Heavy tetraquark confining potential in Coulomb gauge QCD

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We present an analytic nonperturbative solution of the Yakubovsky equation for tetraquark states in the case of equal separations and energies, and demonstrate a direct connection between the tetraquark confinement potential and the temporal gluon propagator. To this end we employ a leading-order heavy quark mass expansion of the Coulomb gauge QCD action, and use the dressed two-point functions of the Yang-Mills sector only. As a result, we find a bound state energy that rises linearly with distance and a string tension twice as large as in a $q\bar{q}$-system.

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I. INTRODUCTION

Exotic states in the heavy quark sector are an increasingly fascinating subject to study. With the discovery and confirmation of many new XYZ-states at BaBar, Belle, the LHC and Beijing spectrometer, interpretations of some of these clearly favor states go beyond the time-honored classification of hadrons into mesons and baryons [1,2], opening up the exciting possibility of the identification of tetraquark, meson molecule or hybrid states. The idea of tetraquarks has been around for quite some time. For the light quark sector, Jaffe proposed that the light scalar nonet including $f_0(980)$ and $a_0(980)$ can be interpreted as a $qq\bar{q}\,\bar{q}$ state instead of $q\bar{q}$ [3,4]. Indeed, the mass ordering and decay patterns of these states nicely fit this picture. In the heavy quark sector, charged states like the $Z_c^+(3900)$ or the $Z_b^+(10610)$ and their cousins cannot be identified with ordinary quarkonia and therefore strongly suggest an interpretation in terms of tetraquarks.

Theoretically, tetraquarks can be described by a generalized Bethe-Salpeter equation for four particle states, originally proposed by Yakubovsky [5] (see also Refs. [6,7] for pedagogical introductions). In a covariant setting, this equation, rounded off to account for quantum-field theoretical effects [8], has been solved under the approximation that the $4q$ state is described by a coupled system of two-body equations with meson and diquark constituents [9]. Based on previous investigations, which showed that a rainbow-ladder kernel is most robust in meson Bethe-Salpeter calculations [10–12], results of tetraquark masses have been obtained by employing a phenomenologically validated one gluon-exchange interaction. A complete classification of tetraquark states in terms of spin flavor, color and spatial degrees of freedom has been constructed in [13]. Other investigations of tetraquark states include large N-limit calculations [14], effective theory studies [15–17] and relativistic quark models [18].

From a fundamental perspective, tetraquarks offer interesting insights into the underlying structure of the strong interaction. The relationship between the nonperturbative scale associated with confinement (the string tension) and the gluon sector is of crucial importance in understanding the low-energy properties of QCD. On the lattice, Wilson loops exhibit an area law at intermediate distances that corresponds to a linearly rising potential, whereas the corresponding coefficient, the so-called Wilsonian string tension, can be explicitly related to a hadronic scale [19]. Within continuous functional approaches, investigations carried out in the Coulomb gauge have shown that in the heavy quark sector (and at least under truncation) one can identify a direct connection between the temporal Yang-Mills Green's function and the potential that confines quarks, both in the two- and three-body case [20,21]. In the Hamiltonian formalism, the temporal Wilson loop gives the physical string tension [22,23], whereas the Coulomb string tension is an upper bound to the Wilson loop string tension [24,25].

While the potential that confines two and three quarks has been relatively extensively studied with continuous methods as well as on the lattice (see for example Ref. [26] for a review), the interaction between quarks in a $4q$ system has received little attention. On the lattice, the problem of van der Waals forces has been investigated, and it has been shown that a flux tube recombination takes place, i.e., around a level-crossing point, the confining potential flips between the disconnected "two-meson" Ansatz and the state where the quarks and antiquarks are connected by a double-Y shaped flux tube, and this implies that the van der Waals forces are absent at long distances [27]. Continuum studies that have investigated the absence of long range forces in tetraquarks include Refs. [28,29].

In this work we will study the nature of the confining force in tetraquarks using a framework gauge fixed to the Coulomb gauge. The realization of confinement in the Coulomb gauge centers around the Gribov-Zwanziger scenario, which conjectures that the confining potential is provided by the temporal gluon propagator, whereas the spatial propagator is suppressed at long distances [30]. In addition, this gauge possesses a number of features that recommend it as an appropriate tool to study the low-energy sector of QCD: within the first order formalism, the total charge of the system is conserved and vanishing, the system reduces naturally to the physical degrees of freedom [31] and the problem of divergent energy integrals disappears [32]. In the Coulomb gauge, the Dyson-Schwinger equations for both Yang-Mills and quark sectors have been derived, and perturbative results have been obtained [33–36]. On the lattice, one important result (which shall be extensively used in this work) is that the temporal gluon propagator is energy independent, and it behaves like $1/\vec{q}^4$ for vanishing $\vec{q}$ [37–39].[1] The lattice results agree with the analytical findings obtained from the Hamiltonian approach to the Yang-Mills theory [25,41,42].

