$. The symmetry-breaking terms in H depend on real constants ϵ_i and scalar densities u^i, where $i = 0, 8, 15$. We shall assume that the 16 scalar densities u^i ($i = 0, 1, \ldots, 15$) together with the corresponding pseudoscalar densities v^i ($i = 0, 1, \ldots, 15$) transform according to the $(4, 4^*) \oplus (4^*, 4)$ representation of SW(4). This is a direct generalization of the GMOR model for the SW(3) theory.[5] Furthermore, in terms of the quark model the $(4, 4^*) \oplus (4^*, 4)$-breaking terms can be interpreted as the quark-mass terms, which arise naturally in unified gauge theories when the weak gauge group is broken spontaneously. Thus it seems appropriate to confine our attention to the $(4, 4^*) \oplus (4^*, 4)$ symmetry-breaking model.

The SW(4) generators F^i and F_5^i ($i = 1, \ldots, 15$) are defined in terms of the vector and the axial-vector current densities as

$$\begin{aligned} F^i(t) &= \int_{x_0 = t} d^3x\; V_0^i(x)\,, \\ F_5^i(t) &= \int_{x_0 = t} d^3x\; A_0^i(x)\,. \end{aligned} \tag{2.2}$$

We shall refer to F^{15} as the charm generator.

We assume that in the limit the explicit symmetry-breaking terms are turned off ($\epsilon_i \to 0$), the SW(4) symmetry of the Hamiltonian is realized in the Nambu-Goldstone manner with the vacuum state invariant only under the usual SU(3) subgroup. The spectra of ordinary particles then follow the well-recognized pattern of SU(3) classification. In the limit $\epsilon_i \to 0$, however, there appear zero-mass Goldstone bosons. For the pseudoscalar mesons, these are the usual octet π, K, and η_8, an SU(3) triplet of charmed particles consisting of the isospin doublet (P_9, P_{10}) and the isosinglet P_{13}, a corresponding charge-conjugate SU(3) triplet consisting of the $I=\frac{1}{2}$ mesons (P_{11}, P_{12}) and the isosinglet P_{14}, and finally an SU(3) singlet P_{15}. For the scalar mesons, these are a charm-carrying SU(3) triplet containing the $I=\frac{1}{2}$ mesons (S_9, S_{10}) and the $I=0$ particle S_{13}, the corresponding conjugate SU(3) triplet (S_{11}, S_{12}) and S_{14}, and the SU(3) singlet S_{15}. Note that if we further assume that the vacuum is also invariant under charm, so that $F_{15}|0\rangle = 0$, the invariance group of the vacuum is enlarged to SU(3)⊗U(1), and the SU(3) singlet S_{15} would not appear as a Goldstone boson.

The fact that the u^i and v^i form a $(4, 4^*) \oplus (4^*, 4)$ representation of SW(4) gives the equal-time commutators of the densities with the generators

$$\begin{aligned}
&[F^i(t), u^j(x)]_{x_0=t} = if_{ijk}u^k(x) ,\\
&[F^i(t), v^j(x)]_{x_0=t} = if_{ijk}v^k(x) ,\\
&[F_5^i(t), u^j(x)]_{x_0=t} = id_{ijk}v^k(x) ,\\
&[F_5^i(t), v^j(x)]_{x_0=t} = -id_{ijk}u^k(x) ,
\end{aligned} \tag{2.3}$$

where $i=1, \ldots, 15$, while j and k run from 0 to 15. The f_{ijk} and d_{ijk} can be calculated by using

$$\begin{aligned}
&\mathrm{Tr}(\lambda_k[\lambda_i, \lambda_j]) = 4if_{ijk} ,\\
&\mathrm{Tr}(\lambda_k\{\lambda_i, \lambda_j\}) = 4d_{ijk} ,
\end{aligned} \tag{2.4}$$

where the λ are the 4×4 matrices. These have been calculated in Ref. 3, and for completeness, we reproduce them in Table I. Notice that $d_{ij0} = (1/\sqrt{2})\delta_{ij}$ because the λ matrices are normalized by

$$\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij} . \tag{2.5}$$

The divergences of the currents follow from the local generalization of the usual equations of motion

$$\partial^\mu J_\mu^i = i[F^i, H] . \tag{2.6}$$

We find the partial conservation laws

$$\partial^\mu V_\mu^i = \epsilon_8 f_{i8k}u^k + \epsilon_{15} f_{i15k}u^k , \tag{2.7a}$$

$$\partial^\mu A_\mu^i = \epsilon_0 d_{i0k}v^k + \epsilon_8 d_{i8k}v^k + \epsilon_{15} d_{i15k}v^k . \tag{2.7b}$$

Note that the vector currents for $i=1$, 2, 3, 8, and 15 are conserved.

Our purpose in this section is to derive restrictions on the parameters which give the symmetry breaking. To do this we relate the parameters to the spectral function using exactly the method of Mathur and Okubo.[6] We write the usual spectral-function representation for the commutator:

$$\begin{aligned}
\langle 0|[A_\mu^i(x), A_\nu^j(y)]|0\rangle
&= \int_0^\infty dm^2 \left[\left(g_{\mu\nu} - \frac{1}{m^2}\partial_\mu\partial_\nu\right)\rho_{ij}^{(1)}(m, A)\right.\\
&\qquad \left. - \frac{1}{m^2}\rho_{ij}^{(0)}(m, A)\partial_\mu\partial_\nu\right]\Delta(x-y, m) .
\end{aligned} \tag{2.8}$$

Taking the divergence of both sides with respect to x, setting $x^0=y^0$, and integrating over x, we relate the integral of the scalar spectral function $\rho_{ij}^{(0)}(m, A)$ to the vacuum expectation value of the scalar densities that appear in H. Performing the same procedure for the commutator of two vector currents, we obtain

$$\begin{aligned}
I_{ij} &\equiv \int_0^\infty dm^2 \rho_{ij}^{(0)}(m, A)\\
&= -(\epsilon_0 d_{j0k} + \epsilon_8 d_{j8k} + \epsilon_{15} d_{j15k})\\
&\quad \times (\xi_0 d_{ik0} + \xi_8 d_{ik8} + \xi_{15} d_{ik15}) ,
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
K_{ij} &\equiv \int_0^\infty dm^2 \rho_{ij}^{(0)}(m, V)\\
&= -(\epsilon_8 f_{8jk} + \epsilon_{15} f_{15jk})(\xi_8 f_{ik8} + \xi_{15} f_{ik15}) ,
\end{aligned} \tag{2.10}$$

where we have defined

$$\xi_i \equiv \langle 0|u^i(0)|0\rangle , \quad i=0, 8, 15 . \tag{2.11}$$

Now, because the integrals of the spectral weight functions are positive definite for $i=j$, we have restrictions on the symmetry-breaking parameters. It is convenient to define the ratios

$$\begin{aligned}
&a \equiv (\tfrac{2}{3})^{1/2}\frac{\epsilon_8}{\epsilon_0} , \quad b \equiv (\tfrac{2}{3})^{1/2}\frac{\xi_8}{\xi_0} ,\\
&e \equiv \frac{1}{\sqrt{3}}\frac{\epsilon_{15}}{\epsilon_0} , \quad f \equiv \frac{1}{\sqrt{3}}\frac{\xi_{15}}{\xi_0} , \quad \gamma \equiv -\tfrac{1}{2}\epsilon_0\xi_0 .
\end{aligned} \tag{2.12}$$

The following quantities are independent and greater than or equal to zero:

$$I_{33} = \gamma(1+a+e)(1+b+f) , \tag{2.13a}$$

$$I_{44} = \gamma(1-\tfrac{1}{2}a+e)(1-\tfrac{1}{2}b+f) , \tag{2.13b}$$

$$I_{88} = \gamma[(1-a+e)(1-b+f)+2ab] , \tag{2.13c}$$

$$I_{99} = \gamma(1+\tfrac{1}{2}a-e)(1+\tfrac{1}{2}b-f) , \tag{2.13d}$$

$$I_{13,13} = \gamma(1-a-e)(1-b-f) , \tag{2.13e}$$

$$I_{15,15} = \gamma(1-2e-2f+7ef+\tfrac{1}{2}ab) , \tag{2.13f}$$

$$K_{44} = \tfrac{9}{4}\gamma ab , \tag{2.13g}$$

TABLE I. Nonvanishing values of f_{ijk} and d_{ijk}.

