Revisiting the implications of *CPT* and unitarity for baryogenesis and leptogenesis

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In the context of Grand Unified Theories (GUT) baryogenesis models, a well-known theorem asserts that *CPT* conservation and the unitarity of the S matrix require that the lowest order contribution that leads to the generation of a nonzero net *CP* violation via the decay of a heavy particle must be to $\mathcal{O}(\alpha_{\not{B}}^3)$, where $\alpha_{\not{B}}$ is a baryon number (B) violating coupling. We revisit this theorem [which holds for lepton number (L) violation, and hence for leptogenesis as well] and examine its implications for models where the particle content allows the heavy particle to also decay via modes which conserve B (or L) in addition to modes which do not. We systematically expand the S matrix order by order in B/L violating couplings, and show, in such cases, that the net *CP* violation is nonzero even to $\mathcal{O}(\alpha_{\not{B}}^2)$, without actually contradicting the theorem. By replacing a B/L violating coupling (usually constrained to be small) by a relatively unconstrained B/L conserving one, our result may allow for sufficient *CP* violation in models where it may otherwise have been difficult to generate the observed baryon asymmetry. As an explicit application of this result, we construct a model in low-scale leptogenesis.

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I. INTRODUCTION

The asymmetry in the Universe between baryonic and antibaryonic matter is expressed in terms of the ratio,

$$Y_{\Delta B} \equiv \frac{n_B - n_{\bar{B}}}{s}, \tag{1}$$

where n_B and $n_{\bar{B}}$ represent the baryon and antibaryon densities, respectively, and $s = g_*(2\pi^2/45)T^3$ is the entropy density in which g_* is the number of relativistic degrees of freedom in the plasma and T is the temperature. The current estimate for this asymmetry has been determined independently from (i) the abundances of light nuclei due to big bang nucleosynthesis (BBN) and, (ii) analyses of the cosmic microwave background radiation (CMB). Its values (at 95% C.L.) [1,2],

$$\begin{aligned} Y_{\mathrm{BBN}} &= (8.10 \pm 0.85) \times 10^{-11}, \\ Y_{\mathrm{CMB}} &= (8.79 \pm 0.44) \times 10^{-11}, \end{aligned} \tag{2}$$

confirm that we exist in a Universe that is baryon dominated [3,4]. The consistency between these independent measurements of the baryon asymmetry is all the more impressive because their respective epochs are separated by about 6 orders of magnitude in temperature, putting its existence on a firm experimental footing.

At variance with this, however, is the fact that a largely symmetric Universe, in terms of matter and antimatter, is expected from our present theoretical understanding of the early Universe and the extremely tiny amount of matter-antimatter asymmetry present in the quark sector of fundamental particle interactions. While B violation, the first of the well-known Sakharov conditions [5] for the generation of the asymmetry, may well be realized at high temperatures in the early Universe [6], the second condition of *CP* violation [7] requires a mechanism beyond the Kobayashi-Maskawa complex phase [8] of the Standard Model (SM). Similarly, the third Sakharov condition of departure from thermal equilibrium may require extending the physics of the Standard Model. The latter allows nonequilibrium processes to occur at the electroweak phase transition [9,10], but these may not be sufficiently first order and thus, unable to generate the requisite asymmetry [11]. It is thus fair to say that while several interesting theories have been proposed to explain the dynamical generation of this asymmetry, the actual mechanism by which this occurs in nature remains to be established.

Baryogenesis is a class of mechanisms that attempt to explain the asymmetry by postulating its dynamic generation

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in the early Universe, during the period between the end of cosmological inflation and reheating, and prior to the onset of nucleosynthesis, via interactions of particles and antiparticles asymmetric in their rates (see Refs. [12–14] for detailed reviews). Examples of mechanisms which have been proposed include (a) GUT baryogenesis models [15–24], (b) electroweak baryogenesis [10,12,13], (c) the Affleck-Dine mechanism [25], and (d) spontaneous baryogenesis [26,27]. In recent times, however, much attention has been focussed on achieving baryogenesis via leptogenesis [28]. This involves the initial generation of an asymmetry in the lepton-antilepton content of the Universe and its subsequent conversion to baryon asymmetry by means of sphaleron interactions that violate baryon (B) and lepton (L) numbers simultaneously, while conserving $B - L$ (see Refs. [29–35] for reviews on the subject).

Our work focuses on the constraints that are imposed on models of baryogenesis (including baryogenesis via leptogenesis) by the fundamental invariances of CPT and unitarity in quantum field theories.

The general consequences of CPT invariance and unitarity of the S matrix in the context of the generation of baryon asymmetry in GUT models have been explored in the past [23,36]. In particular, as first pointed out by Nanopoulos and Weinberg [23], while calculating the CP asymmetry generated in B violating heavy particle decays, the leading contribution to the asymmetry involves processes which are to the third order or higher in the B violating coupling $\alpha_{\not{B}}$. Thus, the (amplitude-level) contribution of graphs to the first order in B/L (i.e., B or L) violation (and to all orders in B/L conserving interactions) vanishes as a consequence of CPT invariance and unitarity of the S matrix in the theory. Henceforth, we shall refer to this result as the Nanopoulos-Weinberg theorem. Its importance lies in it being a general result that applies to any particle physics model that attempts to dynamically generate the baryon asymmetry and the requisite CP violation with interaction vertices that break B or L. The most significant application has been to nonequilibrium decays of heavy particles which constitute the spectrum of theories beyond the Standard Model.

Although widely applicable, the Nanopoulos-Weinberg theorem was formulated in the context of massive gauge boson decays associated with GUTs. An important input in proving this theorem was that, in the models considered in Ref. [23], all decay modes of these heavy bosons were B violating. Such an assumption is, of course, completely justified when formulating a minimal model satisfying the requirements for GUT-based baryogenesis. However, we note that in the present context of efforts to carry physics beyond the Standard Model, a wide range of possible models with varying particle content which can provide the seeds for B/L generation have been studied in the literature. In this wider framework, the heavy particle which leads to a CP asymmetry by its decay may have access to decay modes which conserve B (or L) in addition to those which violate it. Our work pertains to such cases, and points out a facet of the theorem which may guide the building of baryogenesis and leptogenesis models which have not received adequate attention so far.

