Analysis of the $\Lambda_b \to \Lambda_c$ mode via the Isgur-Wise approach and hyperspherical coordinates

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The hyperradial Schrödinger equation is considered with the Killingbeck potential which is a combination of harmonic and Cornell terms. Having calculated the wave function of a three-body baryonic system with the aid of the hyperspherical approach, we investigate the Isgur-Wise function for Lambda baryons and related concepts including the differential decay width, charge radius and the curvature parameters. In particular, the decay width of $\Lambda_b \to \Lambda_c$ transition is reported.

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I. INTRODUCTION

To analyze the particle physics phenomena, various approaches have been proposed with their own capabilities, failures or limitations. In particular, the Isgur-Wise (IW) formalism works well and is quite economical for the analysis of heavy-quark systems. Within this technique, the decay properties of heavy quark systems is expressed in terms of universal form factors which are functions of $y = v.v'$, where v and v' are the four velocities of initial and final states [1,2]. In many cases of heavy quark limit, such as $\Lambda_b \to \Lambda_c$ transition, only one universal form factor, called the Isgur-Wise function (IWF) in the jargon, should be calculated [3]. Having calculated the IWF, we obtain invaluable knowledge on various significant quantities including branching ratio, decay width and Cabibbo-Kobayashi-Maskawa (CKM) matrix [4–6].

Until now, different authors have studied various aspects of the approach. Yaouanc *et al.* obtained the bounds on the so-called curvature parameter of the IWF for $\Lambda_b \to \Lambda_c \ell \bar{v}_\ell$ decay mode [7]. Baryonic IWFs were studied by Jugeau [8]. Study of baryonic weak decays in the light-front quark model was presented by Hong-Wei Ke *et al.* [9]. Using the QCD sum rules within the framework of heavy quark effective theory, the slope parameter and decay branching ratios were calculated for Lambda transition in Ref. [10]. Ebert and co-authors applied the relativistic quark model to the problem and thereby calculated the semileptonic decay rates of heavy baryons [11]. The IWF for Lambda decays was also reported by Jenkins *et al.* [12]. Viet developed the heavy diquark model for baryons and characterized the matrix elements of weak currents of baryons via universal IWF [13]. The analysis of the heavy baryon transitions was also addressed by Körner *et al.* where they discussed the decay rates, angular decay distributions, lifetimes and polarization effects [14]. In an interesting study, Cardarelli and Simula investigated the corresponding form factors of heavy baryons in a light-front constituent quark model [15].

The aim of this work is to investigate the IWF for Lambda baryon transition. In our paper, we propose a different approach to obtain the analytical solutions and make use of the baryon hyperspherical coordinates to calculate the system wave function. We then consider the Lambda transition and comment on the differential decay width. Section IV includes the numerical results and the decay width of $\Lambda_b \to \Lambda_c$ transition is reported in Tables I–III. Conclusions are given in Sec. V.

TABLE I. Parameters of IWF for Lambda baryons.

Baryon	ρ^2	c
Λ_b ($m = 5.62$)	1.1458	0.3065
Λ_c ($m = 2.286$)	1.0364	0.2501

TABLE II. Decay width of $\Lambda_b \to \Lambda_c$ transition versus $B(\Lambda_c \to ab)$.

Model	$\Gamma(\Lambda_b \to \Lambda_c \ell \bar{v})$ [ours]	$\Gamma(\Lambda_b \to \Lambda_c \ell \bar{v})$ [16]
First model	$8.32B(\Lambda_c \to ab) \times 10^{10}\ \mathrm{s}^{-1}$	$5.7B(\Lambda_c \to ab) \times 10^{10}\ \mathrm{s}^{-1}$
Second model	$5.1B(\Lambda_c \to ab) \times 10^{10}\ \mathrm{s}^{-1}$	

TABLE III. Decay width of $\Lambda_b \to \Lambda_c$ transition for $|V_{bc}| = 0.04$.