i	j	k	f_{ijk}	i	j	k	d_{ijk}
1	2	3	1	2	10	11	$\frac{1}{2}$
1	4	7	$\frac{1}{2}$	3	3	8	$1/\sqrt{3}$
1	5	6	$-\frac{1}{2}$	3	3	15	$1/\sqrt{6}$
1	9	12	$\frac{1}{2}$	3	4	4	$\frac{1}{2}$
1	10	11	$-\frac{1}{2}$	3	5	5	$\frac{1}{2}$
2	4	6	$\frac{1}{2}$	3	6	6	$-\frac{1}{2}$
2	5	7	$\frac{1}{2}$	3	7	7	$-\frac{1}{2}$
2	9	11	$\frac{1}{2}$	3	9	9	$\frac{1}{2}$
2	10	12	$\frac{1}{2}$	3	10	10	$\frac{1}{2}$
3	4	5	$\frac{1}{2}$	3	11	11	$-\frac{1}{2}$
3	6	7	$-\frac{1}{2}$	3	12	12	$-\frac{1}{2}$
3	9	10	$\frac{1}{2}$	4	4	8	$-1/(2\sqrt{3})$
3	11	12	$-\frac{1}{2}$	4	4	15	$1/\sqrt{6}$
4	5	8	$\frac{1}{2}\sqrt{3}$	4	9	13	$\frac{1}{2}$
4	9	14	$\frac{1}{2}$	4	10	14	$\frac{1}{2}$
4	10	13	$-\frac{1}{2}$	5	5	8	$-1/(2\sqrt{3})$
5	9	13	$\frac{1}{2}$	5	5	15	$1/\sqrt{6}$
5	10	14	$\frac{1}{2}$	5	9	14	$-\frac{1}{2}$
6	7	8	$\frac{1}{2}\sqrt{3}$	5	10	13	$\frac{1}{2}$
6	11	14	$\frac{1}{2}$	6	6	8	$-1/(2\sqrt{3})$
6	12	13	$-\frac{1}{2}$	6	6	15	$1/\sqrt{6}$
7	11	13	$\frac{1}{2}$	6	11	13	$\frac{1}{2}$
7	12	14	$\frac{1}{2}$	6	12	14	$\frac{1}{2}$
8	9	10	$1/(2\sqrt{3})$	7	7	8	$-1/(2\sqrt{3})$
8	11	12	$1/(2\sqrt{3})$	7	7	15	$1/\sqrt{6}$
8	13	14	$-1/\sqrt{3}$	7	11	14	$-\frac{1}{2}$
9	10	15	$(\frac{2}{3})^{1/2}$	7	12	13	$\frac{1}{2}$
11	12	15	$(\frac{2}{3})^{1/2}$	8	8	8	$-1/\sqrt{3}$
13	14	15	$(\frac{2}{3})^{1/2}$	8	8	15	$1/\sqrt{6}$
i	j	0	0	8	9	9	$1/(2\sqrt{3})$
				8	10	10	$1/(2\sqrt{3})$
i	j	k	d_{ijk}	8	11	11	$1/(2\sqrt{3})$
				8	12	12	$1/(2\sqrt{3})$
1	1	8	$1/\sqrt{3}$	8	13	13	$-1/\sqrt{3}$
1	1	15	$1/\sqrt{6}$	8	14	14	$-1/\sqrt{3}$
1	4	6	$\frac{1}{2}$	9	9	15	$-1/\sqrt{6}$
1	5	7	$\frac{1}{2}$	10	10	15	$-1/\sqrt{6}$
1	9	11	$\frac{1}{2}$	11	11	15	$-1/\sqrt{6}$
1	10	12	$\frac{1}{2}$	12	12	15	$-1/\sqrt{6}$
2	2	8	$1/\sqrt{3}$	13	13	15	$-1/\sqrt{6}$
2	2	15	$1/\sqrt{6}$	14	14	15	$-1/\sqrt{6}$
2	4	7	$-\frac{1}{2}$	15	15	15	$-(\frac{2}{3})^{1/2}$
2	5	6	$\frac{1}{2}$	i	j	0	$\frac{1}{\sqrt{2}}\delta_{ij}$
2	9	12	$-\frac{1}{2}$				

$$K_{99}=\tfrac{1}{4}\gamma(a+4e)(b+4f)\,, \qquad (2.13\text{h})$$

$$K_{13,13}=\gamma(a-2e)(b-2f)\,. \qquad (2.13\text{i})$$

There is one nondiagonal I_{ij} which is not zero (although this one is not positive definite we will need it in later sections):

$$I_{8,15}=\frac{1}{\sqrt{2}}\gamma(a+b-ab+af+be)\,. \qquad (2.14)$$

This can be combined with $I_{8,8}$ and $I_{15,15}$ into positive-definite combinations, for example, $I_{8+15,8+15}$.

Requiring all of the quantities in (2.13) to be positive gives restrictions which are definitely nontrivial. It is easy to see, for example, that the two solutions proposed in Ref. 3,

$$a=-0.053,\quad e=-0.943,\quad f=0.265 \qquad (2.15\text{a})$$

or

$$a=-1.17,\quad e=0.32,\quad f=-0.99 \qquad (2.15\text{b})$$

do not satisfy the restrictions given by (2.13).

Solving for the general restrictions given by (2.13) is obviously very complicated because one must work in a four-dimensional space whose axes are labeled by the values of a, b, e, and f. The form of I_{ij} and K_{ij} simplify a great deal, however, if we replace a, b, and γ, by α, β, and δ, where

$$\begin{aligned} a &\equiv \alpha(1+e)\ , \\ b &\equiv \beta(1+f)\ , \\ \delta &\equiv \gamma(1+e)(1+f)\ . \end{aligned} \tag{2.16}$$

The usual SW(3) quantities scale such that all of the dependence on e and f can be lumped into δ:

$$I_{33} = \delta(1+\alpha)(1+\beta)\ , \tag{2.17a}$$

$$I_{44} = \delta(1-\tfrac{1}{2}\alpha)(1-\tfrac{1}{2}\beta)\ , \tag{2.17b}$$

$$I_{88} = \delta(1-\alpha-\beta+3\alpha\beta)\ , \tag{2.17c}$$

$$K_{44} = \tfrac{9}{4}\,\delta\alpha\beta\ , \tag{2.17d}$$

$$I_{99} = \delta\left(\tfrac{1}{2}\alpha + \frac{1-e}{1+e}\right)\left(\tfrac{1}{2}\beta + \frac{1-f}{1+f}\right), \tag{2.17e}$$

$$I_{13,13} = \delta\left(-\alpha + \frac{1-e}{1+e}\right)\left(-\beta + \frac{1-f}{1+f}\right), \tag{2.17f}$$

$$I_{15,15} = \delta\left[\tfrac{1}{2}\alpha\beta - 2 + 3\,\frac{(1+3ef)}{(1+e)(1+f)}\right]\ , \tag{2.17g}$$

$$K_{99} = \tfrac{1}{4}\delta\left(\alpha + \frac{4e}{1+e}\right)\left(\beta + \frac{4f}{1+f}\right)\ , \tag{2.17h}$$

$$K_{13,13} = \delta\left(\frac{2e}{1+e} - \alpha\right)\left(\frac{2f}{1+f} - \beta\right)\ , \tag{2.17i}$$

$$I_{8,15} = \frac{1}{\sqrt{2}}\,\delta(\alpha+\beta-\alpha\beta)\ . \tag{2.18}$$

In Sec. III we will solve I_{33}, I_{44}, and K_{44} for α and β. Given regions of allowed values of α and β, it is then possible to find the regions of allowed values of e and f. This procedure yields considerable simplification in discussing the allowed domains.

III. SOLUTION OF I_{33}, I_{44}, K_{44}

The equations for I_{33}, I_{44}, and K_{44} have a form, when written in terms of α, β, and δ, which is identical to the form they had in the SW(3) theory written in terms of the SW(3) parameters a, b, and γ (which are different from the a, b, γ used in Sec. II.) The allowed regions for the parameters δ, α, β are therefore exactly the same as the allowed regions of γ, a, b given in Ref. 6 and the same discussions of the subgroup structure of the group SW(3) can be given. There is no reason for us to repeat that material here.

If we saturate the integrals in (2.17) with the lowest-lying singularities, we can express the left-hand sides of (2.17) [as shown in (2.9) and (2.10)] in terms of masses and decay constants and the resulting equations can be solved, with some additional assumptions, for specific values of δ, α, β. This is also clearly done in Ref. 7; however, to be specific about what is being assumed we will repeat that discussion.

Define

$$\begin{aligned} \langle 0|A_\mu^a(0)|P(k)\rangle &= \frac{1}{\sqrt{2}}\, f_P^a\, ik_\mu\ , \\ \langle 0|V_\mu^a(0)|S(k)\rangle &= \frac{1}{\sqrt{2}}\, f_S^a\, ik_\mu\ , \end{aligned} \tag{3.1}$$

where P or S is a pseudoscalar or scalar meson. Where no confusion can result, we will drop the SU(4) label from the decay constant, for example,

$$f_\pi^3 = f_\pi,\ f_K^4 = f_K,\ f_\kappa^4 = f_\kappa\ .$$

For other particles, however, where mixing is involved, we will need the notation of (3.1).

The equations for I_{33}, I_{44}, and K_{44}, when saturated with mesons, become

$$f_\pi^{\,2} m_\pi^{\,2} = 2\delta(1+\alpha)(1+\beta)\ , \tag{3.2a}$$

$$f_K^{\,2} m_K^{\,2} = 2\delta(1-\tfrac{1}{2}\alpha)(1-\tfrac{1}{2}\beta)\ , \tag{3.2b}$$

$$f_\kappa^{\,2} m_\kappa^{\,2} = \tfrac{9}{2}\delta\,\alpha\beta\ . \tag{3.2c}$$

We also assume the GMOR mass relation holds; in our notation

$$\frac{m_K^{\,2}}{m_\pi^{\,2}} = \frac{1-\frac{1}{2}\alpha}{1+\alpha}\ . \tag{3.3}$$

Equation (3.3) gives a value for α. This together with (3.2a) and (3.2b) gives values for β and δ if we assume a value for f_K/f_π. The value $f_K/f_\pi = 1.13$, for instance, implies

$$\alpha = -0.89,\ \ \beta = -0.15,\ \ \delta = 5.3 f_\pi^{\,2} m_\pi^{\,2}\ ,$$

while $f_K/f_\pi = 1.07$ gives

$$\alpha = -0.89,\ \ \beta = -0.09,\ \ \delta = 5.0 f_\pi^{\,2} m_\pi^{\,2}\ .$$

Alternatively, we could proceed as in Ref. 7 by writing spectral representations for the Fourier transform of vacuum expectation values of scalar and pseudoscalar densities. If we then assume asymptotic SW(2) symmetry [where the SW(2) group is generated by F^4, F^5, and $\frac{1}{2}F^3 + (\sqrt{3}/2)F^8$] in the form

$$\lim_{q\to\infty} q^2[\Delta_{ab}^P(q) - \Delta_{ab}^S(q)] = 0\ , \tag{3.4}$$

we have the sum rule

$$\int_0^\infty dm^2 \rho_{ab}(m, P) = \int_0^\infty dm^2 \rho_{ab}(m, S)\ . \tag{3.5}$$

