PHYSICAL REVIEW D **89,** 124035 (2014)

Note on Kerr/CFT correspondence in a first order formalism

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(Received 28 April 2014; published 27 June 2014)

In symmetry-based approaches to black hole entropy, we calculate the central charge of the Virasoro algebra in the first order formulation of gravity for both Palatini and Holst actions. In these calculations, we make use of the near-horizon extremal Kerr metric and the Kerr/conformal field theory correspondence. For the Palatini action the results obtained in the second order formulation are reproduced. We also argue that the Holst term does not contribute to the charge algebra no matter what geometry/boundary conditions one is considering.

DOI: 10.1103/PhysRevD.89.124035 PACS numbers: 04.70.Bw, 04.70.Dy

I. INTRODUCTION

The symmetry-based approaches or the dual holographic description of the black hole entropy has its origin in the work of Brown and Henneaux [1]. The essential point is to argue that the quantum theory of black holes should be a holographic dual of a 2D conformal field theory (CFT) living at the spacetime boundary. One then expects that the states in some unitary representation of the CFT are the microstates of the black hole. The actual construction of the CFT is still elusive. So far, one has been able to only count the microstates rather than explicitly construct a dual CFT. Even the location of the boundary is debatable. In [1], the boundary is taken at asymptotic infinity, while in some other calculations the boundary is taken as the black hole horizon (see [2] and references therein). The main idea behind this approach is to identify the 2D conformal symmetry group, isomorphic to $Diff(S^1)$, that is expected to be the symmetry group of the holographic quantum theory of black holes. The states of this quantum theory, which are possibly the black hole microstates in question, would then furnish a representation for this symmetry group. The representation is expected to be characterized by the appropriate black hole parameters, such as the horizon area, charges and angular momentum. The black hole entropy would then be equal to the logarithm of the dimension of such a representation (for example, the number of quantum states for a fixed area, angular momentum and charges). From the outset, the aim is to calculate only the number of microstates knowing that from the symmetry group alone it would be impossible to label a complete set of microstates. When one looks for symmetries near the horizon or at asymptotic infinity, the usual notion of symmetries represented by exact Killing vectors is not enough. One, therefore, uses an extended notion of symmetries in terms of approximate Killing vectors that gives a larger set of vector fields. The set of such approximate Killing fields is determined by certain falloff conditions on the metric.

The Kerr/CFT correspondence originally initiated by [3] is parallel to the idea of Brown and Henneaux except for the fact that the background spacetime is now the near-horizon extremal Kerr (NHEK), which is topologically $AdS_2 \times S^2$, rather than AdS_3 as in the case of Brown and Henneaux. The appropriate boundary in NHEK is a timelike boundary.

In symmetry-based approaches the issue of the correct Poisson brackets of charges (in the second order formulation) is not completely resolved and is tied to the choice of boundary conditions. There exists more than one way of calculating the Poisson brackets of charges [4–6] and it seems that their algebra having a central extension in one calculation may have different or no central extension at all in some other calculations. On the other hand the symplectic structure in the first order formulation is clean and studied in detail in the context of asymptotic symmetries [7,8] and laws of black hole mechanics [9–13]. Therefore, it seems justified to apply the first order symplectic structure to a well-studied case—the Kerr/CFT correspondence.

In the first order formulation, apart from studying the bulk symplectic structure one also studies the boundary symplectic structure. Depending on the boundary conditions, one may need to add a boundary symplectic current to avoid leakage of any flux across the boundary. This ensures that the symplectic structure is hypersurface independent. This subtle issue is apparently overlooked in the existing calculations of Poisson brackets of charges in the second order formulation. This has already been pointed out in [14] for instance.

We also study the effect of adding the Holst term to the action. It is already known [15] that in presence of the Holst term different values of the Immirzi parameter yield nonequivalent quantum gravity theories and a particular choice is to be made to recover the Bekenstein-Hawking (BH) entropy from the exact counting of states in the quantum theory [16]. An intriguing question is does the semiclassical symmetry-based approach retain any imprint of the Immirzi parameter? We argue that if one works with a hypersurface-independent symplectic structure then the

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Holst term will never contribute, no matter what geometry/boundary conditions one is considering. This is in agreement with some recent calculations [17] where it is claimed that the semiclassical limit of black hole entropy in loop quantum gravity (LQG) does not depend on particular choices of the Immirzi parameter.

In the context of Wald entropy the Holst term in the presence of a negative cosmological constant has been studied in [18]. It has been shown that the Immirzi parameter does not play a role for anti–de Sitter (AdS)-Schwarzschild and AdS-Kerr spacetimes but makes a nontrivial contribution to the entropy and mass for AdS-Taub-Nut spacetime (Similar results have been obtained using Euclidean path integrals in [19]). In the recent past some attempts have been made to compare the Wald entropy and the entropy from symmetry-based approaches from some alternative construction of the Poisson bracket algebra [6]. Our results show that Wald entropy and entropy from symmetry-based approaches might not always match.

The Kerr/CFT correspondence has been generalized to an isolated-horizon CFT correspondence in [20]. In this case the metric in the neighborhood of an axisymmetric extremal isolated horizon has been used and a calculation similar to the one in the Kerr/CFT correspondence has been carried out. However, a study of the "near-horizon" symmetries of an isolated horizon is still missing. Since isolated horizons are studied primarily in the first order formulation, our exercise might shed some light on a symmetry-based approach to isolated horizons.

In this note, we start with the NHEK metric and redo the calculations of the Poisson brackets of charges in the first order formulation of gravity (all of the calculations that have appeared till now have been in the second order formulation). We also study the effect of adding the Holst term to the action. This gives some insight into what role the Holst term plays in the semiclassical regime.

