PHYSICAL REVIEW D **89,** 126003 (2014)

BPS $M2$-branes in $AdS_4 \times Q^{1,1,1}$ and their dual loop operators

Jun-Bao Wu[1,*] and Meng-Qi Zhu[2,†]

[1]*Institute of High Energy Physics, and Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, 19B Yuquan Road, Beijing 100049, People's Republic of China*

[2]*Department of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, 5 Yiheyuan Road, Beijing 100871, People's Republic of China*

(Received 27 February 2014; published 5 June 2014; corrected 11 June 2014)

In this paper, we first compute the Killing spinors of $AdS_4 \times Q^{1,1,1}$ and its certain orbifolds. Based on this, two classes of $M2$-brane solutions are found. The first class of solutions includes $M2$-branes dual to Wilson loops in the fundamental representation as a special case. The second class includes the candidates of the holographic description of vortex loops in the dual field theories.

DOI: 10.1103/PhysRevD.89.126003 PACS numbers: 11.25.Tq, 11.25.Yb

I. INTRODUCTION

Many examples of the AdS_4/CFT_3 correspondence have been established since the seminal paper [1], which itself was inspired by Refs. [2–6]. In this correspondence, certain three-dimensional superconformal Chern-Simons-matter theories are proposed to be dual to M theory on $AdS_4 \times X_7$. The three-dimensional theory has $\mathcal{N} = 1$ (2,3) supersymmetry when X_7 is a weak G_2 (Sasaki-Einstein, 3-Sasaki) manifold. Loop operators play an important role in the studies of this AdS_4/CFT_3 duality, as they do in the case of the AdS_5/CFT_4 correspondence. The 1/6-BPS Wilson loops in the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory were first studied in detail in Refs. [7–9]. Later, a highly nontrivial 1/2-BPS Wilson loop was constructed in Ref. [10]. An interesting explanation on the origin of these Wilson loops was given in [11] based on [12]. Some exact results for Wilson loops were obtained based on powerful tools of supersymmetric localization [13]. The Wilson loops with quite less supersymmetries were studied in [14–16].

It is certainly interesting to generalize these studies on loop operators to AdS_4/CFT_3 correspondences with less supersymmetries, since now the dynamics is less constrained by supersymmetries. In Chern-Simon-matter theories with $\mathcal{N} = 2$ supersymmetries, BPS Wilson loops can be constructed when the loop is a straight line or a circle [17]. This point is different from the four-dimensional $\mathcal{N} = 1$ gauge theories, although they have the same number of supercharges. Half-BPS Wilson loops in generic three-dimensional $\mathcal{N} = 2$ Chern-Simons-matter theories were studied in detail in [18]. The geometry of the matrix models obtained from localization was connected to the geometry of $M2$-brane solutions in the holographic description based on results from differential geometry. There also exists a vortex loop, a kind of disordered operator, in these theories. The holographic dual of the vortex loop in the ABJM theory was studied in [19]. The vortex loops in generic $\mathcal{N} = 2$ Chern-Simons-matter theories were studied by using localization in [20,21] based on [22,23].

The aim of the current paper is to study BPS $M2$-branes in a concrete example, with duality to loop operators in mind. The first reason why we picked up the Sasaki-Einstein manifold $Q^{1,1,1}$ is that the metric of this manifold is very simple, though its isometry group is small. The second, less obvious reason is that the Killing spinor equation is easy to solve on this manifold.[1] We further discussed the Killing spinors of certain orbifolds of $AdS_4 \times Q^{1,1,1}$ by using Lie-Lorentz derivation of spinors with respect to Killing vectors [25–27]. Based on these results, we found two classes of $M2$-branes. The world volumes of these $M2$-branes all have the topology $AdS_2 \times S^1$. The AdS_2 factor is embedded to the AdS_4 part of the background geometry, so these $M2$-branes are candidates for the holographic duals of loop operators. In the first class, the S^1 is embedded in $Q^{1,1,1}$. This class includes the $M2$-branes dual to Wilson loops in the fundamental representation. We think that our study here is complementary to the results in Ref. [18] based on more abstract mathematical tools. In the second class of $M2$-branes, this S^1 has a nontrivial profile in both AdS_4 and $Q^{1,1,1}$. These $M2$-branes are similar to the $M2$-branes in $AdS_4 \times S^7/Z_k$ dual to vortex loops in the ABJM theory [19].

We also noticed that there had been much research about M theory on $AdS_4 \times Q^{1,1,1}$ and its various orbifolds. This is another reason why we choose to study $M2$-branes in this background. Various field theory duals were proposed and checked in Refs. [28–32].[2] Localization was performed to obtained a matrix model [34] for the field theory proposed in Refs. [31,32]. Superconformal indices were computed in

*wujb@ihep.ac.cn
†mqzfly@pku.edu.cn

[1]A similar thing was noticed for the five-dimensional Sasaki-Einstein manifold $T^{1,1}$ [24].