The one-meson saturation of this equation together with the vacuum one-meson matrix element of the divergence conditions (2.7) gives a new equation

$$(1-\tfrac{1}{2}\alpha)\beta m_\kappa^{\,2} = \alpha(1-\tfrac{1}{2}\beta) m_K^{\,2}\ . \tag{3.6}$$

This result, together with the equations for α and β from (3.3), (3.2a), and (3.2b), gives a relation between m_K and f_K/f_π,

$$m_K^2 = \frac{f_K^2(m_K^2 - m_\pi^2)}{f_K^2 - f_\pi^2} \quad . \tag{3.7}$$

If we choose to make the additional assumptions of (3.4) and of the one-meson saturation of (3.5) then we can avoid the need of assuming a value for f_K/f_π to determine α and β. Instead, we can assume a value for m_K and use (3.7). The value $f_K/f_\pi = 1.13$ corresponds to $m_K \approx 1020$ MeV. In the following sections, whenever we need explicit values for α and β we shall use $\alpha = -0.89$, $\beta = -0.15$.

Given that α and β lie within certain ranges, we can then solve the remaining I_{ij} and K_{ij} for the allowed regions of e and f. The boundaries of the allowed e and f regions will be functions of α and β. Figure 1 shows such a plot for the range of α and β that corresponds to the physical SW(3) domain, $-1 \leq \alpha \leq 0$, $-1 \leq \beta \leq 0$.

In the next sections we will try to further restrict e and f by studying those I_{ij} in (2.17) which have explicit e, f dependence. Section IV will discuss the relation of e and f to the masses of charmed particles. In Secs. V and VI we will discuss the restriction on e and f which comes from the I_{ij} which we can saturate with known particles I_{88}, $I_{8,15}$, $I_{15,15}$ and, in the second half of Sec. V, $I_{0,0}$, $I_{0,8}$, $I_{0,15}$. We have not yet written down the $I_{0,i}$ but the e,f dependence in all six of these I_{ij} comes only in one combination which we name $1/R$:

$$\frac{1}{R} \equiv \frac{1+3ef}{(1+e)(1+f)} \quad . \tag{3.8}$$

Thus it is not possible, without having information about charmed particles, to determine e and f separately. What we will find are values for R. Figures 2 and 3 show the lines on the e-f graph for various values of R. In order that the combination in (3.8) cross allowed values of e and f, it is necessary for R to lie in the region

$$0 \leq R \leq \frac{4}{2+|\alpha|+|\beta|-|\alpha||\beta|} \quad . \tag{3.9}$$

In Fig. 1 the value $e=-1$ is of special interest. It is easy to check that the Hamiltonian density (2.1) will be SU(3)⊗SU(3)-invariant if $a=0$ and $e=-1$, or equivalently [see Eq. (2.16)] for arbitrary α as long as $e=-1$. Following arguments

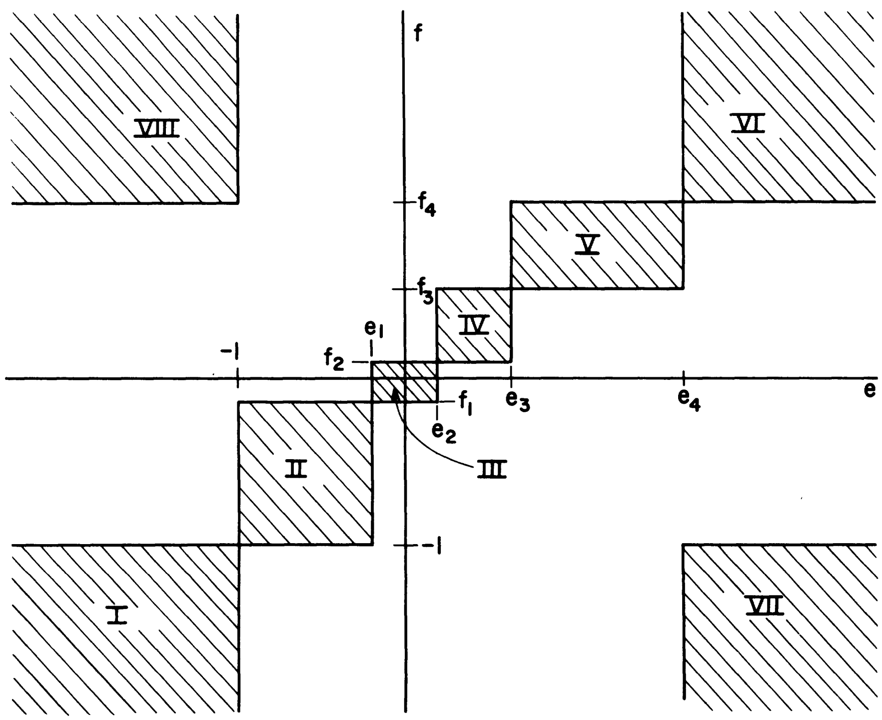

FIG. 1. The allowed values of the parameters e and f when α and β are between -1 and 0. The boundaries are given by

$$e_1 = \frac{-|\alpha|}{2+|\alpha|},\ e_2 = \frac{|\alpha|}{4-|\alpha|},\ e_3 = \frac{1-\frac{1}{2}|\alpha|}{1+\frac{1}{2}|\alpha|},\ e_4 = \frac{1+|\alpha|}{1-|\alpha|},\ f_1 = \frac{-|\beta|}{2+|\beta|},\ f_2 = \frac{|\beta|}{4-|\beta|},\ f_3 = \frac{1-\frac{1}{2}|\beta|}{1+\frac{1}{2}|\beta|},\ f_4 = \frac{1+|\beta|}{1-|\beta|}\ .$$

For the GMOR value $\alpha = -0.89$, one gets $e_1 = -0.31$, $e_2 = 0.29$, $e_3 = 0.38$, and $e_4 = 17.2$.

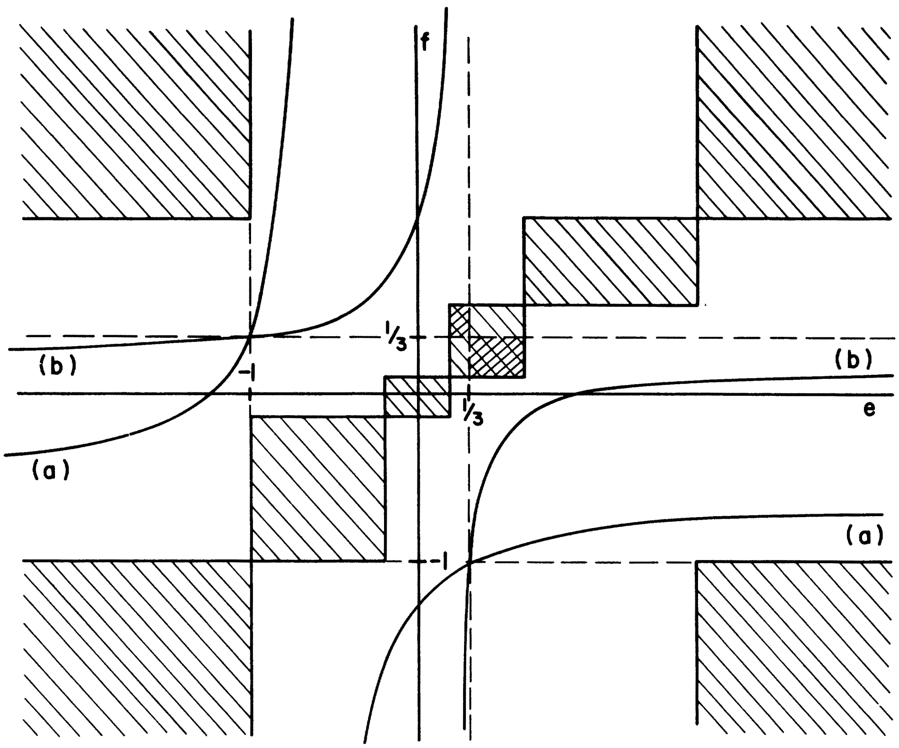

FIG. 2. The curves show typical values of e and f for (a) $R<0$ and (b) $R>\frac{4}{3}$, where $1/R=(1+3ef)/(1+e)(1+f)$. When $f\leq -1$ the values of e are between -1 and $+\frac{1}{3}$. The same is true for $f\geq +\frac{1}{3}$, while for $-1\leq f\leq \frac{1}{3}$ the values of e are $e\leq -1$ or $e\geq \frac{1}{3}$. Thus, for these values of R, the curve for $1/R$ only intersects the allowed regions in the cross-hatched area of region IV and it only intersects this area when $\frac{4}{3}\leq R\leq 4/2+|\alpha|+|\beta|-|\alpha||\beta|$. It is assumed in this figure that $-1\leq\alpha\leq 0$ and $-1\leq\beta\leq 0$ and the boundaries of the allowed regions are shown in Fig. 1.

similar to those in Ref. 6, one can show that the SU(3) ⊗SU(3) subsymmetry would be realized as a Goldstone symmetry if $f\neq -1$. In such a case it would be natural to assume that at $e=-1$ the vacuum is only SU(3) symmetric and the pseudoscalar mesons π, K, and η are massless. The charmed particles, on the other hand, would be massive. These considerations suggest that the "physical" solution for e must be close to -1 (with $f\neq -1$) since then (1) SU(3) ⊗SU(3) would be an approximate symmetry of the Hamiltonian with low-lying π, K, and η, which is consistent with the usual ideas, and (2) the ratio of the charmed-particle mass to the uncharmed-particle mass will be large, which is consistent with the fact that the charmed particles have not yet been observed. Note that if the Hamiltonian is exactly SU(3) ⊗ SU(3)-invariant with the vacuum SU(3)-invariant, this ratio would be infinite. We shall discuss this mass ratio in detail in Sec. IV.