In what follows, we revisit the impact of CPT and unitarity on asymmetry generating interactions by looking at the S matrix order by order in B/L violating couplings, and determine the leading order in these couplings at which the net CP violation generated is nonzero. Specifically, we study the generic scenarios where the parent particle has access to (a) only B/L violating decay modes and (b) both B/L conserving and violating ones. The essential upshot of our considerations is that in models where a consistent and natural scheme of B/L number assignment leads to the presence of both B/L violating and conserving decay modes of a heavy particle, the net CP violation to $\mathcal{O}(\alpha_{\not{B}}^2)$ (calculated with graphs to only first order in B/L violation) is nonzero. We emphasize that our result is in no way contradictory to the Nanopoulos-Weinberg theorem, but rather a useful reanalysis and extension, which might be helpful while considering the building of various models to achieve baryogenesis and leptogenesis.

This paper is organized as follows: in Sec. II, we review the constraints imposed by CPT invariance and unitarity of the S matrix on the possible generation of CP violation in the decays of heavy particles. In Sec. III, we find expressions for the B/L asymmetries generated in different schemes of B/L assignment for the decaying particle and demonstrate their equivalence. We also explore the consequences of the reformulation of the Nanopoulos-Weinberg theorem by constructing an example model of leptogenesis in the same section. The last section contains our conclusions.

II. *CP* VIOLATION IN HEAVY PARTICLE DECAY

A. General implications of *CPT* invariance and S-matrix unitarity

We first briefly review the general implications of CPT conservation and unitarity of the S matrix for various interactions [23,36].

Let us assume that the initial state of a system represented by i (which represents all the quantum numbers of the system at this state) proceeds via interactions to a final state f. The probability of a transition to a state f from the state i is given by $|S_{fi}|^2$, where

$$S_{fi} = \langle f|S|i\rangle \tag{3}$$

is the so-called S-matrix element. This S matrix can be decomposed as follows:

$$S_{fi} = \delta_{fi} + \imath T_{fi}, \tag{4}$$

where T_{fi} represents the fith element of the T *matrix*, which represents the probability amplitude of transition of a system in the initial state i to a distinct final state f, i.e., without transitioning to itself. The S matrix must be unitary,

$$SS^\dagger = 1 = S^\dagger S. \tag{5}$$

Written out in terms of the elements after inserting a complete set of states wherever necessary, this gives

$$\sum_f |S_{fi}|^2 = 1, \quad \text{and} \tag{6a}$$

$$\sum_f |S_{if}|^2 = 1. \tag{6b}$$

Equivalently, in terms of the T matrix, this can be expressed as

$$\sum_n T^*_{ni} T_{nf} = -\imath (T_{if} - T^*_{fi}), \tag{7a}$$

which implies, for $i = f$,

$$\sum_n |T_{ni}|^2 = 2\Im(T_{ii}), \tag{7b}$$

where, in going from Eq. (7a) to (7b) we have denoted the *imaginary part* of the complex quantity T_{if} by $\Im(T_{if})$. It is easy to show, along the same lines but starting from Eq. (6b) instead of (6a), that, also,

$$\sum_n |T_{in}|^2 = 2\Im(T_{ii}). \tag{8}$$

Further, conservation of *CPT* ensures that the probability of transition of an initial state i to a final state f is equivalent to that of the transition of the corresponding *CP* conjugate states $\bar{f}$ to $\bar{\imath}$:

$$T_{fi} = T_{\bar{\imath}\bar{f}}. \tag{9}$$

The consequence of unitarity as expressed in Eqs. (7) and (8), along with *CPT* invariance, ensures that

$$\sum_{\bar{f}} |T_{\bar{f}\bar{\imath}}|^2 = \sum_f |T_{fi}|^2. \tag{10}$$

Therefore, the probability of a system in a state i transitioning to all possible final states f is identical to the probability of the system in the *CP* conjugate state $\bar{\imath}$ transitioning to all possible final states $\bar{f}$. This is an important consequence of *CPT* conservation and unitarity and it tells us, among other things, that the *total* decay width of a particle and its *CP* conjugate (antiparticle) are necessarily identical.

CP violating amplitudes and unitarity. As opposed to constraints on sums over all final states, as considered above, we now pose the question: what constraint does unitarity impose on individual *CP* violating amplitudes? If the particular interaction that generates the transition amplitude T_{fi} is *CP* nonconserving, then the difference between the probabilities of the *CP* conjugate processes $i \to f$ and $\bar{\imath} \to \bar{f}$, or equivalently between $i \to f$ and $f \to i$, is finite and nonzero. Indeed, using Eq. (7b) in the form

$$T_{if} = \imath \sum_n T_{in} T^*_{fn} + T^*_{fi}, \tag{11}$$

it is straightforward to obtain an expression for the difference in the probabilities for the *CP* conjugate interactions

$$\begin{aligned} |T_{\bar{f}\bar{\imath}}|^2 - |T_{fi}|^2 = |T_{if}|^2 - |T_{fi}|^2 \\ = -2\Im\left(\sum_n (T_{in} T^*_{fn}) T_{fi}\right) \\ + \left|\sum_n T_{in} T^*_{fn}\right|^2. \end{aligned} \tag{12}$$

This equation implies that *CP* violating differences are generated by the interference of tree and loop graphs, where the intermediate states in the loop are on shell [36], leading to a nonzero imaginary part in the amplitude.

At this juncture, it is appropriate to recall the result of the Nanopoulos-Weinberg theorem, which examined the net baryon excess ΔB produced in the decays of superheavy X bosons and their antiparticles. The conclusion derived there [23] was that graphs to first order in B violating interactions but to arbitrary order in baryon conserving interactions make no contribution to a net ΔB. In particular, it was shown that when decay amplitudes are calculated using graphs to first order in B violating interactions, *CPT* invariance requires that the decay rate for a particle X into all final states with a given baryon number B equal the rate for the corresponding decay of the antiparticle $\bar{X}$ into all states with baryon number $-B$. Therefore, this theorem indicates that one must consider graphs to at least second order in B violating interactions.