[2]An old proposal can be found in Ref. [33].

Refs. [35,36]. Some other membranes and five-branes in this background were studied in [37–39]. Some spinning membranes dual to local operators were found in [40]. The Penrose limit of $\mathrm{AdS}_4 \times Q^{1,1,1}$ was studied in [41,42]. Some supergravity solutions related to $\mathrm{AdS}_4 \times Q^{1,1,1}$ were discussed in Ref. [43].

In the next section, we will solve the Killing spinor equations on $\mathrm{AdS}_4 \times Q^{1,1,1}$. Two classes of BPS $M2$-brane solutions will be discussed in Sec. 3.

II. KILLING SPINORS OF $\mathrm{AdS}_4 \times Q^{1,1,1}$

The metric on $\mathrm{AdS}_4 \times Q^{1,1,1}$ is

$$ds^2 = R^2(ds_4^2 + ds_7^2), \tag{1}$$

$$ds_4^2 = \frac{1}{4}(\cosh^2 u(-\cosh^2\rho dt^2 + d\rho^2) + du^2 + \sinh^2 u d\phi^2), \tag{2}$$

$$ds_7^2 = \sum_{i=1}^{3}\frac{1}{8}(d\theta_i^2 + \sin^2\theta_i d\phi_i^2) + \frac{1}{16}\left(d\psi + \sum_{i=1}^{3}\cos\theta_i d\phi_i\right)^2, \tag{3}$$

with $\theta_i \in [0,\pi], \phi_i \in [0,2\pi](i=1,2,3), \psi \in [0,4\pi]$. The four-form field strength on this background is

$$H_4 = \frac{3R^3}{8}\cosh^2 u \sinh u \cosh \rho dt \wedge d\rho \wedge du \wedge d\phi. \tag{4}$$

Two kinds of Z_k orbifolds of $Q^{1,1,1}$ were considered in the literature. In the first case [28,29], the orbifold is obtained via the identification $(\phi_1,\phi_2) \sim (\phi_1 + \frac{2\pi}{k}, \phi_2 + \frac{2\pi}{k})$. In the second case [30], the identification is $\phi_1 \sim \phi_1 + \frac{2\pi}{k}$. We will denote the first orbifold as $Q^{1,1,1}/Z_k$ and the second orbifold as $Q^{1,1,1}/Z'_k$ from now on. Flux quantization gives

$$R = 2\pi l_p\left(\frac{N}{6vol(Q^{1,1,1}/Z_k)}\right)^{1/6} \tag{5}$$

$$= l_p\left(\frac{2^8\pi^2 kN}{3}\right)^{1/6}, \tag{6}$$

where we have used

$$vol(Q^{1,1,1}/Z_k) = \frac{\pi^4}{8k}. \tag{7}$$

In order to find the Killing spinors, we find it very useful to introduce the following one-forms[3]:

[3]Such a trick was used for $T^{1,1}$ in Ref. [24].

$$\sigma_I^1 = d\theta_I, \tag{8}$$

$$\sigma_I^2 = \sin\theta_I d\phi_I, \tag{9}$$

$$\sigma_I^3 = \cos\theta_I d\phi_I, \tag{10}$$

with $I = 1,2$ and

$$w^1 = -\cos\psi \sin\theta_3 d\phi_3 + \sin\psi d\theta_3, \tag{11}$$

$$w^2 = \sin\psi \sin\theta_3 d\phi_3 + \cos\psi d\theta_3, \tag{12}$$

$$w^3 = d\psi + \cos\theta_3 d\phi_3, \tag{13}$$

which satisfy

$$d\sigma_I^i + \frac{1}{2}\epsilon^{ijk}\sigma_I^j \wedge \sigma_I^k = 0, \tag{14}$$

$$dw^i + \frac{1}{2}\epsilon^{ijk}w^j \wedge w^k = 0. \tag{15}$$

Using these one-forms, we can reexpress the above metric on $Q^{1,1,1}$ as

$$ds_7^2 = \sum_{I=1}^{2}\frac{1}{8}[(\sigma_I^1)^2 + (\sigma_I^2)^2] + \frac{1}{8}(w^1)^2 + \frac{1}{8}(w^2)^2 + \frac{1}{16}(\sigma_1^3 + \sigma_2^3 + w^3)^2. \tag{16}$$