It may be remarked parenthetically that when SU(3) ⊗ SU(3) symmetry is realized in a Goldstone fashion, as e is varied continuously across the point $e=-1$, the value of f will jump discontinuously between the regions I and II of Fig. 1. This is a general feature of Goldstone symmetries as emphasized in Ref. 6. In concluding this section we would also like to mention that just as in Ref. 6, the boundaries of the other allowed domains in Fig. 1 correspond to the realization of various other subgroups of SW(4). This discussion is relegated to the Appendix.

IV. MASS FORMULAS

We may use the method[5] of GMOR to relate the masses of the would-be Goldstone pseudoscalar and scalar bosons in terms of the symmetry-breaking parameters α and e. However, in the limit of no explicit symmetry breaking, we have assumed that the vacuum state is only SU(3)-invariant, so without further assumptions, the masses of the particles belonging to different SU(3) representations are unrelated. In particular, for the pseudoscalar octet, the pseudoscalar triplet (isospin doublet P_9 and P_{10}, and isoscalar P_{13}), and the analogous scalar triplet (S_9, S_{10}, S_{13}), we get

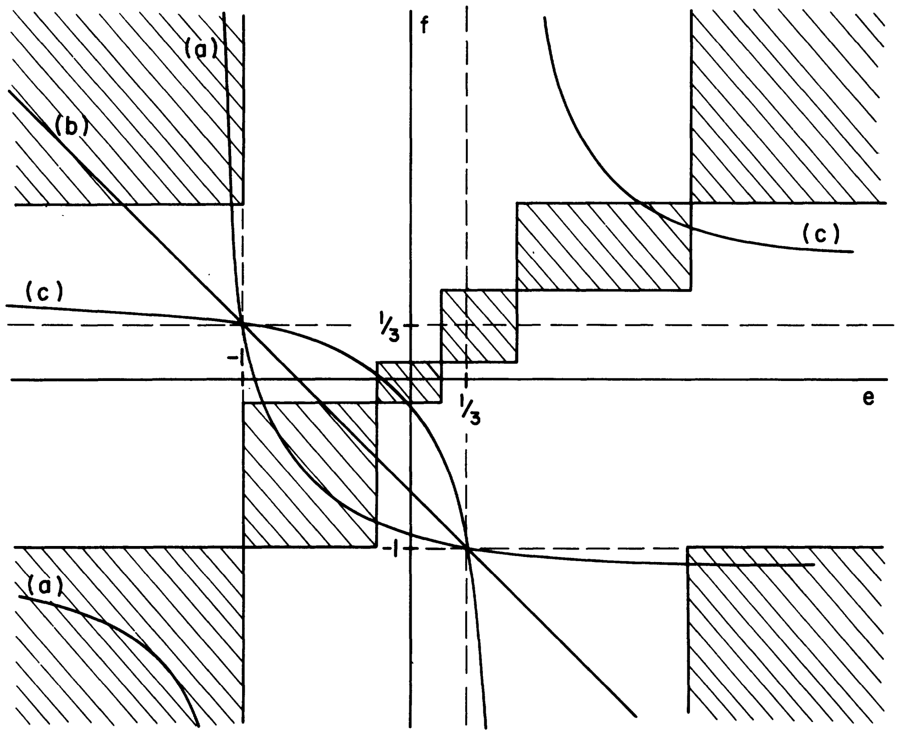

FIG. 3. The curves show typical values of e and f for $0 \leq R \leq \frac{4}{3}$. The curve (a) is for $0 < R < \frac{1}{3}$, (b) for $R = \frac{1}{3}$, and (c) for $\frac{1}{3} < R < \frac{4}{3}$. The lines $e = -1$, $f = -1$ are the $R = 0$ lines and the $e = \frac{1}{3}$, $f = \frac{1}{3}$ lines are the limiting curves as R approaches $\frac{4}{3}$. The boundaries of the allowed regions are shown in Fig. 1.

$$
\begin{aligned}
m_\pi^2 &= (1+\alpha)m_8^2 , \\
m_K^2 &= (1-\tfrac{1}{2}\alpha)m_8^2 , \\
m_9^2(P) &= \left(1+\tfrac{1}{2}\alpha\frac{1+e}{1-e}\right) m_3^2(P) , \\
m_{13}^2(P) &= \left(1-\alpha\frac{1+e}{1-e}\right) m_3^2(P) , \\
m_9^2(S) &= \left(1+\alpha\frac{1+e}{4e}\right) m_3^2(S) , \\
m_{13}^2(S) &= \left(1-\alpha\frac{1+e}{2e}\right) m_3^2(S) ,
\end{aligned}
\qquad (4.1)
$$

where m_3 and m_8 are the triplet and octet masses in the SU(3) limit. Note with reference to Fig. 1 that at $e=e_3$, $m_9(P)$ vanishes as discussed in the Appendix. Similarly, $m_{13}(P)$ vanishes at $e=e_4$, $m_9(S)$ at $e=e_2$, and $m_{13}(S)$ at $e=e_1$.

If one assumes that in the limit of no explicit symmetry breaking, the vacuum state is SU(4)-invariant (generators: $F^1, \ldots, F^{15}$) instead of being SU(3)-symmetric, the scalar mesons would not be Goldstone particles, and the masses of $m_9(P)$ and $m_{13}(P)$ can be related to m_π and m_K. However, in this case we shall have $m_3(P)=m_8$, and the positivity of $m_9^2(P)$ and $m_{13}^2(P)$ requires $|\alpha(1+e)/(1-e)|$ to be smaller than unity, so that one would expect the charmed-particle masses to be of the same order of magnitude as the uncharmed-pseudoscalar mesons.

It should be pointed out that in general the positivity requirement on the squared masses for the charmed particles in Eq. (4.1) considerably restricts the allowed domain of the parameters α and e in much the same fashion as a similar[6] requirement on m_π^2 and m_K^2 restricts $-1 \leq \alpha \leq 2$. For the GMOR solution $\alpha = -0.89$, the allowed region of e is bounded by $-1 \leq e \leq e_1 = -0.31$. Note that for $e > e_1$, $m_{13}^2(S)$ becomes negative. What this means is that one cannot use the perturbation formula like the one for $m_{13}^2(S)$ in Eq. (4.1) beyond the point $e=e_1$. This is an exceptional point where a subsymmetry is realized (see Appendix) with S_{13} as a Goldstone particle, so that as we move across e_1, there is a discontinuity in the solution. However, as discussed in Sec. III, we do expect the physical solution for e to lie in the region near $e = -1$.

In the rest of this section, we do not pursue Eqs. (4.1) anymore, but instead obtain relations between charmed and uncharmed particles based on the idea of asymptotic symmetries. This is an extension of the asymptotic SW(2)-symmetry discussion in Ref. 7 and in Sec. III, which leads to the relation

(3.6) between the masses of K and κ. If we define

$$Q^i = F^4,\ F^5,\ \tfrac{1}{2}F^3 + \frac{\sqrt{3}}{2}F^8,\ F^9,\ F^{10},\ F^{13},\ F^{14},\ \frac{1}{2\sqrt{3}}F^3 - \tfrac{1}{6}F^8 + \tfrac{2}{3}\sqrt{2}F^{15},$$

then the Q^i generate an SU(3) group, and together with the corresponding axial-charge octet Q_5^i, generates a chiral SU(3) ⊗ SU(3) group. This chiral group does not contain isospin SU(2) as a subgroup and is physically distinct from the usual chiral symmetry group. Note that the asymptotic SW(2) group discussed before [see Eq. (3.4)] is a subgroup of this group. If we now assume asymptotic symmetry under this chiral SU(3) ⊗ SU(3) group, we get a sum rule similar to Eq. (3.5). Defining the matrix elements

$$\begin{aligned} \langle 0|v^i(0)|P(k)\rangle &\equiv g_P^i, \\ \langle 0|u^i(0)|S(k)\rangle &= g_S^i, \end{aligned} \tag{4.2}$$

we obtain in the one-meson saturation approximation, the sum rules ($g_P^4 \equiv g_K$)

$$g_K^2 = (g_S^9)^2 = (g_S^{13})^2 . \tag{4.3}$$

The divergence relations (2.7) together with the definitions (3.1) and (4.2) imply

$$\begin{aligned} \epsilon_0(1-\tfrac{1}{2}a+e)g_K &= f_K m_K^2, \\ \tfrac{1}{2}\epsilon_0(a+4e)g_S^9 &= f_S^9 m_9^2(S), \\ \epsilon_0(a-2e)g_S^{13} &= f_S^{13} m_{13}^2(S). \end{aligned} \tag{4.4}$$

Substituting Eq. (4.3) into Eq. (4.4), we get

$$\begin{aligned} \frac{m_9^4(S)(f_S^9)^2}{m_K^4 f_K^2} &= \frac{1}{4}\frac{(a+4e)^2}{(1-\frac{1}{2}a+e)^2}, \\ \frac{m_{13}^4(S)(f_S^{13})^2}{m_K^4 f_K^2} &= \frac{(a-2e)^2}{(1-\frac{1}{2}a+e)^2}. \end{aligned} \tag{4.5}$$

If we use Eqs. (2.13b), (2.13h), and (2.13i) in the pole-dominated form $I_{44} = \frac{1}{2}m_K^2 f_K^2$, $K_{99} = \frac{1}{2}m_9^2(S)(f_S^9)^2$, $K_{13,13} = \frac{1}{2}m_{13}^2(S)(f_S^{13})^2$, we may eliminate the decay constants to obtain the mass relations

$$\frac{m_9^2(S)}{m_K^2} = \frac{(1-\frac{1}{2}\beta)[\alpha+4e/(1+e)]}{(1-\frac{1}{2}\alpha)[\beta+4f/(1+f)]}, \tag{4.6a}$$

$$\frac{m_{13}^2(S)}{m_K^2} = \frac{(1-\frac{1}{2}\beta)[\alpha-2e/(1+e)]}{(1-\frac{1}{2}\alpha)[\beta-2f/(1+f)]}, \tag{4.6b}$$

expressed in terms of the parameters α, β, e, and f defined in Eq. (2.16).