We note, however, that in this paper the authors considered models where the superheavy boson giving nonzero contribution to the net baryon asymmetry had only B violating decay modes. This assumption was incorporated in the proof of this theorem by demanding that in the absence of B violating interactions, the wave function of X, ψ_X is a one-particle state. As noted in the Introduction, over the past two decades, many classes of models for baryogenesis (and leptogenesis) have appeared in the literature, with particle spectra involving not just heavy GUT scale gauge bosons, but also beyond Standard Model (BSM) scalars and Majorana fermions, with B/L and *CP* violating interactions. It is thus reasonable and relevant to relax this particular assumption in the wider context of BSM models and their particle content. By introducing decay modes which are not always B/L violating, it is expected that the result in [23] will be modified when subjected to the same constraints of CPT and unitarity. It is the study of this modification and its consequences for present day B/L violating models which is the main objective of this paper.

In Sec. III, to begin with we shall implement this assumption at the S-matrix level by demanding that in the case where the heavy boson X decays only via B violating interactions, the S-matrix elements $(S_0)_{fX} = \delta_{fX}$, where S_0 denotes the part of the S matrix which contains only B conserving interactions. Expanding the S-matrix order by order in B violating couplings $\alpha_{\not B}$, we then show that the net CP violation generated is zero to $\mathcal{O}(\alpha_{\not B}^2)$, which of course, is tantamount to rederiving the result in [23] (case 1 in Sec. III below). Next, we relax the assumption and examine the consequences (case 2, Sec. III).

III. SYSTEMATIC EXPANSION OF THE S MATRIX IN B/L VIOLATING COUPLINGS

We first split the S matrix into two parts,

$$S = S_0 + \imath\tilde{T}, \tag{13}$$

where S_0 includes the identity element of the total S matrix and also processes represented by graphs with only B conserving interactions. $\tilde{T}$ contains processes described by graphs with B violating interactions to first order or higher and B conserving interactions to all orders. Using this expansion in Eq. (5) we arrive at the following relation between S_0 and $\tilde{T}$:

$$\tilde{T} = S_0\tilde{T}^\dagger S_0 + \imath S_0\tilde{T}^\dagger\tilde{T} = S_0\tilde{T}^\dagger S. \tag{14}$$

In terms of the elements of the S and T matrices, we therefore see that

$$\tilde{T}_{Xf} = \sum_{i,j}(S_0)_{Xi}(\tilde{T}^\dagger)_{ij}S_{jf}. \tag{15}$$

From Eq. (15) we get

$$|\tilde{T}_{Xf}|^2 = \sum_{i,j,k,m}(S_0)_{Xi}\tilde{T}^*_{ji}S_{jf}(S_0)^*_{Xk}\tilde{T}_{mk}S^*_{mf}. \tag{16}$$

Denoting all B violating coupling constants by $\alpha_{\not B}$, we expand the quantity $\tilde{T}$ in a perturbation series in this coupling constant,

$$\tilde{T} = \alpha_{\not B}\tilde{T}_1 + \alpha_{\not B}^2\tilde{T}_2 + \cdots, \tag{17}$$

where the quantities $\tilde{T}_1$ and $\tilde{T}_2$ themselves do not contain any factors of the B violating coupling constant $\alpha_{\not B}$. Thus,

$$S = S_0 + \imath(\alpha_{\not B}\tilde{T}_1 + \alpha_{\not B}^2\tilde{T}_2) + \mathcal{O}(\alpha_{\not B}^3), \tag{18a}$$

i.e.,

$$S_{\bar{f}\bar{X}} = S_{Xf} = (S_0)_{Xf} + \imath(\alpha_{\not B}\tilde{T}_1 + \alpha_{\not B}^2\tilde{T}_2)_{Xf} + \mathcal{O}(\alpha_{\not B}^3), \tag{18b}$$

where in Eq. (18b) we have used CPT conservation, as usual, to rewrite $S_{\bar{f}\bar{X}}$ as S_{Xf}.

A. Case 1: Where the initial particle decays only by B violating interactions

If the initial particle X and its CP conjugate particle $\bar{X}$ decay only via B violating interactions, i.e.,

$$(S_0)_{\bar{f}\bar{X}} = (S_0)_{Xf} = \delta_{Xf}, \tag{19}$$

we get, using Eq. (19) in (16),

$$\sum_{f\in B}|\tilde{T}_{Xf}|^2 = \sum_{f\in B}\sum_{j,m}\tilde{T}^*_{jX}S_{jf}\tilde{T}_{mX}S^*_{mf} \tag{20a}$$

$$= \sum_{f\in B}\Big((\tilde{T}^*_{fX}\tilde{T}_{fX}) - \imath\sum_m\tilde{T}^*_{fX}\tilde{T}_{mX}\tilde{T}^*_{mf} + \imath\sum_m\tilde{T}^*_{mX}\tilde{T}_{mf}\tilde{T}_{fX} + \sum_{j,m}\tilde{T}^*_{jX}\tilde{T}_{jf}\tilde{T}_{mX}\tilde{T}^*_{mf}\Big), \tag{20b}$$

where $\sum_{f\in B}$ represents the sum over all final states f with a given baryon number B. In going from Eq. (20a) to (20b), we have expanded S in accordance with Eq. (13) and summed over the $\delta_{\alpha\beta}$ as appropriate. We can carry over the first sum in the rhs of Eq. (20b) to the other side of the equality, and use CPT as required, to obtain the important difference in the partial decay widths of the CP conjugate processes violating baryon numbers by $\Delta B = B - B(X)$ and $\Delta\bar{B} = -B - B(\bar{X})$ units, respectively:

$$\sum_{\bar{f}\in -B}|\tilde{T}_{\bar{f}\bar{X}}|^2 - \sum_{f\in B}|\tilde{T}_{fX}|^2 = \sum_{f\in B}\Big(-\imath\sum_m\tilde{T}^*_{fX}\tilde{T}_{mX}\tilde{T}^*_{mf} + \imath\sum_m\tilde{T}_{fX}\tilde{T}^*_{mX}\tilde{T}_{mf} + \sum_{j,m}\tilde{T}^*_{jX}\tilde{T}_{jf}\tilde{T}_{mX}\tilde{T}^*_{mf}\Big). \tag{21}$$

We now expand $\tilde{T}$ order by order in $\alpha_{\not B}$ according to Eq. (17) and evaluate this difference. The results of the calculation to $\mathcal{O}(\alpha_{\not B}^2)$ and $\mathcal{O}(\alpha_{\not B}^3)$ are enumerated below.