II. THE NHEK METRIC AND BOUNDARY CONDITIONS

A. Boundary conditions

The NHEK geometry which has an $SL(2,R)\times U(1)$ isometry group has been studied in detail in [21]. We would not go into the details of the NHEK geometry except for the fact that in some global coordinate system the NHEK metric takes the form,

$$ds^2 = 2GJ\Omega^2\left(-(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + \right. \\ \left. + d\theta^2 + \Lambda^2(d\phi^2 + r dt^2)\right). \tag{1}$$

The tetrads can then be obtained such that they satisfy $g_{\mu\nu} = \eta_{IJ} e^I_\mu e^J_\nu$, where η_{IJ} is the Minkowski metric. It then follows that the tetrads can be taken to be

$$e^0 = N\sqrt{1+r^2}dt, \qquad e^1 = \frac{Ndr}{\sqrt{1+r^2}}, \\ e^2 = Nd\theta, \qquad e^3 = N\Lambda(d\phi + rdt), \tag{2}$$

$$N = (2JG\Omega^2)^{\frac{1}{2}}, \qquad \Omega^2 = \frac{1+\cos^2\theta}{2}, \qquad \Lambda = \frac{2\sin\theta}{1+\cos^2\theta}. \tag{3}$$

The range of the coordinates is $0 \le \theta < \pi$ and $0 \le \phi < 2\pi$ and the boundary at $r \to \infty$ is a timelike boundary.

The connection can be calculated from the torsion-free condition, $de^I + \omega^I{}_J \wedge e^J = 0$. It can be recast in the form,

$$\omega^{IJ}_\mu = e^{I\nu}\nabla_\mu e^J_\nu, \tag{4}$$

where ∇_μ is the usual covariant derivative compatible with the metric. It then follows that

$$\omega^{10} = \frac{1}{2} r(\Lambda^2 - 2)dt + \frac{1}{2}\Lambda^2 d\phi, \\ \omega^{20} = -\frac{\sqrt{1+r^2}\frac{dN}{d\theta}}{N}dt, \qquad \omega^{30} = \frac{1}{2}\frac{\Lambda}{\sqrt{1+r^2}}dr, \\ \omega^{21} = -\frac{\frac{dN}{d\theta}}{N\sqrt{1+r^2}}dr, \qquad \omega^{31} = \tfrac{1}{2}\Lambda\sqrt{1+r^2}dt, \\ \omega^{32} = \frac{r}{N}\frac{d(N\Lambda)}{d\theta}dt + \frac{1}{N}\frac{d(N\Lambda)}{d\theta}d\phi. \tag{5}$$

Under the boundary conditions assumed in [3]:

$$\begin{pmatrix} h_{tt} = O(r^2) & h_{t\phi} = O(1) & h_{t\theta} = O(\frac{1}{r}) & h_{tr} = O(\frac{1}{r^2}) \\ h_{\phi t} = h_{t\phi} & h_{\phi\phi} = O(1) & h_{\phi\theta} = O(\frac{1}{r}) & h_{\phi r} = O(\frac{1}{r}) \\ h_{\theta t} = h_{t\theta} & h_{\theta\phi} = h_{\phi\theta} & h_{\theta\theta} = O(\frac{1}{r}) & h_{\theta r} = O(\frac{1}{r^2}) \\ h_{rt} = h_{tr} & h_{r\phi} = h_{\phi r} & h_{r\theta} = h_{\theta r} & h_{rr} = O(\frac{1}{r^3}) \end{pmatrix}. \tag{6}$$

The asymptotic symmetry-generating vector fields can be calculated using

$$\mathcal{L}_\xi g_{\mu\nu} = h_{\mu\nu} \tag{7}$$

and then equating the terms of the same orders in r on both sides. It then follows that symmetry-generating vector fields are of the form,

$$\xi_A = (-r\epsilon'(\phi) + O(1))\partial_r + \left(C + O\left(\frac{1}{r^3}\right)\right)\partial_t + \\ + \left(\epsilon(\phi) + O\left(\frac{1}{r^2}\right)\right)\partial_\phi + O\left(\frac{1}{r}\right)\partial_\theta, \tag{8}$$

where the higher order terms generate trivial diffeomorphisms. The relevant subalgebra isomorphic to a $Diff(S^1)$ is then

$$\xi = \epsilon \frac{\partial}{\partial \phi} - r\epsilon' \frac{\partial}{\partial r}, \tag{9}$$

with $\epsilon(\phi) = -e^{-im\phi}$.

B. Asymptotic expansion of tetrads

The tetrads and the connection can be expanded in a power series:

$$\begin{aligned} e^I &= {}^0e^I + \frac{{}^1e^I}{r} + \frac{{}^2e^I}{r^2} + \cdots, \\ \omega^{IJ} &= {}^0\omega^{IJ} + \frac{{}^1\omega^{IJ}}{r} + \frac{{}^2\omega^{IJ}}{r^2} + \cdots. \end{aligned} \tag{10}$$

Unlike the asymptotically flat case [7] where ${}^0e^I$ is just the Minkowski tetrad and fixed in the phase space, here it does vary because of the boundary conditions imposed. So rather than taking the ANHEK (from here on ANHEK would mean the asymptotic form of the NHEK metric) tetrad as the zeroth order one, we take the following:

$$\begin{aligned} &{}^0e^0 = NA(t,\theta,\phi)rdt, \qquad {}^0e^1 = \frac{Ndr}{r} + NB(t,\theta,\phi)d\phi, \\ &{}^0e^2 = Nd\theta, \qquad {}^0e^3 = \frac{N\Lambda}{C(t,\theta,\phi)}d\phi + N\Lambda C(t,\theta,\phi)rdt. \end{aligned} \tag{11}$$

