

**SUSY models under siege: LHC constraints and electroweak fine-tuning**Howard Baer,<sup>1,\*</sup> Vernon Barger,<sup>2,†</sup> Dan Mickelson,<sup>1,‡</sup> and Maren Padeffke-Kirkland<sup>1,§</sup><sup>1</sup>*Department of Physics and Astronomy, University of Oklahoma, Norman, Oklahoma 73019, USA*<sup>2</sup>*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA*

(Received 14 April 2014; published 23 June 2014)

Recent null results from LHC8 supersymmetry (SUSY) searches along with the discovery of a standard model (SM)-like Higgs boson with mass  $m_h \approx 125.5$  GeV indicates sparticle masses in the TeV range, causing tension with conventional measures of electroweak fine-tuning. We propose a simple *fine-tuning rule* which should be followed under any credible evaluation of fine-tuning. We believe that overestimates of electroweak fine-tuning by conventional measures all arise from violations of this rule. We show that to gain accord with the fine-tuning rule, then both the Higgs mass and the traditional  $\Delta_{\text{BG}}$  fine-tuning measures reduce to the model-independent electroweak fine-tuning measure  $\Delta_{\text{EW}}$ . This occurs by combining dependent contributions to  $m_Z$  or  $m_h$  into independent units. Then, using  $\Delta_{\text{EW}}$ , we evaluate EW fine-tuning for a variety of SUSY models including mSUGRA, NUHM1, NUHM2, mGMSB, mAMSB, hyper-charged AMSB, gaugino AMSB and nine cases of mixed moduli-anomaly (mirage) mediated SUSY breaking models while respecting LHC Higgs mass and  $B$ -decay constraints (we do not impose LHC8 sparticle mass constraints due to the possibility of compressed spectra within many of these models). We find mSUGRA, mGMSB and the AMSB models all to be highly fine-tuned. The NUHM1 model is moderately fine-tuned while NUHM2 which allows for radiatively driven naturalness (RNS) allows for fine-tuning at a meager 10% level in the case where  $m(\text{Higgsinos}) \sim 100\text{--}200$  GeV and the TeV-scale top squarks are well mixed. Models with RNS may or may not be detectable at LHC14. A  $\sqrt{s} \sim 500$  GeV  $e^+e^-$  collider will be required to make a definitive search for the requisite light Higgsinos.

DOI: 10.1103/PhysRevD.89.115019

PACS numbers: 12.60.Jv, 14.80.Ly

**I. INTRODUCTION**

It has long been claimed that electroweak naturalness requires that the superpartners of the standard model (SM) fields exist with masses of order the weak scale [1–42]  $m(\text{sparticles}) \sim m_{\text{weak}} \sim m_Z$ . Already at LEP2, the lack of signal for chargino pairs called into question whether there might exist a “little hierarchy problem” [43] characterized by  $m(\text{sparticle}) \gg m_Z$ . This viewpoint has seemingly been strengthened by

- (i) the lack of any signal for sparticles at LHC8 [44,45] which requires  $m_{\tilde{g}} \gtrsim 1.8$  TeV for models with  $m_{\tilde{q}} \sim m_{\tilde{g}}$  and  $m_{\tilde{q}} \gtrsim 1.3$  TeV for models with  $m_{\tilde{q}} \gg m_{\tilde{g}}$  and
- (ii) the rather large value of  $m_h \approx 125.5$  GeV [46,47] which requires multi-TeV top squarks with small mixing or TeV-scale top squarks with large mixing [48–51].

If indeed weak scale supersymmetry (SUSY) is highly fine-tuned in the electroweak sector, then likely SUSY is not as we know it since the twin requirements of parsimony and naturalness cannot be met simultaneously [52]. Abandoning parsimony is not a step lightly taken since the further one

strays from known physics the more likely one is to be wrong. But before jumping to such strong conclusions—which may well guide support for and construction of future HEP experimental facilities—it is worthwhile to scrutinize the available measures of electroweak fine-tuning (EWFT) in SUSY models. Indeed, in a recent paper we have claimed that *conventional measures tend to overestimate EWFT in supersymmetric models*, often by several orders of magnitude [38].

To ascertain when a claim of fine-tuning is legitimate, we propose a simple *fine-tuning rule* which may act as a guide:

*When evaluating fine-tuning, it is not permissible to claim fine-tuning of dependent quantities one against another.*

We believe the overestimates of EWFT by conventional measures referred to above all come from violations of this rule.

To be explicit, most theories contain several, perhaps many, parameters. Some of these may be set equal to measured values, while others may be undetermined or at least constrained, but may vary over a wide range of values. The parameters are frequently introduced to parametrize our ignorance of more fundamental physics, and their variation allows one to encompass a wide range of possibilities. We can think of each parameter as a dial, capable of being adjusted to a specific, or alternatively a

\*baer@nhn.ou.edu

†barger@pheno.wisc.edu

‡dmickelso@nhn.ou.edu

§m.padeffke@ou.edu

wide, range of values. If some contribution to a measured quantity (e.g.  $m_h^2$  or  $m_Z^2$  in this paper) in a theory blows up, and we have an adjustable parameter which may be dialed independently to compensate, then we may legitimately evaluate fine-tuning: is a huge, unnatural cancellation required? Alternatively, if as a consequence of one contribution blowing up, a related dial/parameter is driven to large, opposite-sign compensating values, then any claimed fine-tuning would violate our rule (the quantities would be *dependent*) and some regrouping of terms into independent quantities should be found. We will meet some clarifying examples in the subsequent sections of this paper.

### A. Simple electroweak fine-tuning

In most supersymmetric models based on high scale input parameters—i.e. SUSY models with soft term boundary conditions imposed at a scale  $\Lambda \gg m_{\text{weak}}$  where  $\Lambda$  may range as high as  $m_{\text{GUT}} \approx 2 \times 10^{16}$  GeV or even the reduced Planck mass  $M_P \approx 2 \times 10^{18}$  GeV—the soft SUSY breaking terms are input at the scale  $\Lambda$  and then evolved to the electroweak scale  $m_{\text{weak}}$  via renormalization group (RG) running.<sup>1</sup> At the weak scale, the scalar potential is minimized and checked to ensure that EW symmetry is properly broken. The value of  $\mu$  is then fixed in terms of the weak scale soft SUSY breaking terms  $m_{H_u}^2$  and  $m_{H_d}^2$  by requiring that the measured value of  $m_Z \approx 91.2$  GeV is obtained,

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \approx -m_{H_u}^2 - \Sigma_u^u - \mu^2, \quad (1)$$

where  $\Sigma_u^u$  and  $\Sigma_d^d$  are radiative corrections that arise from the derivatives of  $\Delta V$  evaluated at the minimum. The radiative corrections  $\Sigma_u^u$  and  $\Sigma_d^d$  include contributions from various particles and sparticles with sizable Yukawa and/or gauge couplings to the Higgs sector. Expressions for the  $\Sigma_u^u$  and  $\Sigma_d^d$  are given in the Appendix of Ref. [37].

Already at this point: if  $-m_{H_u}^2$  (weak) in the right-hand side of Eq. (1) is large positive ( $\gg m_Z^2$ ), then the value of  $\mu$  must be fine-tuned by hand to ensure the measured value of  $m_Z^2$  is obtained. Since most researchers these days run automated computer codes [53] to calculate the weak scale spectrum of SUSY and Higgs particles, this represents a *hidden* fine-tuning that ought to be accounted for.

Alternatively, if soft SUSY breaking terms *and*  $\mu$  are input parameters, then much higher values of  $m_Z \gg 91.2$  GeV are expected from scans over the SUSY model parameter space. For example, in Fig. 1 we plot the value of  $m_Z$  which is generated from a scan over phenomenological

<sup>1</sup>Here we differentiate the superpotential Higgs/Higgsino mass term  $\mu$  from the soft breaking terms, as do most model builders, and we return to the SUSY  $\mu$  problem later.

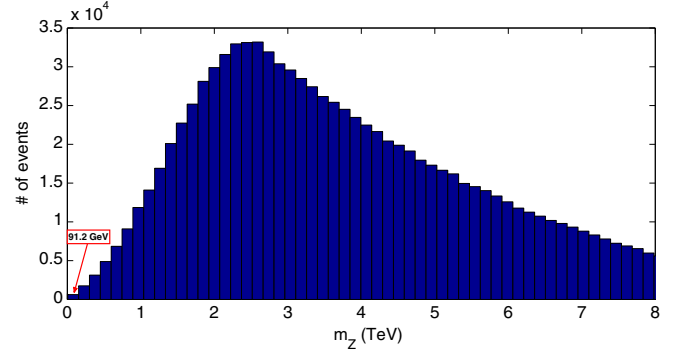


FIG. 1 (color online). Plot of value of  $m_Z$  generated from a scan over pMSSM model parameter space while *not* implementing the  $m_Z^2$  constraint.

MSSM (pMSSM) parameter space [54].<sup>2</sup> The 20-dimensional pMSSM parameter space then includes

$$M_1, M_2, M_3, \quad (2)$$

$$m_{Q_1}, m_{U_1}, m_{D_1}, m_{L_1}, m_{E_1}, \quad (3)$$

$$m_{Q_3}, m_{U_3}, m_{D_3}, m_{L_3}, m_{E_3}, \quad (4)$$

$$A_t, A_b, A_\tau, \quad (5)$$

$$m_{H_u}^2, m_{H_d}^2, \mu, B. \quad (6)$$

The usual strategy is to use the EW minimization conditions [55] to trade the bilinear parameter  $B$  for the ratio of Higgs vacuum expectation values  $\tan\beta \equiv v_u/v_d$  and to exchange  $m_{H_u}^2$  and  $m_{H_d}^2$  for  $m_Z^2$  and  $m_A^2$  [55]. This procedure reduces the number of free parameters to 19 (since  $m_Z$  is fixed) but hides the fine-tuning embedded in Eq. (1) since now  $m_{H_u}^2$  is an output.

Here we will avoid the  $m_Z^2$  constraint and scan over the 20-dimensional pMSSM space for the range of scalar and gaugino mass soft terms from 0–10 TeV,  $-10$  TeV  $< A_i < 10$  TeV,  $\mu$ : 0–3 TeV and  $\tan\beta$ : 3–60, while requiring the lightest neutralino  $\tilde{Z}_1$  as the lightest SUSY particle (LSP) and  $m_{\tilde{W}_1} > 103.5$  GeV (in accord with LEP2 constraints).<sup>3</sup> Our results are shown in Fig. 1. Here, we see that the most probable value of  $m_Z$  is  $\sim 2.5$  TeV with a large spread to both higher and lower values. It is highly unlikely to generate the measured value  $m_Z = 91.2$  GeV: this is the essence of the little hierarchy problem.

Alternatively, the fact that  $m_Z = 91.2$  GeV along with  $m_h \approx 125.5$  GeV tells us from Eq. (1) that to naturally generate the measured value of  $m_Z$  (and  $M_W$ ) and  $m_h$ , then

<sup>2</sup>The pMSSM, or phenomenological MSSM, is the MSSM defined with weak scale input parameters where all  $CP$  violating and flavor violating soft terms have been set to zero. Also, usually first/second generation soft terms are set equal to each other to avoid flavor violations.

<sup>3</sup>This limit diminishes to  $\sim 91.9$  GeV in the case of a winolike weakly interacting massive particle (WIMP).

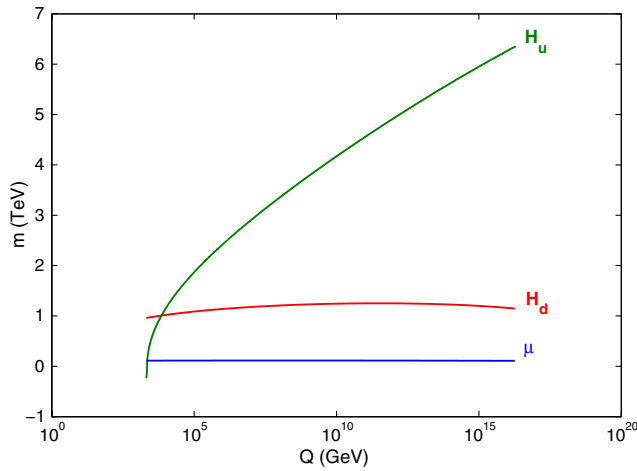


FIG. 2 (color online). Renormalization group evolution of  $\text{sign}(m_{H_u}^2)\sqrt{|m_{H_u}^2|}$ ,  $\sqrt{m_{H_d}^2}$  and  $\mu$  versus energy scale  $Q$  for the radiatively driven naturalness benchmark point from Ref. [56]. The value  $m_A = 1 \text{ TeV} \approx m_{H_d}(\text{weak})$  and  $\mu(\text{weak}) = 110 \text{ GeV}$ .

- (i)  $|\mu| \sim m_Z \sim 100\text{--}200 \text{ GeV}$ ,
- (ii)  $m_{H_u}^2$  should be driven to small negative values such that  $-m_{H_u}^2 \sim 100\text{--}200 \text{ GeV}$  at the weak scale, and
- (iii) the radiative corrections are not too large:  $\Sigma_u^u \lesssim 100\text{--}200 \text{ GeV}$ .

The first two of these conditions are shown in Fig. 2 as soft term and  $\mu$  RG running versus  $Q$  for a radiatively driven natural SUSY benchmark point from Ref. [56] where  $\mu = 110 \text{ GeV}$  and  $\Delta_{\text{EW}} = 16$ .