To $\mathcal{O}(\alpha_{\not B}^2)$: It is easy to see that each of the three sums in the rhs of Eq. (21) gives a contribution that is at least to $\mathcal{O}(\alpha_{\not B}^3)$. Hence, the $\mathcal{O}(\alpha_{\not B}^2)$ contribution to the lhs is zero. Since the tree graph must contain one B violating vertex, an $\mathcal{O}(\alpha_{\not B}^2)$ contribution to the difference in $|T_{fX}|^2$ can only come from the interference of such a tree graph with a loop graph also containing, at most, one B violating vertex.

Thus, this result is consistent with the results of the Nanopoulos-Weinberg theorem, and shows that graphs to the first order in $\alpha_{\not B}$ do not contribute to the *CP* violating difference.

To $\mathcal{O}(\alpha_{\not B}^3)$ and higher: The $\mathcal{O}(\alpha_{\not B}^3)$ contribution to the *CP* violating difference comes from the first two sums in the rhs [37] and is given by

$$\sum_{\bar f\in -B}|\tilde T_{\bar f\bar X}|^2-\sum_{f\in B}|\tilde T_{fX}|^2 = \alpha_{\not B}^3\sum_{f\in B}\sum_m 2\Im((\tilde T_1)^*_{fX}(\tilde T_1)_{mX}(\tilde T_1)^*_{mf}) + \mathcal{O}(\alpha_{\not B}^4). \tag{22}$$

The leading contribution in $\alpha_{\not B}$ to the *CP* violating difference is, therefore, to the third order and, as is evident from Eq. (22), comes due to the interference of a tree level graph with its only vertex being B violating and a loop graph with two B violating vertices.

B. Case 2: Where the initial particle can decay both through B conserving and B violating interactions

We now study, in a similar context, the case where the initial particle X may decay via B conserving and B violating channels to the final states. This translates, in terms of S-matrix elements, to the condition

$$(S_0)_{\bar f\bar X}=(S_0)_{Xf}=\delta_{Xf}+\imath(T_0)_{Xf}, \tag{23}$$

with $(T_0)_{Xf}\neq 0$, and likewise for $(S_0)_{fX}$. We carry out a similar calculation as in Sec. III A, using Eq. (23) in (16). We expand $\sum_{\bar f\in -B}|\tilde T_{\bar f\bar X}|^2-\sum_{f\in B}|\tilde T_{fX}|^2$ order by order in $\alpha_{\not B}$ and read out the $\mathcal{O}(\alpha_{\not B}^2)$ terms in the expansion. Thus, to $\mathcal{O}(\alpha_{\not B}^2)$, we find that

$$\sum_{\bar f\in -B}|\tilde T_{\bar f\bar X}|^2-\sum_{f\in B}|\tilde T_{fX}|^2 = -\imath\alpha_{\not B}^2\sum_{f\in B}(\tilde T_1)_{fX}\sum_m((T_0)_{Xm}(\tilde T_1)^*_{fm}+(\tilde T_1)^*_{mX}(T_0)_{mf}) + \imath\alpha_{\not B}^2\sum_{f\in B}(\tilde T_1)^*_{fX}\sum_m((T_0)^*_{mf}(\tilde T_1)_{mX}+(T_0)^*_{Xm}(\tilde T_1)_{fm}). \tag{24}$$

We would now like to compare the *CP* violating difference in case 2 given by Eq. (24) with that found for case 1 given by Eq. (22). Since, to start with, we have assumed that $(T_0)_{Xm}\neq 0$, and since T_0 contains only B conserving interactions, we have $B(X)=B(m)$. But, as B has to be finally violated in the decay of X, $B(X)\neq B(f)$. Therefore, we arrive at the conclusion that $B(m)\neq B(f)$ which implies that $(T_0)_{mf}=0=(T_0)^*_{mf}$. Using this result, we find that

$$\sum_{\bar f\in -B}|\tilde T_{\bar f\bar X}|^2-\sum_{f\in B}|\tilde T_{fX}|^2 = \alpha_{\not B}^2\sum_{f\in B}\sum_m 2\Im((\tilde T_1)^*_{fX}(T_0)_{mX}(\tilde T_1)^*_{mf}) + \mathcal{O}(\alpha_{\not B}^3), \tag{25}$$

i.e., a nonzero contribution to $\mathcal{O}(\alpha_{\not B}^2)$, unlike in Eq. (22), where we had only obtained nonzero contributions to $\mathcal{O}(\alpha_{\not B}^3)$ and higher. Here we have used the approximate equality of $(\tilde T_1)^*_{ij}$ and $(\tilde T_1)_{ji}$, since their difference is higher order in $\alpha_{\not B}$ and similarly for $(T_0)^*_{ij}$ and $(T_0)_{ji}$. Note the very similar form of Eqs. (22) and (25). The important difference, however, is that a baryon number violating vertex in case 1 has been replaced by a baryon number conserving vertex in case 2, thereby inducing a corresponding change in the transition amplitudes $\alpha_{\not B}\tilde T_1\to T_0$ in the respective expressions. Since B/L violating couplings in almost all models are constrained to be small, this replacement allows for the possibility of generating a higher degree of *CP* violation than would perhaps have been possible with B/L violating decays alone. The example below demonstrates this explicitly.