Now the vielbeins of the 11-dimensional metric are

$$e^{\underline{0}} = \frac{R}{2}\cosh u \cosh \rho dt, \qquad e^{\underline{1}} = \frac{R}{2}\cosh u\, d\rho, \tag{17}$$

$$e^{\underline{2}} = \frac{R}{2}du, \qquad e^{\underline{3}} = \frac{R}{2}\sinh u d\phi, \tag{18}$$

$$e^{\underline{4}} = \frac{R}{2\sqrt{2}}\sigma_1^1, \qquad e^{\underline{5}} = \frac{R}{2\sqrt{2}}\sigma_1^2, \tag{19}$$

$$e^{\underline{6}} = \frac{R}{2\sqrt{2}}\sigma_2^1, \qquad e^{\underline{7}} = \frac{R}{2\sqrt{2}}\sigma_2^2, \tag{20}$$

$$e^{\underline{8}} = \frac{R}{2\sqrt{2}}w^1, \qquad e^{\underline{9}} = \frac{R}{2\sqrt{2}}w^2, \tag{21}$$

$$e^{\underline{\sharp}} = \frac{R}{4}(\sigma_1^3 + \sigma_2^3 + w^3). \tag{22}$$

The spin connections with respect to these vielbeins are

$$\omega^{\underline{01}} = \frac{2}{R}\frac{\tanh\rho}{\cosh u}e^{\underline{0}}, \qquad \omega^{\underline{02}} = \frac{2}{R}\tanh u e^{\underline{0}}, \tag{23}$$

$$\omega^{\underline{12}} = \frac{2}{R}\tanh u e^{\underline{1}}, \qquad \omega^{\underline{23}} = -\frac{2}{R}\coth u e^{\underline{3}}, \tag{24}$$

$$\omega^{\underline{45}} = \frac{1}{R}(-2\sqrt{2}\cot\theta_1 e^{\underline{5}} + e^{\underline{\sharp}}),$$
$$\omega^{\underline{67}} = \frac{1}{R}(-2\sqrt{2}\cot\theta_2 e^{\underline{7}} + e^{\underline{\sharp}}), \tag{25}$$

$$\omega^{\underline{89}} = \frac{1}{R}(2\sqrt{2}\cot\theta_1 e^{\underline{5}} + 2\sqrt{2}\cot\theta_2 e^{\underline{7}} - 3e^{\underline{\sharp}}), \tag{26}$$

$$\omega^{\underline{4\sharp}} = \frac{1}{R}e^{\underline{5}}, \qquad \omega^{\underline{5\sharp}} = -\frac{1}{R}e^{\underline{4}}, \tag{27}$$

$$\omega^{\underline{6\sharp}} = \frac{1}{R}e^{\underline{7}}, \qquad \omega^{\underline{7\sharp}} = -\frac{1}{R}e^{\underline{6}}, \tag{28}$$

$$\omega^{\underline{8\sharp}} = \frac{1}{R}e^{\underline{9}}, \qquad \omega^{\underline{9\sharp}} = -\frac{1}{R}e^{\underline{8}}. \tag{29}$$

And H_4 can now be written as

$$H_4 = \frac{6}{R}e^{\underline{0}} \wedge e^{\underline{1}} \wedge e^{\underline{2}} \wedge e^{\underline{3}}. \tag{30}$$

The Killing spinors of $\mathrm{AdS}_4 \times Q^{1,1,1}$ satisfy the following equation:

$$\nabla_{\underline{m}}\eta + \frac{1}{576}(3\Gamma_{\underline{npqr}}\Gamma_{\underline{m}} - \Gamma_{\underline{m}}\Gamma_{\underline{npqr}})H^{\underline{npqr}}\eta = 0. \tag{31}$$

Our convention about the product of the 11 Γ matrices is

$$\Gamma_{\underline{0123456789\sharp}} = 1. \tag{32}$$

Using the vielbeins and the spin connections given above, we find that the solution to the above equation is

$$\eta = e^{\frac{u}{2}\Gamma_{\underline{2}}\hat{\Gamma}} e^{\frac{\rho}{2}\Gamma_{\underline{1}}\hat{\Gamma}} e^{\frac{t}{2}\Gamma_{\underline{0}}\hat{\Gamma}} e^{\frac{\phi}{2}\Gamma_{\underline{23}}}\eta_0, \tag{33}$$

where η_0 is independent of all the coordinates and satisfies the projection conditions

$$\Gamma^{\underline{45}}\eta_0 = \Gamma^{\underline{67}}\eta_0 = \Gamma^{\underline{89}}\eta_0, \tag{34}$$

and $\hat{\Gamma}$ is defined as

$$\hat{\Gamma} = \Gamma_{\underline{0123}}. \tag{35}$$

The Killing spinors of $Q^{1,1,1}$ were also studied in [44,45]. The Killing spinors of AdS_4 were given in this coordinate system in [7,19].

The above projection conditions show that the background on $\mathrm{AdS}_4 \times Q^{1,1,1}$ is 1/4 BPS; i.e., eight supercharges are preserved. These supercharges correspond to four super-Poincaré charges and four superconformal charges in the dual three-dimensional superconformal field theory.