Formally, these conditions arise from requiring the *electroweak fine-tuning measure*  $\Delta_{\text{EW}}$  to be not too large, where

$$\Delta_{\text{EW}} \equiv \max_i |C_i| / (m_Z^2/2) \quad (7)$$

may be constructed with  $C_{H_d} = m_{H_d}^2 / (\tan^2 \beta - 1)$ ,  $C_{H_u} = -m_{H_u}^2 \tan^2 \beta / (\tan^2 \beta - 1)$  and  $C_\mu = -\mu^2$ . Also,  $C_{\Sigma_u^{(k)}} = -\Sigma_u^{(k)} \tan^2 \beta / (\tan^2 \beta - 1)$  and  $C_{\Sigma_d^{(k)}} = \Sigma_d^{(k)} / (\tan^2 \beta - 1)$ , where  $k$  labels the various loop contributions included in Eq. (1).

The largest of the radiative corrections comes from the top squark sector  $\Sigma_u^u(\tilde{t}_{1,2})$ . These radiative corrections can be minimized for large stop mixing from a large trilinear  $A_t$  parameter, which also raises up the value of  $m_h$  to the 125 GeV regime for top squark masses in the 1–4 TeV range [36].

An advantage of  $\Delta_{\text{EW}}$  is that it is model-independent in the sense that any model which yields the same weak scale mass spectrum will generate the same value of  $\Delta_{\text{EW}}$ .

## B. Fine-tuning of the Higgs mass

### 1. SM case

An alternative measure of EWFT is to require that the (regularized) divergent radiative corrections  $\delta m_h^2$  to the squared Higgs mass  $m_h^2$  are not too large: say  $\delta m_h^2 \lesssim m_h^2$ .

In the SM we have

$$m_{H_{\text{SM}}}^2 = 2\mu^2 + \delta m_{H_{\text{SM}}}^2, \quad (8)$$

where the tree-level squared mass  $2\mu^2$  and the quadratically divergent radiative corrections

$$\delta m_{H_{\text{SM}}}^2 \approx \frac{3}{4\pi^2} \left( -\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_W} + \lambda \right) \Lambda^2 \quad (9)$$

are *independent* (here,  $\lambda_t$  is the SM top Yukawa coupling,  $g$  is the  $SU(2)_L$  gauge coupling,  $\lambda$  is the SM Higgs quartic coupling and  $\Lambda$  is the effective theory energy cutoff scale). Thus, by the EWFT rule, this is a legitimate fine-tuning evaluation. For large  $\Lambda$ , the large radiative corrections must be balanced by a fine-tuning of  $2\mu^2$  such that  $m_{H_{\text{SM}}}^2$  maintains its physical value. Alternatively, to maintain naturalness, then  $\delta m_{H_{\text{SM}}}^2 \sim m_{H_{\text{SM}}}^2$  which requires  $\Lambda \lesssim 1 \text{ TeV}$ ; i.e. the SM is only valid below about the  $\Lambda \sim 1 \text{ TeV}$  scale.

### 2. MSSM case

In the MSSM, it is found that

$$m_h^2 \approx \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2, \quad (10)$$

where now  $\mu^2$  is the *supersymmetric* Higgs/Higgsino bilinear term which gives mass to both SM particles (the gauge and Higgs bosons) and the SUSY partner Higgsinos. In addition,  $m_{H_u}^2$  is the soft SUSY breaking (SSB) up-Higgs mass term. If we assume the MSSM is valid up to the grand unified theory (GUT) scale, then the value of  $\delta m_{H_u}^2$  can be found by integrating the renormalization group equation [57],

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2} \left( -\frac{3}{5} g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10} g_1^2 S + 3f_t^2 X_t \right), \quad (11)$$

where  $t = \ln(Q^2/Q_0^2)$ ,  $S = m_{H_u}^2 - m_{H_d}^2 + \text{Tr}[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 + \mathbf{m}_E^2]$  and where  $X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$ . By neglecting gauge terms and  $S$  ( $S = 0$  in models with scalar soft term universality but can be large in models with nonuniversality), and also neglecting the  $m_{H_u}^2$  contribution to  $X_t$  and the fact that  $f_t$  and the soft terms evolve under  $Q^2$  variation, then this expression may be readily integrated from  $m_{\text{SUSY}}$  to the cutoff  $\Lambda$  to obtain

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2} (m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln(\Lambda^2/m_{\text{SUSY}}^2). \quad (12)$$

Here,  $\Lambda$  may be taken as high as  $m_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV}$  or even the reduced Planck mass  $m_P \approx 2.4 \times 10^{18} \text{ GeV}$ . Also, we take  $m_{\text{SUSY}}^2 \approx m_{\tilde{t}_1} m_{\tilde{t}_2}$ . By requiring [58–61]  $\Delta_{\text{HS}} \sim \delta m_{H_u}^2 / (m_h^2/2) \lesssim 10$  one then expects  $m_{\tilde{t}_{1,2}, \tilde{b}_1} \lesssim 600 \text{ GeV}$ . Using the  $\Delta_{\text{HS}}$  measure along with  $m_h \approx 125 \text{ GeV}$  then

one finds some popular SUSY models to be electroweak fine-tuned to 0.1% [38].

Two pitfalls occur within this approach, which are *different* from the case of the SM.

- (i) The first is that  $m_{H_u}^2(\Lambda)$  and  $\delta m_{H_u}^2$  are *not* independent: the value of  $m_{H_u}^2$  feeds directly into the evaluation of  $\delta m_{H_u}^2$  via the  $X_t$  term. It also feeds indirectly into  $\delta m_{H_u}^2$  by contributing to the evolution of the  $m_{Q_3}^2$  and  $m_{U_3}^2$  terms. In fact, the larger the value of  $m_{H_u}^2(\Lambda)$ , then the larger is the canceling correction  $\delta m_{H_u}^2$ . Thus, this fine-tuning measure fails under the fine-tuning rule.
- (ii) The second is that whereas  $SU(2)_L \times U(1)_Y$  gauge symmetry can be broken at tree level in the SM, in the SUGRA case where SUSY is broken in a hidden sector via the superHiggs mechanism then  $m_{H_u}^2 \sim m_{3/2}^2 > 0$  and EW symmetry is not even broken until one includes radiative corrections. For SUSY models valid up to some high scale  $\Lambda \gg m_{\text{weak}}$ , EW symmetry is broken radiatively by  $m_{H_u}^2$  being driven to large negative values by the large top quark Yukawa coupling [62].

By combining dependent terms, then we have a regrouping [36,37]

$$m_h^2|_{\text{phys}} = \mu^2 + (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2), \quad (13)$$

where now  $\mu^2$  and  $(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2)$  are each independent so each should be comparable to  $m_h^2$  in order to avoid

$$\begin{aligned} m_Z^2 \approx & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & - 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 \\ & + 0.053m_{E_3}^2 + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 \\ & - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned} \quad (15)$$

where all terms on the right-hand side are understood to be GUT scale parameters.

Then, the proposal is that the variation in  $m_Z^2$  with respect to parameter variation should be small,

$$\Delta_{\text{BG}} \equiv \max_i [c_i] \quad \text{where } c_i = \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i} \right| = \left| \frac{a_i}{m_Z^2} \frac{\partial m_Z^2}{\partial a_i} \right|, \quad (16)$$

where the  $a_i$  constitute the fundamental parameters of the model. Thus,  $\Delta_{\text{BG}}$  measures the fractional change in  $m_Z^2$  due to fractional variation in high scale parameters  $a_i$ . The  $c_i$  are known as *sensitivity coefficients* [35].

The requirement of low  $\Delta_{\text{BG}}$  is then equivalent to the requirement of no large cancellations on the right-hand side of Eq. (15) since (for linear terms) the logarithmic derivative just picks off coefficients of the relevant parameter. For instance,  $c_{m_{Q_3}^2} = 0.73(m_{Q_3}^2/m_Z^2)$ . If one allows

fine-tuning. It is often claimed that under such a regrouping, then the SM Higgs mass would also not be fine-tuned. But here we see that in the MSSM case—since the  $m_{H_u}^2$  and  $\delta m_{H_u}^2$  terms are dependent—the situation is different from the SM and one must lump dependent terms together. The regrouping in Eq. (13) of contributions to  $m_h^2$  into independent terms leads back to the  $\Delta_{\text{EW}}$  measure.

### C. $\Delta_{\text{BG}}$ and model dependence

The more traditional measure  $\Delta_{\text{BG}}$  was proposed by Ellis *et al.* [1] and later investigated more thoroughly by Barbieri and Giudice [2]. The starting point is to express  $m_Z^2$  in terms of weak scale SUSY parameters as in Eq. (1),

$$m_Z^2 \approx -2m_{H_u}^2 - 2\mu^2, \quad (14)$$

where the partial equality obtains for moderate-to-large  $\tan\beta$  values and where we assume for now the radiative corrections are small. An advantage of  $\Delta_{\text{BG}}$  over the previous large-log measure is that it maintains the correlation between  $m_{H_u}^2(\Lambda)$  and  $\delta m_{H_u}^2$  by replacing  $m_{H_u}^2(m_{\text{weak}}) = (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2)$  by its expression in terms of high scale parameters. To evaluate  $\Delta_{\text{BG}}$ , one needs to know the explicit dependence of  $m_{H_u}^2$  and  $\mu^2$  on the fundamental parameters. Semianalytic solutions to the one-loop renormalization group equations for  $m_{H_u}^2$  and  $\mu^2$  can be found for instance in Refs. [63]. For the case of  $\tan\beta = 10$ , then [35,64,65]

$m_{Q_3} \sim 3$  TeV (in accord with requirements from the measured value of  $m_h$ ), then one obtains  $c_{m_{Q_3}^2} \sim 800$  and so  $\Delta_{\text{BG}} \geq 800$ . In this case, SUSY would be electroweak fine-tuned to about 0.1%. If instead one sets  $m_{Q_3} = m_{U_3} = m_{H_u} \equiv m_0$  as in models with scalar mass universality, then the various scalar contributions to  $m_Z^2$  largely cancel and  $c_{m_0^2} \sim -0.017m_0^2/m_Z^2$ : the contribution to  $\Delta_{\text{BG}}$  from scalars drops by a factor of  $\sim 50$ .

The above argument illustrates the extreme model dependence of  $\Delta_{\text{BG}}$  for multiparameter SUSY models. The value of  $\Delta_{\text{BG}}$  can change radically from theory to theory even if those theories generate exactly the same weak scale sparticle mass spectrum. The model dependence of  $\Delta_{\text{BG}}$  arises due to a violation of the fine-tuning rule: one must combine dependent terms into independent quantities before evaluating EW fine-tuning.



### D. When is $\Delta_{\text{BG}}$ a reliable measure of naturalness?

In Ref. [38], it was argued that in an ultimate theory, where all soft parameters are correlated, then  $\Delta_{\text{BG}}$  should be a reliable measure of naturalness. In fact, most supersymmetric theories with SUSY breaking generated in a hidden sector fulfill this requirement. For instance, in supergravity theories with hidden sector SUSY breaking via the superHiggs mechanism, then all soft breaking parameters are expected to be some multiple of the gravitino mass  $m_{3/2}$ . (For example, in string theory with dilaton-dominated SUSY breaking [66,67], then we expect  $m_0^2 = m_{3/2}^2$  with  $m_{1/2} = -A_0 = \sqrt{3}m_{3/2}$ .) For any fully specified hidden sector, we expect each SSB term to be some multiple of  $m_{3/2}$ : e.g.

$$m_{H_u}^2 = a_{H_u} m_{3/2}^2, \quad (17)$$

$$m_{Q_3}^2 = a_{Q_3} m_{3/2}^2, \quad (18)$$

$$A_t = a_{A_t} m_{3/2}, \quad (19)$$

$$M_i = a_i m_{3/2}, \quad (20)$$

$$\dots \quad (21)$$

Here, the coefficients  $a_i$  *parametrize our ignorance* of the exact model for SUSY breaking. By using several adjustable parameters, we cast a wide net which encompasses a large range of hidden sector SUSY breaking possibilities. But this does not mean that each SSB parameter is expected to be independent of the others.<sup>4</sup> It just means we do not know how SUSY breaking occurs, and how the soft terms are correlated: it is important not to confuse parameters which ought to be related to one another in any sensible theory of SUSY breaking with independently adjustable soft SUSY breaking terms.

Now, plugging the soft terms (17)–(21) into Eq. (15), one arrives at the expression

$$m_Z^2 = -2.18\mu^2 + a m_{3/2}^2. \quad (22)$$

The value of  $a$  is just some number which is the sum of all the coefficients of the terms  $\propto m_{3/2}^2$ . For now, we assume  $\mu$  is independent of  $m_{3/2}$  as will be discussed shortly.

In this case, we can compute the sensitivity coefficients<sup>5</sup>:

<sup>4</sup>As emphasized by Soni and Weldon in 1983, the soft SUSY breaking terms in SUGRA models are always scaled by  $m_{3/2}$  [68].