IV. A TOY MODEL IN BARYOGENESIS

To illustrate the main idea of this paper (i.e., case 2 in Sec. III B), we consider a toy model for baryogenesis, following an example in Kolb and Turner [38]. The model involves two superheavy bosons X and Y, whose B violating out-of-equilibrium decays can generate the necessary B and *CP* violations. For the following discussion we assign baryon numbers of X and Y to be $B(X)=B(Y)=0$. The relevant terms in the Lagrangian are given by

$$\mathcal{L}=g_{0X}Xf_1^\dagger f_1+g_{0Y}Yf_1^\dagger f_1+g_1Xf_2^\dagger f_1+g_2Yf_2^\dagger f_1+\mathrm{H.c.} \tag{26}$$

Here, f_i $(i=1,2)$ denote fermions carrying distinct nonzero baryon numbers and equal $U(1)_{\rm em}$ charges. Both bosons have zero $U(1)_{\rm em}$ charge. In the above Lagrangian, g_{0X} and g_{0Y} are B conserving real couplings, while g_1 and g_2 are B violating and complex. Now, consider the B violating process

$$X\to\bar f_1+f_2, \tag{27}$$

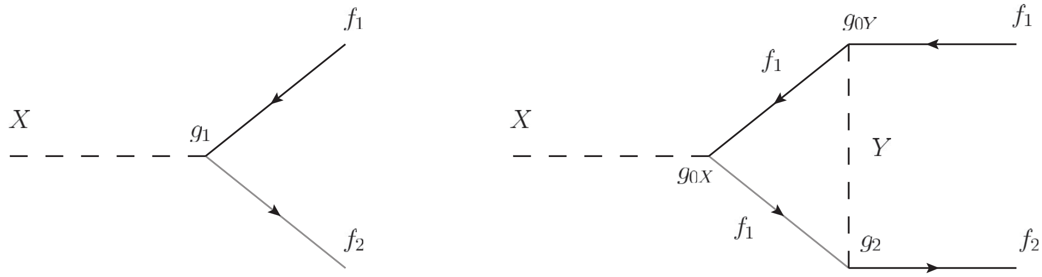

FIG. 1. Tree level and one-loop diagrams for the decay $X \to \bar{f}_1 + f_2$. Similar diagrams also apply for the decay of the Y boson.

where the leading CP violating contribution to the decay width comes from the interference of the tree and loop diagrams in Fig. 1. These interference terms in the decays of X and $\bar{X}$ are given by

$$\Gamma(X \to \bar{f}_1 + f_2) = g_1 g_{0X} g_2^* g_{0Y} I_{XY} + \text{c.c.}$$
$$\Gamma(\bar{X} \to f_1 + \bar{f}_2) = g_1^* g_{0X} g_2 g_{0Y} I_{XY} + \text{c.c.}, \qquad (28)$$

where I_{XY}, denoting the loop factor, can have a nonzero imaginary part when f_1 and f_2 are lighter than the X boson and can go on shell inside the loop. The resulting CP violation in X decays will then be

$$\epsilon_X = \frac{4 g_{0X} g_{0Y} \Im(g_1 g_2^*) \Im(I_{XY})}{\Gamma_X}, \qquad (29)$$

which is nonzero in general [here, $\Gamma_X = \Gamma(X \to \bar{f}_1 + f_2) + \Gamma(\bar{X} \to f_1 + \bar{f}_2)$]. Similarly, the decays of the Y boson will lead to a CP violation as well, which is given by

$$\epsilon_Y = \frac{4 g_{0X} g_{0Y} \Im(g_2 g_1^*) \Im(I'_{YX})}{\Gamma_Y}. \qquad (30)$$

As long as the X and Y bosons have different masses, the total CP asymmetry is nonzero, and the resulting B asymmetry is as follows:

$$\Delta B = (B_2 - B_1) \times [4 g_{0X} g_{0Y} \Im(g_1 g_2^*)] \times \left[\frac{\Im(I_{XY})}{\Gamma_X} - \frac{\Im(I'_{YX})}{\Gamma_Y}\right]. \qquad (31)$$

Thus, as expected from our general arguments in the previous section, once a heavy particle has both B conserving and violating modes of decay, we can generate a B asymmetry which involves graphs of only first order in B violation, and therefore, the interference term is only second order in such couplings [in the above example, ΔB is proportional to $\Im(g_1 g_2^*)$]. In the Appendix, we reexpress the standard Nanopoulos-Weinberg example in terms of our formulation, where, by considering a boson, X, that does not have any B conserving decay mode, we verify that the B asymmetry consequently generated is indeed zero up to second order in B violation. Therein we also discuss an example from Kolb and Turner [38], where additional B violating decay modes of X help generate an asymmetry at higher orders in B violation.

V. A MODEL IN LOW-SCALE LEPTOGENESIS

After considering the above toy model in baryogenesis which demonstrates the primary result of our paper in a very simple example, in this section we give a very brief sketch of a more realistic model in leptogenesis, inspired by the work of Kayser and Segre [39]. In particular, our goal here is to construct an ElectroWeak-Symmetry-Breaking-scale (EWSB) leptogenesis model utilizing both the idea of introducing scalar quartic couplings in the loop graphs as in Ref. [39], as well as having both L conserving and L violating decay modes as discussed in case 2 in Sec. III B.

We introduce two right-handed Majorana neutrinos N_1 and N_2 with masses in the electroweak scale such that $M_{N_1} > M_{N_2}$. Additionally, we introduce another scalar doublet, Φ_2 (apart from the SM Higgs Φ_1); henceforth, h will represent the SM-like Higgs boson, while H^+ will represent the charged Higgs boson from the extended Higgs sector. This leads to the following possible decay modes for the N_1:

$$N_1 \to \ell^- + H^+ \qquad (32)$$

$$N_1 \to N_2 + h. \qquad (33)$$

Consequently, the decay in Eq. (32) will arise out of a Yukawa-type interaction and the decay in Eq. (33) can arise from a coupling of the form $N_1 N_2 S$ (all of which are SM gauge singlets) after electroweak symmetry breaking, whereby the singlet S can mix with the neutral components of the doublet scalars. Due to the Majorana nature of the heavy right-handed neutrinos, depending on the L-number assignment, either the decay in Eq. (32), or its conjugate process ($N_1 \to \ell^+ H^-$), or both will violate L number, while the decay in Eq. (33) will always be L conserving [since $L(N_1) = L(N_2)$].

The final ingredient in our model is a quartic coupling between the scalar doublets, of the form $\lambda(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)$, which, after EWSB, will give rise to trilinear scalar couplings. With this understanding, we consider the diagrams for the process in Eq. (32), as shown in Fig. 2.