Γ (in $10^{10}\ \mathrm{s}^{-1}$) first model	Γ (in $10^{10}\ \mathrm{s}^{-1}$) second model	Λ_c decay modes (Br%) [17]
0.37	0.23	e^+ anything (4.5)
4.16	2.57	p anything (50)
2.91	1.79	Λ anything (35)
0.83	0.51	$\Sigma^\pm$ anything (10)

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II. THE HYPERSPHERICAL APPROACH AND THE BARYON WAVE FUNCTION

Let us consider the three-body baryonic system. Then the configuration of three particles is described by the two Jacobi vectors ρ and λ defined as [18,19]

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \qquad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3). \tag{1}$$

Instead of $\vec{\rho}$ and $\vec{\lambda}$, one can introduce the hyperspherical coordinates, which are given by the angles $\Omega_\rho = (\theta_\rho, \phi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$ together with the hyperradius x and the hyperangle ξ defined respectively by

$$x = \sqrt{\rho^2 + \lambda^2}, \qquad \xi = \tan^{-1}\left(\frac{\rho}{\lambda}\right). \tag{2}$$

Therefore, the Hamiltonian of the system is expressed as

$$H = \frac{p_\rho^2}{2m} + \frac{p_\lambda^2}{2m} + V(x). \tag{3}$$

The corresponding kinetic energy operator of a three-body problem takes the form ($\hbar = c = 1$) [18,19]

$$-\frac{1}{2m}(\Delta_\rho + \Delta_\lambda) = -\frac{1}{2m}\left(\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{L^2(\Omega_\rho, \Omega_\lambda, \xi)}{x^2}\right), \tag{4}$$

where the eigenfunctions of L^2 are hyperspherical harmonics [18]

$$L^2(\Omega_\rho, \Omega_\lambda, \xi) Y_{[\gamma],\ell_\rho,\ell_\lambda}(\Omega_\rho, \Omega_\lambda, \xi) = \gamma(\gamma + 4) Y_{[\gamma],\ell_\rho,\ell_\lambda}(\Omega_\rho, \Omega_\lambda, \xi) \tag{5}$$

with the grand angular momentum $\gamma = 2n + \ell_\rho + \ell_\lambda$ and ℓ_ρ, ℓ_λ are the angular momenta associated with the ρ, λ variables. Any three-body state can be expanded in the hypersherical harmonics basis [18]

$$\psi(\vec{\rho}, \vec{\lambda}) = \sum_{\gamma,\ell_\rho,\ell_\lambda} N_\gamma \psi_{\upsilon,\gamma}(x) Y_{[\gamma],\ell_\rho,\ell_\lambda}(\Omega_\rho, \Omega_\lambda, \xi), \tag{6}$$

where the hyperradial wave function $\psi_{\upsilon,\gamma}(x)$ is a solution of the equation

$$\left(\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right)\psi_{\upsilon,\gamma}(x) = -2m[E - V(x)]\psi_{\upsilon,\gamma}(x). \tag{7}$$

Choosing the combination of Cornell and harmonic terms,

$$V(x) = ax^2 + bx - \frac{c}{x}, \tag{8}$$

which is often called the Killingbeck potential, and applying the transformation $\psi_{\upsilon,\gamma}(x) = x^{-\frac{5}{2}}\varphi_{\upsilon,\gamma}(x)$, the hyperradial Schrödinger equation appears as [18]

$$\varphi''_{\upsilon,\gamma}(x) + \left[2mE - 2ma\,x^2 - 2mbx + \frac{2mc}{x} - \frac{(2\gamma+3)(2\gamma+5)}{4x^2}\right]\varphi_{\upsilon,\gamma}(x) = 0. \tag{9}$$

By proceeding on the basis of the ansatz approach of Ref. [18], we get the ground-state wave function as

$$\psi_{0\gamma}(x) = N_0 x^\gamma \exp\left(-\frac{m\omega_0}{2}x^2 - \frac{2mc}{(2\gamma+5)}x\right). \tag{10}$$

We suppose $\Lambda_b \to \Lambda_c$ transition and continue with two versions of IWF.