<sup>5</sup>In mAMSB, the soft terms are also written as multiples of  $m_{3/2}$  or  $m_{3/2}^2$ . In mGMSB, the soft terms are written as multiples of messenger scale  $\Lambda_m$ . The argument proceeds in an identical fashion in these cases.

$$c_{m_{3/2}^2} = |a (m_{3/2}^2/m_Z^2)| \text{ and} \quad (23)$$

$$c_{\mu^2} = |-2.18(\mu^2/m_Z^2)|. \quad (24)$$

For  $\Delta_{\text{BG}}$  to be  $\sim 1$ – $10$  (natural SUSY with low fine-tuning), then Eq. (24) implies

$$(i) \mu^2 \sim m_Z^2$$

and also Eq. (23) implies

$$(ii) a \cdot m_{3/2}^2 \sim m_Z^2.$$

The first of these conditions implies light Higgsinos with mass  $\sim 100$ – $200$  GeV, the closer to  $m_Z$  the better. The second condition can be satisfied if  $m_{3/2} \sim m_Z$  [2] (which now seems highly unlikely due to a lack of LHC8 SUSY signal<sup>6</sup> and the rather large value of  $m_h$ ) or if  $a$  is quite small: in this latter case, the SUSY soft terms conspire such that there are large cancellations amongst the various coefficients of  $m_{3/2}^2$  in Eq. (15): this is what is called radiatively driven natural SUSY [36,37] since in this case a large high scale value of  $m_{H_u}^2$  can be driven radiatively to small values  $\sim -m_Z^2$  at the weak scale.

Furthermore, we can equate the value of  $m_Z^2$  in terms of weak scale parameters with the value of  $m_Z^2$  in terms of GUT scale parameters:

$$\begin{aligned} m_Z^2 &\simeq -2\mu^2(\text{weak}) - 2m_{H_u}^2(\text{weak}) \\ &\simeq -2.18\mu^2(\text{GUT}) + a m_{3/2}^2. \end{aligned} \quad (25)$$

Since  $\mu$  hardly evolves under RG running (the factor 2.18 is nearly 2), then we have the Barbieri-Giudice (BG) condition for low fine-tuning as

$$-m_{H_u}^2(\text{weak}) \sim a m_{3/2}^2 \sim m_Z^2; \quad (26)$$

i.e. the value of  $m_{H_u}^2$  must be driven to *small* negative values  $\sim -m_Z^2$  at the weak scale. These are exactly the conditions required by the model-independent EWFT measure  $\Delta_{\text{EW}}$ : i.e. we have

$$\lim_{n_{\text{SSB}} \rightarrow 1} \Delta_{\text{BG}} \rightarrow \Delta_{\text{EW}}, \quad (27)$$

where  $n_{\text{SSB}}$  is the number of *independent* soft SUSY breaking terms. Of course, this approach also reconciles the Higgs mass fine-tuning measure (with appropriately regrouped independent terms) with the  $\Delta_{\text{BG}}$  measure (when applied to models with a single independent soft breaking term such as  $m_{3/2}$ ).

<sup>6</sup>For instance, in simple SUGRA models, then the scalar masses  $m_0 = m_{3/2}$ . Since LHC requires rather high  $m_0$ , then we would also expect rather large  $m_{3/2}$ .

### E. The $\mu$ parameter and some solutions to the $\mu$ problem

One of the central problems of supersymmetric theories concerns the origin of the superpotential  $\mu$  term:  $W \ni \mu H_u H_d$ . Since this term is supersymmetric (it may be present even in the absence of SUSY breaking), its value might be expected to be  $\mu \sim M_P$ . However, phenomenology dictates instead that  $\mu \sim m_{\text{weak}}$ . A variety of solutions to the SUSY  $\mu$  problem arise in the literature. Here we comment briefly on three of them.

#### 1. NMSSM

In the next-to-minimal supersymmetric standard model (NMSSM) [69], one assumes some symmetry forbids the usual  $\mu$  term, but then a visible sector gauge singlet superfield  $S$  is added with superpotential

$$W \ni \lambda_S S H_u H_d. \quad (28)$$

The scalar component of  $S$  develops a vev  $\langle S \rangle \sim m_{3/2}$  which generates a  $\mu$  term:  $\mu = \lambda_S \langle S \rangle \sim m_{3/2}$ . In addition to the  $\mu$  term, one obtains new physical Higgs particles along with a singlino. An additional contribution to the Higgs mass is also generated which some authors find appealing.

A drawback to this scenario is that introduction of true gauge singlets may lead back to destabilizing the gauge hierarchy via tadpole diagrams [70,71]. This destabilization can be avoided by introducing  $S$  as a composite object [72] although this leads to possibly recondite models and a movement away from parsimony.

#### 2. Giudice-Masiero

In the Giudice-Masiero (GM) mechanism [73], it is assumed that the usual  $\mu$  term is forbidden by some symmetry which is applicable to the visible sector but which is not respected by hidden sector fields. In such a case, then there may exist a (nonrenormalizable) coupling of Higgs doublets to the hidden sector such as

$$K \ni \lambda h_m H_u H_d / M_P, \quad (29)$$

where  $h_m$  is a hidden sector field. When  $h_m$  develops a SUSY breaking vev  $\langle F_h \rangle \sim m_s^2$  where  $m_s$  is the hidden sector mass scale (with  $m_{3/2} \sim m_s^2 / M_P$ ), then a  $\mu$  term is generated with

$$\mu \sim \lambda \langle F_h \rangle / M_P \sim \lambda m_{3/2}. \quad (30)$$

Thus, in the GM solution,<sup>7</sup> we expect  $\mu \sim m_{3/2}$ . If we expect  $m_{3/2} \gg 1$  TeV scale due to a lack of LHC signal,

<sup>7</sup>The impact of the GM solution on fine-tuning has recently been considered in Ref. [34].

then we would arrive at high EW fine-tuning unless  $\lambda$  was tiny.<sup>8</sup>

#### 3. Kim-Nilles

The Kim-Nilles (KN) mechanism [75] arises as a by-product of the Peccei-Quinn (PQ) solution to the strong  $CP$  problem and is the supersymmetric extension of the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axion model [76]. In KN, the  $H_u$  and  $H_d$  superfields carry PQ charges  $Q_u$  and  $Q_d$  so the usual  $\mu$  term is forbidden by PQ symmetry. An additional visible sector field  $P$  carrying PQ charge  $-(Q_u + Q_d)/2$  is then required so that the superpotential term

$$W_{\text{DFSZ}} \ni \lambda P^2 H_u H_d / M_P \quad (31)$$

is present. The PQ symmetry is broken, for instance, by a superpotential [77]

$$W_{\text{PQ}} \ni \lambda_S S (\text{PQ} - f_a^2) \quad (32)$$

(the PQ charge of  $Q$  and  $S$  is  $-Q_P$  and 0, respectively) which leads to  $\langle P \rangle \sim \langle Q \rangle \sim f_a$ . The axion-axino-saxion fields are combinations of the  $P$  and  $Q$  fields. A  $\mu$  term is then generated with

$$\mu \sim \lambda f_a^2 / M_P. \quad (33)$$

Originally, Kim-Nilles had sought to identify the PQ scale  $f_a$  with the hidden sector SUSY breaking scale  $m_s$ . However, now we see that in fact the developing little hierarchy  $\mu \ll m_{3/2}$  is nothing more than a reflection of an apparent mismatch between the PQ breaking scale and the hidden sector SUSY breaking scale  $f_a \ll m_s$ . Guided by electroweak naturalness, we expect  $\mu \sim 100\text{--}200$  GeV so that with  $\lambda \sim 1$ , then we expect

$$f_a \sim 10^{10} \text{ GeV}. \quad (34)$$

In this case, since the axion mass  $m_a \sim 6.2 \mu\text{eV} (10^{12} \text{ GeV} / f_a)$  then *the Higgs mass tells us where to look for the axion*:  $m_a \sim 620 \mu\text{eV}$  with DFSZ couplings. Furthermore, in such

<sup>8</sup>In a recent paper [74], the authors argue that no-scale SUSY models contain only one free parameter  $m_{3/2}$ , and where  $\mu \sim m_{3/2}$  is generated via GM mechanism so that  $m_Z^2 = a \cdot m_{3/2}^2$  where  $a$  is some constant. In such a case, it is a tautology that  $\Delta_{\text{BG}} = c_{m_{3/2}} = |\partial \ln m_Z^2 / \partial \ln m_{3/2}^2| = 1$  and there is no fine-tuning. However, in this case the authors do not produce an explicit hidden sector-visible sector coupling which produces exactly the right  $\mu$  value which is required to generate  $m_Z = 91.2$  GeV. In the absence of an explicit hidden sector model, then one must regard instead  $\mu$  as a free parameter which parametrizes our ignorance of the hidden sector, so that there are actually two free parameters with  $m_Z^2 \sim -2.18\mu^2 + a \cdot m_{3/2}^2$ . Then as usual large  $\mu$  will require high fine-tuning.

a scenario then one expects dark matter to consist of a DFSZ axion along with a Higgsino-like WIMP: i.e. *two* dark matter particles [78].

## II. NUMERICAL PROCEDURE

In the following section, we will evaluate EW fine-tuning for a variety of SUSY models using the case where all three measures agree since as shown above the Higgs mass fine-tuning and the BG measure both reduce to  $\Delta_{EW}$  once dependent contributions to  $m_Z^2$  or  $m_h^2$  are combined into independent terms.

For each model, we generate random sets of parameter values over the range listed in each subsection, and then generate supersymmetric sparticle and Higgs mass spectra using the ISASUGRA [79] subprogram of ISAJET [80]. We require of each solution that

- (i) electroweak symmetry be radiatively broken,
- (ii) the neutralino  $\tilde{Z}_1$  is the lightest MSSM particle,
- (iii) the light chargino mass obeys the model independent LEP2 limit  $m_{\tilde{W}_1} > 103.5$  GeV [81] ( $m_{\tilde{W}_1} > 91.9$  GeV in the case of a winolike chargino), and
- (iv)  $m_h = 125.5 \pm 2.5$  GeV.

We do not impose any LHC sparticle search limits since our general scan can produce compressed spectra which in many cases can easily elude LHC gluino and squark searches. We also do not impose WIMP dark matter constraints since for cases with a standard thermal WIMP underabundance, the WIMP abundance might be augmented by late decaying cosmological relics (e.g. axinos, saxions, moduli, ...) or in the case of an overabundance, the WIMPs might decay to yet lighter particles (e.g. into light axino LSPs) or be diluted by late time entropy injection [82].

We will also calculate the values of  $BF(b \rightarrow s\gamma)$  [83,84] and  $BF(B_s \rightarrow \mu^+\mu^-)$  [85] for each point generated (we also calculate other  $B$  decay observables which turn out to be far less constraining). The measured value of  $BF(b \rightarrow s\gamma)$  is found to be  $(3.55 \pm 0.26) \times 10^{-4}$  [86]. For comparison, the SM prediction [87] is  $BF^{SM}(b \rightarrow s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$ . Also, recently both the LHCb Collaboration [88] and CMS [89] have measured events interpreted as  $B_s \rightarrow \mu^+\mu^-$ . Their combined branching fraction is determined to be  $BF(B_s \rightarrow \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}$  which is in rough accord with the SM prediction of  $(3.2 \pm 0.2) \times 10^{-9}$ . Here, SUSY model points with

$$(i) \quad BF(b \rightarrow s\gamma) = (3.03 - 4.08) \times 10^{-4}$$

and

$$(i) \quad BF(B_s \rightarrow \mu^+\mu^-) = (1.5 - 4.3) \times 10^{-9}$$

will be labeled as satisfying  $B$ -physics constraints.

## III. ELECTROWEAK FINE-TUNING IN VARIOUS SUSY MODELS

### A. mSUGRA/CMSSM

First we scan over the paradigm mSUGRA [90] or CMSSM [3] model with parameter ranges given by

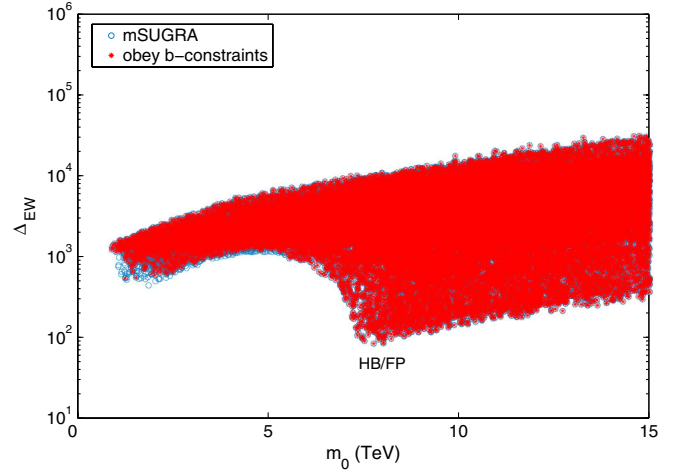


FIG. 3 (color online). Plot of  $\Delta_{EW}$  vs.  $m_0$  from a scan over mSUGRA/CMSSM parameters space whilst maintaining  $m_h = 125.5 \pm 2.5$  GeV and whilst obeying  $B$ -decay constraints. The location of the hyperbolic branch/focus point regions is labelled as HB/FP.

- (i)  $m_0$ : 0–15 TeV,
- (ii)  $m_{1/2}$ : 0–2 TeV,
- (iii)  $-2.5 < A_0/m_0 < 2.5$ :
- (iv)  $\tan\beta$ : 3–60,

and for both signs of  $\mu$ .<sup>9</sup> The results of this scan have been shown previously in Ref. [91] for all  $\tan\beta$  and in Ref. [38] for  $\tan\beta = 10$ . We present it here for completeness so that the reader may more readily compare these results against other SUSY models, and because now we also impose more restrictive  $B$ -decay constraints.