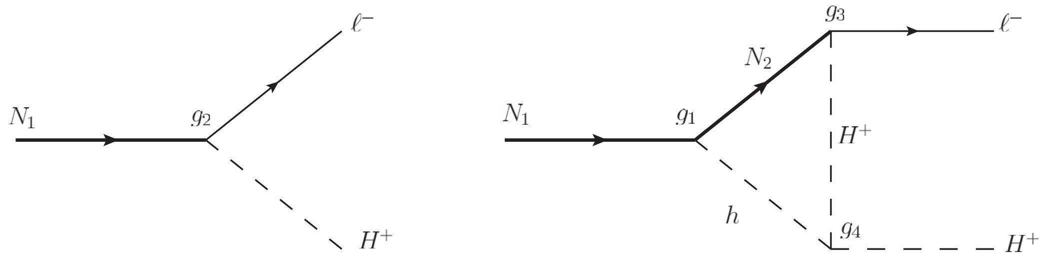

FIG. 2. Tree level and one-loop diagrams for the decay $N_1 \to \ell^- H^+$.

The coupling notations also follow Fig. 2. The relevant interference term is given by

$$\Gamma(N_1 \to \ell^+ H^-) = g_2 g_1 g_3^* g_4 I_{N_1} + \text{c.c.} \tag{34}$$

Here, the Yukawa couplings g_2 and g_3 are complex in general, while g_1 and g_4 are real. The kinematic loop factor has been denoted by I_{N_1}. The resulting *CP* violation is then

$$\epsilon_{N_1} = \frac{4 g_1 g_4 \Im(g_2 g_3^*) \Im(I_{N_1})}{\Gamma_{N_1}}, \tag{35}$$

where

$$\Gamma_{N_1} = \Gamma(N_1 \to \ell^- H^+) + \Gamma(N_1 \to \ell^+ H^-) \propto |g_2|^2. \tag{36}$$

Therefore, with the simplifying assumption that $|g_2| \sim |g_3|$, we see that the magnitude of the Yukawa coupling actually cancels out from the *CP* violation:

$$\epsilon_{N_1} \simeq \frac{4 g_1 g_4 \delta \Im(I_{N_1})}{(M_{N_1}/8\pi)}, \tag{37}$$

where the factor $\delta = \sin(\phi_2 - \phi_3)$ comes from the difference of phases of g_2 and g_3. We have also used $\Gamma_{N_1} \sim |g_2|^2 M_{N_1}/8\pi$ in writing the above expression. Now, let us estimate the magnitudes of the various terms in Eq. (37):

(1) g_1: The $N_1 N_2 S$ coupling is dimensionless, and assumed to be of $\mathcal{O}(1)$. Therefore, g_1 is essentially determined by the mixing of the singlet S with the neutral components of the Higgs doublets. For simplicity, we assume that S dominantly mixes with the SM-like lighter Higgs state recently discovered at the LHC. In that case, the measurement of the Higgs properties puts an upper bound on this mixing $\sin\alpha < 10^{-2}$. Hence, we can safely take $g_1 \simeq 10^{-2}$.

(2) g_4: Since g_4 arises after EWSB from the Higgs quartic coupling discussed above, we have $g_4 \sim \lambda v \sin\beta$, with $v = 246$ GeV, and $\tan\beta = v_1/v_2$ denotes the ratio of the vacuum expectation values of the neutral *CP*-even components of Φ_1 and Φ_2, respectively. In an electroweak scale model for leptogenesis, M_{N_1} is also of the order of v. And therefore, the factor of $M_{N_1}/8\pi$ in the denominator will roughly cancel out the factor $v\sin\beta$ in the numerator.

(3) $\Im(I_{N_1})$: This loop factor is found to be

$$\Im(I_{N_1}) \approx \frac{1}{\pi}\frac{M_{N2}}{M_{N1}^2}\frac{1}{1+\xi^2}\log\left(\frac{1-\xi+\xi^2}{\xi}\right), \tag{38}$$

where $\xi = (M_{H^\pm}/M_{N1})^2$. Considering the present constraints on a charged Higgs boson mass, we can safely take $M_{H^+} \simeq 300$ GeV. This leads to a value for the loop factor $\mathcal{O}(10^{-3}\text{–}10^{-4})$, for $M_{N_2} < M_{H^+} < M_{N_1}$, and $M_{N_1} \sim 500$ GeV.

(4) δ: This phase factor has a maximum value of 1.

Therefore, for our order of magnitude estimate, we finally obtain

$$\epsilon_{N_1} \sim 10^{-5}\lambda. \tag{39}$$

For generating a sufficient lepton asymmetry (which is converted to the required baryon asymmetry by the sphaleron processes), one requires $\epsilon_{N_1} = \mathcal{O}(10^{-6})$. Thus, we need a quartic coupling of $\lambda = 0.1$, which is a likely value (especially in the light of the recent Higgs mass measurement, whereby the SM Higgs quartic can be estimated to be ∼0.13). It is to be noted that the phase factor δ vanishes if $\phi_2 = \phi_3$. Hence, the couplings of the two right-hand neutrino mass eigenstates N_1 and N_2 to the charged lepton ℓ should have different phases in order to obtain a nonzero ϵ_{N_1}.

This rather schematic discussion illustrates the feasibility of having models of electroweak scale leptogenesis where the amount of *CP* violation is not directly related to the neutrino Yukawa couplings, which, in most low energy (TeV) leptogenesis models, are usually constrained to be small, but rather to the relatively unconstrained quartic Higgs couplings in a two Higgs doublet model. A detailed study of the model is beyond the scope of the present paper, and we leave it to future work. However, it is important to emphasize the role played by the L conserving decay channel here—the absence of such a channel would have entailed one to look for leptogenesis involving graphs with higher order L violating couplings within the purview of this model, possibly requiring two or more loops and therefore suppressing the generated *CP* violation significantly.