We retain the terms that go like $rdt, \frac{dr}{r}, d\phi, d\theta$ at the zeroth order. We assume certain regularity conditions to hold on A, B, C to ensure that the tetrads do not become degenerate for any values of θ and ϕ. We note that the asymptotic metric calculated with this is

$$\begin{aligned} ds^2 = 2GJ\Omega^2 \bigg(&-(A(t,\theta,\phi)^2 - \Lambda^2 C(t,\theta,\phi)^2)r^2dt^2 + \frac{dr^2}{r^2} \\ &+ 2\frac{B(t,\theta,\phi)}{r}drd\phi + d\theta^2 + 2r\Lambda^2 dtd\phi \\ &+ \left(\frac{\Lambda^2}{C(t,\theta,\phi)^2} + B(t,\theta,\phi)^2\right)d\phi^2\bigg), \end{aligned} \tag{12}$$

which is in agreement with the falloff conditions. Moreover with the replacement,

$$\begin{aligned} &C(t,\theta,\phi) = A(t,\theta,\phi) = 1 + \eta F(t,\phi), \\ &B(t,\theta,\phi) = \eta\partial_\phi F(t,\phi) \end{aligned} \tag{13}$$

correctly reproduces the asymptotic constraints [3] at linear order in η and leading order in r. For completeness we spell out these conditions. For perturbations $h_{\mu\nu}$ about the NHEK metric the asymptotic constraints imply

$$\begin{aligned} h_{\phi\phi} &= \Lambda^2\Omega^2 f(t,r,\phi), \\ h_{tt} &= r^2(1-\Lambda^2)\Omega^2 f(t,r,\phi), \\ h_{r\phi} &= -\frac{\Omega^2}{2r}\partial_\phi f(t,r,\phi). \end{aligned} \tag{14}$$

Any other contribution to the tetrad consistent with the boundary conditions enter ${}^1e^I$ and higher order terms in the asymptotic expansion. A typical form of ${}^1e^I$ would be

$$\begin{aligned} &{}^1e^0 = A_1(t,\theta,\phi)rdt + A_2(t,\theta,\phi)d\phi, \\ &{}^1e^1 = B_1(t,\theta,\phi)\frac{dr}{r} + B_2(t,\theta,\phi)d\theta + B_3(t,\theta,\phi)d\phi, \\ &{}^1e^2 = C_1(t,\theta,\phi)\frac{dr}{r} + C_2(t,\theta,\phi)d\theta + C_3(t,\theta,\phi)d\phi, \\ &{}^1e^3 = D_1(t,\theta,\phi)rdt + D_2(t,\theta,\phi)d\phi. \end{aligned} \tag{15}$$

One can check that this in agreement with the boundary conditions.

III. PALATINI ACTION

A. Symplectic structure

The Palatini action in first order gravity is given by

$$S = -\frac{1}{16\pi G}\int_{\mathcal{M}} (\Sigma_{IJ} \wedge F^{IJ}), \tag{16}$$

where $\Sigma_{IJ} = \frac{1}{2}\epsilon_{IJKL}e^K \wedge e^L$, ω^{IJ} is a Lorentz $SO(3,1)$ connection and F^{IJ} is a curvature 2-form corresponding to the connection given by $F^{IJ} = d\omega^{IJ} + \omega^I{}_K \wedge \omega^{KJ}$. The action might have to be supplemented with boundary terms to make the variation well defined. But that does not affect the symplectic structure $\Omega(\delta_1, \delta_2)$, since δ_1, δ_2 are independent variations (i.e. they commute).

On shell the variation of the Lagrangian gives $\delta L = d\Theta(\delta)$ where $16\pi G\Theta(\delta) = -\Sigma_{IJ} \wedge \delta\omega^{IJ}$. One then constructs the symplectic structure Ω on the space of solutions. One first constructs the symplectic current $J(\delta_1,\delta_2) = \delta_1\Theta(\delta_2) - \delta_2\Theta(\delta_1)$, which is closed on shell. The symplectic structure is then given by

$$\Omega(\delta_1,\delta_2) = \int_M J(\delta_1,\delta_2) = -\frac{1}{8\pi G}\int_M (\delta_{[1}\Sigma_{IJ} \wedge \delta_{2]}\omega^{IJ}), \tag{17}$$

where M is a Cauchy surface.

A point to note here is that there can be nontrivial contributions from the boundary symplectic structure. We consider the symplectic current 3-form for the Palatini action. It follows that on shell

$$dJ = 0. \tag{18}$$

This implies that when integrated over a closed region of spacetime bounded by $M_1 \cup M_2 \cup B$ (where B is a portion of the boundary of spacetime given by $r \to \infty$ in our case),

$$\int_{M_1} J - \int_{M_2} J + \int_{r\to\infty} J = 0, \tag{19}$$

where M_1, M_2 are the initial and final Cauchy surfaces that asymptote to constant time slices.

If the third term vanishes then the bulk symplectic structure is already hypersurface independent. If the third term does not vanish and turns out to be exact i.e.

$$\int_{r\to\infty} J = \int_{r\to\infty} dj, \tag{20}$$

then the symplectic structure given by $\int_M J - \int_{S_\infty} j$ (where S_∞ is the 2-surface at the intersection of the hypersurface M with the boundary) is hypersurface independent and $j(\delta_1, \delta_2)$ is the "boundary symplectic current". The hypersurface-independent symplectic structure is then given by

$$\tilde{\Omega}(\delta_1, \delta_2) = -\frac{1}{8\pi G}\int_M (\delta_{[1}\Sigma_{IJ} \wedge \delta_{2]}\omega^{IJ}) - \int_{S_\infty} j(\delta_1, \delta_2). \tag{21}$$