The value of  $\Delta_{EW}$  is shown vs  $m_0$  in Fig. 3 where blue dots comprise all solutions while red dots also respect  $B$ -decay constraints. For low  $m_0$ , the value of  $\Delta_{EW}$  is around  $10^3$ , indicating EWFT at the  $\Delta_{EW}^{-1} \sim 0.1\%$  level. As  $m_0$  increases, the value of  $\Delta_{EW}$  can drop sharply into the  $10^2$  range for  $m_0 \sim 7$ –10 TeV. This is the case of the hyperbolic branch/focus-point region (denoted HB/FP) where  $\mu$  becomes small [6,10]. The value of  $\Delta_{EW}$  does not drop to arbitrarily small values because at such large  $m_0$  values then the top squark masses become  $\sim 5$ –10 TeV and the radiative corrections  $\Sigma_u^H(\tilde{t}_{1,2})$  become large. In fact, as  $m_0$  increases beyond 7 TeV, then the minimum of  $\Delta_{EW}$  also increases due to the increasing radiative corrections. With such a high minimum value of  $\Delta_{EW}$ , we would expect mSUGRA/CMSSM probably does not describe nature.

### B. NUHM1

The NUHM1 model [92] is inspired by  $SO(10)$  SUSY GUT models where the Higgs doublets live in the 10-dimensional fundamental representation while the

<sup>9</sup>Our convention for  $\mu$  gives a positive contribution to  $(g-2)_\mu$  when  $\mu > 0$ .

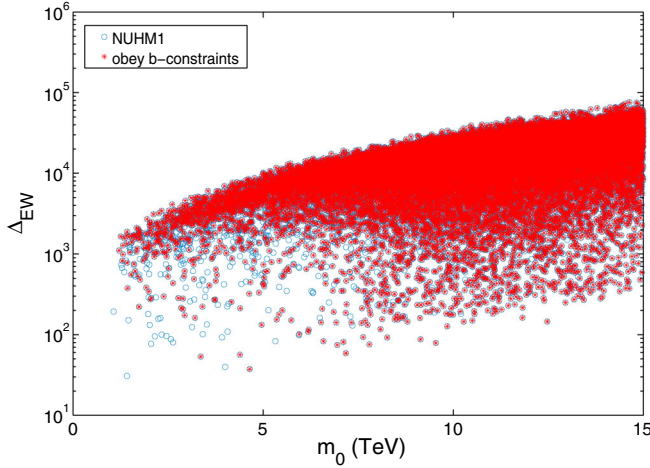


FIG. 4 (color online). Plot of  $\Delta_{EW}$  vs  $m_0$  from a scan over NUHM1 parameters space while maintaining  $m_h = 125.5 \pm 2.5$  GeV.

matter scalars inhabit the 16-dimensional spinor representation. In this case, the parameter set is expanded by one and now we scan over

- (i)  $m_0$ : 0–15 TeV,
- (ii)  $m_{H_u} = m_{H_d} \equiv m_H$ : 0–15 TeV,
- (iii)  $m_{1/2}$ : 0–2 TeV,
- (iv)  $-2.5 < A_0/m_0 < 2.5$ :
- (v)  $\tan\beta$ : 3–60.

By increasing  $m_H \gg m_0$ , then  $m_{H_u}^2$  is only driven to small instead of large negative values, while if  $m_{H_u}^2$  is increased too much, then  $m_{H_u}^2$  is never driven negative and electro-weak symmetry is not broken. If  $m_H$  is taken smaller than  $m_0$ , even with  $m_H^2 < 0$  as a possibility, then  $m_{H_d} \sim m_A$  can be decreased while  $m_{H_u}^2$  is driven to very large negative values. In the former case, where  $m_{H_u}^2$  is driven to small negative values, then  $\mu$  also decreases—since its value is set to yield the measured  $Z$  mass via Eq. (1). In such cases, we expect reduced values of  $\Delta_{EW}$ .

In the scan results shown in Fig. 4, this is indeed borne out, as we see that the minimal value of  $\Delta_{EW}$  reaches as low as  $\sim 30$ , which is much less fine-tuned than mSUGRA. Values of  $\Delta_{EW}$  in the 30–50 range which obey  $B$ -decay constraints and  $m_h \sim 125$  can be found for  $m_0 \sim 3$ –10 TeV. With such large  $m_0$  values, then the top squarks also tend to be in the 3–10 TeV regime and the top squark radiative corrections prevent  $\Delta_{EW}$  from reaching below  $\sim 30$ .

### C. NUHM2

The NUHM2 model [93] is inspired by  $SU(5)$  SUSY GUTs where each of the MSSM Higgs doublets live in separate 5 and  $\bar{5}$  representations, or by  $SO(10)$  SUSY GUTs with  $D$ -term scalar mass splitting. In such a case, we expand the mSUGRA parameter space to include  $m_{H_u}^2$  and  $m_{H_d}^2$  as soft SUSY breaking terms which are independent of  $m_0$ . Using weak scale mass relations, then  $m_{H_u}^2$  and  $m_{H_d}^2$

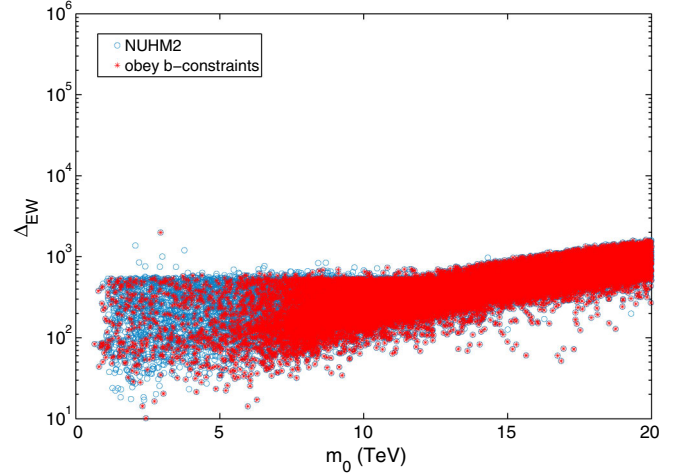


FIG. 5 (color online). Plot of  $\Delta_{EW}$  vs  $m_0$  from a scan over NUHM2 parameters space while maintaining  $m_h = 125.5 \pm 2.5$  GeV.

can be traded for the more convenient weak scale parameters  $\mu$  and  $m_A$ .

In the case of NUHM2, we scan in accord with Ref. [37]

- (i)  $m_0$ : 0–20 TeV,
- (ii)  $m_{1/2}$ : 0.3–2 TeV,
- (iii)  $-3 < A_0/m_0 < 3$ ,
- (iv)  $\mu$ : 0.1–1.5 TeV,
- (v)  $m_A$ : 0.15–1.5 TeV,
- (vi)  $\tan\beta$ : 3–60,

with results shown in Fig. 5. Here, we see that  $\Delta_{EW}$  can reach values as low as 10, corresponding to  $\Delta_{EW}^{-1} \sim 10\%$  EWFT. Even lower values  $\sim 7$  have been generated in Ref. [38] for a fixed  $\tan\beta = 10$  value. The key here is that low  $\mu$  values  $\sim 100$ –200 GeV can be input by hand while top squarks can occur in the 1–5 TeV regime with large mixing, which also acts to reduce the radiative corrections  $\Sigma_u^u(\tilde{t}_{1,2})$  [36]. The required GUT scale values of  $m_{H_u}$  are about  $1.2m_0$  while  $m_{H_d}(m_{GUT})$  can be anywhere in the TeV range [37]. As  $m_0$  increases beyond about 7 TeV, then the min of  $\Delta_{EW}$  slowly increases due to increasing top squark radiative corrections. For the model examined in Ref. [37] with split generations, then 2–4 TeV top squarks are allowed in accord with 10–30 TeV first/second generation scalars: this situation offers at least a partial decoupling solution to the SUSY flavor and  $CP$  problems [94].

### D. mGMSB

In minimal GMSB [95–97], a sector of “messenger” fields is hypothesized which communicates between the hidden SUSY breaking sector and the visible/MSSM sector. Visible sector scalar fields acquire a mass  $m_i^2 \propto (\alpha_i/4\pi)^2 \Lambda^2$  while gauginos acquire a mass  $M_i = (\alpha_i/4\pi)\Lambda$  where  $\Lambda$  parametrizes the induced SUSY breaking scale in the messenger sector. The trilinear SSB  $a$  terms are suppressed by an additional loop factor and hence are expected



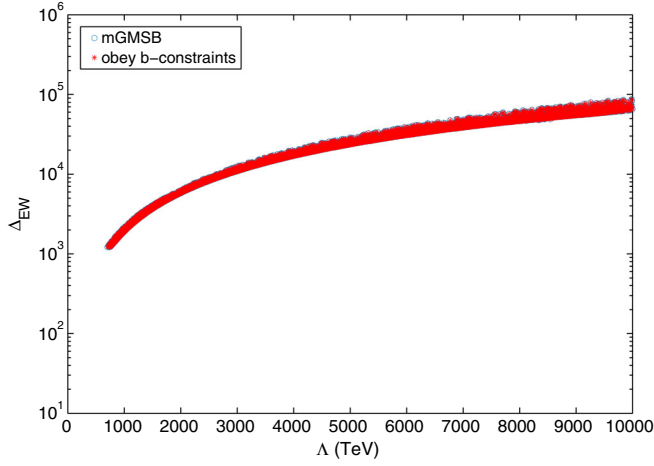


FIG. 6 (color online). Plot of  $\Delta_{EW}$  vs  $\Lambda$  from a scan over mGMSB parameters space while maintaining  $m_h = 125.5 \pm 2.5$  GeV.

to be small. This latter effect leads to only small amounts of stop mixing: consequently huge stop masses are required in mGMSB in order to generate  $m_h \sim 125$  GeV. Furthermore, the hierarchy of mass values in mGMSB

$$M_1 < m_E < M_2 < m_L = m_{H_u} = m_{H_d} \ll M_3 < m_{\tilde{q}} \quad (35)$$

means that the  $m_{H_u}^2$  boundary condition is already suppressed at the messenger scale  $M_{mes}$ , and then is strongly driven to large negative values due to the large values of  $m_{Q_3}$  and  $m_{U_3}$  contributing to the  $X_t$  term in Eq. (11). The upshot is that for allowed parameter ranges,  $m_{H_u}^2$  is driven to large negative values at the weak scale, and the value of  $\mu$  must be large positive (fine-tuned) to obtain the measured value of  $m_Z$ .

Our results are shown in Fig. 6 where we plot  $\Delta_{EW}$  vs  $\Lambda$  from a scan over values

- (i)  $\Lambda$ :  $10^2$ – $10^4$  TeV,
- (ii)  $M_{mes} = 2\Lambda$ ,
- (iii)  $\tan\beta$ : 3–60,
- (iv)  $\text{sign}(\mu) = \pm$ .

From the plot, we see that requiring  $m_h$ : 123–128 GeV then requires  $\Lambda \gtrsim 500$  TeV, which results in very heavy top squarks and large fine-tuning, with the minimum of  $\Delta_{EW}$  at  $10^3$ , or 0.1% EWFT. Here, we would conclude that at least minimal GMSB is not likely to describe nature.

### E. mAMSB

In anomaly-mediated SUSY breaking models [98], it is assumed that the SUSY breaking sector is sequestered from the visible sector—perhaps in extra spacetime dimensions—so that the dominant soft SUSY breaking contribution comes from the superconformal anomaly. In this case, gaugino masses  $M_i = c_i(g_i^2/16\pi^2)m_{3/2}$  with  $c_i = (33/5, 1, -3)$  for the  $U(1)$ ,  $SU(2)$  and  $SU(3)$  groups,

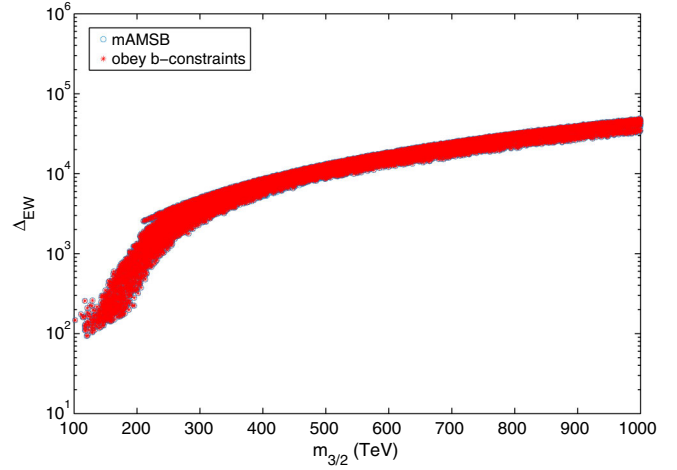


FIG. 7 (color online). Plot of  $\Delta_{EW}$  vs  $m_{3/2}$  from a scan over mAMSB parameters space while maintaining  $m_h = 125.5 \pm 2.5$  GeV.

respectively. Thus, multi-TeV values of  $m_{3/2}$  are required which also ameliorates the so-called cosmological gravitino problem [99]. Also, the lightest gauginos are winolike with a neutral wino as LSP. Because of tachyonic slepton masses in pure AMSB, an additional universal contribution  $m_0^2$  is invoked in order to gain a phenomenologically viable spectrum of matter scalars. Since the trilinear  $a$  parameter is small, mAMSB has trouble generating  $m_h \sim 125$  GeV unless top squarks are in the multi-TeV regime.

We scan over mAMSB parameter space according to

- (i)  $m_{3/2}$ : 20–1000 TeV,
- (ii)  $m_0$ : 0–10 TeV,
- (iii)  $\tan\beta$ : 3–60,
- (iv)  $\text{sign}(\mu) = \pm$ .

Our results are shown in Fig. 7. Here, we see that the minimal value of  $\Delta_{EW}$  occurs at  $m_{3/2} \sim 100$  TeV and has a value  $\sim 100$ , or 1% EWFT.