This paper is a natural continuation of previous works including one of the authors [20,21], where meson and baryon bound states have been investigated via Bethe-Salpeter and Faddeev equations, respectively. Based on a leading-order expansion in the heavy quark mass originally developed within heavy quark effective theory (HQET) [43], a direct connection between the temporal gluon propagator and the string tension has been derived.[2] Here, we follow the same approach and consider the Yakubovsky equation for four-quark states [5] in the Coulomb gauge at the leading order in the mass expansion, in the symmetric case (i.e., the separation between quarks are equal), at equal energies, and by including only two particle irreducible contributions. We will employ lattice results for the temporal gluon propagator, and, in addition, we will use previous findings, namely that the kernel of the Bethe-Salpeter equation reduces nonperturbatively to the ladder truncation. In this setting, we will provide an exact analytical solution to the Yakubovsky equation, which then naturally leads to the confining potential of a $4q$ system.

The organization of this paper is as follows. In Sec. II we briefly survey the results obtained for heavy quark systems. We review the main steps of the expansion of QCD action in powers of the inverse quark mass, and discuss the results obtained for the heavy quark propagator and the corresponding (temporal) quark-gluon vertex. In Sec. III we present the Yakubovsky equation for tetraquark states. Similar to the case of meson and baryon states, we establish (at least under truncation) a direct relation between the physical string tension and the temporal component of the gluon propagator. A short summary and conclusions are presented in Sec. IV.

[1]Another possibility considered in Ref. [40], which avoids the ambiguities related to the definition of the A_0A_0 propagator, is to extract the Coulomb string tension from the exponential falloff of the time-like link-link correlator.

[2]The heavy quark limit has also been recovered under a (perturbative) leading order truncation of Dyson-Schwinger equations [44,45].

II. EXPANSION IN THE HEAVY QUARK MASS

In this section we outline the results obtained within the heavy quark limit that are relevant for this work, and direct the reader to Ref. [20] for a full account. We employ the standard notations and conventions: spatial indices are labeled with the roman letters i, j, …, and the superscripts a, b, … denote color indices in the adjoint representation; flavor, Dirac spinor and (fundamental) color indices are commonly denoted with an index, $\alpha, \beta \ldots$. We work in Minkowsky space, with the metric $g_{\mu\nu} = \mathrm{diag}(1, -\vec{1})$. The Dirac γ-matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, where the notation γ^i refers to the spatial component and the minus sign arising from the metric has been explicitly taken into account. f^{abc} are the structure constants of the $SU(N)$ group, with the Hermitian generators $[T^a, T^b] = if^{abc}T^c$ normalized via $\mathrm{Tr}(T^aT^b) = \delta^{ab}/2$, and the Casimir factor $C_F = (N^2-1)/2N$.

The idea that underlies the heavy quark mass expansion is the so-called heavy quark decomposition, i.e., the (full) quark field q_α is separated into two components via the spinors h and H as follows:

$$
\begin{aligned}
q_\alpha(x) &= e^{-imx_0}[h(x) + H(x)]_\alpha, \\
h_\alpha(x) &= e^{imx_0}\frac{\mathbb{1}+\gamma^0}{2}q_\alpha(x), \\
H_\alpha(x) &= e^{imx_0}\frac{\mathbb{1}-\gamma^0}{2}q_\alpha(x)
\end{aligned}
\tag{2.1}
$$

(similarly for the antiquark field). In our Coulomb gauge functional approach, this particular type of heavy quark transform adapted from HQET [43] can be simply regarded as an arbitrary decomposition. This is then inserted into the QCD generating functional, and, after integrating out the H-fields, an expansion in the heavy quark mass is performed (throughout this work we shall use the established terminology "mass expansion" instead of "expansion in the *inverse* quark mass"). We mention here that the quark and antiquark sources are kept in all steps of the calculation, such that the full gap and Yakubovsky equations can be employed, whereas the kernels, propagators and vertices are replaced with their expressions at the leading order in the mass expansion. The decomposition Eq. (2.1), along with the Yang-Mills truncation, leads to the suppression of the spatial gluon propagator at the leading order in the mass expansion, which in turn means that at leading order the attached gluons couple to the constituent quarks of the four-body state via a temporal quark-gluon vertex. We refer the reader to Ref. [20] for a detailed discussion regarding source terms and the expansion of the QCD action in the parameter $1/m$.