Similar considerations may be made for the validity of asymptotic SU(2) ⊗ SU(2) symmetry generated by $Q^i \pm Q_5^i$ ($i=1,2,3$), where $Q^i = F^9$, F^{10} and $\frac{1}{2}F^3 + (1/2\sqrt{3})F^8 + (\sqrt{2}/\sqrt{3})F^{15}$, with the corresponding axial charges Q_5^i. In the manner discussed above, this would lead to a relation between $m_9^2(P)$ and $m_9^2(S)$ or equivalently between $m_9^2(P)$ and m_K^2 if we use Eq. (4.6a). We obtain

$$\frac{m_9^2(P)}{m_K^2} = \frac{(1-\frac{1}{2}\beta)[\frac{1}{2}\alpha+(1-e)/(1+e)]}{(1-\frac{1}{2}\alpha)[\frac{1}{2}\beta+(1-f)/(1+f)]} \tag{4.7a}$$

Finally, assuming another asymptotic SU(2) ⊗ SU(2) symmetry generated by charges F^{13}, F^{14}, and $-(1/\sqrt{3})F^8 + (\sqrt{2}/\sqrt{3})F^{15}$ and the corresponding axial charges, we can relate $m_{13}^2(P)$ to $m_{13}^2(S)$ and hence to m_K^2 if we use Eq. (4.6b). We get

$$\frac{m_{13}^2(P)}{m_K^2} = \frac{(1-\frac{1}{2}\beta)[(1-e)/(1+e)-\alpha]}{(1-\frac{1}{2}\alpha)[(1-f)/(1+f)-\beta]}. \tag{4.7b}$$

It should be remarked that we have avoided using asymptotic SU(4) symmetry, since already in the W(3) or SW(3) theory of Ref. 7 we know that an exact asymptotic SU(3) symmetry for pseudoscalar densities (with pole dominance) leads to inconsistencies. We have also not discussed here the 8th and 15th components of the scalar or pseudoscalar meson multiplets, since this is complicated by the mixing problem, discussed in Sec. V.

It is clear from Eqs. (4.6) and (4.7) that for values of α and β discussed in Sec. III, the ratio of charmed- to uncharmed-particle masses will be large, if the physical solution for e and f lies in the domain II of Fig. 1, with e rather close to -1 and f away from -1.

V. MASS MIXING

We now turn to the $I_{8,8}$, $I_{8,15}$, $I_{15,15}$ equations to try to further restrict e and f. Following (3.1) we define

$$\begin{aligned} \langle 0|A_\mu^8(0)|\eta(k)\rangle &= \frac{1}{\sqrt{2}} ik_\mu f_\eta^8, \\ \langle 0|A_\mu^8(0)|X(k)\rangle &= \frac{1}{\sqrt{2}} ik_\mu f_X^8, \\ \langle 0|A_\mu^{15}(0)|\eta(k)\rangle &= \frac{1}{\sqrt{2}} ik_\mu f_\eta^{15}, \\ \langle 0|A_\mu^{15}(0)|X(k)\rangle &= \frac{1}{\sqrt{2}} ik_\mu f_X^{15}, \end{aligned} \tag{5.1}$$

where the states η and X are the physical mixtures

$$\begin{aligned} |\eta\rangle &= \cos\theta|P_8\rangle + \sin\theta|P_{15}\rangle, \\ |X\rangle &= -\sin\theta|P_8\rangle + \cos\theta|P_{15}\rangle. \end{aligned} \tag{5.2}$$

Using (5.1) in saturating $I_{8,8}$, $I_{8,15}$, and $I_{15,15}$ we have

$$(f_\eta^8)^2 m_\eta^2 + (f_X^8)^2 m_X^2 = 2\delta(1-\alpha-\beta+3\alpha\beta), \tag{5.3a}$$

$$f_\eta^8 f_\eta^{15} m_\eta^2 + f_X^8 f_X^{15} m_X^2 = \sqrt{2}\,\delta(\alpha+\beta-\alpha\beta), \tag{5.3b}$$

$$(f_\eta^{15})^2 m_\eta^2 + (f_X^{15})^2 m_X^2 = 2\delta(\tfrac{1}{2}\alpha\beta - 2 + 3/R), \tag{5.3c}$$

where, in the last equation, we have used the definition (3.8).

We thus have five unknowns: f_η^8, f_η^{15}, f_X^8, f_X^{15}, and $1/R$. We have only the three equations in (5.3); to solve for the unknowns we need two more equations. We generate these by assuming exact SU(3) symmetry for the matrix elements of A_μ^8 and A_μ^{15}, i.e.,

$$\langle 0|A_\mu^8(0)|P_{15}\rangle = 0 \,, \qquad \langle 0|A_\mu^{15}(0)|P_8\rangle = 0 \,. \tag{5.4}$$

We are thus assuming that SU(3)-breaking effects appear only through the mixing angle, a hypothesis which has frequently been adopted in the literature. We can now solve for the unknowns and in particular for $1/R$. Equations (5.4) give

$$f_\eta^{15} = f_X^{15}\tan\theta \,, \qquad f_X^8 = -f_\eta^8 \tan\theta \,. \tag{5.5}$$

The mixing angle θ is known from the SU(3) mass relation

$$4m_K^2 + m_\pi^2 = 3m_8^2 = 3(m_\eta^2\cos^2\theta + m_X^2\sin^2\theta) \,. \tag{5.6}$$

Using (5.5) in (5.3) we find, after some algebra, the value for $1/R$:

$$\frac{1}{R} = \tfrac{1}{3}\left[2 - \tfrac{1}{2}\alpha\beta + \frac{(\alpha+\beta-\alpha\beta)^2}{1-\alpha-\beta+3\alpha\beta}\,\frac{4m_K^2 - m_\pi^2}{6\sin^2\theta} \times \frac{m_\eta^2\tan^2\theta + m_X^2}{(m_X^2 - m_\eta^2)^2}\right] \,. \tag{5.7}$$

Substituting $\alpha = -0.89$, $\beta = -0.15$, $\theta = 10^\circ$ (Ref. 8), $m_X = 960$ MeV, we find

$$R = 0.35 \,. \tag{5.8}$$

This value for R gives a curve which is almost the curve (b) in Fig. 3. Thus the allowed value of e is never close to -1. The nearest points to $e=-1$ that the curve $R=0.35$ crosses in the regions shown in Figs. 1 or 3 are $e=-2.14$, $f=1.35$ in Region VIII and a curve from $e=-0.58$, $f=-0.07$ to $e=-0.31$, $f=-0.34$ through Region II. Thus the closest we get to $e=-1$ corresponds to the point $e=-0.58$, $f=-0.07$. With the GMOR solution $\alpha = -0.89$, $\beta=-0.15$, we may compute from Eqs. (4.6) and (4.7) the following mass values of the charmed particles: $m_9(P) \simeq 750$ MeV, $m_{13}(P) \simeq 800$ MeV, $m_9(S) \simeq 1600$ MeV, and $m_{13}(S) \to \infty$.[9] The ∞ value for $m_{13}(S)$ arises because the denominator in Eq. (4.6b) vanishes at $f=f_1=-0.07$. The other mass values are rather low and thus unlikely.

We have up till now considered the mixing between the pseudoscalar mesons P_8 and P_{15} to generate the physical states of η and X. We would now like to enlarge our original symmetry group from SW(4) to W(4) ≡ U(4) ⊗ U(4) to see whether or not this enlargement leads to a possible solution of e closer to -1. The enlarged symmetry group, with vacuum still invariant under SU(3) (in the limit of no explicit symmetry breaking), is motivated by the following two considerations:

(a) For some time now there has been another possible candidate for a pseudoscalar SU(3) singlet meson referred to as the E meson whose mass is approximately 1422 MeV.[10] Enlarging the symmetry group to W(4) can provide a natural place for the E meson, since we now have a 3-particle mixing of P_0, P_8, and P_{15} to generate the physical states E, η, and X.

(b) In the context of renormalizable gauge theories, if the hadrons are incorporated through quarks, SW(4) symmetry of strong interactions seems to require invariance under the larger group W(4).[11] Since our original motivation for considerations of the SW(4) symmetry itself is derived from the hadronic structure of unified gauge theories, it is of considerable interest to investigate the enlarged W(4) theory.