### F. HCAMSB

An alternative setup for AMSB, known as hypercharged anomaly-mediation (HCAMSB), has been advocated in Ref. [100]. It is a string motivated scenario which uses a similar construction to the one envisioned for AMSB. In HCAMSB, SUSY breaking is localized at the bottom of a strongly warped hidden region, geometrically separated from the visible region where the MSSM resides. The warping suppresses contributions due to tree-level gravity mediation so that anomaly mediation can become the dominant source of SUSY breaking in the visible sector. Possible exceptions to this sequestering mechanism are gaugino masses of  $U(1)$  gauge symmetries. Thus, in the MSSM, the mass of the bino [the gaugino of  $U(1)_Y$ ] can be the only soft SUSY breaking parameter not determined by anomaly mediation. Depending on its size, the bino mass  $M_1$  can lead to a small perturbation to the spectrum of

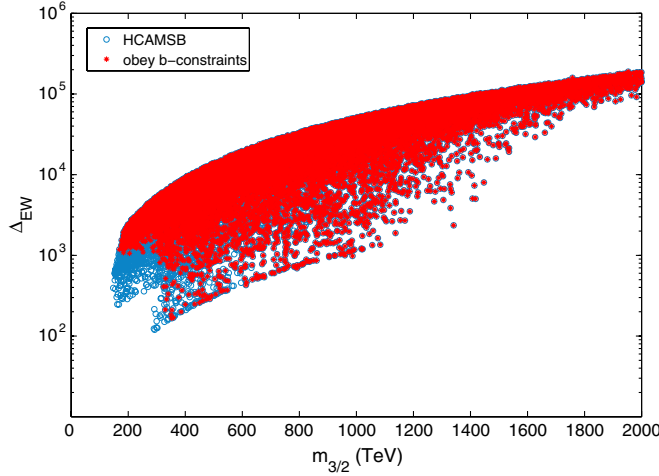


FIG. 8 (color online). Plot of  $\Delta_{EW}$  vs.  $m_{3/2}$  from a scan over HCAMSB parameters space whilst maintaining  $m_h = 125.5 \pm 2.5$  GeV.

anomaly mediation, or it can be the largest soft SUSY breaking parameter in the visible sector. As a result of RG evolution, its effect on other soft SUSY breaking parameters can dominate the contribution from anomaly mediation. In extensions of the MSSM, additional  $U(1)$ s can also communicate SUSY breaking to the MSSM sector.

In HCAMSB, the SSB terms are of the same form as AMSB except for the  $U(1)_Y$  gaugino mass,

$$M_1 = \tilde{M}_1 + \frac{b_1 g_1^2}{16\pi^2} m_{3/2}, \quad (36)$$

where  $\tilde{M}_1 = am_{3/2}$ . The large  $U(1)_Y$  gaugino mass can cause  $m_{H_u}^2$  to first run to large positive values before it is driven negative so that EW symmetry is broken. This potentially leads to lower fine-tuning since then  $m_{H_u}^2$  may be driven to just small negative values.

We scan over the HCAMSB parameter space

- (i)  $m_{3/2}$ : 25–2000 TeV,
- (ii)  $\alpha$ :  $-0.25$ – $0.25$ ,
- (iii)  $\tan\beta$ : 3–60,
- (iv)  $\text{sign}(\mu) = \pm$

with the LEP2 chargino mass limit reduced to  $m_{\tilde{W}_1} > 91.9$  GeV as appropriate for a winolike LSP. Our results are shown in Fig. 8 where we plot  $\Delta_{EW}$  vs  $m_{3/2}$ . Here, we find a minimal value of  $\Delta_{EW} \sim 100$  for  $m_{3/2} \sim 400$  TeV.

### G. inoAMSB

A third alternative related to AMSB is known as gaugino-AMSB or inoAMSB for short [101,102]. The main assumption here is that the high energy theory which generates SUSY breaking is of the sequestered type [98] so that the classical gaugino and scalar masses and  $A$  terms are highly suppressed relative to the gravitino mass scale. Nevertheless, in contrast to what is usually advocated in

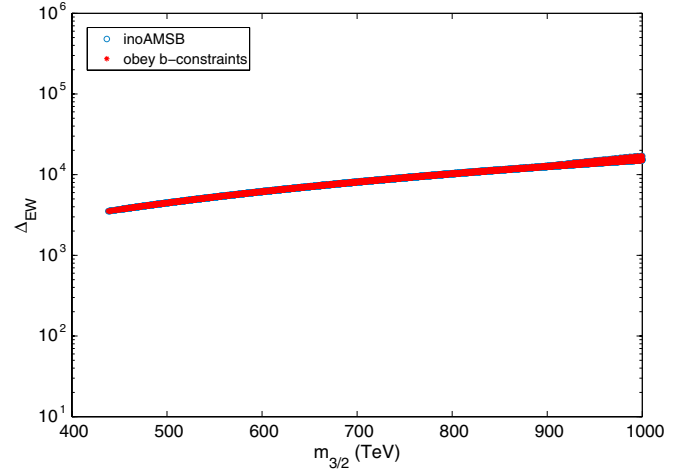


FIG. 9 (color online). Plot of  $\Delta_{EW}$  vs  $m_{3/2}$  from a scan over inoAMSB parameters space while maintaining  $m_h = 125.5 \pm 2.5$  GeV.

AMSB, it has been argued [66] that only *gaugino masses* are generated by Weyl anomalies. In inoAMSB, the scalar masses are then generated by RG running as in what is often called gaugino mediation [103] or simple no-scale SUSY breaking models [104]. The inoAMSB model then avoids both the generic flavor changing neutral current problems of gravity mediated scenarios and also the tachyonic slepton problem of the traditional AMSB construct. The inoAMSB model then depends on just two parameters: the gravitino mass  $m_{3/2}$ , which sets the scale for all sparticle masses, and  $\tan\beta$  while  $m_0 = A_0 \equiv 0$ .

We scan over the inoAMSB parameter space

- (i)  $m_{3/2}$ : 100–1000 TeV,
- (ii)  $\tan\beta$ : 3–60,
- (iii)  $\text{sign}(\mu) = \pm$

with the LEP2 chargino mass limit given by  $m_{\tilde{W}_1} > 91.9$  GeV as appropriate for a winolike LSP.

Our results are shown in Fig. 9 where we plot  $\Delta_{EW}$  vs  $m_{3/2}$ . We find a minimal value of  $\Delta_{EW} \sim 350$  for  $m_{3/2} \sim 420$  TeV so this model is highly electroweak fine-tuned over the parameter range which generates a value of  $m_h$  in accord with measurement. The reason is that by setting  $m_{H_u}^2 = 0$  at  $Q = m_{GUT}$ , then it can only be driven to large negative values resulting in high EW fine-tuning.

### H. Mixed moduli-anomaly mediation

These models, known as mixed moduli-anomaly mediated SUSY breaking (MMAMSB), or mirage mediation, are based on the Kachru-Kalosh-Linde-Trivedi (KKLT) construction [105] of string compactification with fluxes, which produce the necessary de Sitter vacuum. In the KKLT construct, one first introduces nonzero fluxes in the Type IIB string theory compactified on a Calabi-Yau manifold. Because of the nonzero fluxes, the complex structure moduli and the dilaton are completely fixed but the size modulus  $T$

remains a flat direction. To fix this, KKLT invoked non-perturbative effects, such as gaugino condensation on D7 branes. At this stage, all moduli are fixed, but one ends up with supersymmetric vacua and negative vacuum energy. The final step in the construction is to include anti  $D$ -branes yielding the desired de Sitter vacua (with positive vacuum energy) and breaking supersymmetry. Because of the presence of branes and fluxes, the models have generically warped compactifications. Because of the warping, the addition of the anti  $D$ -brane breaks supersymmetry by a very small amount.

The phenomenology of KKLT-inspired models is distinctive in that moduli fields and the Weyl anomaly make comparable contributions to SUSY breaking effects in the observable sector of fields [106]. The contribution of each can be parametrized by  $\alpha$  which yields pure AMSB for  $\alpha = 0$  but which tends to pure moduli (gravity) mediation as  $\alpha$  becomes large. The phenomenology also depends on the so-called modular weights which in turn depend on the location of various fields in the extra dimensions:  $n_i = 0(1)$  for matter fields located on D7 (D3) branes; fractional values  $n_i = 1/2$  are also possible for matter living at brane intersections. It is claimed that MMAMSB models have the potential to be minimally EW fine-tuned [107,108].

The parameter space of MMAMSB models is given by

- (i)  $m_{3/2}$ : 10–100 TeV,
- (ii)  $\alpha$ :  $-15 \rightarrow 15$ ,
- (iii)  $\tan\beta$ : 3–60,
- (iv)  $\text{sign}(\mu) = \pm$ ,

along with

- (i)  $n_H, n_m = 0, 1/2 \text{ or } 1$ .

Many of the following results can be understood by inspection of the  $\alpha$  vs  $m_{3/2}$  plane plots available for each modular weight combination and shown in Ref. [109].

### I. Cases with $n_H = 0$

Our first results for MMAMSB are shown in Fig. 10 in the  $\Delta_{EW}$  vs  $m_{3/2}$  frame for cases with (a)  $(n_H, n_m) = (0, 0)$ , (b)  $(n_H, n_m) = (0, \frac{1}{2})$  and (c)  $(n_H, n_m) = (0, 1)$ . For the case with  $(n_H, n_m) = (0, 0)$ , we find a minimal value of  $\Delta_{EW} \approx 437$  which occurs at  $m_{3/2} \sim 35$  TeV. At this point,  $m_{\tilde{g}} \sim m_{\tilde{q}} \sim 1.8$  TeV which might be expected to be ruled out by LHC8 searches. However, the compressed spectra with gaugino masses  $M_1, M_2, M_3 \sim 800, 1000, 1800$  GeV leads to softer visible energy than might otherwise be expected. The value of  $\mu \sim 1200$  GeV produces the large value of  $\Delta_{EW}$ . While much less tuned spectra are possible, they only occur with very low  $m_h$  values and so are ruled out by the LHC8 Higgs discovery.

The case with  $(n_H, n_m) = (0, \frac{1}{2})$  has a minimal value of  $\Delta_{EW} = 314$  and so also is EW fine-tuned. In this case, the minimum occurs for  $m_{3/2} = 95$  TeV which leads to  $m_{\tilde{g}} = 3.6$  TeV. The large  $\mu = 1.1$  TeV value leads to high EW fine-tuning. Here, the LSP is the lightest Higgsino with mass  $m_{\tilde{Z}_1} \sim \mu$ . Models exist with  $m_h \sim 125$  GeV and much lower fine-tuning reaching to  $\Delta_{EW} \sim 30$  (blue points) but

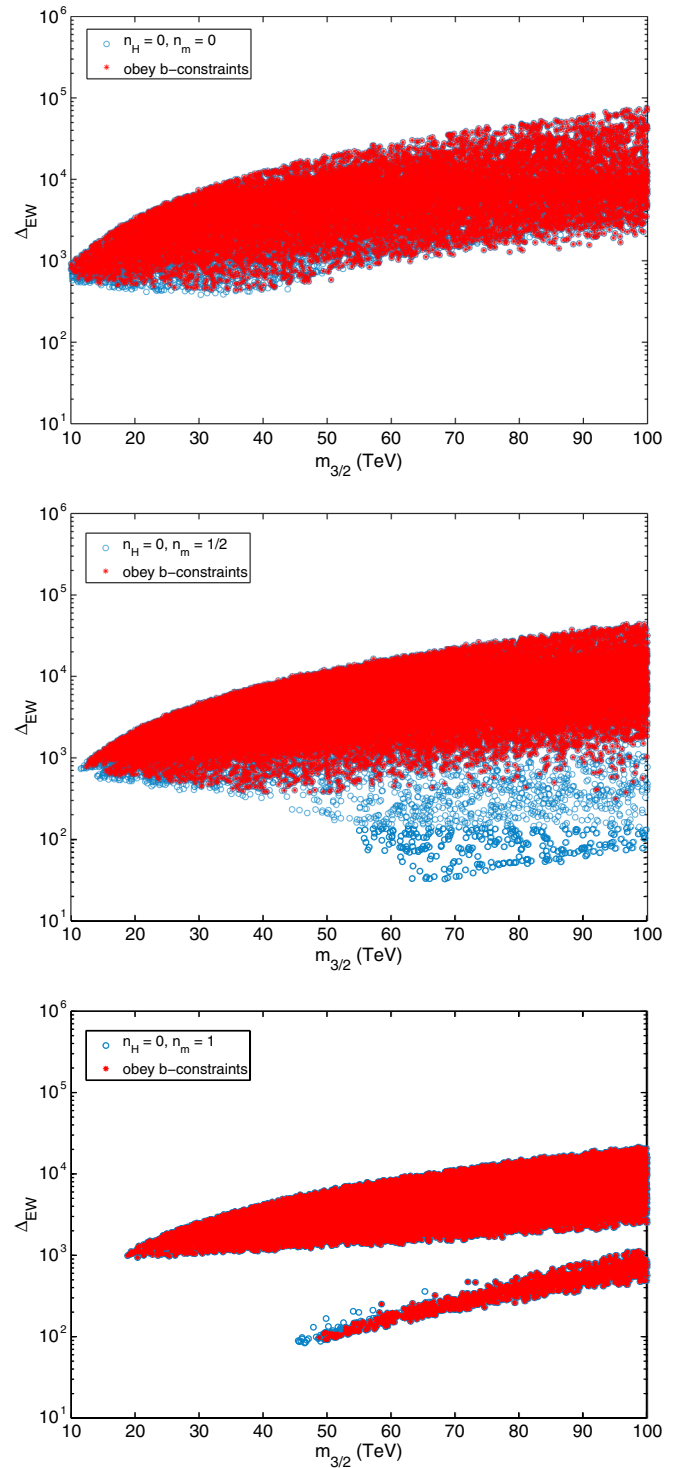


FIG. 10 (color online). Plot of  $\Delta_{EW}$  vs  $m_{3/2}$  from a scan over MMAMSB parameter space with  $n_H = 0$  while maintaining  $m_h = 125.5 \pm 2.5$  GeV.

these all violate  $B$ -decay constraints due to rather light top squarks.