VI. REMARKS AND CONCLUSION

We have expanded the interaction amplitude in a perturbation series in the B/L violating coupling $\alpha_{\not B}$, in order to show the nontrivial implication of the Nanopoulos-Weinberg theorem in the case where B/L assignments are naturally and consistently such that the initial particle may decay by B/L conserving interactions in addition to B/L violating interactions. In particular, it turns out that in such cases, the asymmetry generated due to B/L violating decays may be augmented by B/L conserving interactions in the loop graphs, in a way that deceptively appears contrary to the consequences of the Nanopoulos-Weinberg theorem. This reinterpretation of the theorem has significant implications for models of baryogenesis and leptogenesis by opening up channels which allow for the generation of *CP* violation that might have been earlier ignored with the intention of subscribing to the theorem's stringent requirements. Additionally, the replacement of a B/L violating coupling by a B/L conserving one, as discussed above, may allow for enhanced generation of *CP* violation since the former is typically constrained by experiment to be small. As is well known, the generation of "sufficient" *CP* asymmetry remains an issue not just in the Standard Model but in most extensions of it as well. We have illustrated our main result by constructing a toy model in baryogenesis from out-of-equilibrium decays of heavy bosons.

In addition to setting up new models for B/L genesis employing B/L conserving channels as we have shown, it might be an interesting exercise to reanalyze some currently proposed models of baryogenesis and leptogenesis in the light of this interpretation. As an example of this approach, we have considered a recently proposed model of leptogenesis which generates a *CP* asymmetry only at the two-loop level. By studying a simple variation of this model obtained by slightly altering its particle content in a way which allows B/L conserving decays, we have shown that it is possible to generate sufficient *CP* asymmetry at the one-loop level.

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APPENDIX: EXAMPLES IN BARYOGENESIS TO DEMONSTRATE THE NANOPOULOS-WEINBERG THEOREM

Typically, the contribution to baryon asymmetry generated by the particle X with baryon number $B(X) = B_X$ and total decay width Γ_X, due to its transition to final states f with $B(f) = B_f \neq B_X$, is given by

$$\epsilon_X = \sum_f (B_f - B_X) \frac{\Gamma(X \to f) - \Gamma(\bar{X} \to \bar{f})}{\Gamma_X} \propto \sum_f (B_f - B_X) \sum_m \Im(T^*_{fX} T_{mX} T^*_{mf}). \tag{A1}$$

We consider two examples to illustrate the implications of the Nanopoulos-Weinberg theorem. First, we consider a model in which a heavy scalar boson X with baryon number $B_X = 0$ can decay via a B violating interaction to a pair of fermions f_1 and f_2, while another scalar heavy boson Y can decay only via separate B conserving interactions to both the fermions. The Lagrangian for the model is given below:

$$\mathcal{L}_a = g_1 X f_2^\dagger f_1 + g_2 Y f_1^\dagger f_1 + g_3 Y f_2^\dagger f_2 + \text{H.c.} \tag{A2}$$

The possible tree and one-loop diagrams for the decay process $X \to \bar{f}_1 f_2$ are shown in Fig. 3. Both the tree and one-loop graph have one B violating vertex each (*vertex with coupling constant* g_1 *in both graphs*). One can easily calculate the asymmetry generated, ϵ_X, in the decay of X due to the interference of the two graphs and find that

$$\Gamma(X \to \bar{f}_1 f_2) = |g_1|^2 g_2 g_3 (I_{XY} + I^*_{XY}), \text{and} \tag{A3a}$$

$$\Gamma(\bar{X} \to f_1 \bar{f}_2) = |g_1|^2 g_2 g_3 (I_{XY} + I^*_{XY}), \tag{A3b}$$

which means that

$$\epsilon_X \propto \Gamma(X \to \bar{f}_1 f_2) - \Gamma(\bar{X} \to f_1 \bar{f}_2) = 0. \tag{A3c}$$

Here, we have represented only the contribution to the decay width arising due to the interference between a one-loop graph and a tree graph by Γ. The kinematic factor arising out of the integral over the loop momentum is denoted by I_{XY}, which can be complex if the fermions in the loop are kinematically allowed to go on shell. As a result of Eq. (A3c), the asymmetry generated due to X decays in this model, which is proportional to the *CP* violation, also becomes zero. This is, clearly, what we expect from the Nanopoulos-Weinberg theorem, as the only contributions

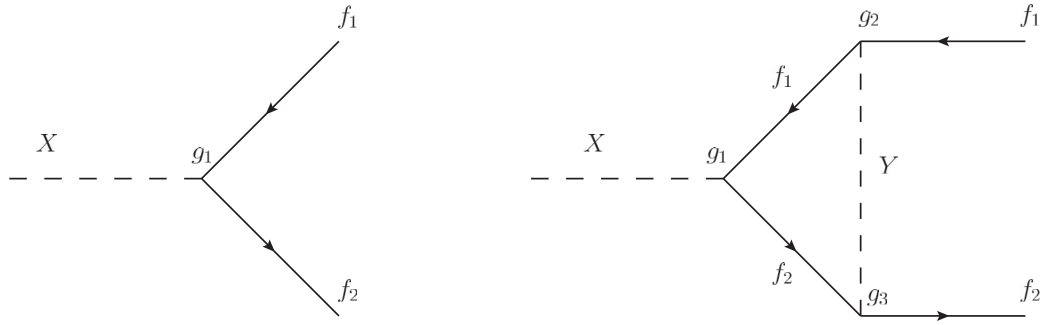

FIG. 3. Tree and one-loop graphs for the decay $X \to \bar{f}_1 f_2$ due to the Lagrangian $\mathcal{L}_a$.

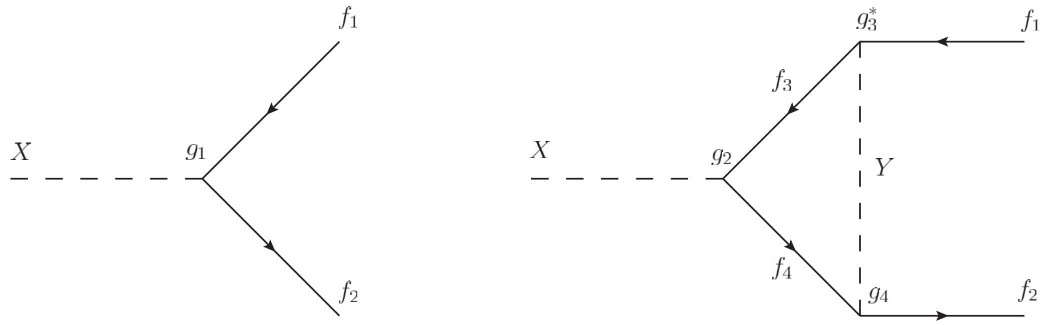

FIG. 4. Tree and one-loop graphs for the decay $X \to \bar{f}_1 f_2$ due to the Lagrangian $\mathcal{L}_b$.

to the B violating decay $X \to \bar{f}_1 f_2$ come from processes represented by graphs to the first order in B violation.