III. ISGUR-WISE FUNCTION

A. The first model

IWF is defined as the overlap of the wave function of two hadrons. In the heavy-baryon transitions, the function has the form [20,21]

$$\xi(\omega) = \sqrt{\frac{2}{\omega+1}}\langle\psi_{\Lambda_c}|\psi_{\Lambda_b}\rangle, \tag{11}$$

where ω is referred to the zero recoil point. Substituting Eq. (10) into Eq. (11) gives

$$\xi(\omega) = 16\pi^2 N_{\rm nor}\sqrt{\frac{2}{\omega+1}}\int_0^\infty x^{2\gamma+5}\exp\left(-\frac{\omega_0}{2}(m_{\Lambda_c} + m_{\Lambda_b})x^2 - \frac{2c}{(2\gamma+5)}(m_{\Lambda_c} + m_{\Lambda_b})x\right)dx, \tag{12}$$

where $N_{\rm nor}$, m_{Λ_c}, m_{Λ_b} are normalization constant, masses of Λ_c and Λ_b baryons, respectively. By using the values of potential parameters in Ref. [18], at $\omega = 1$, the IWF is normalized when $N_{\rm nor} = 0.0001986$, which on the other hand indicates $\xi(\omega)|_{\omega=1} = 1$. In this case, the IWF as a function of the ω parameter has the form

$$\xi(\omega) = 1.0\sqrt{\frac{2}{1+\omega}}. \tag{13}$$

The slope (charge radius) and the convexity of the IWF are defined via [22]

$$\rho^2 = -\frac{d\zeta(\omega)}{d\omega}\bigg|_{\omega=1}, \qquad c = \frac{d^2\zeta(\omega)}{d\omega^2}\bigg|_{\omega=1}. \tag{14}$$

We measured the slope for Lambda transition as 0.2500. The convexity parameter is also obtained as 0.1875. By using the obtained IWF of Eq. (13), we can calculate the decay width for the semileptonic decay of $\Lambda_b \to \Lambda_c$. The differential decay width for this transition has the form [16]

$$\frac{d\Gamma}{d\omega} = \frac{2}{3} m_{\Lambda_c}^4 m_{\Lambda_b} A \xi^2(\omega)\sqrt{\omega^2-1}[3\omega(\eta+\eta^{-1}) - 2 - 4\omega^2]. \tag{15}$$

Hence,

$$\Gamma = A\int_1^{1.43} \frac{204.6350191\sqrt{\omega^2-1}(8.595612783\omega - 2 - 4\omega^2)}{1+\omega}\, d\omega = 62.41960610A \tag{16}$$

with $A = \frac{G_F^2}{(2\pi)^3}|V_{cb}|^2 B(\Lambda_c \to ab)$, where $B(\Lambda_c \to ab)$ is the branching ratio for the decay $\Lambda_c \to a(\frac{1}{2}^+) + b(0^-)$. In addition, $\eta = \frac{m_{\Lambda_b}}{m_{\Lambda_c}}$ and V_{cb} is the Kobayashi-Maskawa matrix element [16]. In Fig. 1, we have plotted $\frac{d\Gamma}{A d\omega}$ vs ω for model 1. In this model, we obtain $\Gamma(\Lambda_b \to \Lambda_c \ell \bar{\nu}) = 8.32\, B(\Lambda_c \to ab) \times 10^{10}\ \mathrm{s}^{-1}$.

B. The second model

At the zero recoil point, IWF can be parametrized in a Taylor series as [3,9,23]

$$\xi(\omega) = 1 - \rho^2(\omega-1) + c(\omega-1)^2 + \cdots. \tag{17}$$

Because of the dominance of slope and curvature parameters, the higher terms in the IWF are often neglected. Summing up the above equations, we state that the IWF in hyperspherical coordinates is written as

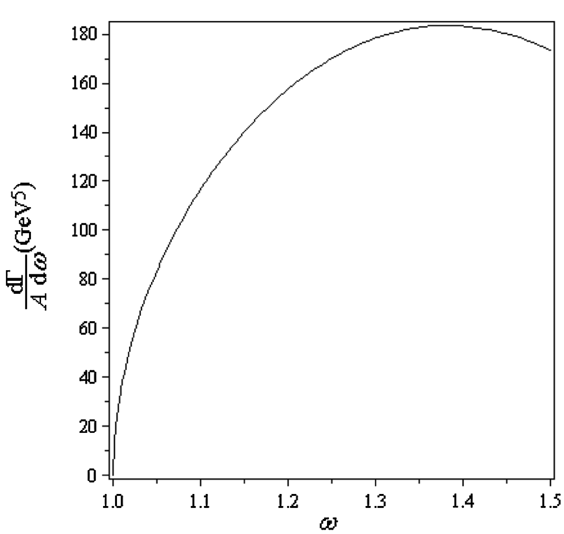

FIG. 1. The differential decay width for $\Lambda_b \to \Lambda_c \ell \bar{\nu}$ in model 1.