Before we provide our solution for the heavy quark propagator, it is appropriate to briefly discuss our truncation scheme, which has also been employed in [20,21]. In the context of the heavy mass expansion, we restrict ourselves to dressed two-point functions of the Yang-Mills sector (i.e., the nonperturbative gluon propagators) and set all the pure Yang-Mills vertices and higher n-point functions occurring in the quark equations to zero. This truncation is justified by the fact that the total number of loops containing Yang-Mills vertices is drastically reduced: on the one hand, these vertices only contribute at the second order perturbatively, since the leading-order perturbative corrections containing purely temporal vertices vanish (temporal Yang-Mills vertices are zero at the tree level [34]), and, on the other hand, loops containing spatial Yang-Mills vertices are suppressed by the mass expansion. Physically, the main consequence of setting the Yang-Mills vertices to zero is the exclusion of the non-Abelian part of the color charge screening mechanism, and the potential glueball contributions of the gluon field. On the other hand, the fact the confining potential stems from the ladder Bethe-Salpeter kernel [20] implies that the gluon does contain the nonperturbative effects attached to the dynamical dressing of color charge (such as glueball states), whereas the quark-gluon vertices correspond to naked charges. Hence, we expect that the inclusion of the non-Abelian corrections would not alter the linear behavior of the bound state energy but would lead to a smaller value of the string tension σ via shifting the position of the pole.

From the full Coulomb gauge gap equation (i.e., the first order formalism without mass expansion [36]), supplemented by the Slavnov-Taylor identity, we find the following solution for the heavy quark propagator:

$$W_{\bar{q}q\alpha\beta}(k_0) = \frac{-i\delta_{\alpha\beta}}{[k_0 - m - \mathcal{I}_r + i\varepsilon]} + \mathcal{O}(1/m), \qquad (2.2)$$

with

$$\mathcal{I}_r = \frac{1}{2} g^2 C_F \int_r \frac{đ\vec{\omega} D_{\sigma\sigma}(\vec{\omega})}{\vec{\omega}^2} + \mathcal{O}(1/m). \qquad (2.3)$$

The constant $\mathcal{I}_r$ is implicitly regularized, under the assumption that the order of the integration is set such that the temporal integral is performed first, and the spatial integral is regularized and finite. The nonperturbative temporal gluon propagator entering $\mathcal{I}_r$ is given by

$$W^{ab}_{\sigma\sigma}(\vec{k}) = \delta^{ab} \frac{i}{\vec{k}^2} D_{\sigma\sigma}(\vec{k}^2). \qquad (2.4)$$

Following lattice results [38], and also continuum investigations [37], we assume that the dressing function $D_{\sigma\sigma}$ is energy independent and diverges like $1/\vec{k}^2$ in the infrared. From the Slavnov-Taylor identity, combined with the solution Eq. (2.2), one easily finds that the temporal quark-gluon vertex remains nonperturbatively bare,

$$\Gamma^a_{\bar{q}q\sigma\alpha\beta}(k_1, k_2, k_3) = [gT^a]_{\alpha\beta} + \mathcal{O}(1/m), \qquad (2.5)$$

whereas the spatial vertex is subleading in the heavy mass expansion [20]. The heavy quark propagator Eq. (2.2) has a few remarkable properties which we shall discuss here briefly. First, as a result of the mass expansion, this propagator has a single pole in the complex k_0-plane, as opposed to the conventional Feynman quark propagator. Therefore, it is necessary to explicitly define the Feynman prescription. It then follows that the closed quark loops vanish due to energy integration,

$$\int \frac{dk_0}{[k_0 - m - \mathcal{I}_r + i\varepsilon][k_0 + p_0 - m - \mathcal{I}_r + i\varepsilon]} = 0, \qquad (2.6)$$

meaning that the theory is quenched at the lowest order in the heavy quark mass expansion. A further observation is that the propagator Eq. (2.2) is diagonal in the outer product of the fundamental color, flavor and spinor spaces, due to the decoupling of the spin degree of freedom in the heavy quark limit [43]. Finally, the position of the pole in the heavy quark propagator does not have a physical meaning since the quark cannot be on shell. As soon as the regularization is removed, the poles are shifted to infinity, and this implies that only the relative energy plays a role in a hadronic system (or, if a single quark is considered, one needs infinite energy to create it from the vacuum). Indeed, it has long been known that the absolute energy does not have a physical meaning, and that only the relative energy, which in the case of tetraquarks is derived from the Yakubovsky equation, must be considered [46].