Now we turn to consider the Killing spinors of the orbifolds $\mathrm{AdS}_4 \times Q^{1,1,1}/Z_k$ and $\mathrm{AdS}_4 \times Q^{1,1,1}/Z'_k$. For this purpose, we compute the Lie-Lorentz derivative of the above Killing spinor η with respect to the Killing vector $K_i \equiv \frac{\partial}{\partial\phi_i}$ defined as

$$\mathcal{L}_{K_i}\eta \equiv (K_i)^{\underline{m}}\nabla_{\underline{m}}\eta + \frac{1}{4}(\nabla_{\underline{m}}(K_i)_{\underline{n}})\Gamma^{\underline{mn}}\eta. \tag{36}$$

After some calculations, we find

$$\mathcal{L}_{K_i}\eta = 0, \tag{37}$$

for each i. This result tells us that η is also the Killing spinor of $\mathrm{AdS}_4 \times Q^{1,1,1}/Z_k$ and $\mathrm{AdS}_4 \times Q^{1,1,1}/Z'_k$. In other words, the supersymmetries are not broken by this orbifolding.

III. PROBE MEMBRANE SOLUTIONS IN $\mathrm{AdS}_4 \times Q^{1,1,1}$

In this section, we will find two classes of probe $M2$-brane solutions in $\mathrm{AdS}_4 \times Q^{1,1,1}$. The bosonic part of the $M2$-brane action is

$$S_{M2} = T_2\left(\int d^3\xi\sqrt{-\det g_{mn}} - \int P[C_3]\right), \tag{38}$$

where g_{mn} is the induced metric on the membrane, T_2 is the tension of the $M2$-brane:

$$T_2 = \frac{1}{(2\pi)^2 l_p^3}, \tag{39}$$

and $P[C_3]$ is the pullback of the bulk 3-form gauge potential to the world volume of the membrane. The gauge choice for the background 3-form gauge potential C_3 in the case at hand is

$$C_3 = \frac{R^3}{8}(\cosh^3 u - 1)\cosh\rho dt \wedge d\rho \wedge d\phi. \tag{40}$$

From the variation of this action, the membrane equation of motion is

$$\frac{1}{\sqrt{-g}}\partial_m(\sqrt{-g}g^{mn}\partial_n X^{\underline{N}})G_{\underline{MN}} + g^{mn}\partial_m X^{\underline{N}}\partial_n X^{\underline{P}}\Gamma^{\underline{Q}}_{\underline{NP}}G_{\underline{QM}}$$
$$= \frac{1}{3!\sqrt{-g}}\epsilon^{mnp}(P[H_4])_{\underline{M}mnp}. \tag{41}$$

We always use the indices from the beginning (middle) of the alphabet to refer to the frame (coordinate) indices and the underlined indices to refer to the target space ones. Also notice that ϵ^{mnp} is a tensor density on the world volume of the membrane.

We are mainly interested in BPS $M2$-branes. The supersymmetry projector equation reads

$$\Gamma_{M2}\eta = \eta, \tag{42}$$

with

$$\Gamma_{M2} = \frac{1}{\sqrt{-g}}\partial_\tau X^{\mu_1}\partial_\xi X^{\mu_2}\partial_\sigma X^{\mu_3} e^{m_1}_{\mu_1} e^{m_2}_{\mu_2} e^{m_3}_{\mu_3}\Gamma_{\underline{m}_1\underline{m}_2\underline{m}_3}, \tag{43}$$

where τ, ξ, σ are coordinates on the world volume of the $M2$-brane.

A. BPS $M2$-branes dual to Wilson loops revisited

In this class of solutions, the world volume of the $M2$-brane has the topology $\mathrm{AdS}_2 \times S^1$ with $\mathrm{AdS}_2 \in \mathrm{AdS}_4$ and $S^1 \in M_7$. From now on, by M_7 we mean either $Q^{1,1,1}/Z_k$ or $Q^{1,1,1}/Z'_k$. This class includes $M2$-branes dual to BPS Wilson loops in gauge theories as a special case, and this case was studied in [18]. In that paper, the authors started with general discussions on BPS Wilson loops in the fundamental representation in $\mathcal{N}=2$ Chern-Simons-matter theories and the dual $M2$-brane solutions. They also included $M2$-branes in $\mathrm{AdS}_4 \times Q^{1,1,1}/Z_k$ as one of the explicit examples. They used a different coordinate system for the AdS_4 part, and for the $Q^{1,1,1}/Z_k$ part they used some results in differential geometry which appeared in their general discussions. We will use the explicit results of Killing spinors obtained in the previous section.

The ansatz of these solutions is

$$t=\tau, \quad \rho=\xi, \quad \psi=\psi(\sigma), \quad \phi_i=\phi_i(\sigma), \quad i=1,2,3, \tag{44}$$

with u, ϕ, θ_i $(i=1,2,3)$ being constants. Here τ, ξ, σ are three coordinates on the world volume of the $M2$-brane. We consider the case that $\sigma \in [0, 2\pi]$ is a compact direction (i.e., we always identify $\sigma + 2\pi$ with σ).