$$\xi(\omega) = \int_0^\infty 16\pi^2 x^5 |\psi(x)|^2 \cos(px)dx. \tag{18}$$

Now, writing the expansion of $\cos(px)$,

$$\cos(px) = 1 - \frac{p^2x^2}{2!} + \frac{p^4x^4}{4!} + \cdots, \tag{19}$$

and considering $p^2 = 2m^2(\omega-1)$ (where p^2 is the square of virtual momentum transfer), we obtain

$$\xi(\omega) = \int_0^\infty 16\pi^2 x^5 |\psi(x)|^2 dx - 16\pi^2 m^2(\omega-1) \times \int_0^\infty |\psi(x)|^2 x^7 dx + \frac{8}{3}\pi^2 m^4 (\omega-1)^2 \int_0^\infty |\psi(x)|^2 x^9 dx. \tag{20}$$

By a simple comparison of Eqs. (17) and (20), the slope and curvature parameters are presented as

$$\rho^2 = 16\pi^2 m^2 \int_0^\infty |\psi(x)|^2 x^7 dx,$$

$$c = \frac{8}{3}\pi^2 m^4 \int_0^\infty |\psi(x)|^2 x^9 dx. \tag{21}$$

Table I shows the calculated parameters of IWF for heavy Lambda baryons. The normalization constant of the wave function can be also calculated from

$$16\pi^2 N_0^2 \int_0^\infty x^{2\gamma+5} e^{-m\omega x^2 - \frac{4mc}{(2\gamma+5)}x} dx = 1. \tag{22}$$

It is evident that in this model at $\omega = 1$, $\xi(1) = 1.0$. Recalling Eq. (15), the differential decay width for Lambda transition is calculated as [16]

$$\Gamma = A\int_1^{1.43} 102.3175095(2.1458 - 1.1458\omega + 0.3065(\omega-1)^2)^2\sqrt{\omega^2-1}(8.595612783\omega - 2 - 4\omega^2)d\omega$$
$$= 38.54437473A\ . \tag{23}$$

In Fig. 2, we have plotted $\frac{d\Gamma}{Ad\omega}$ vs ω for model 2. We have calculated the total decay width for semileptonic transition of the Lambda baryon as $\Gamma(\Lambda_b \to \Lambda_c \ell\bar{\nu}) = 5.1B(\Lambda_c \to ab) \times 10^{10}\ \mathrm{s}^{-1}$.

IV. RESULT AND DISCUSSION

In our numerical results, we have used the values of the quark masses from Ref. [16] as $m_u = m_d = 0.65$ GeV and $m_b = 5.02$ GeV, $m_c = 1.58$ GeV. It seems that the slope of the IWF should have a larger value for baryons than the case of mesons. Then IWF drops faster in the case of baryons. Guo and Muta reported the slope of IWF for Lambda baryon as 1.4 [16] which is in agreement with the second model of the present paper. In Table I, we have reported the slope of the IWF for the Lambda baryon as $\rho^2 = 1.1458$. In the case of Lambda transition, the slope parameter is reported as $\rho^2 = 2.01$ by Huang *et al.* [10]. They also showed that $\xi(1) = 0.29$ [10]. Ivanov *et al.* reported 1.04, 1.09, 1.12, 1.22 for the charge radius of the Λ_b baryon [24] and 1.05, 1.09, 1.12, 1.22, 1.32 for the charge radius of the Σ_b baryon [24]. The UKQCD collaboration computed $\rho^2 = 1.1 \pm 1.0$ for $\Lambda_b \to \Lambda_c \ell\bar{\nu}$ decays [25]. We have presented the total decay width for semileptonic transition of the Lambda baryon in the two mentioned models. As we see in Table II, the obtained value $\Gamma(\Lambda_b \to \Lambda_c \ell\bar{\nu}) = 5.1B(\Lambda_c \to ab) \times 10^{10}\ \mathrm{s}^{-1}$ in the second model is in agreement with the result of Guo and Muta which reports $\Gamma(\Lambda_b \to \Lambda_c \ell\bar{\nu}) = 5.7B(\Lambda_c \to ab) \times 10^{10}\ \mathrm{s}^{-1}$ [16]. Moreover, it is consistent with the reported value of the first model $\Gamma(\Lambda_b \to \Lambda_c \ell\bar{\nu}) = 8.32B(\Lambda_c \to ab) \times 10^{10}\ \mathrm{s}^{-1}$. Table III shows our calculation of decay width of the $\Lambda_b \to \Lambda_c$ transition for $|V_{bc}| = 0.04$. The decay width of the $\Lambda_b^0 \to \Lambda_c^+ e^- \bar{\nu}_e$ process is reported as 5.39 (in $10^{10}\ \mathrm{s}^{-1}$) in Ref. [24]. The UKQCD collaboration obtained the decay rate integrated over the ω parameter as $\int_1^{1.2} d\omega\ \frac{d\Gamma}{d\omega}(\Lambda_b \to \Lambda_c + \ell\bar{\nu}) = 1.4\ ^{+5}_{-4'}\ |V_{cb}|^2 10^{13}\ \mathrm{s}^{-1}$ [25]. Our obtained results in Table III are acceptable when compared with the result of the UKQCD collaboration. It has to be mentioned that when $|V_{bc}| = 0.04$ the reported value of the UKQCD collaboration will be $2.240000000^{+5}_{-4} \times 10^{10}\ \mathrm{s}^{-1}$ [25] which is in the vicinity of our values in Table III. In Figs. 1 and 2, the behavior of differential decay width of $\Lambda_b \to \Lambda_c \ell\bar{\nu}$ vs ω is shown. Figure 3 gives a comparative view of IWF in the two applied models. By comparing the results with Refs. [10,16,24,25] it seems that the second model yields a more physically motivated result and is therefore more suitable to investigate hadrons including a single heavy quark.