Since the heavy quark mass expansion breaks the charge conjugation symmetry, the antiquark and quark propagators are not equivalent. The Feynman prescription for the antiquark propagator is derived from the observation that the Bethe-Salpeter equation must have a physical interpretation of bound states—in this case, the quark and the antiquark are not connected by a primitive vertex, and hence they do not create a virtual quark-antiquark pair (closed loop) but a system composed of two separate unphysical particles. For the antiquark propagator, we obtain (the derivation is similar to the quark propagator)

$$W_{q\bar{q}\alpha\beta}(k_0) = \frac{-i\delta_{\alpha\beta}}{[k_0 + m - \mathcal{I}_r + i\varepsilon]} + \mathcal{O}(1/m), \qquad (2.7)$$

and the corresponding vertex is given by

$$\Gamma^a_{q\bar{q}\sigma\alpha\beta}(k_1, k_2, k_3) = -[gT^a]_{\beta\alpha} + \mathcal{O}(1/m). \qquad (2.8)$$

A last important consequence of the heavy quark mass expansion is the reduction of the interaction kernels from both Bethe-Salpeter and Yakubovsky equations to the ladder exchange, since the crossed box contributions cancel due to energy integration over multiple quark propagators with the same Feynman prescription (see Ref. [20] for a detailed discussion and calculation).

III. YAKUBOVSKY EQUATION FOR TETRAQUARK STATES

The Yakubovsky equation is a four-body bound state equation which has been successfully used to describe tetraquark states. It embodies two-, three- and four-quark irreducible diagrams. Since the irreducible three- and four-body forces all involve pure Yang-Mills vertices which, as discussed in the previous section, are neglected in our approach, it follows that there are no 3PI and 4PI contributions at this level of approximation. Consequently, it is appropriate to employ only the (anti)diquark and meson kernels that also appear in the corresponding Bethe-Salpeter equations. In a covariant setting, a similar approximation has been successfully applied to study tetraquark bound states [9].

In this approximation, and by using the formulation [8], where the correct multiplicity is taken into account via the inclusion of two-pair kernels, the Yakubovsky equation reads

$$
\begin{aligned}
&\Gamma_{\alpha\beta\gamma\delta}(p_1,p_2,p_3,p_4)\\
&\quad=\int \text{đ}k\{S^{(qq)}_{\beta\alpha;\alpha'\beta'}(p_1,p_2;k)\Gamma_{\alpha'\beta'\gamma\delta}(p_1+k,p_2-k,p_3,p_4)+S^{(aa)}_{\delta\gamma;\gamma'\delta'}(p_3,p_4;k)\Gamma_{\alpha\beta\gamma'\delta'}(p_1,p_2,p_3+k,p_4-k)\\
&\qquad+S^{(qa)}_{\gamma\beta;\beta'\gamma'}(p_2,p_3;k)\Gamma_{\alpha\beta'\gamma'\delta}(p_1,p_2+k,p_3-k,p_4)+S^{(qa)}_{\delta\alpha;\alpha'\delta'}(p_1,p_4;k)\Gamma_{\alpha'\beta\gamma\delta'}(p_1-k,p_2,p_3,p_4+k)\\
&\qquad+S^{(qa)}_{\gamma\alpha;\alpha'\gamma'}(p_1,p_3;k)\Gamma_{\alpha'\beta\gamma'\delta}(p_1+k,p_2,p_3-k,p_4)+S^{(qa)}_{\delta\beta;\beta'\delta'}(p_2,p_4;k)\Gamma_{\alpha\beta'\gamma\delta'}(p_1,p_2+k,p_3,p_4-k)\}\\
&\qquad-\int \text{đ}k\text{đ}q\{S^{(qq)}_{\beta\alpha;\alpha'\beta'}(p_1,p_2;k)S^{(aa)}_{\gamma\delta;\delta'\gamma'}(p_3,p_4;q)\Gamma_{\alpha'\beta'\gamma'\delta'}(p_1+k,p_2-k,p_3+q,p_4-q)\\
&\qquad+S^{(qa)}_{\gamma\alpha;\alpha'\gamma'}(p_1,p_3;k)S^{(qa)}_{\delta\beta;\beta'\delta'}(p_2,p_4;q)\Gamma_{\alpha'\beta'\gamma'\delta'}(p_1+k,p_2-q,p_3-k,p_4+q)\\
&\qquad+S^{(qa)}_{\delta\alpha;\alpha'\delta'}(p_1,p_4;k)S^{(qa)}_{\gamma\beta;\beta'\gamma'}(p_2,p_3;q)\Gamma_{\alpha'\beta'\gamma'\delta'}(p_1+k,p_2+q,p_3-q,p_4-k)\}.
\end{aligned}
\tag{3.1}
$$