For the case with  $(n_H, n_m) = (0, 1)$ , then the lowest  $\Delta_{EW}$  value is found to be  $\sim 91$ , a considerable improvement but still 9 times greater than the min from NUHM2. In this case,

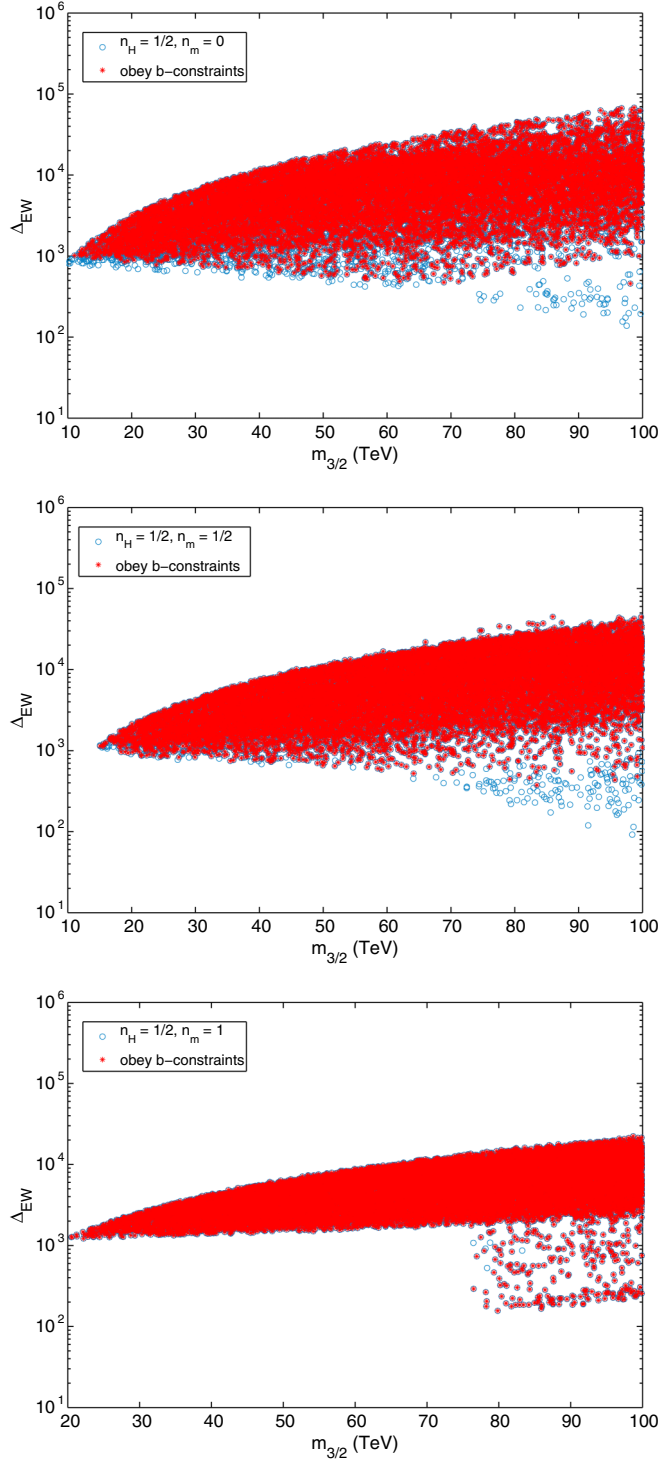


FIG. 11 (color online). Plot of  $\Delta_{EW}$  vs  $m_{3/2}$  from a scan over MMAMSB parameter space with  $n_H = 1/2$  while maintaining  $m_h = 125.5 \pm 2.5$  GeV.

the solutions form two distinct branches—the upper with  $\alpha < 0$  while the lower has  $\alpha > 0$ . The lowest  $\Delta_{EW} = 91$  point actually has  $\mu \sim 150$  GeV, but with  $m_{3/2} \sim 50$  TeV, then the top squarks have mass  $m_{\tilde{t}_{1,2}} \sim 2.1, 2.8$  TeV and not enough mixing so the values of  $\sum_u \tilde{t}_{1,2}$  dominate the fine-tuning.

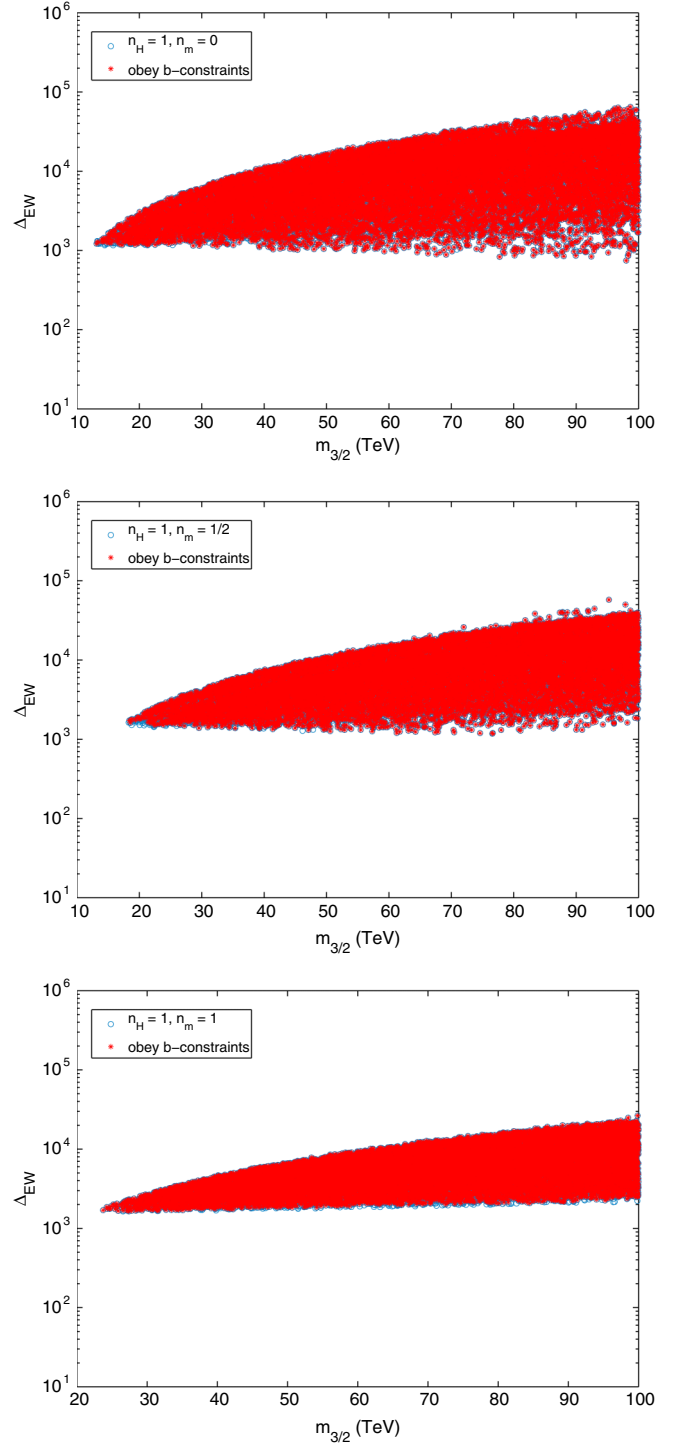


FIG. 12 (color online). Plot of  $\Delta_{EW}$  vs  $m_{3/2}$  from a scan over MMAMSB parameters space with  $n_H = 1$  while maintaining  $m_h = 125.5 \pm 2.5$  GeV.

### J. Cases with $n_H = 1/2$

Results for MMAMSB for cases with (a)  $(n_H, n_m) = (\frac{1}{2}, 0)$ , (b)  $(n_H, n_m) = (\frac{1}{2}, \frac{1}{2})$  and (c)  $(n_H, n_m) = (\frac{1}{2}, 1)$  are shown in Fig. 11 in the  $\Delta_{EW}$  vs  $m_{3/2}$  plane. For frame (a), we find a minimum  $\Delta_{EW} = 457$  at  $m_{3/2} = 98$  TeV where a



spectrum with  $m_{\tilde{g}} \sim m_{\tilde{q}} \sim 2$  TeV but with  $\mu = 1.4$  TeV. The LSP is a neutral Higgsino with mass  $\sim 1.34$  TeV and  $\Omega_{\tilde{Z}_1} h^2 \sim 0.15$ . Even lower  $\Delta_{\text{EW}}$  solutions reaching values of  $\sim 100$  occur at very high  $m_{3/2}$ , but these blue points are excluded by  $B$ -decay constraints.

For the  $(n_H, n_m) = (\frac{1}{2}, \frac{1}{2})$  case in frame (b), the lowest value of  $\Delta_{\text{EW}}$  is found to be 375 at  $m_{3/2} = 83$  TeV. Here again,  $\mu \simeq 1.25$  TeV which gives a Higgsino-like LSP and rather compressed spectra.

For the  $(n_H, n_m) = (\frac{1}{2}, 1)$  case shown in frame (c), then  $\Delta_{\text{EW}}$  can reach as low as  $\sim 100$  at the high  $m_{3/2} \sim 80$  TeV point. For this point,  $\mu$  drops as low as 589 GeV and the LSP is again Higgsino-like with a thermal underabundance of neutralino dark matter. The gluino and squark masses cluster around 3.5–4.5 TeV, beyond LHC reach.

### K. Cases with $n_H = 1$

The MMAMSB cases with (a)  $(n_H, n_m) = (1, 0)$ , (b)  $(n_H, n_m) = (1, \frac{1}{2})$  and (c)  $(n_H, n_m) = (1, 1)$  are shown in Fig. 12. For the first case with (1,0), then the min of  $\Delta_{\text{EW}}$  is 859 at  $m_{3/2} = 90$  TeV. The large EWFT is generated by the large  $\mu = 1.9$  TeV value. Even so, the LSP is mainly bino with mass  $m_{\tilde{Z}_1} \sim 1.7$  TeV.

For frame (b) with  $(n_H, n_m) = (1, \frac{1}{2})$ , then the min of  $\Delta_{\text{EW}}$  is 1178 at  $m_{3/2} = 76$  TeV where  $m_{\tilde{g}} \sim 3.9$  TeV and the binolike LSP has mass  $\sim 1.8$  TeV.

Finally, frame (c) shows the  $(n_H, n_m) = (1, 1)$  case where a min of  $\Delta_{\text{EW}}$  is found to be 1643 at  $m_{3/2} = 27$  TeV. Here, gluino and squark masses tend to exceed 4 TeV while  $\mu \sim 2.6$  TeV which leads to the large EW fine-tuning.

## IV. CONCLUSIONS

In this paper, we have reexamined electroweak fine-tuning in light of recent LHC results on the Higgs discovery with  $m_h \simeq 125.5$  GeV and the lack of any sort of signal for sparticles. This situation has led to various claims that the MSSM is no longer viable, or at least highly fine-tuned in the EW sector. Alternatively, it has been claimed that conventional measures, applied conventionally, overestimate EWFT.

To clarify the situation, we have proposed a rule of fine-tuning: When evaluating fine-tuning, it is not permissible to claim fine-tuning of *dependent* quantities one against another. In the case of Higgs mass fine-tuning, we find that the measure  $\Delta_{\text{HS}}$  violates this rule by measuring nonindependent terms which can lead to large cancellations. Then, Higgs mass fine-tuning can grossly overestimate—often by orders of magnitude—the electroweak fine-tuning. By appropriately combining dependent terms, then  $\Delta_{\text{HS}}$  reduces to the model independent  $\Delta_{\text{EW}}$  measure: the offending large logs are still present, but can now cancel against other nonindependent terms.

We have also examined the traditional measure  $\Delta_{\text{BG}}$ . In this case, the measure appears at first sight to be highly

model dependent. The model dependence is traced to the fact that most users regard the multiple parameters of most popular SUSY models as *independent* when in fact their independence is only an artifact of trying to construct a model which encompasses a wide range of hidden sector SUSY breaking possibilities. If instead one relates the various soft parameters—such as multiples of  $m_{3/2}$  as expected in supergravity models with SUSY broken via the superHiggs mechanism—then it is shown that  $\Delta_{\text{BG}}$  also reduces to the model-independent electroweak measure  $\Delta_{\text{EW}}$ .

For low  $\Delta_{\text{EW}}$ , then it is required that 1.  $\mu \sim 100 - 300$  GeV, 2.  $m_{\tilde{H}_u}^2$  is radiatively driven to small negative values  $\sim m_Z$  and 3. the top-squarks are in the few TeV range with large mixing. The large mixing reduces top-squark radiative contributions to  $\Delta_{\text{EW}}$  while lifting  $m_h$  into the 125 GeV range.