We next consider a model in which both the superheavy bosons X and Y can decay only via B violating interactions to fermion pairs. The interaction Lagrangian for this model is given by

$$\mathcal{L}_b = g_1 X f_2^\dagger f_1 + g_2 X f_4^\dagger f_3 + g_3 Y f_1^\dagger f_3 + g_4 Y f_2^\dagger f_4 + \text{H.c.}, \tag{A4}$$

where each fermion f_i has a different and unique B number B_i. The baryon asymmetry generated out of the decays of the superheavy scalars X and Y in this model has been extensively studied in the literature (see, e.g., [38]). The graphs at the tree and one-loop levels that contribute to the decay $X \to \bar{f}_1 f_2$ are shown in Fig. 4; the loop graph in this case has three B violating vertices.

It is easy to see that the asymmetry generated in this case is nonzero:

$$\epsilon_X^{12} = \frac{4(B_2 - B_1)\Im(I_{XY})\Im(g_1^* g_2 g_3^* g_4)}{\Gamma_X}, \tag{A5}$$

where, as usual, I_{XY} denotes a factor arising out of integration over the loop momentum. One can similarly see that the asymmetry generated due to the decay $X \to \bar{f}_3 f_4$ is given by

$$\epsilon_X^{34} = \frac{4(B_4 - B_3)\Im(I_{XY})\Im(g_1 g_2^* g_3 g_4^*)}{\Gamma_X}. \tag{A6}$$

The total asymmetry due to all possible B violating decays of X is, thus,

$$\begin{aligned}\epsilon_X &= \epsilon_X^{34} + \epsilon_X^{12} \\ &= \frac{4((B_4 - B_3) - (B_2 - B_1))}{\Gamma_X} \\ &\quad \times \Im(I_{XY})\Im(g_1 g_2^* g_3 g_4^*) \neq 0.\end{aligned} \tag{A7}$$

This is also what is expected from the Nanopoulos-Weinberg theorem, since the one-loop contribution to the B violating decays in this case are of the third order in B violation.

[1] F. Iocco, G. Mangano, G. Miele, O. Pisanti, and P. D. Serpico, Phys. Rep. **472**, 1 (2009).
[2] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. Nolta *et al.*, Astrophys. J. Suppl. Ser. **192**, 16 (2011).
[3] G. Steigman, Annu. Rev. Astron. Astrophys. **14**, 339 (1976).
[4] A. G. Cohen, A. De Rujula, and S. Glashow, Astrophys. J. **495**, 539 (1998).
[5] A. Sakharov, Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32 (1967).
[6] V. Kuzmin, V. Rubakov, and M. Shaposhnikov, Phys. Lett. **155B**, 36 (1985).
[7] C, P, and T will denote the charge conjugation, parity, and time-reversal transformations, respectively, henceforth in our work.
[8] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[9] P. Huet and E. Sather, Phys. Rev. D **51**, 379 (1995).
[10] V. Rubakov and M. Shaposhnikov, Usp. Fiz. Nauk **166**, 493 (1996).
[11] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, Nucl. Phys. **B466**, 189 (1996).
[12] J. M. Cline, arXiv:hep-ph/0609145.
[13] A. Riotto and M. Trodden, Annu. Rev. Nucl. Part. Sci. **49**, 35 (1999).
[14] A. Riotto, arXiv:hep-ph/9807454.
[15] A. Y. Ignatiev, N. Krasnikov, V. Kuzmin, and A. Tavkhelidze, Phys. Lett. **76B**, 436 (1978).
[16] M. Yoshimura, Phys. Rev. Lett. **41**, 281 (1978).
[17] D. Toussaint, S. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D **19**, 1036 (1979).
[18] S. Dimopoulos and L. Susskind, Phys. Rev. D **18**, 4500 (1978).
[19] J. R. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Phys. Lett. **80B**, 360 (1979).
[20] S. Weinberg, Phys. Rev. Lett. **42**, 850 (1979).
[21] M. Yoshimura, Phys. Lett. **88B**, 294 (1979).
[22] S. M. Barr, G. Segre, and H. A. Weldon, Phys. Rev. D **20**, 2494 (1979).
[23] D. V. Nanopoulos and S. Weinberg, Phys. Rev. D **20**, 2484 (1979).
[24] A. Yildiz and P. H. Cox, Phys. Rev. D **21**, 906 (1980).
[25] I. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985).
[26] A. G. Cohen and D. B. Kaplan, Phys. Lett. B **199**, 251 (1987).
[27] A. G. Cohen and D. B. Kaplan, Nucl. Phys. **B308**, 913 (1988).
[28] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
[29] C. S. Fong, E. Nardi, and A. Riotto, Adv. High Energy Phys. **2012**, 1 (2012).
[30] E. Nardi, in *Particle Physics at the Tercentenary of Mikhail Lomonosov, 2013*, Chap. 52, pp. 238–243.
[31] A. Pilaftsis, J. Phys. Conf. Ser. **447**, 012007 (2013).
[32] A. Pilaftsis, J. Phys. Conf. Ser. **171**, 012017 (2009).
[33] P. Di Bari, Contemp. Phys. **53**, 315 (2012).
[34] S. Davidson, E. Nardi, and Y. Nir, Phys. Rep. **466**, 105 (2008).
[35] W. Buchmuller, R. Peccei, and T. Yanagida, Annu. Rev. Nucl. Part. Sci. **55**, 311 (2005).
[36] E. W. Kolb and S. Wolfram, Nucl. Phys. **B172**, 224 (1980).
[37] The third sum contributes only to $\mathcal{O}(\alpha_{\not{B}}^4)$ and higher.
[38] E. W. Kolb and M. S. Turner, *The Early Universe* (WestView Press, Boulder, Colorado, 1993), Chap. 6.
[39] B. Kayser and G. Segre, Phys. Lett. B **704**, 570 (2011).