The amplitudes S contain three types of kernels, corresponding to quark-quark, antiquark-antiquark and quark-antiquark pairs. As discussed in the previous section, in the limit of the heavy quark mass these kernels reduce to ladder gluon exchange:

$$
\begin{aligned}
S^{(qq)}_{\alpha\beta;\beta'\alpha'}(p_i,p_j;k)&=g^2T^a_{\alpha\alpha'}T^a_{\beta\beta'}W_{\sigma\sigma}(\vec{k})W_{\bar{q}q}(p_i^0+k_0)W_{\bar{q}q}(p_j^0-k_0)\\
S^{(aa)}_{\alpha\beta;\beta'\alpha'}(p_i,p_j;k)&=g^2T^a_{\alpha'\alpha}T^a_{\beta'\beta}W_{\sigma\sigma}(\vec{k})W^T_{q\bar{q}}(p_i^0+k_0)W^T_{q\bar{q}}(p_j^0-k_0)\\
S^{(qa)}_{\alpha\beta;\beta'\alpha'}(p_i,p_j;k)&=-g^2T^a_{\alpha\alpha'}T^a_{\beta'\beta}W_{\sigma\sigma}(\vec{k})W^T_{q\bar{q}}(p_i^0+k_0)W_{\bar{q}q}(p_j^0-k_0).
\end{aligned}
\tag{3.2}
$$

In the above, we have already replaced the temporal quark-gluon vertices by their expressions, Eqs. (2.5) and (2.8). In our convention, p_1, p_2 denote the quark momenta, p_3, p_4 the antiquark momenta, and $P_0=\sum_{i=1}^4 p_i^0$ is the pole four-momentum (total energy) of the bound tetraquark state. $\Gamma_{\alpha\beta\gamma\delta}$ represents the quark-tetraquark vertex for a particular bound state and its indices denote explicitly only its quark content. Just like the heavy quark propagator, $\Gamma_{\alpha\beta\gamma\delta}$ becomes a Dirac scalar due to the decoupling of the spin in the heavy mass limit. Similar to the homogeneous Bethe-Salpeter equation for mesons and Faddeev equation for baryons, the integral equation (3.1) depends only parametrically on the total energy P_0 (for notational convenience we have dropped the P_0 dependence from $\Gamma_{\alpha\beta\gamma\delta}$). We also note that, as in the case of meson and baryon bound states, the energy independence of the temporal gluon propagator will play a key role in the derivation of the confining potential. The Yakubovsky equation is diagrammatically shown in Fig. 1.

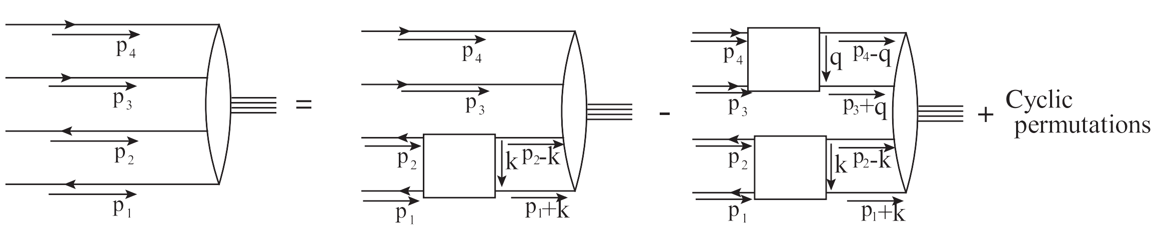

FIG. 1. Yakubovsky equation for four-quark bound states. Solid lines represent the quark propagator, and boxes represent the meson, diquark and antidiquark kernels, respectively. The ellipse depicts the Yakubovsky vertex function corresponding to the bound state represented by a quadruple-line.

Let us now investigate the energy dependence of Eq. (5.75). As shown in the previous section, the Bethe-Salpeter kernel was energy independent, and thus it was straightforward to show that the Bethe-Salpeter vertex itself did not contain an energy dependent part. This observation was then used to calculate the confining potential from the Bethe-Salpeter equation, via a simple analytical integration over the relative energy variable. Unfortunately, this approach cannot be extended to baryon states: despite the instantaneous kernel, a relative energy dependence still remains and thus one cannot assume an energy independent Faddeev vertex.

Before we specify our ansatz for the Yakubovsky vertex, let us shortly recall the energy behavior of the meson and baryon vertices. In the case of mesons it was straightforward to show that the Bethe-Salpeter vertex was energy independent, and consequently the confining potential could be calculated via integrating over the relative energy. On the other hand, the quark-baryon vertex did contain an energy component, similar in structure to the quark propagator [21]. In the case of tetraquarks, we again have a nontrivial relative energy dependence and hence we cannot employ an energy independent vertex. Instead, inspired by the structure of the Faddeev vertex, we assume that the Yakubovsky vertex also obeys a separable ansatz (recall that the dependence on the total energy is implicit):

$$\Gamma_{\alpha\beta\gamma\delta}(p_1,p_2,p_3,p_4) = \Psi_{\alpha\beta\gamma\delta}\Gamma_t(p_1^0,p_2^0,p_3^0,p_4^0)\Gamma_s(\vec{p}_1,\vec{p}_2,\vec{p}_3,\vec{p}_4). \quad (3.3)$$

Γ_t and Γ_s represent the temporal and spatial component, respectively, and $\Psi_{\alpha\beta\gamma\delta}$ denotes the color component, with α, β being quark, and γ, δ antiquark, indices. As we shall see shortly below, the energy component of the Yakubovsky vertex is essential in deriving the tetraquark confining potential.