We also evaluated  $\Delta_{\text{EW}}$  values from a scan over parameters of 16 models: mSUGRA, NUHM1, NUHM2, mGMSB, mAMSB, HCAMSB, inoAMSB and nine cases of mixed moduli-anomaly (mirage) mediated SUSY breaking. Our overall results are summarized in Fig. 13 where we show the range of  $\Delta_{\text{EW}}$  generated on the y axis versus models on the x axis. Only one model—NUHM2—reaches to the rather low  $\Delta_{\text{EW}} \sim 10$  values, indicating just 10% EWFT. This can be so because the freedom in the soft Higgs sector allows arbitrarily low values of  $\mu$  (subject to LEP2 constraints) to be generated while at the same time driving  $m_{\tilde{H}_u}^2$  to just small negative values, while also accommodating TeV-scale top squarks with large mixing. For the remaining models, their inherent constraints make satisfying these conditions with  $m_h \sim 125$  GeV very difficult unless they are highly fine-tuned. The best of the remainder models include NUHM1 which allows for min  $\Delta_{\text{EW}}$  as low as 30. Thus,  $\Delta_{\text{EW}}$  does indeed put SUSY models under siege.

Luckily, at least NUHM2 and its generalizations survive, and even thrive. In the case of the surviving NUHM2 spectra (those with  $\Delta_{\text{EW}} \lesssim 30$ ), a discovery at LHC14 might take place provided  $m_{\tilde{g}} \lesssim 2$  TeV [110]: this reach covers about half of the parameter space [37]. The definitive search for SUSY would have to take place at a linear  $e^+e^-$  collider where  $\sqrt{s}$  could extend beyond  $2m_{\text{Higgsino}}$ —in this case  $\sqrt{s} \sim 500-600$  GeV is required for a thorough search.<sup>10</sup> Such a machine would either discover SUSY or rule out SUSY naturalness [112]. We may also expect an ultimate discovery of a Higgsino-like WIMP and a DFSZ-type axion, since models such as SUSY DFSZ solve the strong  $CP$  fine-tuning problem and the  $\mu$  problem while at the same time allowing naturally for a little hierarchy of

<sup>10</sup>The proposed TLEP  $e^+e^-$  collider with projected maximal  $\sqrt{s} \sim 350$  GeV may not have sufficient energy to thoroughly explore natural SUSY [111].

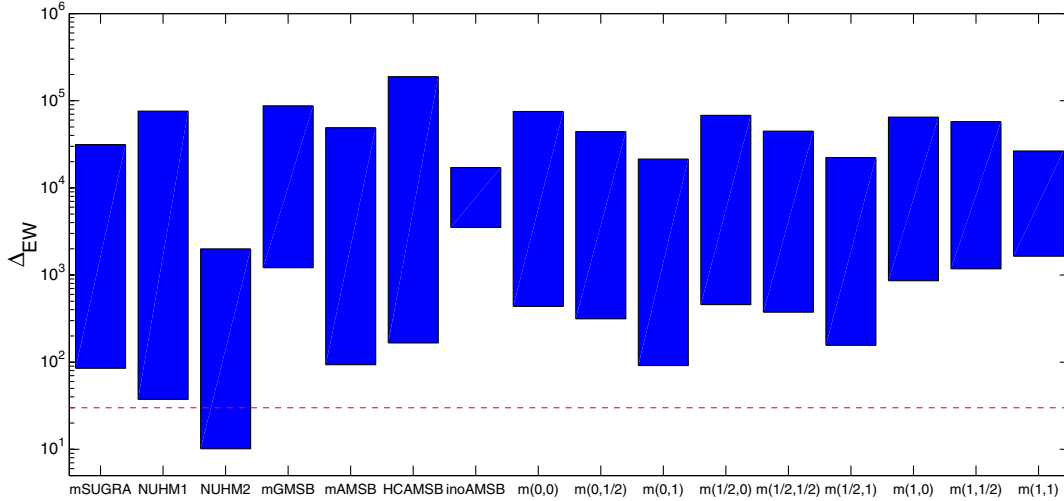


FIG. 13 (color online). Histogram of the range of  $\Delta_{EW}$  values generated for each SUSY model considered in the text. We would consider  $\Delta_{EW} \lesssim 30$ —the lower the better—as acceptable values for EW fine-tuning. This region is located below the dashed red line.

$f_a \ll m_s$ , where  $m_s \sim 10^{11}$  GeV represents the mass scale usually associated with hidden sector SUSY breaking. That hierarchy is then reflected by the hierarchy  $\mu \ll m_{3/2}$  which seems to be what naturalness combined with LHC data is telling us.

We end by confessing that the general features of some of our viewpoints have been articulated previously by Giudice [113]:

*“It may well be that, in some cases, Eq. (16) overestimates the amount of tuning. Indeed, Eq. (16) measures the sensitivity of the prediction of  $m_Z$  as we vary parameters in ‘theory space.’ However, we have no idea*

*how this ‘theory space’ looks like, and the procedure of independently varying all parameters may be too simple-minded.”*

#### ACKNOWLEDGMENTS

We thank P. Huang, D. Mickelson, A. Mustafayev and X. Tata for many previous collaborations leading up to this study. We thank M. Drees for comments on the manuscript. H. B. would like to thank the DESY Helmholtz Alliance group for hospitality while this work was completed. This work was supported in part by the U.S. Department of Energy, Office of High Energy Physics.

- 
- [1] J. R. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, *Mod. Phys. Lett. A* **01**, 57 (1986).
  - [2] R. Barbieri and G. Giudice, *Nucl. Phys.* **B306**, 63 (1988).
  - [3] G. Kane, C. Kolda, L. Roszkowski, and J. Wells, *Phys. Rev. D* **49**, 6173 (1994).
  - [4] G. W. Anderson and D. J. Castano, *Phys. Lett. B* **347**, 300 (1995); *Phys. Rev. D* **52**, 1693 (1995).
  - [5] S. Dimopoulos and G. F. Giudice, *Phys. Lett. B* **357**, 573 (1995).
  - [6] K. L. Chan, U. Chattopadhyay, and P. Nath, *Phys. Rev. D* **58**, 096004 (1998); S. Akula, M. Liu, P. Nath, and G. Peim, *Phys. Lett. B* **709**, 192 (2012); M. Liu and P. Nath, *Phys. Rev. D* **87**, 095012 (2013).
  - [7] P. H. Chankowski, J. R. Ellis, and S. Pokorski, *Phys. Lett. B* **423**, 327 (1998); P. H. Chankowski, J. R. Ellis, M. Olechowski, and S. Pokorski, *Nucl. Phys.* **B544**, 39 (1999).
  - [8] G. L. Kane and S. F. King, *Phys. Lett. B* **451**, 113 (1999); M. Bastero-Gil, G. L. Kane, and S. F. King, *Phys. Lett. B* **474**, 103 (2000).
  - [9] J. A. Casas, J. R. Espinosa, and I. Hidalgo, *J. High Energy Phys.* **01** (2004) 008.
  - [10] J. L. Feng, K. T. Matchev, and T. Moroi, *Phys. Rev. D* **61**, 075005 (2000); J. L. Feng, K. T. Matchev, and T. Moroi, arXiv:hep-ph/0003138; J. L. Feng and D. Sanford, *Phys. Rev. D* **86**, 055015 (2012).
  - [11] R. Harnik, G. D. Kribs, D. T. Larson, and H. Murayama, *Phys. Rev. D* **70**, 015002 (2004).
  - [12] Y. Nomura and B. Tweedie, *Phys. Rev. D* **72**, 015006 (2005); Y. Nomura, D. Poland, and B. Tweedie, *Nucl. Phys.* **B745**, 29 (2006).
  - [13] P. Athron and D. J. Miller, *Phys. Rev. D* **76**, 075010 (2007).
  - [14] S. Cassel, D. M. Ghilencea, and G. G. Ross, *Nucl. Phys.* **B825**, 203 (2010); **B835**110 (2010); S. Cassel,

- D. M. Ghilencea, S. Kraml, A. Lessa, and G. G. Ross, *J. High Energy Phys.* **05** (2011) 120; G. G. Ross and K. Schmidt-Hoberg, *Nucl. Phys.* **B862**, 710 (2012); G. G. Ross, K. Schmidt-Hoberg, and F. Staub, *J. High Energy Phys.* **08** (2012) 074; D. M. Ghilencea and G. G. Ross, *Nucl. Phys.* **B868**, 65 (2013); A. Kaminska, G. G. Ross, and K. Schmidt-Hoberg, *J. High Energy Phys.* **11** (2013) 209.
- [15] R. Dermisek and H. D. Kim, *Phys. Rev. Lett.* **96**, 211803 (2006).
- [16] B. C. Allanach and B. Gripaios, *J. High Energy Phys.* **05** (2012) 062.
- [17] I. Gogoladze, F. Nasir, and Q. Shafi, *Int. J. Mod. Phys. A* **28**, 1350046 (2013); I. Gogoladze, F. Nasir, and Q. Shafi, *J. High Energy Phys.* **11** (2013) 173.
- [18] M. Perelstein and B. Shakya, *J. High Energy Phys.* **10** (2011) 142; *Phys. Rev. D* **88**, 075003 (2013).
- [19] C. Wymant, *Phys. Rev. D* **86**, 115023 (2012).
- [20] S. Antusch, L. Calibbi, V. Maurer, M. Monaco, and M. Spinrath, *Phys. Rev. D* **85**, 035025 (2012); *J. High Energy Phys.* **01** (2013) 187.
- [21] G. D. Kribs, A. Martin, and A. Menon, *Phys. Rev. D* **88**, 035025 (2013).
- [22] E. Hardy, *J. High Energy Phys.* **10** (2013) 133; E. Hardy, *J. High Energy Phys.* **03** (2014) 069.
- [23] J. Galloway, M. A. Luty, Y. Tsai, and Y. Zhao, *Phys. Rev. D* **89**, 075003 (2014).
- [24] N. Craig and K. Howe, *J. High Energy Phys.* **03** (2014) 140.
- [25] H. Baer, V. Barger, and M. Padeffke-Kirkland, *Phys. Rev. D* **88**, 055026 (2013).
- [26] S. Fichet, *Phys. Rev. D* **86**, 125029 (2012).
- [27] J. E. Younkin and S. P. Martin, *Phys. Rev. D* **85**, 055028 (2012).
- [28] K. Kowalska and E. M. Sessolo, *Phys. Rev. D* **88**, 075001 (2013).
- [29] C. Han, K.-i. Hikasa, L. Wu, J. M. Yang, and Y. Zhang, *J. High Energy Phys.* **10** (2013) 216.
- [30] E. Dudas, G. von Gersdorff, S. Pokorski, and R. Ziegler, *J. High Energy Phys.* **01** (2014) 117.
- [31] A. Arvanitaki, M. Baryakhtar, X. Huang, K. van Tilburg, and G. Villadoro, *J. High Energy Phys.* **03** (2014) 022.
- [32] J. Fan and M. Reece, [arXiv:1401.7671](https://arxiv.org/abs/1401.7671).
- [33] T. Gherghetta, B. von Harling, A. D. Medina, and M. A. Schmidt, *J. High Energy Phys.* **04** (2014) 180.
- [34] K. Kowalska, L. Roszkowski, E. M. Sessolo, and S. Trojanowski, *J. High Energy Phys.* **04** (2014) 166.
- [35] J. L. Feng, *Annu. Rev. Nucl. Part. Sci.* **63**, 351 (2013).
- [36] H. Baer, V. Barger, P. Huang, A. Mustafayev, and X. Tata, *Phys. Rev. Lett.* **109**, 161802 (2012).
- [37] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, and X. Tata, *Phys. Rev. D* **87**, 115028 (2013).
- [38] H. Baer, V. Barger, and D. Mickelson, *Phys. Rev. D* **88**, 095013 (2013).
- [39] H. Baer, V. Barger, M. Padeffke-Kirkland, and X. Tata, *Phys. Rev. D* **89**, 037701 (2014).
- [40] S. P. Martin, *Phys. Rev. D* **89**, 035011 (2014).
- [41] A. Fowlie, [arXiv:1403.3407](https://arxiv.org/abs/1403.3407).
- [42] A. Mustafayev and X. Tata, [arXiv:1404.1386](https://arxiv.org/abs/1404.1386).
- [43] R. Barbieri and A. Strumia, *Phys. Lett. B* **433**, 63 (1998).
- [44] G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. D* **87**, 012008 (2013).
- [45] S. Chatrchyan *et al.* (CMS Collaboration), *J. High Energy Phys.* **10** (2012) 018.
- [46] G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **716**, 1 (2012).
- [47] S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **716**, 30 (2012).
- [48] M. S. Carena and H. E. Haber, *Prog. Part. Nucl. Phys.* **50**, 63 (2003).
- [49] L. J. Hall, D. Pinner, and J. T. Ruderman, *J. High Energy Phys.* **04** (2012) 131.
- [50] H. Baer, V. Barger, and A. Mustafayev, *Phys. Rev. D* **85**, 075010 (2012).
- [51] A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi, and J. Quevillon, *Phys. Lett. B* **708**, 162 (2012).
- [52] N. Craig, [arXiv:1309.0528](https://arxiv.org/abs/1309.0528).
- [53] See, e.g., H. Baer, in *Perspectives on Supersymmetry II*, edited by G. Kane (World Scientific, Singapore, 2010), pp. 446–468.
- [54] A. Djouadi, J.-L. Kneur, and G. Moultaka, *Comput. Phys. Commun.* **176**, 426 (2007); J. Conley, S. Gainer, J. Hewett, M. Le, and T. Rizzo, *Eur. Phys. J. C* **71**, 1697 (2011); S. Sekmen, S. Kraml, J. Lykken, F. Moortgat, S. Padhi, L. Pape, M. Pierini, H. B. Prosper, and M. Spiropulu, *J. High Energy Phys.* **02** (2012) 075; M. W. Cahill-Rowley, J. L. Hewett, A. Ismail, and T. G. Rizzo, *Phys. Rev. D* **86**, 075015 (2012); **88**, 035002 (2013); C. Boehm, P. S. B. Dev, A. Mazumdar, and E. Pukartas, *J. High Energy Phys.* **06** (2013) 113.
- [55] H. Baer and X. Tata, *Weak Scale Supersymmetry: From Superfields to Scattering Events* (Cambridge University Press, Cambridge, UK, 2006).
- [56] H. Baer and J. List, *Phys. Rev. D* **88**, 055004 (2013).
- [57] V. D. Barger, M. S. Berger, and P. Ohmann, *Phys. Rev. D* **49**, 4908 (1994).
- [58] R. Kitano and Y. Nomura, *Phys. Lett. B* **631**, 58 (2005); *Phys. Rev. D* **73**, 095004 (2006).
- [59] M. Papucci, J. T. Ruderman, and A. Weiler, *J. High Energy Phys.* **09** (2012) 035.
- [60] C. Brust, A. Katz, S. Lawrence, and R. Sundrum, *J. High Energy Phys.* **03** (2012) 103; R. Essig, E. Izaguirre, J. Kaplan, and J. G. Wacker, *J. High Energy Phys.* **01** (2012) 074.
- [61] J. A. Evans, Y. Kats, D. Shih, and M. J. Strassler, [arXiv:1310.5758](https://arxiv.org/abs/1310.5758).
- [62] L. E. Ibañez and G. G. Ross, *Phys. Lett.* **110B**, 215 (1982); K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, *Prog. Theor. Phys.* **68**, 927 (1982); **71**, 413 (1984); L. Ibañez, *Phys. Lett.* **118B**, 73 (1982); J. Ellis, J. Hagelin, D. Nanopoulos, and M. Tamvakis, *Phys. Lett.* **125B**, 275 (1983); L. Alvarez-Gaumé, J. Polchinski, and M. Wise, *Nucl. Phys.* **B221**, 495 (1983).
- [63] L. E. Ibanez, C. Lopez, and C. Munoz, *Nucl. Phys.* **B256**, 218 (1985); A. Lleyda and C. Munoz, *Phys. Lett. B* **317**, 82 (1993).
- [64] H. Abe, T. Kobayashi, and Y. Omura, *Phys. Rev. D* **76**, 015002 (2007).
- [65] S. P. Martin, *Phys. Rev. D* **75**, 115005 (2007).