For our purposes, the larger group adds three equations:

$$I_{00} = \delta\left[\tfrac{3}{2}\alpha\beta + \frac{1+3ef}{(1+e)(1+f)}\right] \,, \tag{5.9a}$$

$$I_{0,8} = (\tfrac{3}{2})^{1/2}\delta(\alpha+\beta-\alpha\beta) \,, \tag{5.9b}$$

$$I_{0,15} = \frac{\sqrt{3}}{2}\,\delta\left[2 + \alpha\beta - 2\frac{1+3ef}{(1+e)(1+f)}\right] \,. \tag{5.9c}$$

As mentioned before, e and f only enter in the combination (3.8). These expressions are also symmetric under $\alpha \leftrightarrow \beta$.

It is provocative to note that if we only mix η and X with $I_{0,0}$, $I_{0,8}$, and $I_{8,8}$ we get $R \simeq 0.05$, a value much more to our liking. Of course this has meaning only if for some reason P_{15} does not mix with P_8 and P_0.

We now discuss the general mixing problem involving the unmixed states P_0, P_8, and P_{15} that result in the physical states E, η, and X. For this purpose, in addition to Eq. (5.1), we need the definitions:

$$\begin{aligned}
\langle 0|A_\mu^0(0)|\eta(k)\rangle &= \frac{1}{\sqrt{2}}\, ik_\mu f_\eta^0 \,, \\
\langle 0|A_\mu^0(0)|X(k)\rangle &= \frac{1}{\sqrt{2}}\, ik_\mu f_X^0 \,, \\
\langle 0|A_\mu^0(0)|E(k)\rangle &= \frac{1}{\sqrt{2}}\, ik_\mu f_E^0 \,, \\
\langle 0|A_\mu^8(0)|E(k)\rangle &= \frac{1}{\sqrt{2}}\, ik_\mu f_E^8 \,, \\
\langle 0|A_\mu^{15}(0)|E(k)\rangle &= \frac{1}{\sqrt{2}}\, ik_\mu f_E^{15} \,.
\end{aligned} \tag{5.10}$$

The states η, X, and E require three mixing angles which we take to be a standard set of Euler angles by performing the rotations

$$\begin{pmatrix} |E\rangle \\ |\eta\rangle \\ |X\rangle \end{pmatrix} = \begin{pmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} |P_0\rangle \\ |P_8\rangle \\ |P_{15}\rangle \end{pmatrix} . \tag{5.11}$$

Saturating $I_{0,0}$, $I_{0,8}$, $I_{0,15}$, $I_{8,8}$ $I_{8,15}$, $I_{15,15}$, respectively, with η, X, and E and using the definition (3.8), we have

$$(f_\eta^0)^2 m_\eta^2 + (f_X^0)^2 m_X^2 + (f_E^0)^2 m_E^2 = 2\delta(\tfrac{3}{2}\alpha\beta + 1/R), \tag{5.12a}$$

$$f_\eta^0 f_\eta^8 m_\eta^2 + f_X^0 f_X^8 m_X^2 + f_E^0 f_E^8 m_E^2 = \sqrt{6}\,\delta(\alpha+\beta-\alpha\beta), \tag{5.12b}$$

$$f_\eta^0 f_\eta^{15} m_\eta^2 + f_X^0 f_X^{15} m_X^2 + f_E^0 f_E^{15} m_E^2 = \sqrt{3}\,\delta(2+\alpha\beta-2/R), \tag{5.12c}$$

$$(f_\eta^8)^2 m_\eta^2 + (f_X^8)^2 m_X^2 + (f_E^8)^2 m_E^2 = 2\delta(1-\alpha-\beta+3\alpha\beta), \tag{5.12d}$$

$$f_\eta^8 f_\eta^{15} m_\eta^2 + f_X^8 f_X^{15} m_X^2 + f_E^8 f_E^{15} m_E^2 = \sqrt{2}\,\delta(\alpha+\beta-\alpha\beta), \tag{5.12e}$$

$$(f_\eta^{15})^2 m_\eta^2 + (f_X^{15})^2 m_X^2 + (f_E^{15})^2 m_E^2 = 2\delta(\tfrac{1}{2}\alpha\beta - 2 + 3/R). \tag{5.12f}$$

We have 13 unknowns: the nine decay constants f_p^a, the three mixing angles, and 1/R. We have the six equations of (5.12) and the SU(3) mass relation

$$\begin{aligned} 4m_K^2 - m_\pi^2 &= 3m_8^2 \\ &= (m_\eta^2\cos^2\phi + m_X^2\sin^2\phi\cos^2\psi \\ &\quad + m_E^2\sin^2\phi\sin^2\psi) . \end{aligned} \tag{5.13}$$

We can get the additional six equations we need by again assuming exact SU(3) symmetry for the matrix elements

$$\langle 0|A_\mu^a|P_b\rangle = 0, \quad a \neq b \tag{5.14}$$

for a, $b=0$, 8, and 15.

After eliminating all the decay constants we are left with four complicated equations for θ, ϕ, ψ, and 1/R. These four equations can be most easily solved for 1/R if we construct a matrix B, where

$$B = A^T A \tag{5.15}$$

and the matrix A is the diagonal matrix (m_E, m_η, m_X) times the total rotation matrix in (5.11). The four complicated equations can then be written in terms of the elements of the matrix B:

$$B_{22} = m_8^2 , \tag{5.16a}$$

$$\frac{B_{11}}{B_{12}^2} = \gamma_1 , \tag{5.16b}$$

$$\frac{B_{33}}{B_{23}^2} = \gamma_2 , \tag{5.16c}$$

$$\frac{B_{13}}{B_{12}B_{23}} = \gamma_3 , \tag{5.16d}$$

where

$$\gamma_1 = c(3\alpha\beta + 2/R) , \tag{5.17a}$$

$$\gamma_2 = c(3\alpha\beta - 12 + 18/R) , \tag{5.17b}$$

$$\gamma_3 = c(3\alpha\beta + 6 - 6/R) , \tag{5.17c}$$

$$c = \frac{1-\alpha-\beta+3\alpha\beta}{(\alpha+\beta-\alpha\beta)^2} \, \frac{1}{3m_8^2} . \tag{5.17d}$$

The six matrix elements of the symmetric matrix B depend upon only the three unknown angles so there must exist three additional conditions. These are given by

$$\det B = m_\eta^2 m_X^2 m_E^2 , \tag{5.18a}$$

$$\mathrm{Tr}\, B = m_\eta^2 + m_X^2 + m_E^2 , \tag{5.18b}$$

$$\mathrm{Tr}\, B^2 = m_\eta^4 + m_X^4 + m_E^4 . \tag{5.18c}$$

The seven equations (5.16) and (5.18) can now be solved for $1/R$. But this involves solving quadratic equations and the final equation, in which the only unknown in $1/R$, involves the square root of the quantity

$$\begin{aligned} D \equiv\ & K^2(m_\eta^2 + m_X^2 + m_E^2 - m_8^2)^2 \\ & -4K\gamma_1\gamma_2 m_\eta^2 m_X^2 m_E^2 , \end{aligned} \tag{5.19}$$

where

$$K = m_8^2(\gamma_1\gamma_2 - \gamma_3^2) - \gamma_1 - \gamma_2 + 2\gamma_3 .$$

Thus we must require D to be positive. D, however, is only positive for

$$R > 14.3 \tag{5.20a}$$

or

$$\tfrac{4}{3} \leq R \leq \tfrac{5}{3} . \tag{5.20b}$$

Except for $R\approx\frac{4}{3}$ these regions of R are completely outside the allowed values for e and f as given in Fig. 1 and Eq. (3.9).

The final equation has a solution for R in the region (5.20b) at $R\simeq 1.55$. This is clearly outside all of the allowed regions of e and f as shown in Fig. 1. Thus the three-particle mixing of η, X, and E has *no* solution for any of the allowed values of e and f.

VI. $\eta\to\gamma\gamma$ AND $X\to\gamma\gamma$ DECAYS

The solution of the $I_{8,8}$, $I_{8,15}$, $I_{15,15}$ equations, as preformed in Sec. V, requires the additional assumption, beyond one-meson saturation, of exact SU(3) symmetry for the matrix element. This assumption is expressed by (5.4) or (5.14). We can avoid this assumption if we put in more experimental information. In this section we will try to solve the 8, 15 equations by assuming the experimental width for the $\eta\to\gamma\gamma$ decay and assuming it makes sense to calculate this decay and $X\to\gamma\gamma$ in the soft-meson limit.[12]