The color structure of tetraquarks is nontrivial since a singlet can be obtained via two different representations [13]. Interestingly, diquark and antidiquark pairs also contribute to the color singlet tetraquark state, although themselves cannot exist as color singlets (at least for $N=3$ colors). By writing the color function Ψ as

$$\Psi_{\alpha\beta\gamma\delta} = \delta_{\alpha\delta}\delta_{\beta\gamma} + \delta_{\alpha\gamma}\delta_{\beta\delta}, \quad (3.4)$$

and with the Fierz identity for the generators,

$$2[T^a]_{\alpha\beta}[T^a]_{\delta\gamma} = \delta_{\alpha\gamma}\delta_{\delta\beta} - \frac{1}{N}\delta_{\alpha\beta}\delta_{\delta\gamma}, \quad (3.5)$$

we calculate the color factors corresponding to various channels:

$$\begin{aligned}
T^a_{\alpha\alpha'}T^a_{\beta\beta'}\Psi_{\alpha'\beta'\gamma\delta} &= \frac{1}{2}\left(1-\frac{1}{N}\right)\Psi_{\alpha\beta\gamma\delta} \quad \text{(diquark and antidiquark channel)}\\
T^a_{\alpha\alpha'}T^a_{\gamma'\gamma}\Psi_{\alpha'\beta\gamma'\delta} &= \frac{1}{2}\left(1+N-\frac{2}{N}\right)\Psi_{\alpha\beta\gamma\delta} \quad \text{(meson channel)}\\
T^a_{\alpha\alpha'}T^a_{\beta\beta'}T^b_{\delta'\delta}T^b_{\gamma'\gamma}\Psi_{\alpha'\beta'\gamma'\delta'} &= \frac{1}{4}\left(1-\frac{2}{N}+\frac{1}{N^2}\right)\Psi_{\alpha\beta\gamma\delta} \quad \text{(diquark- antidiquark channel)}\\
T^a_{\alpha\alpha'}T^a_{\delta'\delta}T^b_{\beta\beta'}T^b_{\gamma'\gamma}\Psi_{\alpha'\beta'\gamma'\delta'} &= \frac{1}{4}\left(N^2+N-2-\frac{2}{N}+\frac{2}{N^2}\right)\Psi_{\alpha\beta\gamma\delta} \quad \text{(meson-meson channel)}. \quad (3.6)
\end{aligned}$$

Inspecting Eq. (3.1), we notice that the energy and three-momentum integrations separate since the temporal gluon propagator is energy independent, whereas the heavy quark propagator is independent of the spatial momentum. Fourier transforming the spatial part of the vertex

$$\Gamma_s(\vec{p}_1,\vec{p}_2,\vec{p}_3,\vec{p}_4) = \int đ\vec{x}_1 đ\vec{x}_2 đ\vec{x}_3 đ\vec{x}_4 \exp\left(-i\sum_{i=1}^4 \vec{p}_i\cdot\vec{x}_i\right) \times \Gamma_s(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4), \quad (3.7)$$

we find that the convolution product with the temporal gluon propagator is given by

$$\begin{aligned}
&\int đ\vec{k} W_{\sigma\sigma}(\vec{k})\Gamma_s(\vec{p}_1+\vec{k},\vec{p}_2-\vec{k},\vec{p}_3,\vec{p}_4)\\
&= \int đ\vec{x}_1 đ\vec{x}_2 đ\vec{x}_3 đ\vec{x}_4 \exp\left(-i\sum_{i=1}^4 \vec{p}_i\cdot\vec{x}_i\right)\\
&\quad\times W_{\sigma\sigma}(\vec{x}_2-\vec{x}_1)\Gamma_s(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4). \quad (3.8)
\end{aligned}$$