For a vector field X the variation δ_X acts on the fields like a Lie derivative $\mathcal{L}_X$. One can then show that if the equations of motion hold in the bulk, then the bulk symplectic structure $\Omega(\delta, \delta_X)$ contributes only at the boundary ∂M of the Cauchy surface M. Therefore it follows that

$$\tilde{\Omega}(\delta, \delta_X) = \Omega(\delta, \delta_X) - \int_{S_\infty} j(\delta, \delta_X), \tag{22}$$

where

$$\Omega(\delta, \delta_X) = -\frac{1}{16\pi G}\int_{\partial M} [(X.\omega^{IJ})\delta\Sigma_{IJ} - (X.\Sigma_{IJ}) \wedge \delta\omega^{IJ}]. \tag{23}$$

For another vector field X' it immediately follows that

$$\tilde{\Omega}(\delta_{X'}, \delta_X) = \Omega(\delta_{X'}, \delta_X) - \int_{S_\infty} j(\delta_{X'}, \delta_X), \tag{24}$$

where

$$\Omega(\delta_{X'}, \delta_X) = -\frac{1}{16\pi G}\int_{\partial M} [(X.\omega^{IJ})\mathcal{L}_{X'}\Sigma_{IJ} - (X.\Sigma_{IJ}) \wedge \mathcal{L}_{X'}\omega^{IJ}]. \tag{25}$$

It then implies that if the vector fields are Hamiltonian (Sec. III D),

$$\left[H_X, H_{X'}\right] = H_{[X,X']} + \tilde{\Omega}(\delta_{X'}, \delta_X), \tag{26}$$

where the term $H_{[X,X']}$ is added to take into account the nonvanishing of $[\delta_X, \delta_{X'}]$.

B. The boundary symplectic structure

To go ahead with any calculation we first need to find the boundary symplectic structure. The only contributions to the boundary symplectic structure come from $\delta_{[1}\Sigma_{10} \wedge \delta_{2]}\omega^{10}$ and $\delta_{[1}\Sigma_{30} \wedge \delta_{2]}\omega^{30}$. For variations of the form (which corresponds to variations about the ANHEK background obeying the linearized asymptotic constraints)

$$\begin{aligned} A &= C = 1, \\ B &= 0, \\ \delta A &= \delta C, \\ \delta B &= \partial_\phi \delta A, \end{aligned} \tag{27}$$

and using the form of the connection calculated from only the zeroth order tetrad (Appendix B) one can show that

$$\begin{aligned} \int_{r\to\infty} J &= \frac{1}{4\pi G}\int \frac{\partial}{\partial t}\left(\frac{N^2\Lambda}{A}\delta_{[1}A\delta_{2]}B\right) dt \wedge d\theta \wedge d\phi \\ &= \frac{1}{4\pi G}\int_{S_2}\left(\frac{N^2\Lambda}{A}\delta_{[1}A\delta_{2]}B\right) d\theta \wedge d\phi \\ &\quad - \frac{1}{4\pi G}\int_{S_1}\left(\frac{N^2\Lambda}{A}\delta_{[1}A\delta_{2]}B\right) d\theta \wedge d\phi, \end{aligned} \tag{28}$$

where S_1, S_2 are the intersections of M_1, M_2 respectively with the boundary.

To arrive at the above result we first identified the total time derivative and then used restrictions Eq. (27) to see if the other terms vanish.

It then follows that the relevant hypersurface-independent quantity $\tilde{\Omega}(\delta, \delta_X)$ is given by

$$\begin{aligned} \tilde{\Omega}(\delta, \delta_X) = &-\frac{1}{16\pi G}\int_{S_\infty} [(X.\omega^{IJ})\delta\Sigma_{IJ} - (X.\Sigma_{IJ}) \wedge \delta\omega^{IJ}] \\ &- \frac{1}{8\pi G}\int_{S_\infty} \frac{N^2\Lambda}{A}(\delta A\delta_X B - \delta_X A\delta B) d\theta \wedge d\phi. \end{aligned} \tag{29}$$

In general the boundary symplectic structure can have nonzero order-one contributions coming from higher order terms in the asymptotic expansion. Ideally one should do the asymptotic expansion and check the boundary symplectic structure order by order. We must point out that we were unable to find a systematic way to isolate the terms of different orders in ω^{IJ}. However in this case since we will be studying perturbations generated by ξ around the ANHEK background it would suffice to check the zeroth order tetrad.

C. Algebra of charges

We therefore go ahead and calculate $\tilde{\Omega}(\delta_\xi, \delta_{\xi'})$. To calculate the contribution from the bulk it is enough to consider only the NHEK tetrad [Eq. (2)] and connection [Eq. (4)] and not the quantities in the asymptotic expansion. As can be seen the relevant vector field has a nonzero interior product $\xi.e^I$ for $I = 1$ and 3,

$$\xi.e^1 = -\frac{Nr\epsilon'}{\sqrt{1+r^2}}, \qquad \xi.e^3 = \Lambda N\epsilon. \tag{30}$$

We note that $\xi.(\mathcal{X}\wedge\mathcal{Y}) = (\xi.\mathcal{X})\mathcal{Y} - \mathcal{X}(\xi.\mathcal{Y})$ for 1-forms $\mathcal{X}$ and $\mathcal{Y}$. It then follows that $\xi.\Sigma_{IJ}$ restricted to the two surfaces spanned by θ and ϕ survive only for

$$\xi.\Sigma_{10} = \Lambda N^2\epsilon d\theta, \qquad \xi.\Sigma_{20} = \Lambda^2 N^2 \epsilon d\phi,$$
$$\xi.\Sigma_{30} = -\frac{N^2 r\epsilon'}{\sqrt{1+r^2}} d\theta. \tag{31}$$

The nonzero terms for $\xi.\omega^{IJ}$ can be readily calculated from the expression of the connection

$$\xi.\omega^{10} = \frac{1}{2}\Lambda^2\epsilon, \qquad \xi.\omega^{30} = -\frac{1}{2}\frac{\Lambda}{\sqrt{1+r^2}} r\epsilon',$$
$$\xi.\omega^{32} = \frac{2(\Lambda N)'}{N}\epsilon. \tag{32}$$