The periodic conditions for the fields ψ, ϕ_i are

$$\psi(\sigma+2\pi) = \psi(\sigma) + 2\pi n_\psi, \tag{45}$$

$$\phi_1(\sigma+2\pi) = \phi_1(\sigma) + \frac{2\pi n_1}{k}, \tag{46}$$

$$\phi_2(\sigma+2\pi) = \phi_2(\sigma) + \frac{2\pi n_1}{k} + 2\pi n_2, \tag{47}$$

$$\phi_3(\sigma+2\pi) = \phi_3(\sigma) + 2\pi n_3, \tag{48}$$

with $n_i \in \mathbf{Z}$, $i=1,2,3$, when $M_7 = Q^{1,1,1}/Z_k$.

For the case that $M_7 = Q^{1,1,1}/Z'_k$, the corresponding conditions are

$$\psi(\sigma+2\pi) = \psi(\sigma) + 2\pi n_\psi, \tag{49}$$

$$\phi_1(\sigma+2\pi) = \phi_1(\sigma) + \frac{2\pi n_1}{k}, \tag{50}$$

$$\phi_2(\sigma+2\pi) = \phi_2(\sigma) + 2\pi n_2, \tag{51}$$

$$\phi_3(\sigma+2\pi) = \phi_3(\sigma) + 2\pi n_3, \tag{52}$$

with $n_i \in \mathbf{Z}$, $i=1,2,3$.

Now the $M2$-brane action is

$$S_{M2} = \frac{T_{M2}R^3}{4}\int d^3\sigma \cosh^2 u \cosh\rho \times \left[\frac{1}{8}\sum_{i=1}^{3}\sin^2\theta_i\phi_i'^2 + \frac{1}{16}\left(\psi' + \sum_{i=1}^{3}\cos\theta_i\phi_i'\right)^2\right]^{1/2}, \tag{53}$$

where $'$ means $\partial/\partial\sigma$. The equation of motion for u gives

$$u = 0, \tag{54}$$

while the equation of motion from variation of θ_i gives

$$\sin\theta_i\phi_i'\left(\psi' + \sum_{j=1}^{3}\cos\theta_j\phi_j' - 2\cos\theta_i\phi_i'\right) = 0. \tag{55}$$

The equations of motion for ψ, ϕ_i can be solved by

$$\psi = m_\psi\sigma, \qquad \phi_i = m_i\sigma. \tag{56}$$

We also checked that the above three equations are equivalent to the results from the $M2$-brane equations of motion given in Eq. (41).

To compute the on-shell action of the $M2$-brane whose boundary at infinity is an S^1, we switch to the Euclidean AdS_4 with the metric:

$$ds_4^2 = \frac{1}{4}(\cosh^2 u(d\rho + \sinh^2\rho d\psi^2) + du^2 + \sinh^2 u d\phi^2). \tag{57}$$

The on-shell action of the $M2$-brane [Eq. (53)] now becomes

$$S_{M2} = \frac{T_{M2}R^3}{4}\int d\Omega_{EAdS_2} d\sigma \times \left[\frac{1}{8}\sum_{i=1}^{3}\sin^2\theta_i m_i^2 + \frac{1}{16}\left(m_\psi + \sum_{i=1}^{3}\cos\theta_i m_i\right)^2\right]^{1/2}, \tag{58}$$

with

$$\int d\Omega_{EAdS_2} = \int d\rho d\psi \sinh\rho. \tag{59}$$

Using the fact that $\sigma\in[0,2\pi]$, $T_{M2}=1/(4\pi^2 l_p^3)$, and Eq. (6), we get

$$S_{M2}=2\sqrt{\frac{kN}{3}\left(\frac{1}{8}\sum_{i=1}^{3}\sin^2\theta_i m_i^2+\frac{1}{16}\left(m_\psi+\sum_{i=1}^{3}\cos\theta_i m_i\right)^2\right)}\int d\Omega_{EAdS_2}. \tag{60}$$

After adding boundary terms as in Ref. [46], we get

$$S_{M2}=-4\pi\sqrt{\frac{kN}{3}\left(\frac{1}{8}\sum_{i=1}^{3}\sin^2\theta_i m_i^2+\frac{1}{16}\left(m_\psi+\sum_{i=1}^{3}\cos\theta_i m_i\right)^2\right)}. \tag{61}$$

We now search for the BPS $M2$-brane in $\mathrm{AdS}_4\times Q^{1,1,1}$ among these solutions. Γ_{M2} now becomes

$$\Gamma_{M2}=\left(\frac{1}{16}\left(\psi'+\sum_{i=1}^{3}\cos\theta_i\phi_i'\right)^2+\frac{1}{8}\sum_{i=1}^{3}\sin^2\theta_i\phi_i'^2\right)^{-1/2}\times\Gamma_{\underline{01}}\left(\frac{1}{4}\left(\psi'+\sum_{i=1}^{3}\cos\theta_i\phi_i'\right)\Gamma_{\underline{\sharp}}+\frac{1}{\sqrt{2}}\sin\theta_1\phi_1'\Gamma_{\underline{5}}+\frac{1}{\sqrt{2}}\sin\theta_2\phi_2'\Gamma_{\underline{7}}-\frac{1}{\sqrt{2}}\cos\psi\sin\theta_3\phi_3'\Gamma_{\underline{8}}+\frac{1}{\sqrt{2}}\sin\psi\sin\theta_3\phi_3'\Gamma_{\underline{9}}\right). \tag{62}$$