It is straightforward to show that the above structure is preserved also in the convolution product with two gluon propagators, and hence the spatial component of the vertex completely drops from the calculation. Motivated by the symmetry of the system, we further restrict to the case of equal quark separations: $|\vec{r}| = |\vec{x}_i - \vec{x}_j|$ $(i,j=\overline{1,4}; i>j)$. The original equation can be recast into an equation for the temporal component Γ_t:

$$
\begin{aligned}
&\Gamma_t(p_1^0,p_2^0,p_3^0,p_4^0) \\
&= g^2 W_{\sigma\sigma}(\vec{r}) \int đk \bigg\{ \frac{1}{3}[W_{\bar{q}q}(p_1^0+k_0)W_{\bar{q}q}(p_2^0-k_0)\Gamma_t(p_1^0+k_0,p_2^0-k_0,p_3^0,p_4^0)+(3,4)] \\
&\quad -\frac{5}{6}[W^T_{\bar{q}q}(p_3^0-k_0)W_{\bar{q}q}(p_2^0+k_0)\Gamma_t(p_1^0,p_2^0+k_0,p_3^0-k_0,p_4^0)+(1,4)+(2,4)+(1,3)] \bigg\} \\
&\quad -\left(g^2 W_{\sigma\sigma}(\vec{r})\right)^2 \int đkđq \bigg\{ \frac{1}{9} W_{\bar{q}q}(p_1^0+k_0)W_{\bar{q}q}(p_2^0-k_0)W^T_{\bar{q}q}(p_3^0+q_0)W^T_{\bar{q}q}(p_4^0-q_0)\Gamma_t(p_1^0+k_0,p_2^0-k_0,p_3^0+q_0,p_4^0-q_0) \\
&\quad +\frac{43}{18}[W_{\bar{q}q}(p_1^0+k_0)W^T_{\bar{q}q}(p_3^0-k_0)W_{\bar{q}q}(p_2^0+q_0)W^T_{\bar{q}q}(p_4^0-q_0)\Gamma_t(p_1^0+k_0,p_2^0-q_0,p_3^0-k_0,p_4^0+q_0)+(1,4;2,3)] \bigg\},
\end{aligned}
\tag{3.9}
$$

where (i,j) represent the terms attached to the corresponding pairs of (anti)quarks, and can be explicitly read off from Eqs. (3.1) and (3.2).

Now, in order to identify the structure of the solution, it is useful to rewrite the energy integral as follows:

$$
\begin{aligned}
&\int đk_0 W_{\bar{q}q}(\tilde{p}_2^0-k_0-m)W_{\bar{q}q}(\tilde{p}_1^0+k_0-m)\tilde{\Gamma}_t(\tilde{p}_1^0+k_0,\tilde{p}_2^0-k_0,\tilde{p}_3^0,\tilde{p}_4^0) \\
&= -\frac{2}{\tilde{p}_1^0+\tilde{p}_2^0-2\mathcal{I}_r+i\varepsilon}\int đk_0 \frac{\Gamma_t(\tilde{p}_1^0+\tilde{p}_2^0+k_0,-k_0,\tilde{p}_3^0,\tilde{p}_4^0)}{\tilde{p}_1^0+\tilde{p}_2^0+k_0-\mathcal{I}_r+i\varepsilon},
\end{aligned}
\tag{3.10}
$$

where we have introduced the shifted momenta $\tilde{p}_{1,2}^0 = p_{1,2}^0 + m$ for notational convenience. The integration over antiquark propagators leads to an identical formula, except that the mass term has the opposite sign (in this case, $\tilde{p}_{3,4}^0 = p_{3,4}^0 - m$), whereas in the integral over a quark and an antiquark, the mass term completely vanishes. The double integrals can straightforwardly be rewritten in a similar form. Without loss of generality, we can further restrict to equal energies, i.e., $\tilde{p}_i^0 = P_0/4$. Inspired by the energy integral, Eq. (3.10), and noticing the similarities with our previous three-body calculation [21], we make the following ansatz for the tetraquark vertex:

$$
\Gamma_t(\tilde{p}_1^0,\tilde{p}_2^0,\tilde{p}_3^0,\tilde{p}_4^0) = \sum_{\substack{i,j=1,4\\ i<j}} \frac{1}{\tilde{p}_i^0+\tilde{p}_j^0-2\mathcal{I}_r-A(P_0,\mathcal{I}_r)+i\varepsilon},
\tag{3.11}
$$

where $A(P_0,\mathcal{I}_r)$ is a function that needs to be determined. For equal energies, the ansatz takes the simpler form:

$$
\Gamma_t(\tilde{p}_1^0,\tilde{p}_2^0,\tilde{p}_3^0,\tilde{p}_4^0)|_{\tilde{p}_i^0=\frac{P_0}{4}} = \frac{12}{P_0-4\mathcal{I}_r-2A(P_0,\mathcal{I}_r)+i\varepsilon}.
\tag{3.12}
$$