To calculate $\mathcal{L}_\xi \Sigma_{IJ}$ one uses the expression for the action of the Lie derivative on forms

$$\mathcal{L}_\xi \Sigma_{IJ} = d(\xi.\Sigma_{IJ}) + \xi.d\Sigma_{IJ}. \tag{33}$$

On restricting the 2-form to the two surfaces spanned by θ and ϕ one gets the following nonzero components:

$$\mathcal{L}_\xi \Sigma_{10} = \Lambda N^2 \epsilon' d\phi \wedge d\theta,$$
$$\mathcal{L}_\xi \Sigma_{30} = -\frac{N^2 r\epsilon''}{\sqrt{1+r^2}} d\phi \wedge d\theta. \tag{34}$$

$\mathcal{L}_\xi \omega^{IJ}$ can be similarly calculated and their restrictions to the two surfaces have the following form:

$$\mathcal{L}_\xi \omega^{10} = \frac{1}{2}\Lambda^2 \epsilon' d\phi,$$
$$\mathcal{L}_\xi \omega^{30} = -\frac{1}{2}\frac{\Lambda r\epsilon''}{\sqrt{1+r^2}} d\phi. \tag{35}$$

Having calculated all the required terms one can go ahead and calculate $\Omega(\delta_\xi, \delta_{\xi'})$. Putting everything together one gets

$$\Omega(\delta_{\xi_m}, \delta_{\xi_n}) = \frac{1}{8\pi G}\int_{S_\infty} [\Lambda^3 N^2 \epsilon_m \epsilon_n' d\theta \wedge d\phi + N^2 \Lambda \epsilon_m' \epsilon_n'' d\theta \wedge d\phi]. \tag{36}$$

With the substitution $\epsilon_m = -e^{-im\phi}$ as in [3] we get

$$\Omega(\delta_{\xi_m}, \delta_{\xi_n}) = \frac{i(m)\delta_{m+n,0}}{4G}\int_{S_\infty} \Lambda^3 N^2 d\theta + \frac{i(mn^2)\delta_{m+n,0}}{4G}\int_{S_\infty} N^2 \Lambda d\theta. \tag{37}$$

The relevant integrals can be calculated and are given as

$$\int \Lambda^3 N^2 d\theta = 2JG\int_0^\pi \frac{4\sin^2\theta}{(1+\cos^2\theta)^2} d\theta = 8JG,$$
$$\int \Lambda N^2 d\theta = 2JG\int_0^\pi \sin\theta d\theta = 4JG. \tag{38}$$

Therefore,

$$\Omega(\delta_{\xi_m}, \delta_{\xi_n}) = i(m^3 + 2m)J\delta_{m+n,0}. \tag{39}$$

We also need to check whether $j(\delta_\xi, \delta_{\xi'})$ contributes to the central charge. Using the variations in Appendix A we see that for the ANHEK background,

$$\int_{S_\infty} j(\delta_{\xi_m}, \delta_{\xi_n}) = \frac{1}{8\pi G}\int_{S_\infty} N^2 \Lambda(\epsilon_m' \epsilon_n'' - \epsilon_n' \epsilon_m'') d\theta \wedge d\phi$$
$$= 2Jm^3 \delta_{m+n,0}. \tag{40}$$

Therefore it follows that

$$\tilde{\Omega}(\delta_{\xi_m}, \delta_{\xi_n}) = i(-m^3 + 2m)J\delta_{m+n,0}. \tag{41}$$

D. Hamiltonian

To see if the vector fields are Hamiltonian we check whether $\tilde{\Omega}(\delta, \delta_\xi)$ can be written as a total variation. We do this in two steps. First we consider only the bulk symplectic structure $\Omega(\delta_1, \delta_2)$ and then check the contributions from $j(\delta_1, \delta_2)$.

We note that here we need the asymptotic expansions. The only terms that will contribute to the expression of $\Omega(\delta, \delta_\xi)$ are then seen to be $I = 1, J = 0$ and $I = 3, J = 0$. The relevant terms restricted to the 2-surfaces are then of the form

$$\omega^{10} = g_1(t,\theta,\phi)d\phi + g_2(t,\theta,\phi)d\theta,$$
$$\omega^{30} = h_1(t,\theta,\phi)d\phi + h_2(t,\theta,\phi)d\theta,$$
$$\xi.\omega^{10} = g_1(t,\theta,\phi)\epsilon(\phi),$$
$$\xi.\omega^{30} = -\frac{1}{2}(\Lambda)\epsilon'(\phi) + h_1(t,\theta,\phi)\epsilon(\phi), \tag{42}$$

where $g_{1,2}(t,\theta,\phi), h_{1,2}(t,\theta,\phi)$ are functions which depend on Λ, Ω, A, B, C and their derivatives. First we consider the bulk symplectic structure,

$$\Omega(\delta,\delta_\xi) = -\frac{1}{16\pi G}\int_{\partial M}[(\xi.\omega^{IJ})\delta\Sigma_{IJ} - (\xi.\Sigma_{IJ})\wedge\delta\omega^{IJ}]. \tag{43}$$

We note that

$$(\xi.\omega^{10})\delta\Sigma_{10} - (\xi.\Sigma_{10})\delta\omega^{10} = g_1\epsilon\delta\left(\frac{N^2\Lambda}{A}\right)d\theta\wedge d\phi + \left(\frac{N^2\Lambda\epsilon}{A}\right)\delta g_1 d\theta\wedge d\phi, \tag{44}$$

$$(\xi.\omega^{30})\delta\Sigma_{30} - (\xi.\Sigma_{30})\delta\omega^{30} = \left(-\frac{1}{2}\Lambda\epsilon' + h_1\epsilon\right)\delta(N^2B)d\phi\wedge d\theta - (-N^2\epsilon' + N^2B\epsilon)\delta h_1 d\theta\wedge d\phi. \tag{45}$$

It is therefore at once evident that the contribution from the bulk symplectic structure is integrable provided we assume $\delta\epsilon = 0$.