Consider PCAC including the triangle anomaly

$$\partial^\mu A_\mu^3=\frac{1}{\sqrt{2}}f_\pi m_\pi{}^2\Phi_\pi+\frac{e^2}{16\pi^2}S^{(3)}F_{\mu\nu}\tilde{F}^{\mu\nu}\ , \tag{6.1a}$$

$$\partial^\mu A_\mu^8=\frac{1}{\sqrt{2}}f_\eta^8 m_\eta{}^2\Phi_\eta+\frac{1}{\sqrt{2}}f_X^8 m_X{}^2\Phi_X+\frac{e^2}{16\pi^2}S^{(8)}F_{\mu\nu}\tilde{F}^{\mu\nu}\ , \tag{6.1b}$$

$$\partial^\mu A_\mu^{15}=\frac{1}{\sqrt{2}}f_\eta^{15} m_\eta{}^2\Phi_\eta+\frac{1}{\sqrt{2}}f_X^{15} m_X{}^2\Phi_X+\frac{e^2}{16\pi^2}S^{(15)}F_{\mu\nu}\tilde{F}^{\mu\nu}\ , \tag{6.1c}$$

where $S^{(i)}$ is given by the charge matrix Q as

$$S^{(i)}=\mathrm{Tr}\,Q^2\tfrac{1}{2}\lambda_i\ . \tag{6.2}$$

In calculating the decays we will make the low-energy approximation of assuming that the divergence term in (6.1) does not contribute. Then, effectively,

$$\frac{1}{\sqrt{2}}f_\pi m_\pi{}^2\Phi_\pi=-\frac{e^2}{16\pi^2}S^{(3)}F_{\mu\nu}\tilde{F}^{\mu\nu}\ , \tag{6.3}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} f_\eta^8 m_\eta{}^2 & f_X^8 m_X{}^2\\ f_\eta^{15} m_\eta{}^2 & f_X^{15} m_X{}^2\end{pmatrix}\begin{pmatrix}\Phi_\eta\\ \Phi_X\end{pmatrix}=-\frac{e^2}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\begin{pmatrix}S^{(8)}\\ S^{(15)}\end{pmatrix}. \tag{6.4}$$

Multiplying (6.4) by the inverse of the matrix we have

$$\begin{pmatrix}\Phi_\eta\\ \Phi_X\end{pmatrix}=-\frac{e^2}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\frac{\sqrt{2}}{m_\eta{}^2 m_X{}^2(f_\eta^8 f_X^{15}-f_X^8 f_\eta^{15})}\times\begin{pmatrix} f_X^{15} m_X{}^2 & -f_X^8 m_X{}^2\\ -f_\eta^{15} m_\eta{}^2 & f_\eta^8 m_\eta{}^2\end{pmatrix}\begin{pmatrix}S^{(8)}\\ S^{(15)}\end{pmatrix}. \tag{6.5}$$

The 2γ decay widths are given by

$$\Gamma_{\pi\to\gamma\gamma}=m_\pi{}^3 g_\pi{}^2 G\ , \tag{6.6a}$$

$$\Gamma_{\eta\to\gamma\gamma}=m_\eta{}^3 g_\eta{}^2 G\ , \tag{6.6b}$$

$$\Gamma_{X\to\gamma\gamma}=m_X{}^3 g_X{}^2 G\ , \tag{6.6c}$$

where G is a common (dimensionless) factor and, from (6.3) and (6.5),

$$g_\pi=\frac{S^{(3)}}{f_\pi}\ , \tag{6.7a}$$

$$g_\eta=\frac{f_X^{15}S^{(8)}-f_X^8 S^{(15)}}{f_\eta^8 f_X^{15}-f_X^8 f_\eta^{15}}\ , \tag{6.7b}$$

$$g_X=\frac{f_\eta^8 S^{(15)}-f_\eta^{15}S^{(8)}}{f_\eta^8 f_X^{15}-f_X^8 f_\eta^{15}}\ . \tag{6.7c}$$

Again we will assume the one-meson saturation of $I_{8,8}$, $I_{8,15}$, and $I_{15,15}$ as given by (5.3). Using these three equations and taking the ratio

$$\Gamma_{\eta\to\gamma\gamma}/\Gamma_{\pi\to\gamma\gamma}$$

as known, we have four equations for the unknown f_η^8, f_η^{15}, f_X^8, f_X^{15}, and $1/R$. For an additional relation among the decay constants we will use the formula first given by Glashow *et al.*[12]:

$$4f_K{}^2-f_\pi{}^2=3[(f_\eta^8)^2+(f_X^8)^2]\ . \tag{6.8}$$

This can be easily derived[7] by assuming the SU(3) breaking of the decay constants is of the octet type,

$$\int_0^\infty dm^2\frac{\rho_{ab}^{(0)}(m^2)}{m^2}=c_1\delta_{ab}+c_2 d_{8ab}+c_3\delta_{a,15}\delta_{b,15}+c_4(\delta_{a,15}\delta_{b,8}+\delta_{a,8}\delta_{b,15})+c_5 d_{15ab}\ . \tag{6.9}$$

Solving first for the decay constants we have

$$(f_X^8)^2=\frac{2\delta(1-\alpha-\beta+3\alpha\beta)-\frac{1}{3}m_\eta{}^2(4f_K{}^2-f_\pi{}^2)}{m_X{}^2-m_\eta{}^2}\ , \tag{6.10a}$$

$$(f_\eta^8)^2=\frac{\frac{1}{3}m_X{}^2(4f_K{}^2-f_\pi{}^2)-2\delta(1-\alpha-\beta+3\alpha\beta)}{m_X{}^2-m_\eta{}^2}\ , \tag{6.10b}$$

with more complicated expressions for f_η^{15} and f_X^{15}. Then, in terms of f_η^8 and f_X^8, the value of $1/R$ is given by the expression

$$\frac{1}{R}=\tfrac{2}{3}-\tfrac{1}{6}\alpha\beta+\tfrac{1}{6}\frac{(\alpha+\beta-\alpha\beta)^2}{1-\alpha-\beta+3\alpha\beta}$$

$$+\frac{m_\eta{}^2 m_X{}^2(f_X^8)^2}{3(1-\alpha-\beta+3\alpha\beta)}\left[\frac{(1/\sqrt{2})(\alpha+\beta-\alpha\beta)S^{(8)}/S^{(3)}-(1-\alpha-\beta+3\alpha\beta)S^{(15)}/S^{(3)}}{2\delta(1-\alpha-\beta+3\alpha\beta)(m_\pi/m_\eta)^{3/2}(1/f_\pi)(\Gamma_\eta/\Gamma_\pi)^{1/2}-f_\eta^8 m_\eta{}^2 S^{(8)}/S^{(3)}}\right]^2, \tag{6.11}$$

where by Γ_η/Γ_π we mean the experimental 2γ rates. Taking[10]

$$\Gamma_{\eta\to 2\gamma}=1000\pm 250\text{ eV},$$
$$\Gamma_{\pi\to 2\gamma}=7.3\pm 1.5\text{ eV}, \tag{6.12}$$

we have $(\Gamma_\eta/\Gamma_\pi)^{1/2}\simeq 11.7\pm 2.5$. The decay constants are given by (6.10), but because only the square of f_η^8 can be found, the relative sign of the two terms in the denominator of (6.11) is not fixed. Substituting the values $\alpha=-0.89$, $\beta=-0.15$, $f_K/f_\pi=1.13$, and $\delta=5.3\,f_\pi{}^2m_\pi{}^2$, we have

$$\frac{1}{R}=0.74+0.17\left[\frac{S^{(15)}/S^{(3)}+0.34S^{(8)}/S^{(3)}}{(1.53\text{ to }2.36)\pm S^{(8)}/S^{(3)}}\right], \tag{6.13}$$

where the range of numbers in the denominator corresponds to the values of $(\Gamma_\eta/\Gamma_\pi)^{1/2}$ allowed by the experimental errors.

To determine $S^{(3)}$, $S^{(8)}$, and $S^{(15)}$ it is necessary to have a model for the quarks. For example, in the four-quark model p, n, λ, p', with a charge matrix

$$Q=\begin{pmatrix} q & & & \\ & q-1 & & \\ & & q-1 & \\ & & & q\end{pmatrix}, \tag{6.14}$$

we have

$$\frac{S^{(8)}}{S^{(3)}}=\frac{1}{\sqrt{3}},\qquad \frac{S^{(15)}}{S^{(3)}}=-(\tfrac{2}{3})^{1/2}, \tag{6.15}$$

independent of q. Using these values in (6.13) gives

$$1.23\leqslant R\leqslant 1.33. \tag{6.16}$$

For R values in this range the curve of allowed e, f values would pass through Region III of Fig. 1. This gives e values very far from -1. To get values of e closer to -1 we need a value for R closer to zero, that is, we need the denominator of (6.13) to be small.[13] Unfortunately this does not seem possible since $S^{(8)}/S^{(3)}$ is rather model-independent. The charge scheme of Ref. 4, where the quark charges are choosen by analogy to the leptons, also gives (6.15).

Of course it is well known that the value for $S^{(8)}$ given in (6.15) does not give the correct $\eta\to\gamma\gamma$ decay rate[14] so we probably should not be surprised that using $S^{(8)}$ with the experimental value of $\eta\to\gamma\gamma$ requires e to be far from -1. Probably all we should conclude is that SW(4) with $e\approx-1$ does not solve the $\eta\to\gamma\gamma$ problem that existed in the SW(3) theory. One would suspect the fault is with the low-energy approximation.

So far we have not discussed the decay $X\to\gamma\gamma$. Since we know all of the decay constants which enter (6.7c) in terms of the $\eta\to\gamma\gamma$ decay rate we can immediately write a sum rule relating the two decays,

$$\frac{f_\eta^8}{f_\pi}\left(\frac{m_\pi}{m_\eta}\right)^{3/2}\left(\frac{\Gamma_\eta}{\Gamma_\pi}\right)^{1/2}+\frac{f_X^8}{f_\pi}\left(\frac{m_\pi}{m_X}\right)^{3/2}\left(\frac{\Gamma_X}{\Gamma_\pi}\right)^{1/2}=\frac{S^{(8)}}{S^{(3)}}. \tag{6.17}$$

Again f_η^8 and f_X^8 are given by (6.10) and again we cannot determine the signs of f_η^8 and f_X^8. Using (6.12) and (6.15) we can predict

$$1.50\times 10^4\leqslant\frac{\Gamma_X}{\Gamma_\pi}\leqslant 2.84\times 10^4, \tag{6.18a}$$

or

$$2.07\times 10^3\leqslant\frac{\Gamma_X}{\Gamma_\pi}\leqslant 8.37\times 10^3, \tag{6.18b}$$

where the range of values within (6.18a) or (6.18b) is caused by the error in Γ_η/Γ_π and the two solutions (6.18a) or (6.18b) correspond to the uncertainty in the signs of f_η^8 and f_X^8.