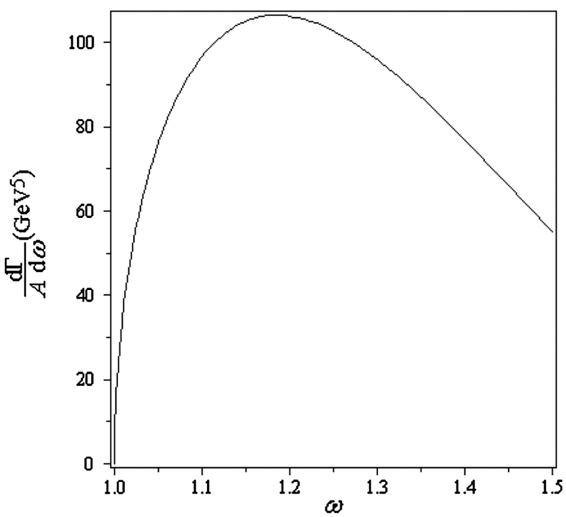

FIG. 2. The diffrential decay width for $\Lambda_b \to \Lambda_c \ell\bar{\nu}$ in model 2.

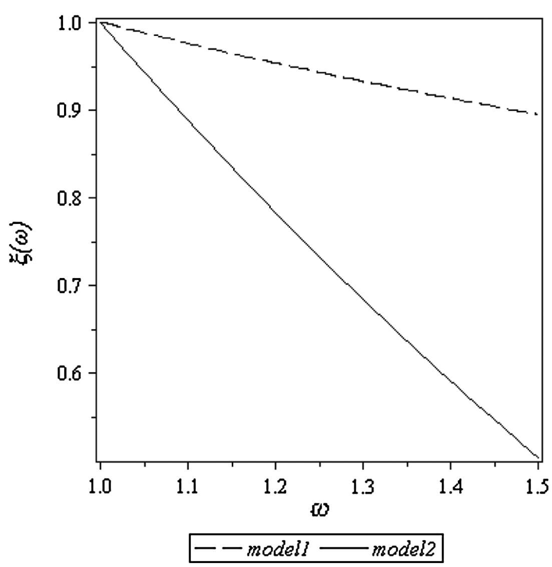

FIG. 3. Comparison of IWF in two models.

V. CONCLUSIONS

We calculated the baryon wave function using the hyperspherical coordinates by considering the

Killingbeck potential which is the combination of Cornell and quadratic potentials. Using the obtained wave function, we investigated the IWF for heavy baryon transition and reported the slope parameter for the Lambda baryon. An analysis of differential decay width for the transition was also discussed and the comparison with other related results indicated the acceptability of the solutions.

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