For the vector fields ξ in question, we also need to check if the charges are still integrable with the addition of the boundary symplectic current. Using the expressions for $\delta_\xi A, \delta_\xi B, \delta_\xi C$ from Appendix A we see that this contribution is equal to

$$\frac{1}{A}(\delta_\xi A\delta B - \delta_\xi B\delta A) = \frac{1}{A}(-\epsilon' A + \epsilon\partial_\phi A)\delta B - \frac{1}{A}(-\epsilon'' + \epsilon\partial_\phi B + \epsilon' B)\delta A. \tag{46}$$

We note that the first and the third term are integrable. So we concentrate on the other terms

$$\begin{aligned}\epsilon\partial_\phi(\log A)\delta B - \epsilon\partial_\phi B\delta(\log A) - \epsilon' B\delta(\log A) &= \partial_\phi[\epsilon(\log A)\delta B] - \epsilon(\log A)\delta\partial_\phi B - \epsilon'(\log A)\delta B \\ &\quad - \epsilon\partial_\phi B\delta(\log A) - \epsilon' B\delta(\log A) \\ &\equiv -\delta(\epsilon(\log A)\partial_\phi B) - \delta(\epsilon' B(\log A)),\end{aligned} \tag{47}$$

where we have omitted the first term, while going from the first to the second expression, as it is a total ϕ derivative and does not contribute to the integral. So it follows that the charges are still integrable. Moreover for the ANHEK background (for which $A = 1$ and $B = 0$) the boundary symplectic structure does not contribute to the Hamiltonian. So, for the given background one can set the Hamiltonian function to be

$$H_\xi = \left[-\frac{1}{16\pi G}\int_{\partial M} N^2\Lambda^3\epsilon d\theta\wedge d\phi\right]. \tag{48}$$

E. Entropy calculations

To calculate the entropy we choose an approach outlined in [5] and used in [22]. The charge for the vector field ξ has been calculated in Sec. III D. It therefore follows that

$$\begin{aligned}H_{[\xi_m,\xi_n]} &= -\frac{1}{16\pi G}\int_{\partial M} N^2\Lambda^3(\epsilon_m\epsilon_n' - \epsilon_n\epsilon_m')d\theta\wedge d\phi \\ &= -\frac{1}{16\pi G}i(m-n)2\pi\delta_{m+n,0}\times 8\pi G \\ &= -2imJ\delta_{m+n,0}.\end{aligned} \tag{49}$$

Putting this in the expression for the Poisson bracket, we get

$$\begin{aligned}[H_{\xi_m}, H_{\xi_n}] &= -iJm^3\delta_{m+n,0}, \\ i[H_{\xi_m}, H_{\xi_n}] &= Jm^3\delta_{m+n,0}.\end{aligned} \tag{50}$$

Comparing this with the Virasoro algebra,

$$i[H_m, H_n] = (m-n)H_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}. \tag{51}$$

It then follows that

$$\begin{aligned}i[H_1, H_{-1}] &= 2H_0 = J, \\ i[H_2, H_{-2}] &= 4H_0 + \frac{c}{2} = 8J.\end{aligned} \tag{52}$$

One can now solve the above system of linear algebraic equations for c and H_0, which gives

$$c = 12J, \qquad H_0 = \frac{J}{2}. \tag{53}$$

Now using the Cardy formula:

$$S = 2\pi\sqrt{\frac{cH_0}{6}} = 2\pi J, \tag{54}$$

which is in accordance with the Bekenstein-Hawking entropy formula. The Planck's constant in the formula can be recovered by the naive quantization $i\hbar[H_m, H_n] \to [H_m, H_n]$.

IV. HOLST ACTION

In the first order formulation, both the Holst and Palatini actions give the same equations of motion, viz. Einstein's equations in spite of the fact that the two actions differ by a term which is not a total derivative. Therefore, NHEK is a solution of both these actions. It is therefore legitimate to check whether under NHEK boundary conditions the use of the Holst action gives a different result from the Palatini action.

The Holst action in the bulk is given by

$$S_H = -\frac{1}{16\pi G}\int_{\mathcal{M}} \Sigma_{IJ} \wedge \left(F^{IJ} + \frac{1}{\gamma}{}^*F^{IJ}\right), \qquad (55)$$

where ${}^*F^{IJ} = \frac{1}{2}\epsilon^{IJ}{}_{KL}F^{KL}$, γ is the Immirzi parameter.

The symplectic current is then given by

$$J_H(\delta_1, \delta_2) = -\frac{1}{8\pi G}\left[\delta_{[1}\Sigma_{IJ} \wedge \delta_{2]}\left(\omega^{IJ} + \frac{1}{\gamma}{}^*\omega^{IJ}\right)\right]. \qquad (56)$$

On half-shell i.e. if the torsion-free conditions hold then the symplectic current simplifies [8] and is then given by

$$J_H(\delta_1, \delta_2) = -\frac{1}{16\pi G}(\delta_{[1}\Sigma_{IJ} \wedge \delta_{2]}\omega^{IJ}) + \frac{1}{8\pi G\gamma} d(\delta_{[1}e_I \wedge \delta_{2]}e^I). \qquad (57)$$

We first note that the first term in the above expression is the usual Palatini term (denoted by J_p in the next expression). To construct the hypersurface-independent symplectic structure we note that on shell,

$$dJ_H = 0. \qquad (58)$$

This implies that when integrated over a closed region of spacetime bounded by $M_1 \cup M_2 \cup B$ (where B is a portion of the boundary of spacetime given by $r \to \infty$ in our case),

$$\left(\int_{M_1} - \int_{M_2} + \int_{r\to\infty}\right) J_P + \left(\int_{M_1} - \int_{M_2} + \int_{r\to\infty}\right) d(\delta_{[1}e_I \wedge \delta_{2]}e^I) = 0, \qquad (59)$$

where M_1, M_2 are the initial and final Cauchy surfaces that asymptote to constant time slices.