- [66] V. S. Kaplunovsky and J. Louis, *Phys. Lett. B* **306**, 269 (1993).
- [67] A. Brignole, L. E. Ibanez, and C. Munoz, *Nucl. Phys. B* **422**, 125 (1994); **B436747(E)** (1995).
- [68] S. K. Soni and H. A. Weldon, *Phys. Lett.* **126B**, 215 (1983).
- [69] For a review, see U. Ellwanger, C. Hugonie, and A. M. Teixeira, *Phys. Rep.* **496**, 1 (2010).
- [70] U. Ellwanger, *Phys. Lett.* **133B**, 187 (1983).
- [71] J. Bagger and E. Poppitz, *Phys. Rev. Lett.* **71**, 2380 (1993); J. Bagger, E. Poppitz, and L. Randall, *Nucl. Phys.* **B455**, 59 (1995).
- [72] L. Randall and M. Reece, *J. High Energy Phys.* **08** (2013) 088.
- [73] G. F. Giudice and A. Masiero, *Phys. Lett. B* **206**, 480 (1988).
- [74] T. Leggett, T. Li, J. A. Maxin, D. V. Nanopoulos, and J. W. Walker, [arXiv:1403.3099](https://arxiv.org/abs/1403.3099).
- [75] J. E. Kim and H. P. Nilles, *Phys. Lett.* **138B**, 150 (1984).
- [76] M. Dine, W. Fischler, and M. Srednicki, *Phys. Lett.* **104B**, 199 (1981); A. P. Zhitnitskii, *Sov. J. Nucl. Phys.* **31**, 260 (1980).
- [77] K. J. Bae, E. J. Chun, and S. H. Im, *J. Cosmol. Astropart. Phys.* **03** (2012) 013.
- [78] K. J. Bae, H. Baer, and E. J. Chun, *Phys. Rev. D* **89**, 031701 (2014); *J. Cosmol. Astropart. Phys.* **12** (2013) 028.
- [79] H. Baer, C. H. Chen, R. Munroe, F. Paige, and X. Tata, *Phys. Rev. D* **51**, 1046 (1995); H. Baer, J. Ferrandis, S. Kraml, and W. Porod, *Phys. Rev. D* **73**, 015010 (2006).
- [80] H. Baer, F. E. Paige, S. D. Protopopescu, and X. Tata, [arXiv:hep-ph/0312045](https://arxiv.org/abs/hep-ph/0312045).
- [81] Joint LEP 2 Supersymmetry Working Group, Combined LEP Chargino Results up to 208 GeV, [http://lepsusy.web.cern.ch/lepsusy/www/inos\\_moriond01/charginos\\_public.html](http://lepsusy.web.cern.ch/lepsusy/www/inos_moriond01/charginos_public.html).
- [82] H. Baer, K. Y. Choi, J. E. Kim, and L. Roszkowski, review on nonthermal dark matter (to be published).
- [83] V. Barger, M. Berger, and R. J. N. Phillips, *Phys. Rev. Lett.* **70**, 1368 (1993); J. Hewett, *Phys. Rev. Lett.* **70**, 1045 (1993).
- [84] H. Baer and M. Brhlik, *Phys. Rev. D* **55**, 3201 (1997); H. Baer, M. Brhlik, D. Castano, and X. Tata, *Phys. Rev. D* **58**, 015007 (1998).
- [85] S. Rai Choudhury and N. Gaur, *Phys. Lett. B* **451**, 86 (1999); K. S. Babu and C. F. Kolda, *Phys. Rev. Lett.* **84**, 228 (2000); our calculation uses the formulas in J. Mizukoshi, X. Tata, and Y. Wang, *Phys. Rev. D* **66**, 115003 (2002).
- [86] D. Asner *et al.* (Heavy Flavor Averaging Group), [arXiv:1010.1589](https://arxiv.org/abs/1010.1589).
- [87] M. Misiak *et al.*, *Phys. Rev. Lett.* **98**, 022002 (2007).
- [88] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **111**, 101805 (2013).
- [89] S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **111**, 101804 (2013).
- [90] A. Chamseddine, R. Arnowitt, and P. Nath, *Phys. Rev. Lett.* **49**, 970 (1982); R. Barbieri, S. Ferrara, and C. Savoy, *Phys. Lett.* **119B**, 343 (1982); N. Ohta, *Prog. Theor. Phys.* **70**, 542 (1983); L. Hall, J. Lykken, and S. Weinberg, *Phys. Rev. D* **27**, 2359 (1983).
- [91] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, and X. Tata, *Phys. Rev. D* **87**, 035017 (2013).
- [92] H. Baer, A. Mustafayev, S. Profumo, A. Belyaev, and X. Tata, *Phys. Rev. D* **71**, 095008 (2005).
- [93] D. Matalliotakis and H. P. Nilles, *Nucl. Phys.* **B435**, 115 (1995); P. Nath and R. L. Arnowitt, *Phys. Rev. D* **56**, 2820 (1997); J. Ellis, K. Olive, and Y. Santoso, *Phys. Lett. B* **539**, 107 (2002); J. Ellis, T. Falk, K. Olive, and Y. Santoso, *Nucl. Phys.* **B652**, 259 (2003); H. Baer, A. Mustafayev, S. Profumo, A. Belyaev, and X. Tata, *J. High Energy Phys.* **07** (2005) 065.
- [94] M. Dine, A. Kagan, and S. Samuel, *Phys. Lett. B* **243**, 250 (1990); A. Cohen, D. B. Kaplan, and A. Nelson, *Phys. Lett. B* **388**, 588 (1996); N. Arkani-Hamed and H. Murayama, *Phys. Rev. D* **56**, R6733 (1997); T. Moroi and M. Nagai, *Phys. Lett. B* **723**, 107 (2013).
- [95] M. Dine, W. Fischler, and M. Srednicki, *Nucl. Phys.* **B189**, 575 (1981); S. Dimopoulos and S. Raby, *Nucl. Phys.* **B192**, 353 (1981); M. Dine and W. Fischler, *Phys. Lett.* **110B**, 227 (1982); C. R. Nappi and B. A. Ovrut, *Phys. Lett.* **113B**, 175 (1982); L. Alvarez-Gaume, M. Claudson, and M. B. Wise, *Nucl. Phys.* **B207**, 96 (1982); M. Dine and A. E. Nelson, *Phys. Rev. D* **48**, 1277 (1993); M. Dine, A. E. Nelson, and Y. Shirman, *Phys. Rev. D* **51**, 1362 (1995); M. Dine, A. E. Nelson, Y. Nir, and Y. Shirman, *Phys. Rev. D* **53**, 2658 (1996); for a review, see G. F. Giudice and R. Rattazzi, *Phys. Rep.* **322**, 419 (1999).
- [96] Y. Kats, P. Meade, M. Reece, and D. Shih, *J. High Energy Phys.* **02** (2012) 115.
- [97] P. Draper, P. Meade, M. Reece, and D. Shih, *Phys. Rev. D* **85**, 095007 (2012).
- [98] L. Randall and R. Sundrum, *Nucl. Phys.* **B557**, 79 (1999); G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, *J. High Energy Phys.* **12** (1998) 027.
- [99] M. Y. Khlopov and A. D. Linde, *Phys. Lett.* **138B**, 265 (1984).
- [100] R. Dermisek, H. Verlinde, and L.-T. Wang, *Phys. Rev. Lett.* **100**, 131804 (2008); H. Baer, R. Dermisek, S. Rajagopalan, and H. Summy, *J. High Energy Phys.* **10** (2009) 078.
- [101] S. P. de Alwis, *J. High Energy Phys.* **03** (2010) 078.
- [102] H. Baer, S. de Alwis, K. Givens, S. Rajagopalan, and H. Summy, *J. High Energy Phys.* **05** (2010) 069; H. Baer, S. P. de Alwis, K. Givens, S. Rajagopalan, and W. Sreethawong, *J. High Energy Phys.* **01** (2011) 005.
- [103] D. E. Kaplan, G. D. Kribs, and M. Schmaltz, *Phys. Rev. D* **62**, 035010 (2000); Z. Chacko, M. A. Luty, A. E. Nelson, and E. Ponton, *J. High Energy Phys.* **01** (2000) 003; for phenomenology, see M. Schmaltz and W. Skiba, *Phys. Rev. D* **62**, 095005 (2000); **62**, 095004 (2000); H. Baer, A. Belyaev, T. Krupovnickas, and X. Tata, *Phys. Rev. D* **65**, 075024 (2002); A. De Simone, J. J. Fan, M. Schmaltz, and W. Skiba, *Phys. Rev. D* **78**, 095010 (2008).
- [104] J. R. Ellis, C. Kounnas, and D. V. Nanopoulos, *Nucl. Phys.* **B247**, 373 (1984); J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos, and K. Tamvakis, *Phys. Lett.* **134B**, 429 (1984); G. A. Diamandis, J. R. Ellis, A. B. Lahanas, and D. V. Nanopoulos, *Phys. Lett. B* **173**, 303 (1986); for a review, see A. Lahanas and D. V. Nanopoulos, *Phys. Rep.* **145**, 1 (1987).



- [105] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, *Phys. Rev. D* **68**, 046005 (2003).
- [106] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, and S. Pokorski, *J. High Energy Phys.* **11** (2004) 076; K. Choi, A. Falkowski, H. P. Nilles, and M. Olechowski, *Nucl. Phys.* **B718**, 113 (2005); K. Choi, K.-S. Jeong, and K. Okumura, *J. High Energy Phys.* **09** (2005) 039; A. Falkowski, O. Lebedev, and Y. Mambrini, *J. High Energy Phys.* **11** (2005) 034; M. Endo, M. Yamaguchi, and K. Yoshioka, *Phys. Rev. D* **72**, 015004 (2005); H. Baer, E.-K. Park, X. Tata, and T. T. Wang, *J. High Energy Phys.* **08** (2006) 041; L. L. Everett, I.-W. Kim, P. Ouyang, and K. M. Zurek, *J. High Energy Phys.* **08** (2008) 102.
- [107] K. Choi, K. S. Jeong, T. Kobayashi, and K.-i. Okumura, *Phys. Lett. B* **633**, 355 (2006); K. Choi, K. S. Jeong, T. Kobayashi, and K.-i. Okumura, *Phys. Rev. D* **75**, 095012 (2007).
- [108] O. Lebedev, H. P. Nilles, and M. Ratz, *arXiv:hep-ph/0511320*.
- [109] H. Baer, E.-K. Park, X. Tata, and T. T. Wang, *J. High Energy Phys.* **06** (2007) 033.
- [110] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong, and X. Tata, *J. High Energy Phys.* **12** (2013) 013.
- [111] M. Koratzinos *et al.*, *arXiv:1305.6498*.
- [112] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong, and X. Tata, *arXiv:1306.3148*.
- [113] G. F. Giudice, in *Perspectives on LHC Physics*, edited by G. Kane and A. Pierce (World Scientific, Singapore, 2008), pp. 155–178.