Plugging this back into Eq. (3.9) and using the result, Eq. (2.6), we are left with an algebraic equation for the function $A(P_0,\mathcal{I}_r)$. As has been emphasized in [20], there are only two possibilities for the bound state energy once all regulators are removed: either it is finite and linear rising with distance (i.e., a confined state), or it is infinite and therefore unphysical. Since we are searching for a confining solution for our tetraquark, the following condition has to be satisfied:

$$
P_0 - 4\mathcal{I}_r = 2C_F i g^2 W_{\sigma\sigma}(\vec{r}).
\tag{3.13}
$$

This condition essentially requires that the Fourier transform integral is convergent, such that the divergences contained within $\mathcal{I}_r$ cancel exactly and the bound state energy remains finite:

$$
\int đ\omega \frac{1}{\vec{\omega}^4}(1-e^{-i\vec{\omega}\vec{r}}) = \frac{|\vec{r}|}{8\pi}.
\tag{3.14}
$$

Replacing $\mathcal{I}_r$ and $W_{\sigma\sigma}$ by their expressions, Eq. (2.3) and Eq. (2.4), respectively, and the gluon dressing function with $D_{\sigma\sigma} = X/\vec{\omega}^2$, we can rewrite Eq. (3.13) as

$$
P_0 \equiv \sigma_{4q}|\vec{r}| = \frac{g^2 C_F X}{4\pi}|\vec{r}|.
\tag{3.15}
$$

Inserting the ansatz (3.12) into Eq. (3.9) we find, after a laborious but fairly straightforward calculation,

$$
A(P_0,\mathcal{I}_r) = \frac{5}{4C_F}(P_0-4\mathcal{I}_r),
\tag{3.16}
$$

which gives for the temporal component of the tetraquark vertex function

$$\Gamma_t(\tilde{p}_1^0,\tilde{p}_2^0,\tilde{p}_3^0,\tilde{p}_4^0)=\sum_{\substack{i,j=1,4\\ i<j}}\frac{1}{\tilde{p}_i^0+\tilde{p}_j^0-\frac{15}{16}P_0+\frac{7}{4}\mathcal{I}_r+i\varepsilon}. \tag{3.17}$$

These results are inline with our previous findings for $\bar{q}q$ and $3q$ systems. From Eq. (3.15) we find that the string tension σ_{4q} corresponding to a tetraquark state, i.e., the coefficient of the four-body linear confining term, is two times bigger than the one of the $\bar{q}q$ system calculated in Ref. [20]:

$$\sigma_{4q}=\frac{g^2C_FX}{4\pi}=2\sigma_{\bar{q}q}. \tag{3.18}$$

For comparison, the string tension for three quark states has the value $\sigma_{3q}=\frac{3}{2}\sigma_{\bar{q}q}$. Just like in the case of meson and baryon states, our results show that in the Coulomb gauge and at the leading order in the mass expansion there is a direct connection between the string tension and the nonperturbative Yang-Mills sector of the theory, at least under the truncation considered here. Notice also that the total mass has disappeared (similar to mesons), since the two quarks and two antiquarks move with opposite (and equal) velocities such that the center of mass is stationary. In fact this is related to our original specification for the Feynman prescription—recall that we have assigned the reversed sign for antiquarks, which corresponds to a particle that moves with opposite velocity. For comparison, in the case of baryons, where the three quarks move in the same direction with equal velocities, the total bound state energy contains three times the quark mass.

IV. SUMMARY AND CONCLUSIONS

In this paper we have derived the four-quark confinement potential in the heavy quark limit of the Coulomb gauge QCD. To this end, we have solved the Yakubovsky equation for tetraquark states in a symmetric configuration and for equal quark energies. We have expanded the QCD action by using a method adapted from HQET, and restricted to the leading order. Further, we have truncated the system such that only nonperturbative propagators of the Yang-Mills sector are included, and all pure Yang-Mills vertices and higher order functions are neglected.

As in the meson and baryon cases, a direct connection between the physical string tension and the Yang-Mills sector of the Coulomb gauge QCD (the temporal gluon propagator) has been established. A bound state energy that raises linearly with the distance has been derived, and the coefficient of the linearly rising term is found to be two times that of a meson system. Since only symmetric configurations have been considered, no statement can be made regarding the shape of the string that confines the quarks. However, the restriction to equal energies does not alter the validity of our statements—clearly the confining potential should hold for any configuration, including that of equal energies.

A possible extension of this work is to include the next order in the mass expansion, and analyze the contribution of the spatial gluon propagator which so far has been neglected. A different line of research is the inclusion of vertex corrections—this should trigger the phenomenon of charge screening which is expected to alter the value of the string tension. Finally, our result serves as a basis for phenomenological descriptions of heavy tetraquarks in terms of potentials.

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