We note that the second term is always zero. So the Immirzi parameter can never appear in the hypersurface-independent symplectic structure calculated from the Holst action. Therefore the Holst term modifies neither the Poisson bracket nor the Hamiltonian no matter what geometry or boundary conditions one is considering. Thus if one uses the Holst action instead of the Palatini action the semiclassical entropy is still the same as that calculated from the Palatini action and is therefore independent of the Immirzi parameter.

V. DISCUSSION

Apart from the motivations pointed out in the Introduction, the first order formalism gives a cleaner calculation. For example, it is evident that the desired central extension comes from terms like $(\xi.\omega^{IJ})d(\xi.\Sigma_{IJ})$ and $d(\xi.\omega^{IJ}) \wedge \xi.\Sigma_{IJ}$. Therefore, from the expressions of the tetrads and connections it is possible to predict which vector field will give an m^3 term.

It seems that in the second order formulation the boundary symplectic structure has been studied only in the context of asymptotically flat geometries. Such studies have not been made in symmetry-based approaches in the second order formulation. Therefore, the symplectic structure given in [4] may not be hypersurface independent for the boundary conditions appropriate for the NHEK geometry. This has been pointed out for Kerr/CFT in [14].

In this case the boundary symplectic structure does not vanish. A nonvanishing boundary symplectic structure implies that the bulk symplectic structure alone is not hypersurface independent. This would precisely give a Hamiltonian calculated from the bulk symplectic structure to be hypersurface dependent. Since for the NHEK background and the vector fields generating the $DiffS^1$, the Hamiltonian calculated from the bulk symplectic structure is already time independent; it was expected that at least for ξ the boundary symplectic structure should not contribute. However the results of Sec. III B show that there is a nontrivial contribution to the central charge from the boundary symplectic structure.

We show, by explicit calculation, that only if the boundary symplectic structure is taken into account i.e. one works with a truly hypersurface-independent symplectic structure, the entropy results match with those obtained in second order formulation. So even though the results do not change we think that the relevance and importance of the boundary symplectic structure has been fully conveyed in this work.

VI. CONCLUSION

We studied the Kerr/CFT correspondence using the symplectic structure in the first order formulation of gravity. The boundary symplectic structure was studied. It was shown that it does not vanish. The results obtained are then in agreement with those already obtained in the second order formulation. We studied the effect of adding the Holst term and showed that it does not contribute.

It is known that the Immirzi parameter labels the nonequivalent quantization in LQG. It is also believed that a fine-tuning of the Immirzi parameter is required in order to reproduce the BH entropy formula. However, recently in [17] it has been argued that the Immirzi parameter is not so relevant in getting the semiclassical value for BH entropy. Our result that the entropy formula is independent of the Immirzi parameter is consistent with the claim that it plays no fundamental role in the quantum theory.

It will be interesting though to see if the Immirzi parameter contributes to Wald entropy for NHEK. Wald prescription for the black hole entropy works only for bifurcate Killing horizons. Hence, a straightforward implementation of this method to the case of extremal Kerr is not possible. A widely accepted approach is to calculate the entropy for a nonextremal black hole and then take the extremal limit or along the lines of [23] for instance.

ACKNOWLEDGMENTS

The author wants to thank Amit Ghosh for useful discussions, David Sloan for clarifying a point from a reference and Alejandro Corichi for a useful clarification.

APPENDIX: A VARIATIONS OF A,B AND C

Note that $\delta_\xi A$ is not equal to $\mathcal{L}_\xi A$. Rather it has to be calculated from the action of $\mathcal{L}_\xi$ on the fields,

$$\mathcal{L}_\xi e^0 = N(-r\epsilon' A dt + r\epsilon\partial_\phi A dt). \tag{A1}$$

It therefore follows that $\delta_\xi A = -\epsilon' A + \epsilon\partial_\phi A$.

Similarly

$$\mathcal{L}_\xi e^3 = N\Lambda\left(\frac{\epsilon'}{C} - \frac{\epsilon\partial_\phi C}{C^2}\right)d\phi + N\Lambda r(-\epsilon' C + \epsilon\partial_\phi C)dt, \tag{A2}$$

which implies

$$\delta_\xi\left(\frac{1}{C}\right) = \frac{\epsilon'}{C} - \frac{\epsilon\partial_\phi C}{C^2}. \tag{A3}$$

Therefore $\delta_\xi C = -\epsilon' C + \epsilon\partial_\phi C$. For consistency one can check that the dt term gives the same variation,

$$\mathcal{L}_\xi e^1 = N(B\epsilon' + \epsilon\partial_\phi B - \epsilon'')d\phi. \tag{A4}$$

Therefore $\delta_\xi B = B\epsilon' + \epsilon\partial_\phi B - \epsilon''$.