Our sum rule is very different from that of Glashow *et al.*[12] who found

$$\left(\frac{m_\pi}{m_X}\right)^5\frac{\Gamma_X}{\Gamma_\pi}+\left(\frac{m_\pi}{m_\eta}\right)^5\frac{\Gamma_\eta}{\Gamma_\pi}\geqslant 1,$$

which in turn implies $\Gamma_X/\Gamma_\pi\geqslant 9\times 10^3$ for all of the usual SU(3) quark models.[7]

VII. SUMMARY

The very general conditions of positivity of the spectral functions give bounds on the symmetry-breaking parameters α, β, e, and f which are definitely nontrivial. In particular the allowed domain structure rules out the solutions of Ref. 3. The additional assumption of meson dominance for I_{33}, I_{44}, and K_{44} leads to the GMOR solution for α and β; $\alpha\approx-1$, $\beta\approx 0$. Further, the experimental condition that the masses of the charmed particles be much larger than the masses of the known particles requires that the parameter e be close to -1 which is also the point where $SU(3)\otimes SU(3)$ is an approximate symmetry of the Hamiltonian. These results require only rather weak assumptions.

Unfortunately, meson dominance of $I_{8,8}$, $I_{8,15}$ with the η and X mesons does not lead to values of e very close to -1. The closest value is on the upper edge of Region II in Fig. 1, where $e=-0.58$. If we enlarge the basic group to $U(4)\otimes U(4)$, meson dominance of I_{00}, I_{08}, I_{88}, $I_{0,15}$, $I_{8,15}$, and $I_{15,15}$ with the η, X, and E mesons, yields no solution for the allowed values of e. Similarly, the experimental value for the two-photon decay of η is inconsistent with $e\approx -1$.

Of course these calculations require stronger assumptions than were required by the results mentioned in the first paragraph; either the mass mixing assumption or, in the case of $\eta\to\gamma\gamma$, the low-energy approximation. It should be mentioned, however, that in seeking a solution close to $e=-1$ [$SU(3)\otimes SU(3)$ limit and therefore SU(3) limit], one might expect that the SU(3) assumption (5.4) or (5.14) would not be unreasonable, and a similar comment may be made about the low-energy approximation for the $\eta\to\gamma\gamma$ decay. Whereas our calculation of η-X mixing does show that a solution exists in the allowed domain II of Fig. 1, this does not get close enough to the $SU(3)\otimes SU(3)$ symmetric point, which seems to be crucial for generating large mass values for the charmed particles.

APPENDIX

Using Eqs. (2.7), it is easy to show that the boundaries of the allowed domains in Fig. 1 are related to various subgroups of SW(4) as follows:

(1) $e=e_1\equiv -|\alpha|/(2+|\alpha|)$ implies $\partial\mu V_\mu^{13,14}=0$ and together with isospin, hypercharge (F^8) and charm (F^{15}) conservation corresponds to the validity of the subgroup $SU_I(2)\otimes SU(2)\otimes U(1)$, where $SU_I(2)$ is the usual isospin group generated by $F^{1,2,3}$, U(1) is the hypercharge group generated by F^8 and SU(2) is generated by $F^{13,14,15}$. If the vacuum is invariant under isospin, hypercharge and charm, then at $e=e_1$, the scalar isosinglets S_{13} and S_{14} are zero-mass Goldstone particles.

(2) $e=e_2\equiv|\alpha|/(4-|\alpha|)$ leads to $\partial\mu V_\mu^{9,10,11,12}=0$ and corresponds to the validity of an SU(3) subgroup of SW(4) generated by $F^{1,2,3,9,10,11,12}$ and $\frac{1}{3}F^8+\frac{2}{3}\sqrt{2}F^{15}$. If the generators $F^{9,10,11,12}$ do not destroy the vacuum state, the scalar isodoublets $S_{9,10}$ and $S_{11,12}$ will be massless.

(3) $e=e_3\equiv(1-\frac{1}{2}|\alpha|)/(1+\frac{1}{2}|\alpha|)$ yields $\partial\mu A_\mu^{9,10,11,12}=0$ and corresponds to the validity of another SU(3) subgroup of SW(4) generated by $F^{1,2,3}$, $\frac{1}{3}F^8+\frac{2}{3}\sqrt{2}F^{15}$, and $F_5^{9,10,11,12}$. This is the chimeral analog of the SU(3) group realized at $e=e_2$. With the vacuum invariance as in (2), we expect the pseudoscalar isodoublets $P_{9,10}$ and $P_{11,12}$ to be massless here.

(4) $e=e_4\equiv(1+|\alpha|)/(1-|\alpha|)$ corresponds to $\partial\mu A_\mu^{13,14}=0$ and implies that the Hamiltonian density is invariant under another subgroup $SU_I(2)\otimes SU'(2)\otimes U(1)$ of SW(4), where SU′(2) is the chimeral analog of the SU(2) group discussed in (1) and is generated by $F_5^{13,14}$ and F^{15}. If the symmetry of the vacuum is generated by $F^{1,2,3,8,15}$, then the pseudoscalar isosinglet mesons P_{13} and P_{14} would be massless.

(5) $e=-1$ implies the usual chiral $SU(3)\otimes SU(3)$ symmetry of the Hamiltonian as discussed in Sec. III.

There are other subgroups of SW(4) which are realized for the special value, $\alpha=0$. For example, the Hamiltonian would be SU(4)-invariant (generated by $F^1,\ldots,F^{15}$) if $\alpha=0$ and $e=0$. Note if $\alpha=0$, the points e_1 and e_2 coincide at $e=0$, the SU(4)-invariant point. Similarly, the Hamiltonian would be invariant under a chimeral SU(4) symmetry, generated by $F^1,\ldots,F^8$, $F_5^9,\ldots,F_5^{14}$, and F^{15}, if $\alpha=0$ and $e=1$. Again for $\alpha=0$, e_3 and e_4 coincide at this point.

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SU(4) × SU(4) as an approximate symmetry of hadrons*

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The (4, 4*) + (4*, 4) model of broken SU(4) × SU(4) as an approximate symmetry of hadrons is investigated. Spectral-function sum rules for scalar and pseudoscalar densities are derived in this model. Sum rules based on octet-type breaking of SU(3) at $q^2 = 0$ and $q^2 = \infty$ are also obtained. It is shown that the $q^2 = 0$ sum rule rules out SU(2) × SU(2) as a good symmetry of the Hamiltonian, when the vacuum is approximately SU(3)-invariant, in the present model. Thus the problem of understanding SU(2) × SU(2) and SU(3) as approximate symmetries of the Hamiltonian persists as in the case of the popular (3, 3*) + (3*, 3) model of broken SU(3) × SU(3). It is shown that the $q^2 = \infty$ sum rule is consistent with the idea that SU(2) × SU(2) is a good symmetry of the Hamiltonian. A mass formula for charmed pseudoscalar mesons is also derived.

I. INTRODUCTION

SU(4)×SU(4) as an approximate symmetry of hadrons has recently been proposed by several authors. The motivation for such an idea comes from the recent developments in unified gauge theories of leptons and several attempts to incorporate hadrons in such theories.[1] In unified gauge theories with hadrons, in order to restore renormalizability and eliminate sizeable strangeness-changing neutral currents, a fourth quark has been introduced carrying charm quantum number in addition to the usual triplet of quarks. It is then natural to consider SU(4) as a possible approximate symmetry[2] of hadrons. This, however, poses problems because the known spectrum of hadrons seems to fall in SU(3) multiplets. A way out of this difficulty has been proposed by Dittner and Eliezer. They suggest SU(4)×SU(4) as an approximate symmetry[3] of the Hamiltonian of hadrons where the symmetry is realized by Goldstone bosons, and in the chiral limit the vacuum is only SU(3)-invariant. In this scheme, Dittner *et al.*[3,4] have been able to obtain a solution for the symmetry-breaking parameters which shows that both SU(2)×SU(2) and SU(3) are good symmetries of the Hamiltonian. Their solution also requires that the masses of the charmed mesons be large (~5 GeV), explaining why such particles, if they exist, have not yet been detected.

The purpose of the present paper is to analyze the breaking of SU(4)×SU(4) down to the isospin group SU(2) following a method[5] recently applied to SU(3)×SU(3). The basic idea of such an approach is to obtain constraints[6,7] on the symmetry-breaking parameters by studying the spectral-function sum rules for the scalar and pseudoscalar densities. In Sec. II of this paper, spectral-function sum rules for the scalar and pseudoscalar densities in the broken SU(4)×SU(4) model are derived. In Sec. III sum rules based on the assumption of octet-type breaking of SU(3) for the two-point functions are derived. It is shown that the broken-SU(3) sum rule for the pseudoscalar density constrains the symmetry-breaking parameters in such a way that SU(2)×SU(2) cannot be a good symmetry of the Hamiltonian if the vacuum is approximately SU(3)-invariant. This is in contradiction with the result of Dittner *et al.*[3,4] who claim SU(2)×SU(2) as well as SU(3) as good symmetries of the Hamiltonian. In Sec. IV sum rules are derived assuming the validity of octet-type breaking of asymptotic SU(3) symmetry[8] for the two-point functions. It is shown that the asymptotic sum rules are consistent