We need the solutions of $\Gamma_{M2}\eta=\eta$ to also satisfy the projection conditions Eq. (34). This leads to, for each i,

$$\sin\theta_i=0, \tag{63}$$

or

$$\phi_i'=0. \tag{64}$$

Now we get

$$\Gamma_{M2}=\mathrm{sgn}\left(m_\psi+\sum_{i=1}^{3}\cos\theta_i m_i\right)\Gamma_{\underline{01\sharp}}. \tag{65}$$

The BPS condition leads to

$$\Gamma_{\underline{01\sharp}}\eta=\mathrm{sgn}\left(m_\psi+\sum_{i=1}^{3}\cos\theta_i m_i\right)\eta. \tag{66}$$

By using the fact that we have $u=0$ on the world volume of this $M2$-brane solution, it is not hard to see that the above condition is equivalent to the condition

$$\Gamma_{\underline{01\sharp}}\eta_0=\pm\eta_0, \tag{67}$$

on the $M2$-brane world volume. This condition is compatible with the projection conditions Eq. (34), and this BPS $M2$-brane is half-BPS with respect to the background.

The $M2$-brane in $\mathrm{AdS}_4\times Q^{1,1,1}/Z_k$ dual to the half-BPS Wilson loop is a special solution of this class [18]. It is given by

$$m_\psi=0,\qquad m_1=m_2=\frac{1}{k},$$
$$m_3=0,\qquad(\theta_1,\theta_2)=(0,0),(0,\pi),(\pi,0),(\pi,\pi). \tag{68}$$

The result for the on-shell action is

$$S_{M2}=-2\pi\sqrt{\frac{N}{3k}}, \tag{69}$$

when $(\theta_1,\theta_2)=(0,0),(\pi,\pi)$, while in the case that $(\theta_1,\theta_2)=(0,\pi),(\pi,0)$

$$S_{M2}=0. \tag{70}$$

The first two solutions give leading contribution to the vacuum expectation value of Wilson loops, which reads

$$\langle W\rangle\sim\exp\left(2\pi\sqrt{\frac{N}{3k}}\right), \tag{71}$$

in the leading order of large N expansion. As mentioned in [18], this is consistent with the result from the matrix model computations in Ref. [34].

Similarly, among the half-BPS $M2$-branes in $\mathrm{AdS}_4\times Q^{1,1,1}/Z_k'$, the one with

$$m_\psi=0,\quad m_1=\frac{1}{k},\quad m_2=m_3=0,\quad \theta_1=0,\pi \tag{72}$$

is dual to half-BPS Wilson loops. For the on-shell action

$$S_{M2} = -\pi\sqrt{\frac{N}{3k}}, \tag{73}$$

we get

$$\langle W\rangle \sim \exp\left(\pi\sqrt{\frac{N}{3k}}\right). \tag{74}$$

B. The second class of solutions

Now we consider the ansatz

$$t = \tau, \qquad \rho = \xi, \qquad \phi = \sigma, \tag{75}$$

$$\psi = \psi(\sigma), \qquad \phi_i = \phi_i(\sigma), \tag{76}$$

with u, θ_i being constant. We also demand that u is nonzero. The $M2$-brane action is now

$$S_{M2} = \frac{T_{M2}R^3}{8}\int d^3\sigma \cosh\rho \times [\cosh^2 u\sqrt{\sinh^2 u + c} - \cosh^3 u + 1], \tag{77}$$

with the definition of c

$$c \equiv \frac{1}{2}\sum_{i=1}^{3}\sin^2\theta_i^2\phi_i'^2 + \frac{1}{4}\left(\psi' + \sum_{i=1}^{3}\cos\theta_i\phi_i'\right)^2. \tag{78}$$

The equation of motion for u gives

$$2\cosh u\sinh u\sqrt{\sinh^2 u + c} + \frac{\cosh^3 u\sinh u}{\sqrt{\sinh^2 u + c}} - 3\sinh u\cosh^2 u = 0. \tag{79}$$

For nonzero u, it has two solutions:

$$c = 1 \tag{80}$$

and

$$c = -\frac{3}{4}\cosh^2 u + 1. \tag{81}$$

From now on we will consider only the first solution, which leads to

$$2\sum_{i=1}^{3}\sin^2\theta_i\phi_i'^2 + \left(\psi' + \sum_{i=1}^{3}\cos\theta_i\phi_i'\right)^2 = 4. \tag{82}$$

Similar to the solutions in the previous subsection, the equation of motion for θ_i gives

$$\sin\theta_i\phi_i'\left(\psi' + \sum_{j=1}^{3}\cos\theta_j\phi_j' - 2\cos\theta_i\phi_i'\right) = 0. \tag{83}$$

And the equations of motion for ψ, ϕ_i can be solved by

$$\psi = m_\psi\sigma, \qquad \phi_i = m_i\sigma. \tag{84}$$

The above equations are equivalent to the results from the $M2$-brane equations of motion given in Eq. (41).