APPENDIX: B FORM OF THE CONNECTION

The form of the connection calculated from the zeroth order tetrad is of the form,

$$\begin{aligned}
{}^0\omega^{10} &= -\frac{1}{2}\frac{2A(t,\theta,\phi)^2 - \Lambda(\theta)^2 C(t,\theta,\phi)^2}{A(t,\theta,\phi)} r dt \\
&\quad - \frac{1}{2}\frac{C(t,\theta,\phi)^2 \frac{\partial}{\partial\theta}B(t,\theta,\phi)}{A(t,\theta,\phi)} d\theta + \frac{1}{2}\frac{\Lambda(\theta)^2}{A(t,\theta,\phi)} d\phi, \\
{}^0\omega^{20} &= -\frac{((\frac{d}{d\theta}N(\theta))A(t,\theta,\phi)^2 + N(\theta)A(t,\theta,\phi)\frac{\partial}{\partial\theta}A(t,\theta,\phi) - N(\theta)\Lambda(\theta)^2 C(t,\theta,\phi)\frac{\partial}{\partial\theta}C(t,\theta,\phi))}{N(\theta)A(t,\theta,\phi)} r dt \\
&\quad - \frac{1}{2}\frac{C(t,\theta,\phi)^2\frac{\partial}{\partial\theta}B(t,\theta,\phi)}{A(t,\theta,\phi)}\frac{dr}{r} + \frac{1}{2}\frac{-B(t,\theta,\phi)(\frac{\partial}{\partial\theta}B(t,\theta,\phi))C(t,\theta,\phi)^3 + 2\Lambda(\theta)^2\frac{\partial}{\partial\theta}C(t,\theta,\phi)}{A(t,\theta,\phi)C(t,\theta,\phi)} d\phi, \\
{}^0\omega^{30} &= \frac{1}{\Lambda(\theta)A(t,\theta,\phi)}\Big(B(t,\theta,\phi)C(t,\theta,\phi)A(t,\theta,\phi)^2 - B(t,\theta,\phi)C(t,\theta,\phi)^3\Lambda(\theta)^2 \\
&\quad - C(t,\theta,\phi)A(t,\theta,\phi)\frac{\partial}{\partial\phi}A(t,\theta,\phi) + C(t,\theta,\phi)^2\Lambda(\theta)^2\frac{\partial}{\partial\phi}C(t,\theta,\phi)\Big) r dt \\
&\quad + \frac{1}{2}\frac{C(t,\theta,\phi)\Lambda(\theta)^2}{\Lambda(\theta)A(t,\theta,\phi)}\frac{dr}{r} + \frac{\Lambda(\theta)\frac{\partial}{\partial\theta}C(t,\theta,\phi)}{A(t,\theta,\phi)} d\theta \\
&\quad - \frac{1}{2}\frac{1}{C(t,\theta,\phi)^2\Lambda(\theta)A(t,\theta,\phi)}\Big(-2C(t,\theta,\phi)^2\Lambda(\theta)^2\frac{\partial}{\partial\phi}C(t,\theta,\phi) + B(t,\theta,\phi)C(t,\theta,\phi)^3\Lambda(\theta)^2\Big) d\phi, \\
{}^0\omega^{21} &= -\frac{\frac{d}{d\theta}N(\theta)}{N(\theta)r} dr - \frac{1}{2}\frac{2B(t,\theta,\phi)\frac{d}{d\theta}N(\theta) + N(\theta)\frac{\partial}{\partial\theta}B(t,\theta,\phi)}{N(\theta)} d\phi, \\
{}^0\omega^{31} &= \frac{1}{2}\frac{C(t,\theta,\phi)\Lambda(\theta)^2}{\Lambda(\theta)} r dt + \frac{1}{2}\frac{C(t,\theta,\phi)\frac{\partial}{\partial\theta}B(t,\theta,\phi)}{\Lambda(\theta)} d\theta,
\end{aligned}$$

$$
\begin{aligned}
{}^{0}\omega^{32} &= \frac{C(t,\theta,\phi)(\Lambda(\theta)\frac{d}{d\theta}N(\theta)+N(\theta)\frac{d}{d\theta}\Lambda(\theta))}{N(\theta)}rdt+\frac{1}{2}\frac{C(t,\theta,\phi)\frac{\partial}{\partial\theta}B(t,\theta,\phi)}{\Lambda(\theta)}\frac{dr}{r}\\
&+\frac{1}{2}\frac{1}{C(t,\theta,\phi)^2N(\theta)\Lambda(\theta)}\left(N(\theta)B(t,\theta,\phi)\left(\frac{\partial}{\partial\theta}B(t,\theta,\phi)\right)C(t,\theta,\phi)^3+2\left(\frac{d}{d\theta}N(\theta)\right)C(t,\theta,\phi)\Lambda(\theta)^2\right.\\
&\left.+2N(\theta)\Lambda(\theta)\left(\frac{d}{d\theta}\Lambda(\theta)\right)C(t,\theta,\phi)-2N(\theta)\Lambda(\theta)^2\frac{\partial}{\partial\theta}C(t,\theta,\phi)\right)d\phi, \qquad \text{(B1)}
\end{aligned}
$$

$$
\begin{aligned}
{}^{1}\omega^{10} &= -\frac{1}{2}\frac{C(t,\theta,\phi)^2\frac{\partial}{\partial t}B(t,\theta,\phi)}{A(t,\theta,\phi)}rdt+\frac{1}{2}\frac{-\frac{\partial}{\partial t}B(t,\theta,\phi)}{A(t,\theta,\phi)}d\phi,\\
{}^{1}\omega^{30} &= \frac{\Lambda(\theta)\frac{\partial}{\partial t}C(t,\theta,\phi)}{A(t,\theta,\phi)}rdt+\frac{1}{2}\frac{C(t,\theta,\phi)(-\frac{\partial}{\partial t}B(t,\theta,\phi))}{\Lambda(\theta)A(t,\theta,\phi)}\frac{dr}{r}\\
&-\frac{1}{2}\frac{1}{(C(t,\theta,\phi))^2\Lambda(\theta)A(t,\theta,\phi)}\left(B(t,\theta,\phi)\left(\frac{\partial}{\partial t}B(t,\theta,\phi)\right)C(t,\theta,\phi)^3-2\Lambda(\theta)^2\frac{\partial}{\partial t}C(t,\theta,\phi)\right)d\phi,\\
{}^{1}\omega^{31} &= \frac{1}{2}\frac{C(t,\theta,\phi)(\frac{\partial}{\partial t}B(t,\theta,\phi))}{\Lambda(\theta)}rdt. \qquad \text{(B2)}
\end{aligned}
$$

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