Now we turn to discuss the BPS condition for the $M2$-branes in $AdS_4 \times Q^{1,1,1}$. Now Γ_{M2} becomes

$$\begin{aligned}\Gamma_{M2} = \frac{1}{\cosh^2 u\sqrt{\sinh^2 u + c}} &\times \cosh\rho\Gamma_{\underline{01}}\left(\sinh u\Gamma_{\underline{3}} + \frac{1}{2}\left(\psi' + \sum_{i=1}^{3}\phi_i'\right)\Gamma_{\underline{\sharp}}\right. \\ &+ \frac{1}{\sqrt{2}}\sin\theta_1\phi_1'\Gamma_{\underline{5}} + \frac{1}{\sqrt{2}}\sin\theta_2\phi_2'\Gamma_{\underline{7}} \\ &\left.- \frac{1}{\sqrt{2}}\cos\psi\sin\theta_3\phi_3'\Gamma_{\underline{8}} + \frac{1}{\sqrt{2}}\sin\psi\sin\theta_3\phi_3'\Gamma_{\underline{9}}\right).\end{aligned} \tag{85}$$

To have BPS branes, we also need for each i to have

$$\sin\theta_i = 0 \tag{86}$$

or

$$\phi_i' = 0. \tag{87}$$

The fact that $c = 1$ now leads to

$$\psi' + \sum_{i=1}^{3}\cos\theta_i\phi_i' = \pm 2. \tag{88}$$

Using these results, we can get

$$\Gamma_{M2} = \Gamma_{\underline{01}}\left(\frac{\sinh u}{\cosh u}\Gamma_{\underline{3}} \pm \frac{1}{\cosh u}\Gamma_{\underline{\sharp}}\right). \tag{89}$$

From Eq. (33), we can get that

$$\Gamma_{M2}\eta = \eta \tag{90}$$

is equivalent to

$$\Gamma_{\underline{01\sharp}}\eta_0 = \pm\eta_0. \tag{91}$$

So when for each $i = 1, 2, 3$ we have either $\sin\theta_i = 0$ or ϕ_i being constant on the world volume, the $M2$-branes in this class are half-BPS. This is similar to the situation in the previous subsection. After some calculations using the metric in Eq. (57), we can get that the on-shell action of the $M2$-brane is

$$S_{M2} = -2\pi\sqrt{\frac{kN}{3}}, \tag{92}$$

with the boundary term included.

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, we found some BPS $M2$-branes in M theory on $\mathrm{AdS}_4 \times Q^{1,1,1}$ and its certain orbifolds. We reproduced the $M2$-branes dual to BPS Wilson loops in the fundamental representation in the field theory side. We also studied a second class of the BPS $M2$-branes which should include the $M2$-branes dual to vortex loops in the field theory side. We also find the explicit solution to the Killing spinor equations in this background.

There are several further directions that are interesting for us. For the holographic dual to BPS Wilson loops in the (anti)fundamental representation, one should search for suitable $D2$ ($D6$)-brane solutions in the IIA string background obtained from the S^1 reduction of the above M-theory background. On the other hand, one can try to find a suitable $M2$-branes (Kaluza-Klein monopoles) solution in the M-theory background directly. To correctly identify the dual brane solutions, we also need a more precise understanding of the loop operators in the field theory side. We would also like to try to generalize our studies here to other Sasaki-Einstein 7-manifolds. We hope to report our progress in these directions in the near future.

ACKNOWLEDGMENTS

The authors are grateful to Bin Chen, Jarah Evslin, Nakwoo Kim, De-Sheng Li, Wei Li, Feng-Li Lin, Zheng-Wen Liu, Jiang Long, Hong Lu, Jian-Feng Wu, Gang Yang, Jie Yang, and Hossein Yavartanoo for various helpful discussions. J. W. thanks Lanzhou University for warm hospitality during the Workshop on String/M-theory, Gravity and Topological Field Theories and Ehwa Womans University for warm hospitality during the IEU Workshop on Solving AdS/CFT. This work was supported in part by the National Natural Science Foundation of China under Contracts No. 11105154 (J. W.), No. 11222549 (J. W.), No. 10925522 (M. Z.), and No. 11021092 (M. Z.). J. W. gratefully acknowledges the support of K. C. Wong Education Foundation and Youth Innovation Promotion Association, CAS, as well.

[1] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, J. High Energy Phys. 10 (2008) 091.
[2] J. Bagger and N. Lambert, Phys. Rev. D **75**, 045020 (2007).
[3] J. Bagger and N. Lambert, Phys. Rev. D **77**, 065008 (2008).
[4] J. Bagger and N. Lambert, J. High Energy Phys. 02 (2008) 105.
[5] A. Gustavsson, Nucl. Phys. **B811**, 66 (2009).
[6] A. Gustavsson, Nucl. Phys. **B807**, 315 (2009).
[7] N. Drukker, J. Plefka, and D. Young, J. High Energy Phys. 11 (2008) 019.
[8] B. Chen and J.-B. Wu, Nucl. Phys. **B825**, 38 (2010).
[9] S.-J. Rey, T. Suyama, and S. Yamaguchi, J. High Energy Phys. 03 (2009) 127.
[10] N. Drukker and D. Trancanelli, J. High Energy Phys. 02 (2010) 058.
[11] K.-M. Lee and S. Lee, J. High Energy Phys. 09 (2010) 004.
[12] D. Berenstein and D. Trancanelli, Phys. Rev. D **78**, 106009 (2008).
[13] A. Kapustin, B. Willett, and I. Yaakov, J. High Energy Phys. 03 (2010) 089.
[14] V. Cardinali, L. Griguolo, G. Martelloni, and D. Seminara, Phys. Lett. B **718**, 615 (2012).
[15] N. Kim, Mod. Phys. Lett. A **28**, 1350150 (2013).
[16] D. Marmiroli, arXiv:1312.2972.
[17] D. Gaiotto and X. Yin, J. High Energy Phys. 08 (2007) 056.
[18] D. Farquet and J. Sparks, J. High Energy Phys. 01 (2014) 083.
[19] N. Drukker, J. Gomis, and D. Young, J. High Energy Phys. 03 (2009) 004.
[20] A. Kapustin, B. Willett, and I. Yaakov, J. High Energy Phys. 06 (2013) 099.
[21] N. Drukker, T. Okuda, and F. Passerini, arXiv:1211.3409.
[22] D. L. Jafferis, J. High Energy Phys. 05 (2012) 159.
[23] N. Hama, K. Hosomichi, and S. Lee, J. High Energy Phys. 03 (2011) 127.
[24] D. Arean, D. E. Crooks, and A. V. Ramallo, J. High Energy Phys. 11 (2004) 035.
[25] Y. Kosmann, Annali di Matematica Pura Applicata **91**, 317 (1972).
[26] J. M. Figueroa-O'Farrill, Classical Quantum Gravity **16**, 2043 (1999).
[27] T. Ortin, Classical Quantum Gravity **19**, L143 (2002).
[28] S. Franco, A. Hanany, J. Park, and D. Rodriguez-Gomez, J. High Energy Phys. 12 (2008) 110.
[29] S. Franco, I. R. Klebanov, and D. Rodriguez-Gomez, J. High Energy Phys. 08 (2009) 033.
[30] M. Aganagic, Nucl. Phys. **B835**, 1 (2010).
[31] F. Benini, C. Closset, and S. Cremonesi, J. High Energy Phys. 02 (2010) 036.
[32] D. L. Jafferis, J. High Energy Phys. 08 (2013) 046.
[33] D. Fabbri, P. Fre', L. Gualtieri, C. Reina, A. Tomasiello, A. Zaffaroni, and A. Zampa, Nucl. Phys. **B577**, 547 (2000).

[34] S. Cheon, H. Kim, and N. Kim, J. High Energy Phys. 05 (2011) 134.
[35] S. Cheon, D. Gang, S. Kim, and J. Park, J. High Energy Phys. 05 (2011) 027.
[36] R. Eager and J. Schmude, arXiv:1305.3547.
[37] I. R. Klebanov, S. S. Pufu, and T. Tesileanu, Phys. Rev. D **81**, 125011 (2010).
[38] C.-h. Ahn, Phys. Lett. B **466**, 171 (1999).
[39] N. Benishti, D. Rodriguez-Gomez, and J. Sparks, J. High Energy Phys. 07 (2010) 024.
[40] N. Kim and J. H. Lee, Int. J. Mod. Phys. A **26**, 1019 (2011).
[41] U. Gursoy, C. Nunez, and M. Schvellinger, J. High Energy Phys. 06 (2002) 015.
[42] C.-h. Ahn, Phys. Lett. B **539**, 281 (2002).
[43] R. Hernandez and K. Sfetsos, J. High Energy Phys. 07 (2002) 045.
[44] P. Hoxha, R. R. Martinez-Acosta, and C. N. Pope, Classical Quantum Gravity **17**, 4207 (2000).
[45] A. Donos and J. P. Gauntlett, J. High Energy Phys. 10 (2012) 120.
[46] N. Drukker, D. J. Gross, and H. Ooguri, Phys. Rev. D **60**, 125006 (1999).